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AN INTRODUCTION TO ANALYTICAL MODELLING OF PLANT TRANSPIRATION FOR GEOTECHNICAL ENGINEERS

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ABSTRACT

Historically, geotechnical engineers have been concerned with groundwater flow, particularly within the saturated zone. More recently, geotechnical engineers have identified the importance of unsaturated flow and the flux boundary condition with respect to water flow across the soil-atmosphere interface. Plant transpiration is an important component of the surface flux rate.

This paper outlines a theoretical approach for the prediction of transpiration rates. A laboratory experiment was conducted to measure evapotranspiration rates and analytical modelling was performed to simulate the laboratory data. The experimental results identify the significance of including the transpiration flux in the surface flux boundary condition. The analytical modelling results show the proposed methodology for the prediction of transpiration to be reasonably accurate.

RESUME

INTRODUCTION

With an ever expanding knowledge base, the science of geotechnical engineering must continually apply theory developed by other disciplines. The evaluation of plant transpiration is one such area where the research has been conducted primarily by Soil Scientists. Geotechnical engineers often overlook the surface flux component related to the vegetative cover. The flux boundary condition at the soil surface is a critical factor for problems in geotechnical engineering related to soil cover system design, saturated/unsaturated groundwater flow, slope stability and volume change in expansive soils. In such engineered systems, the transpiration component may be critical in the success of the design.

The design of soil covers involves the placement of multiple soil layers over hazardous wastes. The arrangement allows for the control of oxygen diffusion and moisture percolation into the underlying waste to reduce leachate production. Vegetation is placed on the soil cover for aesthetic as well as design purposes. Vegetation promotes the integrity of the soil against erosion due to surficial runoff waters. The resulting influence of the vegetative cover on the performance of the cover system with respect to moisture flow must be evaluated. A review of literature on existing transpiration prediction methodologies identifies a science which although has had immense research conducted does not have consensus on an acceptable method to predict transpiration. Methods to predict transpiration vary between highly empirical (de Jong, 1974) to physically based models which are highly theoretical (Federer, 1979). Marker and Mein (1985) state that empirical based models lack accuracy whereas theoretical methods are too complicated for routine use.

METHODOLOGY TO PREDICT TRANSPIRATION

The computer program, SoilCover (MEND, 1993), was developed by a group of researchers at the University of Saskatchewan. SoilCover is a 1-dimensional, transient, heat and mass transfer finite element model. One of the special features of SoilCover is the ability to evaluate bare soil evaporation from unsaturated soil surfaces as given in eq. 1 by Wilson et al. (1994).

[1] $$E = f(u)\,(e_s - e_a)$$

where: E = the actual evaporation from the saturated/unsaturated soil surface (mm/s),
$f(u)$ = the turbulent mixing parameter (units),
e_s = the vapour pressure at the soil surface (units), and
e_a = the vapour pressure in the overlying atmosphere (units).

The partial differential equation describing mass movement through porous media, including its modification to include the transpiratory root uptake flux is presented in Eq. 2 (Wilson, 1990). Wilson (1990) also provides a heat transfer equation which is solved simultaneously with eq. 2.

[2] $$\frac{\partial h_w}{\partial t} = C_w^1 \frac{\partial}{\partial z}\left(k_w \frac{\partial h_w}{\partial z}\right) + C_w^2 \frac{\partial}{\partial z}\left(D_v \frac{\partial P_v}{\partial z}\right) + S$$

where: h_w = the total hydraulic head (m),

k_w = the coefficient of permeability as a function of matric suction (m/s),
D_v = the coefficient of water vapour diffusion through the soil (kg·m/kN·s),
C_w^1 = the modulus of volume change of water with respect to the liquid phase,
$= \left(\frac{1}{\rho_w\, g\, m_w^2} \right)$,
C_w^2 = modulus of volume change of water with respect to the vapour phase,
$= \left(\frac{1}{(\rho_w)^2\, g\, m_w^2} \right)$,
m_w^2 = the slope to the moisture retention curve (1/kPa),
ρ_w = the mass density of water (kg/m^3),
P = the total pressure in the bulk air phase (kPa),
P_v = the actual vapour pressure in the bulk air phase (kPa),
t = the time (s),
z = the vertical position (m), and
S = the root uptake sink term (m/s).

The first term on the right hand side of Eq. 2 describes the flow of liquid phase water according to Darcy's Law. The second term describes the flow of vapour phase water according to Fick's Law. The third term describes the root uptake sink term which occurs as a flux from each influenced node. The methodology to evaluate the sink term will be briefly discussed in the following paragraphs.

The methodology to predict transpiration has been based upon the method proposed by Feddes et al. (1978). The semi-empirical method to evaluate the transpiration flux was selected due to its ability to be adapted to bare soil and partitioned cover evaporation and evapotranspiration.

The first step in evaluating the transpiration flux involves determination of the potential evaporation rate. The term potential evaporation has been well defined in the past, hence only a brief discussion will be presented here. The exact definition of potential evaporation differs between disciplines and various authors. However it is generally agreed to be the maximum potential cummulative sum of bare soil evaporation and plant transpiration (Granger, 1989).

The second step in evaluating the transpiration flux involves determination of the potential transpiration flux. Because the evaporation and transpiration components must be evaluated individually, the potential evaporation flux must be distributed into its evaporation and transpiration components. Ritchie (1972) observed the transpiration component was dependent upon the leaf area index values of the plant canopy. From his research, three degrees of vegetative cover were identified with the resulting influence upon the potential transpiration flux presented in Eqs. 3, 4 and 5.

[3] $$E_p = 0.0 \qquad LAI < 0.1$$

[4] $$E_p = E_o\,(-0.21 + 0.70\, LAI^{1/2}) \qquad 0.1 < LAI < 2.7$$

[5] $$E_p = E_o \qquad 2.7 < LAI$$

where, E_p = the potential transpiration rate per unit time (mm/day),
LAI = the leaf area index of the vegetative cover, and
$= \left(\frac{\text{surface area}_{\text{leaf}}}{\text{surface area}_{\text{soil}}}\right)$
E_o = the potential evaporation rate per unit time (mm/day).

Depending on the ratio of the surface area of the leaves to the soil surface area they cover, the surface flux condition described by either eqs. 3, 4 and 5 assume either: evaporation only, a combined evaporation and transpiration flux or solely transpiration flux conditions. The three phases of vegetative cover are identified as bare soil, partial cover and full cover conditions, respectively. Ritchie (1972) describes the leaf area index values which define lower and upper limits of the partial cover condition as 0.1 and 2.7 respectively.

The mass flux due to transpiration, being a surface potential flux, must be distributed through the soil profile which is occupied by the vegetative root structure. The proposed method of distributing the potential transpiration is presented in Figure 1 as a decreasing uptake rate with depth (Prasad, 1988).

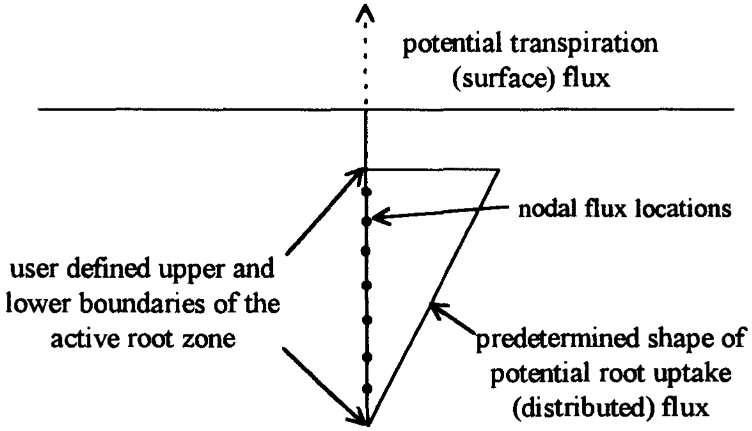

FIGURE 1. The shape function used to calculate the potential root uptake distribution through the active root zone (after Prasad, 1988).

The potential transpiration flux, defined in Figure 1, is distributed into nodal fluxes. The potential nodal flux rates are then dependent upon the potential transpiration flux, the location of the node with respect to the top and bottom of the active root zone and the node spacing. The potential nodal fluxes are then modified to determine the actual nodal root uptake flux as required in Eq. 2. The potential root uptake flux is modified by a reducing term given in Eq. 6, which is in turn based upon the matric suction at that nodal locations by the relationship presented in Figure 2.

[6] $$S = PRU \cdot PLF$$

where, S = the actual nodal root uptake sink term, required in Eq. 1 (m/s),
PRU = the potential root uptake flux (m/s), and

PLF = the plant limiting factor, dependent upon the nodal matric suction as defined in Figure 2.

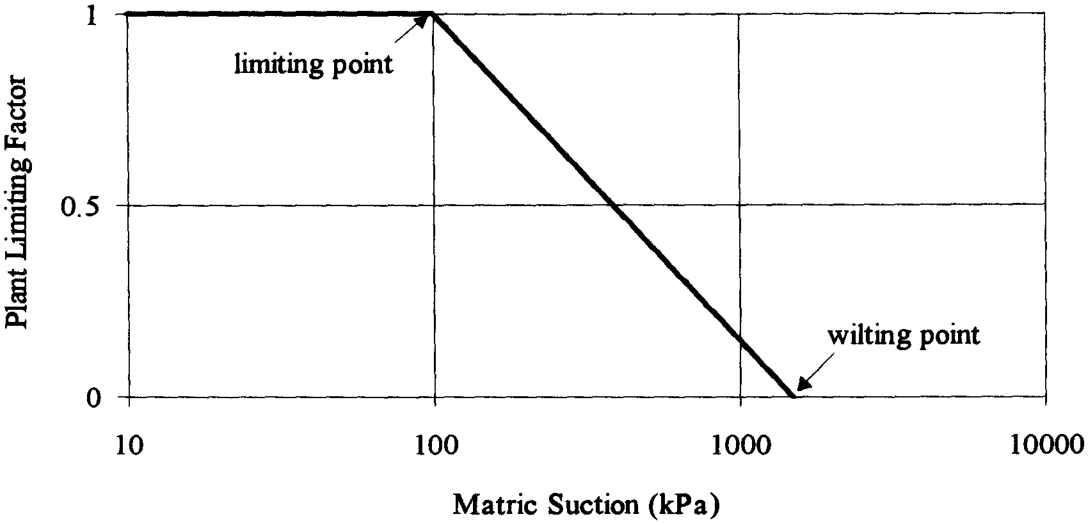

FIGURE 2. Definition of the plant limiting factor from the nodal matric suction. Generally accepted values for the limiting and wilting points range from 50 to 100 kPa and 1500 to 2000 kPa respectively (Feddes et al., 1978).

LABORATORY PROGRAM

A laboratory program was conducted to evaluate the physical responses of a soil profile to plant vegetation as well as to provide a data set for the verification of the transpiration equations presented in eqs. 2 through 6. The experiment consisted of 1-dimensional evapotranspiration from a soil column system placed within an environmental chamber. The soil column surface was vegetated and allowed to evapotranspire in the prevailing controlled atmospheric conditions. The surface flux as well as the subsurface moisture conditions were monitored to evaluate the influence of the vegetative cover. The soil column system, including some of the construction and instrumentation details, is presented in Figure 3.

The surface evapotranspiration flux condition was evaluated directly by measuring the flow of moisture into the soil column and the change in mass of the entire soil column system. The soil profile was instrumented for the measurement of matric suction using thermal conductivity sensors and tensiometers. The soil column was also instrumented for temperature using type T thermocouples. Manual water content profiles were evaluated using the PVC column port holes sealed with rubber stoppers. Climatic characterization included pan (potential) evaporation rates as well as ambient relative humidity and temperature. The vegetative canopy leaf area index was measured directly and the subsurface rooting characteristics were determined on the basis of water content profiles.

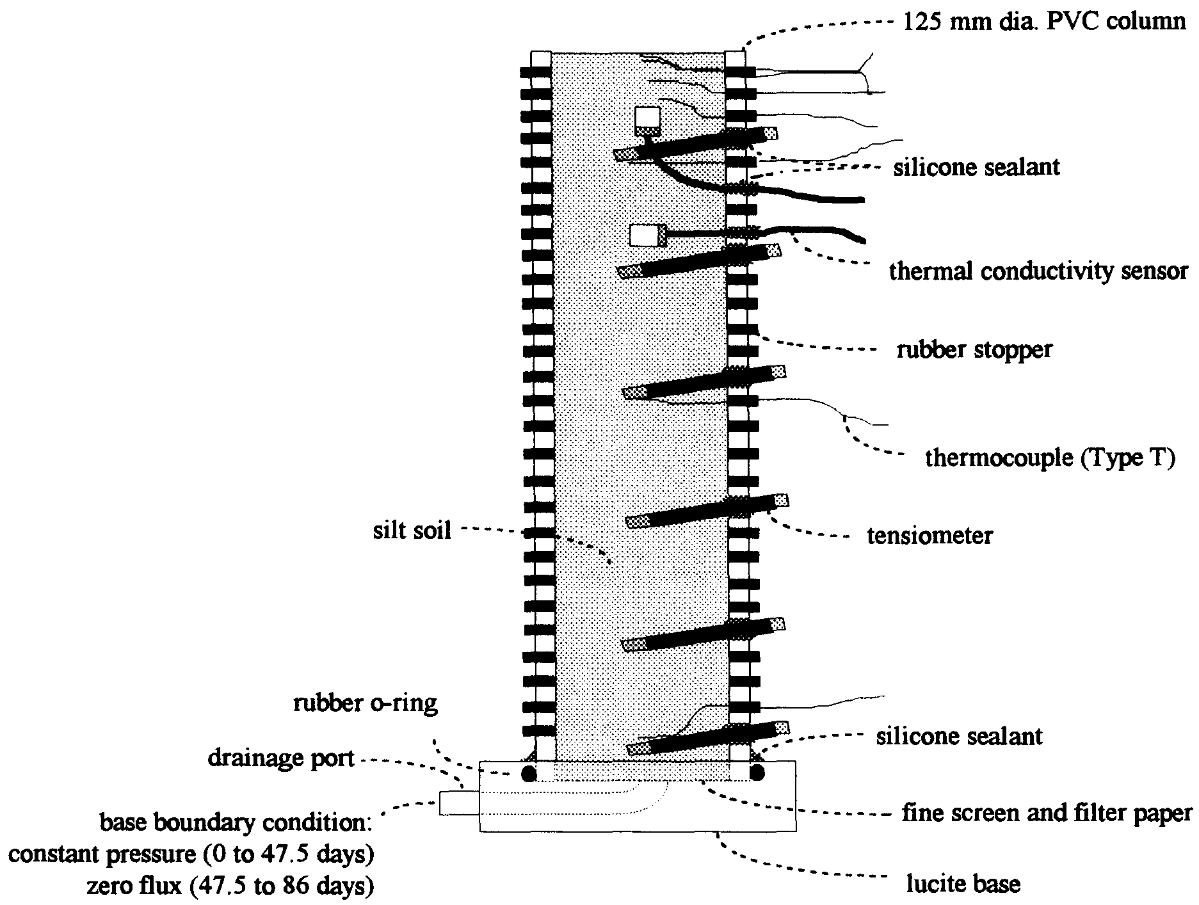

FIGURE 3. Diagram of the PVC column used in the evapotranspiration study, displaying the general construction and instrumentation details.

The soil used in the experiment was characterized as a low plasticity silt material. The soil water characteristic curve for the silt material is presented in Figure 4. The soil was observed to consolidate slightly under desiccation conditions, however was otherwise found to perform well through the experiment.

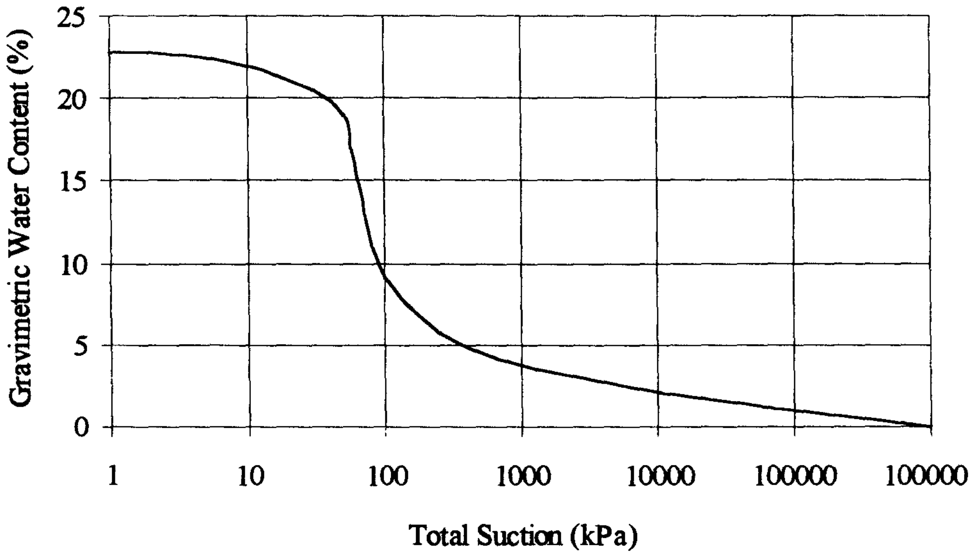

FIGURE 4. Soil-water characteristic curve for the silt material used in the soil column evapotranspiration study.

ANALYTICAL MODELLING RESULTS

The measured surface flux results obtained through the experimental program are presented in Figure 5. Figure 5 also shows the computed rate of evaporation, transpiration and evapotranspiration on the basis of eqs. 1 through 6. Good agreement is apparent between the measured and computed rates of evapotranspiration. The individual components of evaporation and transpiration were not measured, however the computed trends may be discussed in general terms.

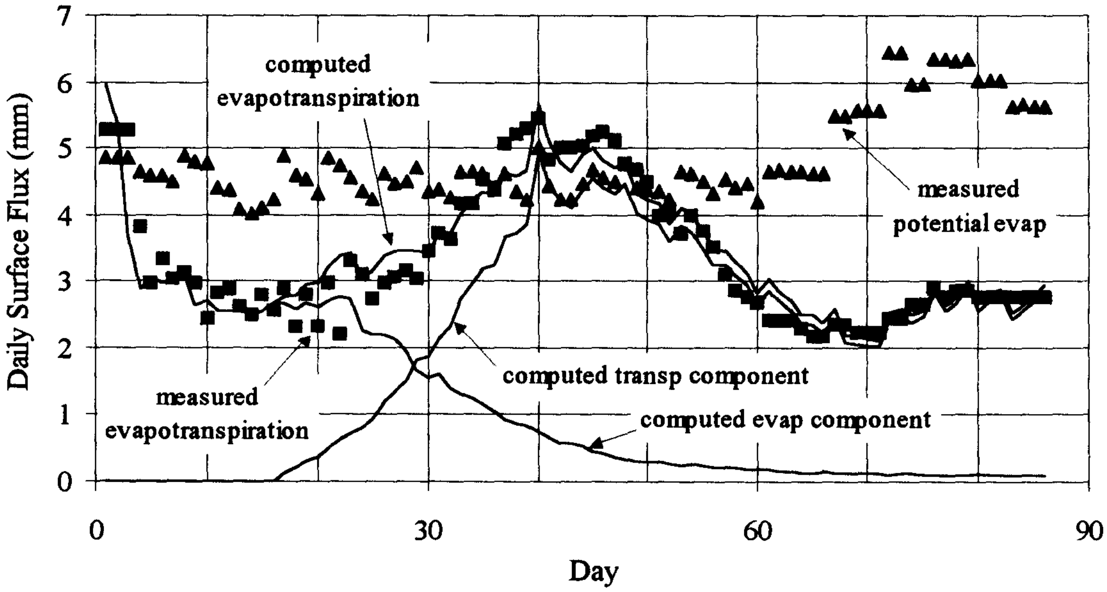

FIGURE 5. Comparison of the measured and predicted evapotranspiration fluxes. Also presented are the simulated evaporation and transpiration flux partitions.

Six surface flux stages were observed to occur through the experimental period, three involving evaporation only and three involving both evaporation and transpiration fluxes. During the first two days from initiation of the experiment, the soil surface was saturated and the flux continued at potential evaporation rates. Between days three and 10, the soil surface was desaturating and the evaporative flux continually decreased. Between days 10 and 17 the soil column system appeared to be near equilibrium for evaporative conditions. After day 17, the vegetative leaf area index exceeded 0.1, causing the transpiration flux to increase. On day 40, the vegetative canopy reached a leaf area index value of 2.7 and the potential transpiration flux was found to equal the potential evaporation flux. Between day 40 and day 47, the total evapotranspiration continued at potential evaporation rates. The flux supplying the column base was discontinued on day 47 which induced moisture limiting conditions. Almost immediately, the surface flux condition responded to the increased matric suctions caused by continuous drying conditions. The limiting conditions for evapotranspiration continued to the conclusion of the experiment on day 86. The increase in evapotranspiration after day 70 coincides with an increase in potential evaporation.

The simulated evaporation component in Figure 5 was found to continuously decrease with increasing transpiration fluxes. The decreasing evaporation flux was directly related to the desaturating surface due to the interception of moisture by the root system before it could reach the soil surface.

The measured and simulated water content profiles on select days are presented in Figure 6. The four days shown represent evaporation flux conditions only (day 16), non-limiting transpiration with full vegetative cover conditions (day 47), and evapotranspiration in moisture limiting conditions (days 72 and 86). The water content profile trends indicate the influence of transpiration and root water uptake. The water content profile on day 16 represents steady state conditions with an evaporative flux. Changes in the water content profiles after day 16 are related to the transpiration flux and the resulting root water uptake patterns.

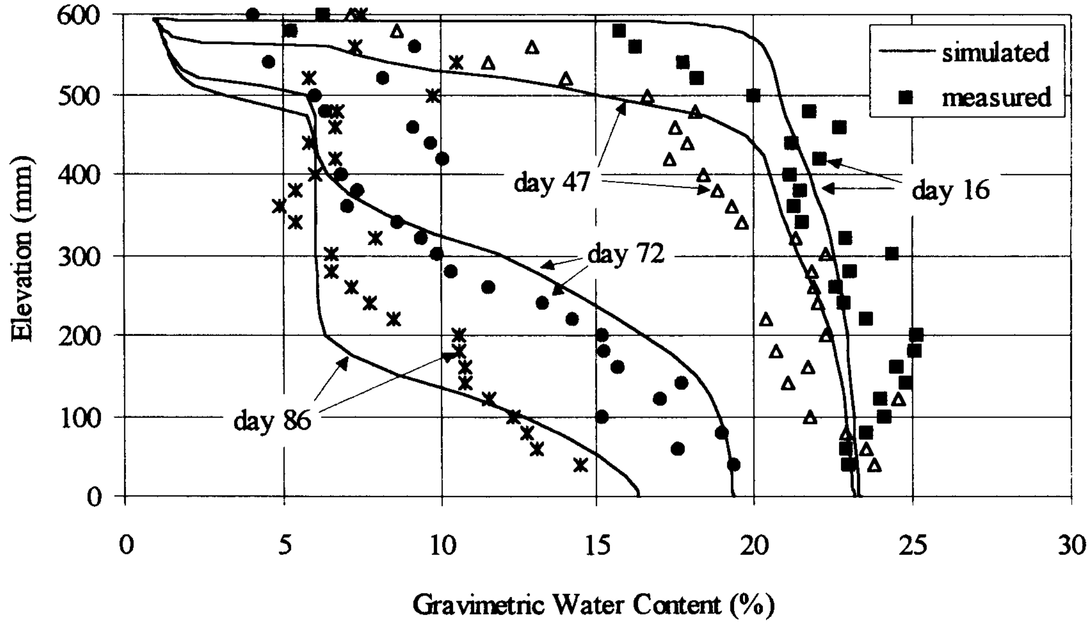

FIGURE 6. Simulated and measured water content profiles on days 16, 47, 72 and 86.

Comparisons of the simulated and measured water content results indicate the computed water uptake patterns were representative of the actual root uptake conditions. The computed root uptake trends on 10 day intervals are presented in Figure 7. The y-axis of Figure 7 are in units mm/day/mm, hence the uptake distribution must be integrated over the entire depth to determine the surface flux in units mm/day.

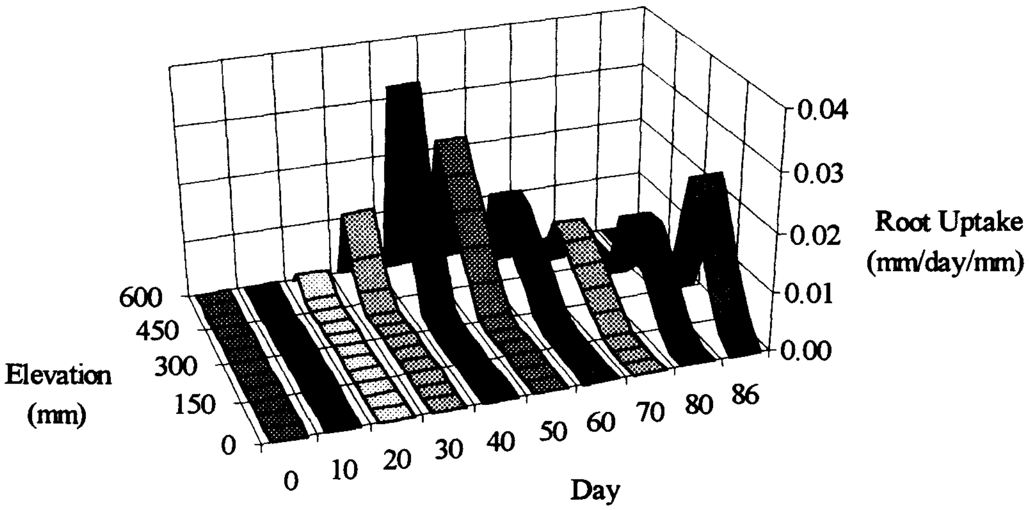

FIGURE 7. Presentation of the simulated root extraction patterns through the soil depth, displaying trends on 10 day intervals.

The various phases of the surface flux conditions are identified in Figure 7. Between initiation and day 17, the surface flux consisted totally of evaporation, hence root uptake was zero. Between days 20 and 40, the root uptake pattern is shown to increase substantially with increasing vegetative cover. After day 50, the matric suctions through the root system increase to moisture limiting conditions, hence the fluxes near the surface progressively decrease to the conclusion of the experiment. Also of interest is the increase in depth of the root system, indicating the progressive growth of the root system.

CONCLUSIONS

An experiment was conducted to evaluate the influence of transpiration on the surface flux and subsurface moisture conditions. The experiment was successful in evaluating the surface flux through six surface flux stages including bare soil evaporation and cummulative evaporation and transpiration fluxes. A good correlation was found between measured and computed evapotranspiration. Unfortunately, the evaporation and transpiration components of the surface flux could not be identified except by theoretical means. Moisture redistribution patterns under the influence of transpiration withdrawal of moisture were also successfully documented.

The proposed methodology to predict the transpiratory flux and root uptake rates appears to properly evaluate the measured data trends observed in the soil column experiment. The methodology follows a simple solution which conforms to geotechnical engineering nomenclature. The terms required for the transpiration flux solution are easy to define and measure and the results do not require a sophisticated biological interpretation.

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