Use of a Kelvin Rheological Model to Simulate the Consolidation of a Saturated Soil Strata

(Presentation in English)

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1. INTRODUCTION

The Kelvin Model is commonly used to illustrate the load transfer process from the water phase to the soil structure during the consolidation of a saturated soil [1]. In the past various consolidation equations have been derived by incorporating the Kelvin model into the Terzaghi's consolidation equation [2]. The mathematics, however, have been solved only for a soil layer of unit or infinitesimal thickness. In this paper, the mathematical formulation for the entire strata is derived, where the Kelvin models for each infinitesimal layer of the entire strata are interconnected.

2. KELVIN MODEL FOR SATURATED SOILS

In the past, attempts to use the multi-element Kelvin model to simulate the consolidation process of an entire strata did not fully satisfy the conservation of mass in that the water does not flow from one dashpot (or layer) to another [3].

For one-dimensional consolidation of a n-layer soil with one way drainage through the top, each layer, except the the bottom-most layer, is modelled using a modified Kelvin type element (Fig. 1). Only the bottom-most layer is truly modelled as a Kelvin element (i.e., one spring and one piston in parallel). In the modified Kelvin elements, the fluid exiting from a lower element enters into the upper element.

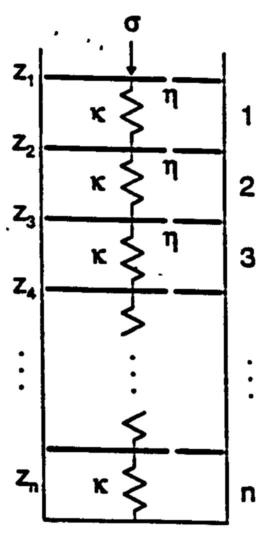

Figure 1 - Modified Kelvin model simulating a column of soil in one-dimensional consolidation.

3. MATHEMATICAL FORMULATION USING MULTI-ELEMENT KELVIN MODEL

The positions of the n pistons at any time, t, (Fig. 1) are denoted by $z_1(t), z_2(t)......, z_n(t)$. The total stress for the first layer can be written as:

$$\sigma = \sigma_{H1} + \sigma_{N1} = (z_1 - z_2)\kappa_1 + \eta_1 \bar{z}_1 \qquad (1)$$

where σ = total stress on the soil, σ_{H1} = stress on the Hookean portion (i.e., spring) of the Kelvin element of the first layer, σ_{N1} = stress in the Newtonian (i.e., dashpot) of the Kelvin element of the first layer, κ_1 = spring constant, and η_1 = viscosity.

A similar equation can be written for the second soil layer as follows:

$$(z_1 - z_2)\kappa_1 = \sigma_{H2} + (\sigma_{N2} - \sigma_{N1}) = (z_2 - z_3)\kappa_2 + \eta_2 \bar{z}_2$$

(2)

Eqs. (1) and (2) show that the summation of the water pressure and the spring stress in the second layer must equal the total stress, σ. In a similar manner, stress versus displacement equations can be written for the remaining (n-2) elements. The n stress versus displacement equations represent a system of linear differential equations which can be expressed in the following matrix form.

$$\begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \\ \bar{z}_3 \\ . \\ \bar{z}_n \end{bmatrix} = \begin{bmatrix} -\frac{\kappa_1}{\eta_1} & \frac{\kappa_1}{\eta_1} & 0 & 0 & \cdots & 0 \\ \frac{\kappa_1}{\eta_2} & -\frac{\kappa_1+\kappa_2}{\eta_2} & \frac{\kappa_2}{\eta_2} & 0 & \cdots & 0 \\ 0 & \frac{\kappa_2}{\eta_3} & -\frac{\kappa_2+\kappa_3}{\eta_3} & \frac{\kappa_3}{\eta_3} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & \frac{\kappa_{n-1}}{\eta_n} & -\frac{\kappa_{n-1}+\kappa_n}{\eta_n} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ . \\ z_n \end{bmatrix} + \begin{bmatrix} \frac{\sigma}{\eta_1} \\ 0 \\ 0 \\ . \\ 0 \end{bmatrix} \qquad (3)$$

For a homogeneous soil, κ_i is equal to κ and η_i is equal to η. Eq. (3), along with the initial condition for Eq. (1) can be written as:

$$\dot{z} = Az + b \qquad z(0) = 0 \tag{4}$$

where:

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \cdot \\ \dot{z}_n \end{bmatrix}, A = \frac{\kappa}{\eta}\begin{bmatrix} -1 & 1 & 0 & 0 & \dots & 0 \\ 1 & -2 & 1 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & 1 & -2 \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \cdot \\ z_n \end{bmatrix}, b = \frac{\sigma}{\eta}\begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ 0 \end{bmatrix}$$

(5)

The closed form solution to Eq. (4) can be expressed as:

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ \\ z_n(t) \end{bmatrix} = \int_0^t e^{A(t-s)} b\,ds$$

(6)

where

$$e^{A(t-s)} = \sum_{i=0}^{\infty} \frac{(t-s)^i}{i!} A^i = I + A(t-s) + \frac{(t-s)^2}{2!} A^2 + \frac{(t-s)^3}{3!} + \dots$$

and I = identity matrix.

The coefficient matrix, A, has n real distinct, negative eigenvalues. The solution has the form:

$$z_1(t) = \sum_{i=1}^{n} \alpha_i (1 - e^{\lambda_i t})$$

(7)

where α_i = constant (i = 1,2,...n) and λ_i = eigenvalue of A ($\lambda_i < 0$; i = 1, 2, ...n).

In the limit as n goes to infinity, Eq. 7 assumes the same form as the solution for the percent consolidation obtained from the Terzaghi's model.

Eqn. (6) implies that the consolidation is primarily governed by the eigenvalues of A. The n eigenvalues of A are uniquely determined by the soil parameters since only κ_i and η_i appear in the coefficient matrix A. The external load (or stress), σ, affects only the magnitude of the strain.

4. EXAMPLE PROBLEM

The consolidation of a Regina Clay specimen was modelled using a 10-layer rheological model. The Regina Clay has the following parameters; coefficient of permeabilty, k_s, of 3.449 x 10^{-10} (m/s) and coefficient of volume change, m_v, of 4.05 x 10^{-3} (1/kPa). The rheological parameters can be obtained from the soil parameters as follows:

$$\eta = \frac{\gamma_w}{k_s}\Delta z \quad \text{and} \quad \kappa = \frac{1}{\Delta z\, m_v}$$

(8)

where γ = unit weight of water. For the Regina Clay, η = 2.84 x 10^7 (kPa.s/m) and κ = 2470 kPa/m). The pore-water pressure in each layer as a result of an external applied load of 49 kPa are shown in Fig. 2.

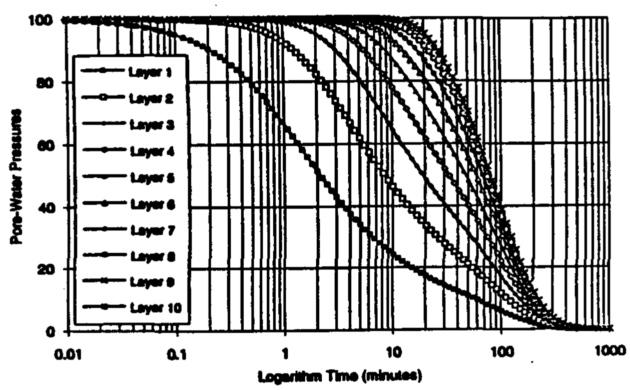

Figure 2 - Pore-water pressures at different layers in the specimen of Regina clay during one-dimensional consolidation.

5. CONCLUSIONS

Solution from the proposed rheological model approaches the classical Terzaghi's solution when the number of layers is large. The rheological parameters can be obtained from the soil parameters. The proposed model can be used to simulate consolidation problems involving multi-layered soil, where each soil could have its own parameters

6. REFERENCES

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