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APPROPRIATE INTERCOLUMN FORCE FUNCTIONS AND LAMBDA VALUES FOR THREE DIMENSIONAL SLOPE STABILITY ANALYSIS

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Appropriate intercolumn force functions and lambda values for three-dimensional slope stability analysis

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ABSTRACT: The formulation of a general limit equilibrium model for three-dimensional slope stability analysis involves the knowledge of five intercolumn force functions and five lambda values (Lam and Fredlund, 1993). This paper presents the investigation of the intercolumn force functions and the lambda values. The findings from the investigation provide insights into the selection of appropriate intercolumn force functions and lambda values for three-dimensional stability analyses, which in many cases, could substantially simplify the solution procedures for the general limit equilibrium model.

1 RECENT DEVELOPMENT

Three-dimensional slope stability analysis using limit equilibrium method of columns is becoming of increased importance to geotechnical engineering as a result of rapid developments in computing capabilities. Several methods of columns have been proposed in the literature. The various method differ in the assumptions used to render the problem solvable. In all cases, assumptions are made with respect to the nature of the intercolumn forces.

Hovland (1977) appears to have been the first to formulate a solution for three-dimensional stability analysis. Hovland's solution is an extension of the two-dimensional Ordinary method in which all intercolumn forces acting on the sides of the columns are ignored (Fellenius, 1936). This assumption allows direct computation of the normal force and the shear force acting on the base of each column, and consequently direct solution to the factor of safety equation.

Chen and Chameau (1982) proposed a formulation in which the intercolumn shear forces in the plane of movement were assumed to be parallel to the base of the column, and the intercolumn forces perpendicular to the plane of movement were assumed to have the same inclination throughout the entire sliding mass. Chen and Chameau's method could be considered primarily an extension of the two-dimensional Ordinary method and partly as an extension of the two-dimensional Spencer method (Spencer, 1967)..

Hungr (1987) proposed a formulation that was an extension to the two-dimensional Bishop's simplified method (Bishop, 1954). The intercolumn forces were assumed to be horizontal. In other words, the vertical intercolumn shear forces of each column were ignored. The factor of safety is obtained by summing moments about the axis of rotation for all columns. Hungr, Salgado and Byrne (1989) proposed an extension to the two-dimensional Janbu's simplified method (Janbu, 1954). The method uses the same assumptions regarding the intercolumn force functions as the Bishop's simplified method, but the factor of safety is obtained by summing horizontal forces of the entire sliding mass.

Lam and Fredlund (1993) proposed a more general formulation for three-dimensional slope stability analysis. The model satisfies both force and moment equilibrium. The model is an extension of the two-dimensional general limit equilibrium (GLE) method (Fredlund and Krahn, 1977). Each intercolumn shear force acting on the sides of the column is related to the intercolumn normal force by a force function and a lambda value. As a result, the solution to such a rigorous model requires information on five intercolumn force functions and the solving of five lambda values.

This paper presents the investigation of the intercolumn force functions and the development of the appropriate intercolumn force functions and lambda values for a three-dimensional limit equilibrium slope stability analysis.

2 FORCES IN A COLUMN

In the three-dimensional general limit equilibrium method proposed by Lam and Fredlund (1993), the earth mass above the slip surface is divided into columns and the forces acting on the various faces of each column are calculated using statics. A free body diagram showing the various forces acting on a single column is presented in Figure 1.

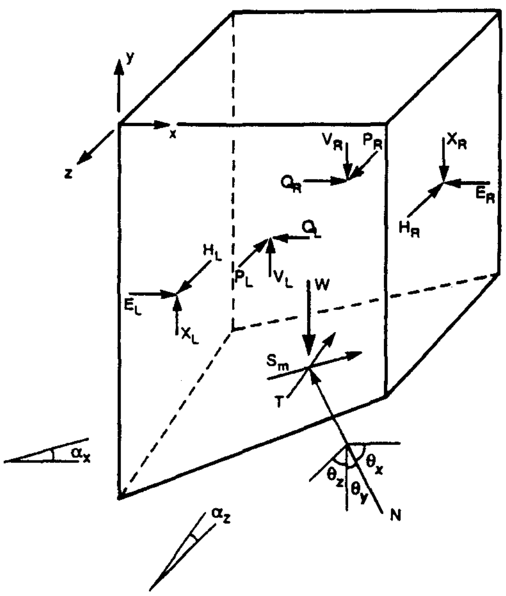

Figure 1. Free body diagram of a column

where E_L, E_R= intercolumn normal force on the left and right front plane of a column; X_L, X_R= intercolumn vertical shear force on the left and right front plane of a column; H_L, H_R= intercolumn horizontal shear force on the left and right front plane of a column; P_L, P_R= intercolumn normal force on the left and right side plane of a column; V_L,V_R= intercolumn vertical shear force on the left and right side plane of a column; Q_L, Q_R= intercolumn horizontal shear force on the left and right side plane of a column; N= normal force at the base of a column; Sm= shear force mobilized at the base of a column along the direction of movement; T= shear force at the base of a column perpendicular to the direction of movement; W= weight of a column; α_x, α_z = angle of the base plane along and perpendicular to the direction of movement; θ_x, θ_y, θ_z= angle between the normal force and the X, Y, Z-axis

The general limit equilibrium formulation is statically indeterminate in that the number of unknowns exceeds the number of equations. The indeterminacy can be viewed as arising from a lack of knowledge regarding the stresses within the soil mass. To reduce the degree of indeterminacy, Lam and Fredlund (1993) made the assumption that the intercolumn shear forces acting on the various faces of the column can be related to their respective normal forces by intercolumn force functions. Mathematically, these force functions can be represented as :

$$\frac{X}{E} = \lambda_1 f(1) \tag{2.1}$$

$$\frac{H}{E} = \lambda_2 f(2) \tag{2.2}$$

$$\frac{V}{P} = \lambda_3 f(3) \tag{2.3}$$

$$\frac{Q}{P} = \lambda_4 f(4) \tag{2.4}$$

$$\frac{T}{N} = \lambda_5 f(5) \tag{2.5}$$

where f(1),f(2),f(3),f(4) and f(5) are the intercolumn force functions; and λ_1, λ_2, λ_3, λ_4 and λ_5 are the percentage of the intercolumn force functions used when solving for the factor of safety. The intercolumn force functions describe the general variation of the direction of the resultants of the normal and intercolumn shear forces, and the lambda values describe the degree of variation.

Based on the above equations the intercolumn shear forces X, H, V, Q and T can be calculated from the normal forces, E, P and N. This approach in calculating the intercolumn shear forces was suggested by Morgenstern and Price in 1965, and has been used in rigorous two-dimensional stability analyses (Fredlund, 1984). This approach of rendering the problem determinate has proven to be successful primarily due to the observation that the computed factor of safety is insensitive to the assumed intercolumn force functions (Morgenstern-Price, 1965, Bishop, 1954; Fredlund and Krahn, 1977).

3 INVESTIGATION OF INTERCOLUMN FORCE FUNCTIONS

The intercolumn force functions were investigated using procedures similar to those taken by Fan, Fredlund and Wilson (1986) in the two-dimensional case. Four example slopes were investigated (Lam, 1991). The geometries of these slopes range from

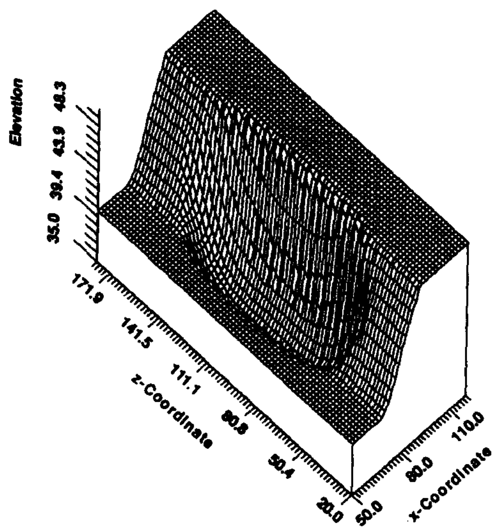

Figure 2. Slip surface of a uniform slope (Example 1)

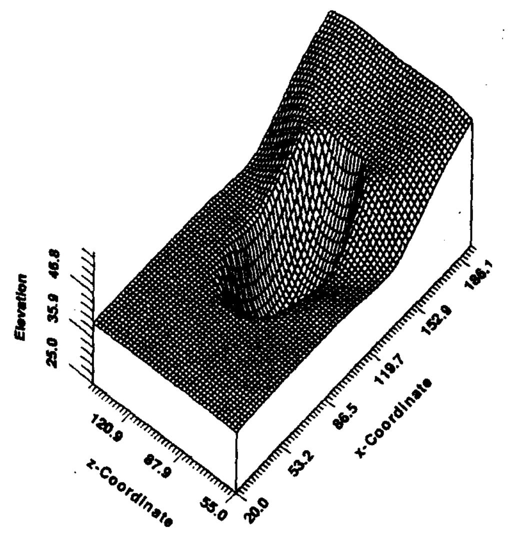

Figure 3. Slip surface of a non-uniform slope (Example 3)

flat (Example 1) to steep (Example 2) uniform slopes as well as non-uniform slopes involving the corner of an embankment (Example 3) and the corner of an excavation (Example 4).

Young's modulus of elasticity, E, and the Poisson's ratio, μ, are taken to be 100,000 kPa and 0.4 respectively. The soil mass is assumed to be under the influence of gravity loading only. Zero force condition is assumed for the top surface of the slope, and zero lateral displacement conditions are assumed at the four sides and the base of the slope. Based on the stress state computed from the finite element analyses, the intercolumn normal and shear forces were determined by summing the forces acting on the sides of the column using Simpson's method of integration.

Figures 4 and 5 illustrate the computed five intercolumn force functions along the center section in the direction of the slope movement for a uniform and a non-uniform slope. The intercolumn force functions have the same form as in the two-dimensional cases (Fan, Fredlund and Wilson,1986). The functions are bell-shaped with the maximum value occurring approximately at the center of the slope. As the slip surface changes in the third dimension (i.e., the Z direction), the shape of the functions also changes in the third dimension. Therefore, the intercolumn force functions can be represented by a three-dimensional surface over the entire slip surface as illustrated in Figure 6.

Results from the above four example problems indicate that the shape and magnitude of the intercolumn force functions are sensitive to the geometry of the slope. The followings can be concluded:

- For slopes with uniform geometries (Figure 2), only the intercolumn force function along the direction of movement (X/E) has values of significant magnitude. The magnitude of all other four functions are zero.
- For slopes with non-uniform geometries (Figure 3), only the intercolumn force functions along the direction of movement (X/E) and perpendicular to the direction of movement (V/P) have values of significant magnitude. The magnitude of all other force functions are insignificant.

The four example problems used in the study are all symmetrical configurations. Therefore, the above conclusions that some of the intercolumn force function are insignificant may not be true for a general three-dimensional slope with non-symmetrical configuration. However, the above findings suggest that it may not be always necessary to define all five intercolumn force functions and to solve for all five lambda values in a three-dimensional stability analysis.

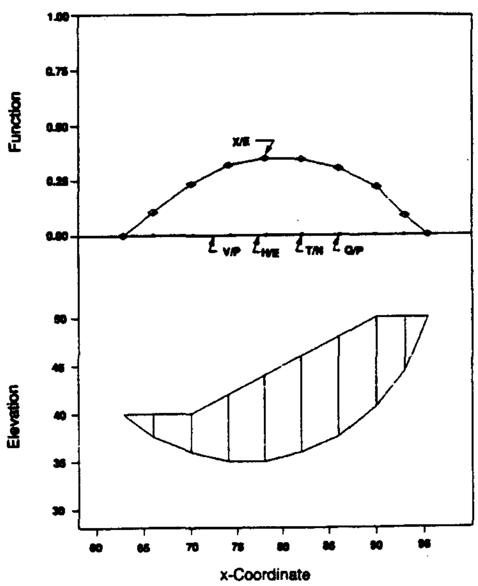

Figure 4. The five intercolumn force functions for a uniform slope along the center section in the direction of the slope movement (Example 1)

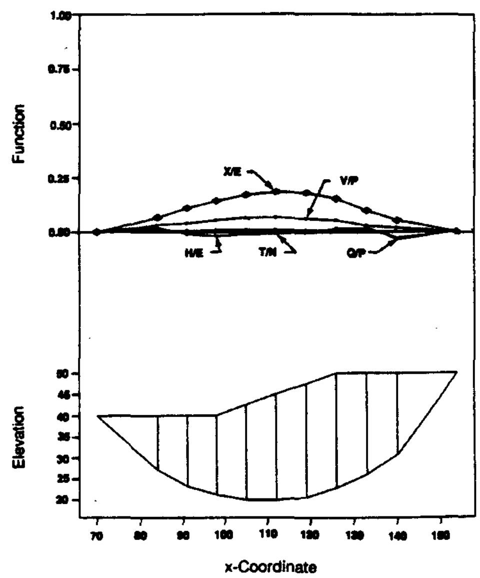

Figure 5. The five intercolumn force functions for a non-uniform slope along the center section in the direction of the slope movement (Example 3)

4 SIGNIFICANCE OF INTERCOLUMN FORCE FUNCTIONS

The general limit equilibrium formulation as proposed by Lam and Fredlund (1993) allows the simulation of other common methods of analysis by making different assumptions about the intercolumn forces. The four example slopes used to determine the intercolumn force functions were analyzed using the proposed model (Lam, 1991). The following methods of analysis were simulated:

- Ordinary method
- Bishop's Simplified method
- Janbu's Simplified method
- GLE method with constant function (i.e., Spencer's method)
- GLE method with a half-sine function
- GLE method with the actual functions determined from stress analysis

The significance of the intercolumn force functions was studied by comparing the computed three-dimensional factors of safety of the various methods with the factor of safety computed using the actual force functions determined from stress analysis (i.e., Reference factor of safety). A comparison of the computed factors of safety between the various methods for the four example slopes are shown in Table 1. The following points are noteworthy:

- The GLE method with the half-sine intercolumn force functions gives the best approximation of the reference factor of safety. The average percentage difference is 0.29%.
- The GLE method with the constant intercolumn function gives the second best approximation of the reference factor of safety. The average percentage difference is 0.35%.
- Bishop's simplified method gives a good approximation of the reference factor of safety, particularly in the case of uniform slopes. The average percentage difference is 1.0%.
- Janbu's simplified method (without the correction factor) significantly under estimates the reference factor of safety. The average percentage difference is 7.8%.
- The Ordinary method gives the poorest approximation of the reference factor of safety, particularly in the case of non-uniform slopes. The average percentage difference is 11.3%.

Furthermore, when both moment and force equilibrium are satisfied, the factor of safety is relatively insensitive to the assumed shape of the intercolumn force functions. This is shown clearly in Table 1. The GLE method provides essentially the same factor of safety for the different intercolumn force functions in all four example problems. However, when only moment or only force equilibrium is satisfied (e.g., Bishop's simplified, Janbu's simplified and the Ordinary method), the factor of safety is sensitive to the assumed nature of the intercolumn forces. The above observations are consistent with the general understanding in the two-dimensional cases.

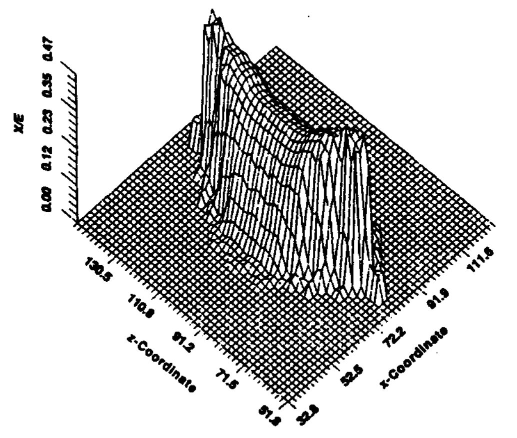

Figure 6. A three-dimensional surface of the intercolumn force function X/E for a uniform slope (Example 1)

Table 1. A comparison of the computed three-dimensional factor of safety of various methods

	Example			
Method	1	2	3	4
GLE, ANSYS fn. (Reference FOS)	1.882	1.512	2.700	2.615
GLE, Half-sine fn.	1.881	1.511	2.691	2.633
GLE, Constant fn.	1.888	1.512	2.688	2.637
Bishop's Simplified	1.881	1.511	2.634	2.577
Janbu's Simplified	1.732	1.471	2.373	2.394
Ordinary	1.736	1.447	2.282	2.157

5 APPROPRIATE INTERCOLUMN FORCE FUNCTIONS AND LAMBDA VALUES

For uniform slopes with symmetric slip surfaces, it is appropriate to define one intercolumn force function (X/E) and solve for one lambda value (λ_1). The solution to the general model can be obtained in the same way as in two-dimensional cases.

For non-uniform slopes with symmetric slip surfaces, it is appropriate to define two intercolumn force functions (X/E and V/P), and solve for two lambda values (λ_1 , λ_2). The solution to the general model can be obtained in the same way as the one-force-function cases. However, the solution procedures are repeated for different values of λ_2. The three-dimensional factor of safety is obtained at the two lambda values when both force and moment equilibrium are satisfied and when the factor of safety is the minimum (Lam, 1991).

For slopes with complex geometries and non-symmetric slip surfaces, the solution to the general limit equilibrium model may require the knowledge of all five intercolumn force functions and the solving for five lambda values. Although it is theoretically possible to implement a five-level nested successive iteration procedure to solve for five lambda values, the procedure would require far too many iterations.

Previous studies (Fredlund, 1984, Fan, Fredlund and Wilson, 1986) have shown that for a simple two-dimensional slope consisting of a frictional material, 100% of the interslice shear forces computed from a stress analysis yield essentially the same factor of safety for both moment and force equilibrium. As the cohesive component of the soil increases, the lambda value decreases. In other words, the lambda values can be visualized as the percentage of intercolumn shear forces required to solve for an equal moment and force equilibrium factor of safety. This observation suggests that there is no theoretical justification for using varying percentages (i.e., five lambda values) of the computed equilibrium stresses on different planes of the columns. To do so would contravene the principle of the conservation of energy.

Therefore, for slopes with complex geometries and non-symmetric slip surfaces, it is proposed that a three-dimensional stress analysis on the slope can be performed and the stress state input into the general model. At this point in the analysis, it is possible to take one of two approaches. One approach is to determine the five intercolumn force functions from the stress state of the slope and solve for the factor of safety using one common value for all five intercolumn force functions (i.e., $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5$). This approach retains the essence of the limit equilibrium method. The second approach is to calculate the activating force from the shear stress, and the resisting force from the normal stress together with the shear strength parameters. A comparison of the resisting and activating forces along the entire slip surface allows the estimation of the overall factor of safety of the sliding mass. As a result, it will no longer be necessary to define the intercolumn force functions and solve for any lambda value. Rather, the overall factor of safety is computed directly from the stress analysis results.

engineer's estimate of stress conditions, and then solve for one common lambda value (i.e., $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5$). It is true that using one common lambda value for all five intercolumn force functions may alter the actual intercolumn shear forces distributions within the soil mass. However, the findings from this study show that as long as the assumed force functions are reasonable, the factor of safety is not sensitive to the actual shape of the force functions when both moment and force equilibrium are satisfied. This observation is well known in two-dimensional cases, and is shown to be true also in three-dimensional cases (Lam, 1991).

6 CONCLUSION

The general limit equilibrium model for three-dimensional stability analysis proposed by Lam and Fredlund (1993) involves the knowledge of five intercolumn force functions and five lambda values. The findings from this study suggest that depending on the geometry and the shape of the slip surface, it is appropriate to consider only one or two intercolumn force functions and solve for either one or two lambda values.

For uniform slopes with symmetric slip surfaces, it is appropriate to define one intercolumn force function (X/E) and solve for one lambda value (λ_1). For non-uniform slopes with symmetric slip surfaces, the solution may require the definition of two intercolumn force functions (X/E and V/P), and solve for two lambda values.

For slopes with non-symmetric slip surfaces, the solution may require the definition of all five intercolumn force functions and solving for five lambda values. Alternatively, the five functions could be determined from a stress analysis and the solution can then be obtained by solving for one common lambda value for all the functions.

Since the factor of safety is not sensitive to the shape of the intercolumn force when both moment and force equilibrium are satisfied, the solution procedure to the rigorous three-dimensional stability model may be further simplified by assuming five reasonable intercolumn force functions and solving for only one common lambda value. This simplified approach to the proposed general method greatly reduces the complexity of the solution procedure, and providing a factor of safety accurate enough for most engineering practice.

Chen, R.H., and Chameau, J.L. 1982. Three-dimensional limit equilibrium analysis of slopes. Geotechnique, 33:31-40.

Fan, K., Fredlund, D.G., and Wilson, G.W. 1986. An interslice force function for limit equilibrium slope stability analysis. Canadian Geotechnical Journal, 23:287-296.

Fellenius, W. 1936. Calculation of the stability of earth dams. Proceedings of the Second Congress on Large Dams, 4:445-463.

Fredlund, D.G., and Krahn, J. 1977. Comparison of slope stability methods of analysis. Canadian Geotechnical Journal, 14:429-439.

Fredlund, D.G., 1984. Analytical methods for slope stability analysis: state-of -the-art. In Proceedings of the 4th International Symposium on Landslides, Toronto, pp.229-250.

Hovland, H.J. 1977. Three-dimensional slope stability analysis method. ASCE Journal of the Geotechnical Engineering Division, 103(GT9):971-986.

Hungr, O. 1987. An extension of Bishop's simplified method of slope stability analysis to three-dimensions. Geotechnique, 37:113-117.

Hungr,O., Salgado, F.M., and Byrne, P.M. 1989. Evaluation of a three-dimensional method of slope stability analysis. Canadian Geotechnical Journal, 26:679-686.

Janbu, N. 1954. Application of composite slip surface for stability analysis. In Proceedings of the European Conference on Stability of Earth Slopes, Stockholm, Sweden, Balkema, Rotterdam, pp.43-49.

Lam, L., 1991. Generalized three-dimensional slope stability analysis using method of columns. Ph.D. Thesis, University of Saskatchewan, Saskatoon, Canada.

Lam, L., and Fredlund, D.G. 1993. A general limit equilibrium model for three-dimensional slope stability analysis. Canadian Geotechnical Journal, 30:905-919.

Morgenstern, N.R., and Price, V. 1965. The analysis of the stability of general slip surfaces. Geotechnique, 15:79-93.

Spencer, E. 1967. A method of analysis of the stability of embankments assuming parallel interslice forces. Geotechnique, 17:11-26.