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Predicting the shear strength function for unsaturated soils using the soil-water characteristic curve

Prédiction de la résistance au cisaillement d'un sol non saturé en utilisant la courbe de rétention d'eau

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ABSTRACT: The shear strength of unsaturated soils can be computed using the soil-water characteristic curve and the saturated shear strength parameters of the soil. Closed-form solutions for the prediction of unsaturated shear strength are developed in this paper using simple soil-water characteristic curve equations available in the literature. These closed-form solutions are not suitable for all types of soils and large ranges of suction. A general form for the shear strength equation of unsaturated soils using a rigorous soil-water characteristic curve equation is also developed in this paper. The proposed models make use of the soil-water characteristic curve and the saturated shear strength parameters to predict the variation of shear strength with respect to suction.

RESUME : On peut calculer la résistance au cisaillement de sols non saturés à l'aide de la courbe caractéristique sol-eau et des paramètres de résistance au cisaillement du sol saturé. On developpe les solutions exactes afin de prédire la résistance au cisaillement des sols non saturés, à l'aide de courbes caractéristiques sol-eau qu'on trouve dans la littérature. Toutefois ces solutions exactes ne conviennent pas à tous les types de sol et pour des gammes étendues de succion. Une forme générale des équations de résistance au cisaillement pour les sols non saturés faisant appel à une équation rigoureuse de la courbe eau-sol est également proposée. Ces modèles permettent de prédire la variation de la résistance au cisaillement en fonction de la succion

1. INTRODUCTION

A theoretical framework for unsaturated soil mechanics has been firmly established over the past two decades. The constitutive equations for volume change, shear strength and flow for unsaturated soil are becoming generally accepted in geotechnical engineering (Fredlund and Rahardjo, 1993). The measurement of soil parameters for the unsaturated soil constitutive models, however, remains a demanding laboratory process. For most practical problems approximate soil properties are adequate for analysis purposes. Hence, empirical procedures to estimate unsaturated soil parameters are sufficient.

Laboratory studies indicate that there is a relationship between the soil-water characteristic curve and the unsaturated shear strength. Closed-form solutions for the prediction of unsaturated shear strength are developed in this paper using soil-water characteristic curve equations proposed by McKee and Bumb (1984) and Brooks and Corey (1964). These closed-form solutions are simple, but are not suitable for all types of soils and large ranges of suction. A more general form of a shear strength equation which is valid for all soils and for all ranges of suction, using a rigorous soil-water characteristic curve equation by Fredlund and Xing (1994), is also developed in this paper.

The models presented herein make use of the soil-water characteristic curve and the saturated shear strength parameters to predict the variation of shear strength with respect to suction.

2. BACKGROUND

The accepted shear strength equation for saturated soils is a linear function of effective stress and is given as follows:

$$\tau_f = c' + (\sigma - u_w)tan\phi' \qquad (1)$$

where: τ_f = shear strength
c' = effective cohesion
ϕ' = effective angle of shearing resistance
σ = total stress
u_w = pore-water pressure

Unlike saturated soils, the shear strength of unsaturated soils cannot be described by a single stress state variable. Fredlund et al. (1978) described the shear strength of unsaturated soil in terms of two stress state variables; net normal stress, $(\sigma_n - u_a)$, and matric suction, $(u_a - u_w)$. The equation is:

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad (2)$$

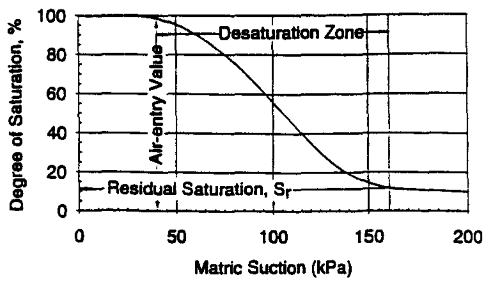
where:
$(\sigma_n - u_a)$ = net normal stress
ϕ^b = angle of shearing resistance with respect to matric suction

2.1 Use of Soil-Water Characteristic Curve

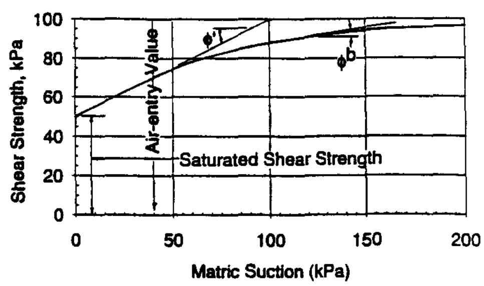
The soil-water characteristic curve may be defined as the variation of suction with the water storage capacity within the macro and micro pores of a soil. The soil-water characteristic curve is generally plotted as the variation of gravimetric water content, w, or volumetric water content, θ, or degree of saturation, S, with suction. The suction may be either the matric suction of the soil (i.e., $(u_a - u_w)$), or total suction (i.e., the matric plus osmotic suction). At high suctions (i.e., greater than about 2,500 kPa), matric suction and total suction can generally be assumed to be similar in magnitude.

Figure 1(a) shows a typical soil-water characteristic curve plotted as the degree of saturation versus matric suction. The air-entry value of the soil, $(u_a - u_w)_b$, and the residual degree of saturation, S_r, are shown in Fig. 1(a). Beyond the value, $(u_a - u_w)_b$, the specimen starts to desaturate. Desaturation continues as suction is increased. The rate of desaturation considerably decreases beyond a particular level of suction. The degree of saturation at this value of suction is called the residual degree of saturation, S_r.

Figure 1(b) shows a typical relationship between shear strength and increasing soil suction. The relationship between the shear strength and the soil-water characteristic curves can be seen by comparing Figures 1(a) and 1(b). The rate of desaturation with respect



a) Matric suction versus degree of saturation



b) Matric suction versus shear strength

Figure 1. Relationship between soil-water characteristic curve and shear strength

to an increase in matric suction, (i.e., $dS/d(u_a - u_w)$) is greatest between the air-entry value and residual suction.

The shear strength contribution due to matric suction is primarily through the water inter-aggregate contact area. As there is little change in water content of the soil below the air-entry value, suction as a stress state variable is as effective as net normal stress in mobilizing the shearing resistance along all the contact area points. This implies that ϕ^b is equal to ϕ'. Above the air entry value, the contribution of shear strength by suction decreases with the desaturation of the soil and results in a nonlinear variation of shear strength with respect to suction. Thus, there is a strong correlation between the shear strength behavior of an unsaturated soil and the soil-water characteristic curve.

3. A MODEL FOR THE SHEAR STRENGTH FUNCTIONS

Let us assume that a shear strength increment, $d\tau$, due to a matric suction increment, $d(u_a - u_w)$, is proportional to the product of the change in suction and the effective water contact area in the soil at the current state:

$$d\tau = CA_w d(u_a - u_w) \qquad (3)$$

where: A_w = effective water contact area
C = constant of proportionality

The effective water contact area, A_w, on a section through the soil is defined as:

$$A_w = \frac{A_{tw} - A_{dw}}{A_{tw}} \qquad (4)$$

where:
A_{tw} = total water inter-aggregate contact area in the saturated state
A_{dw} = unsaturated inter-aggregate contact area

A_w is a dimensionsless number which varies from unity to zero as the soil moves from saturated state to the residual water content (or dry) condition. A_w, reduces as the water is driven out from the soil pores due to suction. At residual conditions, A_{tw}, is approximately equal to A_{dw}. The effective saturation, S_e, of the soil also varies similarly. The effective saturation S_e, is defined as:

$$S_e = \frac{S - S_r}{1 - S_r} \qquad (5)$$

where: S = current degree of saturation
S_r = residual saturation

S_e, is commonly used in the field of soil science and geotechnical engineering and can be estimated from the soil-water characteristic curve data. Due to similarities in behavior of A_w and S_e, the following relationship is proposed.

$$A_w = [S_e]^p \qquad (6)$$

where: p = fitting parameter

Substituting [6] in [3] gives

$$d\tau = C\,[S_e]^p\, d(u_a - u_w) \qquad (7)$$

On integration, [7] takes on the following form:

$$\tau = C_1 + C \int_0^{(u_a - u_w)} \left[S_e\right]^p d(u_a - u_w) \qquad (8)$$

where: C_1 = constant of integration

When matric suction is equal to zero, the second term in [8] equals zero and

$$C_1 = \tau(0) = c' + (\sigma_n - u_a)\tan\phi' \qquad (9)$$

Equation [9] represents the saturated shear strength of the soil.

Below the air-entry value, the shear strength increases at a linear rate equal to $\tan\phi'$; thus,

$$C = \tan\phi' \qquad (10)$$

On substitution of [9] and [10] into [8], the resulting expression for shear strength is:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + \tan\phi' \int_0^{\psi} \left[S_e\right]^p d(u_a - u_w) \qquad (11)$$

where: ψ = variable of integration representing suction present in the soil

The unsaturated shear strength prediction depends on the values of residual conditions and the fitting parameter 'p'. A value of 'p' equal one may be reasonably good for inactive soils like sands, silts and some fine soils, for matric suction ranges of 0 to 500 kPa which is the range in which geotechnical engineers are generally interested.

A fitting parameter like 'p' is essential to obtain a better correlation between predictions and experimental shear strength data. This parameter could be a varying function related to the soil-water characteristic curve. In this paper, the relationships are derived assuming 'p' equals one.

Equation [11] shows that if the saturated shear strength parameters, c' and ϕ', are known and if the effective saturation, S_e, is defined as a mathematical function, and the residual conditions are determined from the soil-water characteristic curve, the variation of shear strength with respect to suction can be predicted.

3.1 Closed Form Solutions for Predicting the Unsaturated Shear Strength Using Simple Soil-Water Characteristic Curve Equations

McKee and Bumb (1984) suggest a exponential relationship to define, S_e, from the soil-water characteristic curve as:

$$S_e = e^{-\left(\frac{\psi - (u_a - u_w)_b}{b_1}\right)} \quad (12)$$

where: b_1 = a fitting parameter

Substituting [12] into [11], with p = 1, gives:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + \tan\phi' \int_0^{(u_a - u_w)_b} d\psi + \tan\phi' \int_0^{(u_a - u_w)} e^{-\left(\frac{\psi - (u_a - u_w)_b}{b_1}\right)} d\psi \quad (13)$$

The closed-form equation for shear strength becomes:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)_b \tan\phi' + b_1\left(1 - e^{-\frac{(u_a - u_w)}{b_1}}\right)\tan\phi' \quad (14)$$

Equation [12] is generally used to describe a soil-water characteristic curve for suction values greater than the air-entry value. Reasonable shear strength prediction can be expected using [14] if a good estimate is made for defining the effective saturation, S_e, from the soil-water characteristic curve. Equation [12] is generally recommended for soils like sands and certain silts which have low-air entry values and low compressibilities or storage capacity. A horizontal line can be assumed in the range from zero to the air-entry value. The air-entry value of the soil however has to be determined from the soil-water characteristic curve data.

Brooks and Corey (1964) describe the effective saturation, S_e as :

$$S_e = \left(\frac{(u_a - u_w)_b}{\psi}\right)^{b_2} \quad (15)$$

where: b_2 = a fitting parameter

Again, Equation [15] is valid for matric suction ranges greater than the air-entry value. Substituting [15] into [11], the closed form solution for predicting the shear strength would the form:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)_b \tan\phi' + \frac{(u_a - u_w)_b^{b_2}}{b_2 - 1}\left(\frac{1}{(u_a - u_w)_b^{b_2 - 1}} - \frac{1}{(u_a - u_w)^{b_2 - 1}}\right)\tan\phi'$$

(for $b_2 \neq 1$) (16)

For the special case of $b_2 = 1$, [11] can be written as:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)_b \tan\phi' + (u_a - u_w)_b \ln\left(\frac{(u_a - u_w)}{(u_a - u_w)_b}\right)\tan\phi'$$

(for $b_2 = 1$) (17)

Though some accuracy is lost, the closed-form solutions are useful for approximating the shear strength. It is important to check whether the soil-water characteristic curve data is reliably represented by [12] and [15] before using the equations.

3.2 Shear Strength Prediction Using the Rigorous Soil-Water Characteristic Curve Equation

A general or rigorous equation describing the soil-water characteristic curve for the entire suction range of (i.e., 0 to 1,000,000 kPa) is given by Fredlund and Xing (1994).

$$\theta = \theta_s \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{1{,}000{,}000}{\psi_r}\right)}\right]\left[\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)}\right]^m \quad (18)$$

In terms of degree of saturation versus suction, [18] takes the form:

$$S = \left[1 - \frac{\ln\left(1 + \frac{\psi}{\psi_r}\right)}{\ln\left(1 + \frac{1{,}000{,}000}{\psi_r}\right)}\right]\left[\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)}\right]^m \quad (19)$$

where:

θ = volumetric water content

θ_s = saturated volumetric water content

S = degree of saturation

ψ = soil suction

e = natural number, 2.71828..

ψ_r = the suction corresponding the residual water content, θ_r

a = an approximation of the air-entry value

n = a parameter which controls the slope at the inflection point in the soil-water characteristic curve

m = a parameter which is related to the residual water content

The general predictive model for shear strength using the rigorous form of soil-water characteristic curve equation [19] would be:

$$\tau = c' + (\sigma - u_a)\tan\phi' + \tan\phi' \int_0^{\psi} \left[\left(\frac{S - S_r}{1 - S_r}\right)\right] d(u_a - u_w) \quad (20)$$

where: S in [20] is defined by [19].

The residual saturation, S_r, and its corresponding suction value is determined from the soil-water characteristic curve data.

The fitting parameters in [19] (i.e., *a*, *n*, and *m*) for the soil-water characteristic curve data can be determined using a nonlinear regression procedure outlined by Fredlund and Xing (1994). Thus, the soil-water characteristic curve data of any soil can be conveniently defined as a mathematical function. Numerical results show that [19] can describe the soil-water characteristic curve reasonably well for most soils in the entire suction range (i.e., 0 to 1,000,000 kPa).

4. EXPERIMENTAL PROGRAM

An experimental program was conducted for determining the shear strength characteristics of specimens of a statically compacted glacial till at different water contents and densities under consolidated drained conditions using multistage direct shear tests. The tests were conducted under different levels of net normal stress and matric suction. Soil-water characteristic curves were developed using a pressure plate apparatus and desiccators with specimens pre-consolidated to the equivalent net normal stresses used in the actual experimental program. The experimental test results were compared with the model developed. There is a good correlation between the measured and predicted values of shear strength (Vanapalli, 1994).

A typical set of test results conducted on a specimen loaded to a net normal stress of 25 kPa is presented. The initial water content of the specimen was 16% and the dry density, γ_d, was 1.78 Mg/m^3. The saturated shear strength parameters are: effective cohesion, c' is zero and the angle of shearing resistance, ϕ' is 23 degrees. The soil-water characteristic curve developed on a preloaded specimen with initial conditions similar to shear strength testing for the range of suction of 0 to 1,000,000 kPa is shown in Fig. 2.

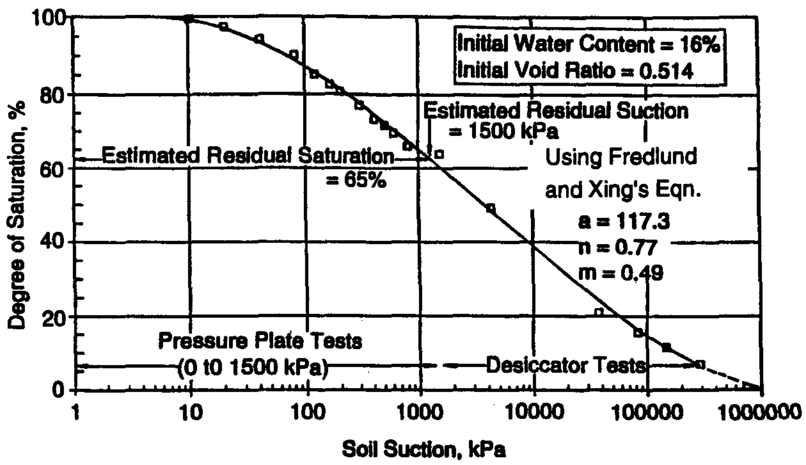


Figure 2 Soil-water characteristic curve for a compacted till specimen with a preload of 25 kPa

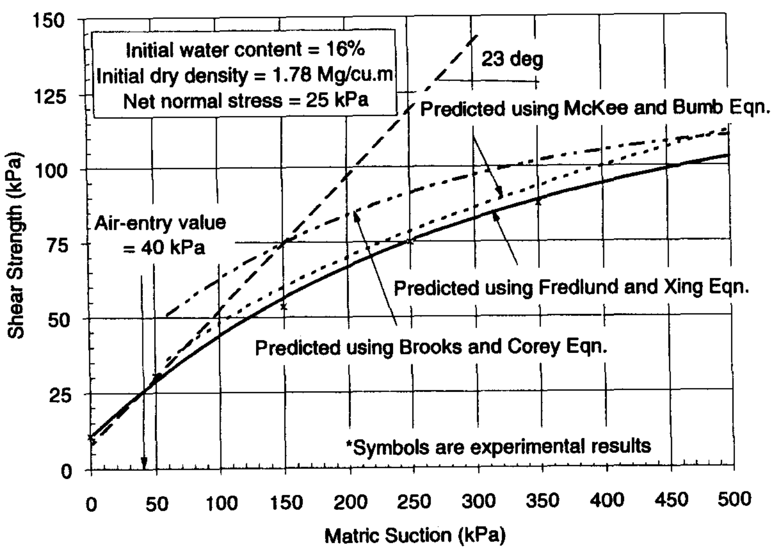


Figure 3 Comparison of experimental and predicted variation of shear strength with matric suction using different models.

The fitting parameters using nonlinear regression analysis for defining the soil-water characteristic curve according to the method of Fredlund and Xing (1994) were : $a = 117.3$, $n = 0.77$, and $m = 0.49$ using [20]. The fitting parameters b_1 and b_2 for equations [12] and [15] are 207 and 0.535 respectively. The residual degree of saturation and suction were 65% and 1,500 kPa respectively from the soil-water characteristic curve data. The variation of shear strength with suction using the models and experimental results are shown in Fig. 3. The experimental results are shown by symbols and the continuous lines are the predicted variation of shear strength with suction using the models. There is good correlation between the experimental results and the predicted values of shear strength using [20] compared to closed-form solutions [14] and [16].

5. CONCLUSIONS

The soil-water characteristic curve data and the saturated shear strength parameters are sufficient to predict the variation of shear strength with respect to matric suction using the models developed in this paper.

Closed form solutions were developed using McKee and Bumb (1984) and Brooks and Cory (1964) equations for predicting the unsaturated shear strength behavior of soils. These solutions are approximate but simple. These solutions would be suitable for sandy and silty soils which desaturate relatively faster and have low air-entry values.

A more general predictive model is developed using the rigorous form of soil-water characteristic curve equation valid for all soils and ranges of suction proposed by Fredlund and Xing (1994). The predicted unsaturated shear strengths are close to the measured experimental values for a glacial till tested under different levels of net normal stress and suction values (Vanapalli, 1994). In this paper only one set of data is presented .

The analytical approach presented in this paper should encourage geotechnical engineers to put the unsaturated shear strength theories into practice.

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