Predicting the permeability function for unsaturated soils using the soil-water characteristic curve

D.G. Fredlund, Anqing Xing, and Shangyan Huang

Department of Civil Engineering, University of Saskatchewan, Saskatoon, SK S7N 0W0, Canada

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The coefficient of permeability for an unsaturated soil is primarily determined by the pore-size distribution of the soil and can be predicted from the soil-water characteristic curve. A general equation, which describes the soil-water characteristic curve over the entire suction range (i.e., from 0 to 10^6 kPa), was proposed by the first two authors in another paper. This equation is used to predict the coefficient of permeability for unsaturated soils. By using this equation, an evaluation of the residual water content is no longer required in the prediction of the coefficient of permeability. The proposed permeability function is an integration form of the suction versus water content relationship. The proposed equation has been best fit with example data from the literature where both the soil-water characteristic curve and the coefficient of permeability were measured. The fit between the data and the theory was excellent. It was found that the integration can be done from zero water content to the saturated water content. Therefore, it is possible to use the normalized water content (volumetric or gravimetric) or the degree of saturation data versus suction in the prediction of the permeability function.

Key words: coefficient of permeability, soil-water characteristic curve, unsaturated soil, water content, soil suction.

Le coefficient de perméabilité d'un sol non saturé est principalement déterminé par la répartition de la taille des pores et il peut être prédit à partir de la courbe caractéristique sol-eau. Une équation générale décrivant la courbe caractéristique sol-eau sur la plage complète des valeurs de succion (soit de 0 à 10^6 kPa) a été proposée par les deux premiers auteurs dans un autre article. Cette équation est utilisée pour prédire le coefficient de perméabilité des sols non saturés. Grâce à cette équation on n'a plus besoin d'évaluer la teneur en eau résiduelle pour prédire le coefficient de perméabilité. La fonction de perméabilité proposée est une forme intégrale de la relation succion-teneur en eau. L'équation a été ajustée à partir d'exemples de données recueillies dans la littérature où l'on a mesuré à la fois la courbe caractéristique sol-eau et le coefficient de perméabilité. L'accord entre les données et la théorie est excellent. On a trouvé que l'intégration peut être faite pour des teneurs en eau allant de zéro à la teneur en eau de saturation. Il est donc possible d'utiliser la teneur en eau normalisée (volumique ou massique) ou les données connues sur le degré de saturation en fonction de la succion pour prédire la fonction de perméabilité.

Mots clés : coefficient de perméabilité, courbe caractéristique sol-eau, sol non saturé, teneur en eau, succion du sol.

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Introduction

There is no engineering soil property that can vary more widely than that of the coefficient of permeability. For saturated soils, the coefficient of permeability can vary more than 10 orders of magnitude when considering soils that range from a gravel to a clay. This wide range in coefficient of permeability has proven to be a major obstacle in analyzing seepage problems.

Soils that become desaturated are even more difficult to analyze. In this case it is possible for a single soil to have a coefficient of permeability that ranges over 10 orders of magnitude. Initial consideration of problems involving unsaturated soils might lead an engineer to conclude that no useful analyses are possible when the soil becomes unsaturated. However, experience has now shown that many important questions can be addressed using seepage analyses on unsaturated soils.

Generally an upper and lower bound can be established on the coefficient of permeability function for the soil. As a result, an upper and lower bound can also be established on other pertinent variables such as mass flows and pore-water pressures. Analyses have shown that the mass of water flowing through a soil is directly proportional to the coefficient of permeability. On the other hand, the pore-water pressures and hydraulic heads are to a large extent independent of the absolute coefficient of permeability values. This observation is particularly of value as analyses are extended into the unsaturated soil zone.

For many geotechnical problems involving unsaturated soils, knowledge of the pore-water pressures or hydraulic heads is of primary interest. These values are relatively insensitive to the saturated coefficient of permeability and the permeability function. This becomes particularly valuable when one considers the difficulties of the work involved in measuring the unsaturated coefficients of permeability for a soil.

The coefficient of permeability has been shown to be a relatively unique function of the water content of a soil during the desorption process and a subsequent sorption process. The function appears to be unique as long as the volume change of the soil structure is negligible or reversible. However, to use this procedure in a seepage analysis, it is necessary to know the relationship between water content and matric suction. This relationship is highly hysteretic with respect to the desorption and sorption processes (Fig. 1). As a result, it appears to be more reasonable to go directly to a permeability function related to matric suction.

There is one permeability function for the desorption process and another function for the sorption process in an unsaturated soil. Both functions have a similar characteristic shape and as such can be fitted with a similar form of mathematical equation. Most engineering problems usually involve either a desorption process or a sorption process.

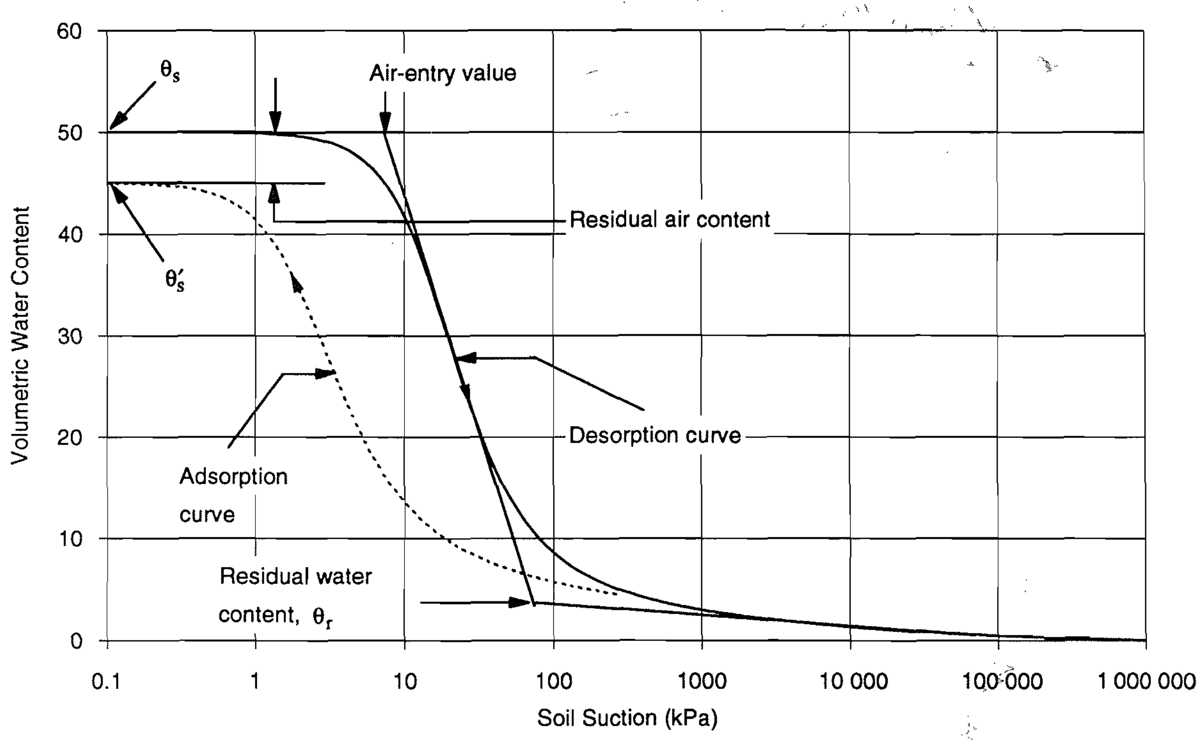

FIG. 1. Typical desorption and adsorption curves for a silty soil. θ_s, saturated volumetric water content; θ_s',

Even in many cases where both desorption and sorption processes are involved, a single equation is appropriate for engineering purposes.

Numerous attempts have been made to predict empirically the permeability function for an unsaturated soil. These procedures make use of the saturated coefficient of permeability and the soil-water characteristic curve for the soil. As more precise equations have been developed for the soil-water characteristic curve, likewise, more reliable predictions have been made for the coefficient of permeability function.

This paper reviews the background of the empirical prediction of the coefficient of permeability function. Using the present knowledge on a mathematical relationship for the soil-water characteristic curve, a new permeability function is predicted. The theoretical basis for the permeability function is shown. The function is based on the entire soil-water characteristic curve (i.e., from saturation down to a water content of zero or a suction of 10^6 kPa) and assumes that the ease of water flow through the soil is a function of the amount of water in the soil matrix.

Definitions

The coefficient of permeability k of an unsaturated soil is not a constant. The coefficient of permeability depends on the volumetric water content θ, which, in turn, depends upon the soil suction ψ. The soil suction may be either the matric suction of the soil, (i.e., $u_a - u_w$, where u_a is pore-air pressure, and u_w is pore-water pressure), or total suction (i.e., matric plus osmotic suctions). Soil suction is one of the two stress state variables that control the behaviour of unsaturated soils. Therefore, it is suggested that the term "permeability function for unsaturated soils" be used to represent the relationship between the coefficient of permeability and soil suction. When the coefficient of permeability at any soil suction $k(\psi)$ is referenced to the saturated coefficient of permeability k_s, the relative coefficient of permeability $k_r(\psi)$ can be written as follows:

TABLE 1. Empirical equations for the unsaturated coefficient of permeability $k(\theta)$

Function	Reference
$k_r = \Theta^n$, where $\Theta = (\theta - \theta_r)/(\theta_s - \theta_r)$, and $n = 3.5$	Averjanov 1950
$k = k_s\left(\dfrac{\theta}{\theta_s}\right)^n$	Campbell 1973
$k = k_s \exp[\alpha(\theta - \theta_s)]$	Davidson et al. 1969

[1] $$k_r(\psi) = \frac{k(\psi)}{k_s}$$

The relative coefficient of permeability as a function of volumetric water content, $k_r(\theta)$, can be defined similarly. The relative coefficient of permeability ($k_r(\psi)$ or $k_r(\theta)$) is a scalar function. The volumetric water content θ can be used in its normalized form, which is also referred to as the relative degree of saturation:

[2] $$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

where:

Θ is the normalized volumetric water content or relative degree of saturation,
θ_s is the saturated volumetric water content, and
θ_r is the residual volumetric water content.

Degree of saturation S, which indicates the percentage of the voids that are filled with water, is often used in place of the normalized water content Θ.

Literature review

There are two approaches to obtain the permeability function of an unsaturated soil: (i) empirical equations, and

TABLE 2. Empirical equations for the unsaturated coefficient of permeability $k(\psi)$

Function	Reference
$k = k_s$ for $\psi \le \psi_{aev}$	Brooks and Corey 1964
$k_r = (\psi/\psi_{aev})^{-n}$ for $\psi \ge \psi_{aev}$	
$k_r = \exp(-\alpha\psi)$	Gardner 1958
$k = k_s/(a\psi^n + 1)$	
$k = a\psi + b$	Richards 1931
$k = k_s$ for $\psi \le \psi_{aev}$	Rijtema 1965
$k_r = \exp[-\alpha(\psi - \psi_{aev})]$ for $\psi_{aev} \le \psi \le \psi_1$	
$k = k_1\left(\dfrac{\psi}{\psi_1}\right)^{-n}$ for $\psi > \psi_1$	
$k = \alpha\psi^{-n}$	Wind 1955

NOTES: ψ_1 is the residual soil suction (i.e., ψ_r), and k_1 is the coefficient of permeability at $\psi = \psi_1$.

(*ii*) statistical models. Several measured permeability data are required to use an empirical equation. A statistical model can be used to predict the permeability function when the saturated coefficient of permeability k_s and the soil-water characteristic curve are available.

Empirical equations

Several empirical equations for the permeability function of unsaturated soils are listed in Tables 1 and 2. These equations can be used in engineering practice when measured data are available for the relationship between the coefficient of permeability and suction, $k(\psi)$, or for the relationship between the coefficient of permeability and the water content, $k(\theta)$. The smallest number of measured points required to use one of the permeability equations in Tables 1 and 2 is equal to the number of fitting parameters in the adopted equation. When the number of measurements exceeds the number of the fitting parameters, a curve-fitting procedure can be applied to determine the fitting parameters. This approach allows a closed-form analytical solution for unsaturated flow problems.

Statistical models

Statistical models have also been used to determine the permeability function for an unsaturated soil using the characteristics of the soil-water characteristic curve. This approach is based on the fact that both the permeability function and the soil-water characteristic curve are primarily determined by the pore-size distribution of the soil under consideration. Figure 2 shows a typical soil-water characteristic curve for a sandy loam and its permeability function. Based on the pore-size distribution, Burdine (1953) proposed the following equation for the relative coefficient of permeability:

$$[3] \quad k_r(\theta) = \frac{k(\theta)}{k_s} = \Theta^q \frac{\int_{\theta_r}^{\theta} \frac{d\theta}{\psi^2(\theta)}}{\int_{\theta_r}^{\theta_s} \frac{d\theta}{\psi^2(\theta)}}$$

where $q = 2$. The square of the normalized water content was used to account for tortuosity. This model is significantly more accurate than the same equation without the correction factor Θ^q.

Childs and Collis-George (1950) proposed a model for predicting the coefficient of permeability based on the random variation of pore size. This model was improved by Marshall (1958) and further modified by Kunze et al. (1968). The calculations are performed by dividing the relation between volumetric water content and suction into n equal water-content increments (Fig. 3). The following permeability function has been slightly modified to use SI units and matric suction instead of pore-water pressure head:

$$[4] \quad k(\theta_i) = \frac{k_s}{k_{sc}} \frac{T_s^2 \rho_w g}{2\mu_w} \frac{\theta_s^p}{n^2} \sum_{j=i}^{m} [(2j+1-2i)\psi_j^{-2}]$$

$$i = 1, 2, \ldots, m$$

where:

$k(\theta_i)$ is the calculated coefficient of permeability for a specified volumetric water content θ_i, corresponding to the ith interval;

i is the interval number that increases with decreasing water content (for example, $i = 1$ identifies the first interval that closely corresponds to the saturated water content θ_s, and $i = m$ identifies the last interval corresponding to the lowest water content θ_L, on the experimental soil-water characteristic curve);

j is a counter from i to m;

k_{sc} is the calculated saturated coefficient of permeability;

T_s is the surface tension of water;

ρ_w is the water density;

g is the gravitational acceleration;

μ_w is the absolute viscosity of water;

p is a constant that accounts for the interaction of pores of various sizes;

m is the total number of intervals between the saturated volumetric water content θ_s and the *lowest* water content θ_L on the experimental soil-water characteristic curve;

n is the total number of intervals computed between the saturated volumetric water content θ_s and *zero* water content (i.e., $\theta = 0$) (note that $n = m\,[\theta_s / (\theta_s - \theta_L)]$; $m \le n$; and $m = n$ when $\theta_L = 0$); and

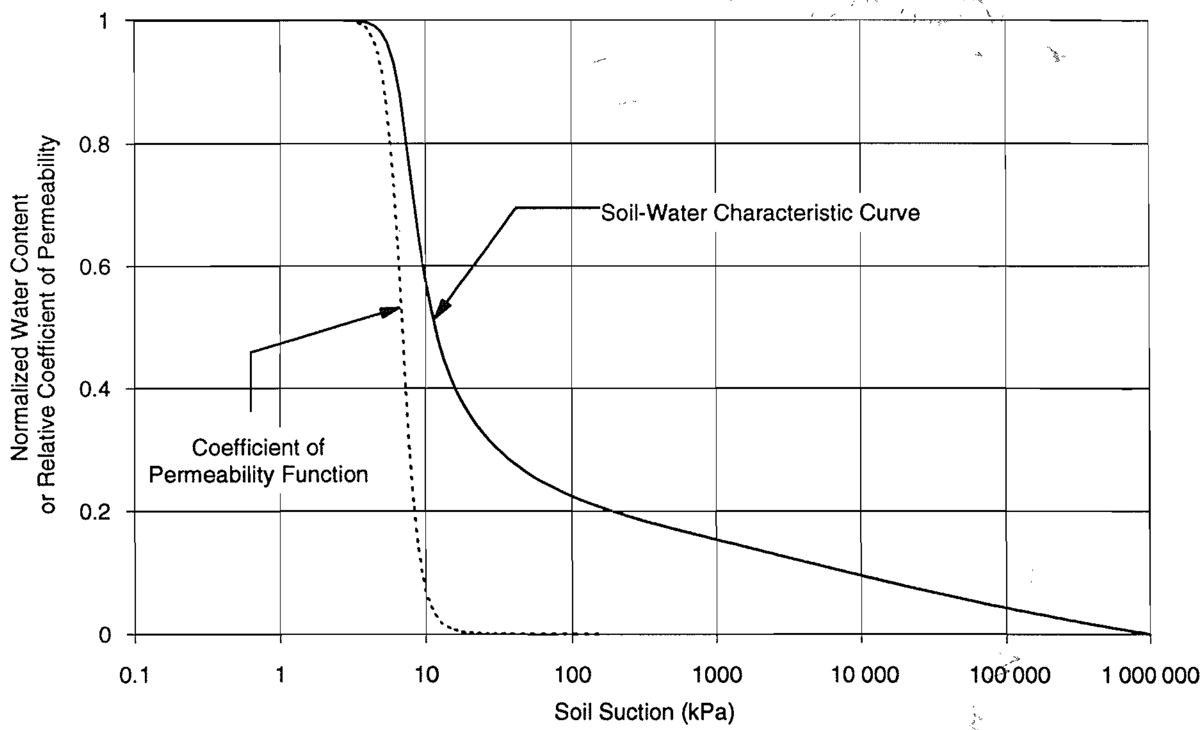

FIG. 2. Typical soil-water characteristic curve and permeability function for a silty soil.

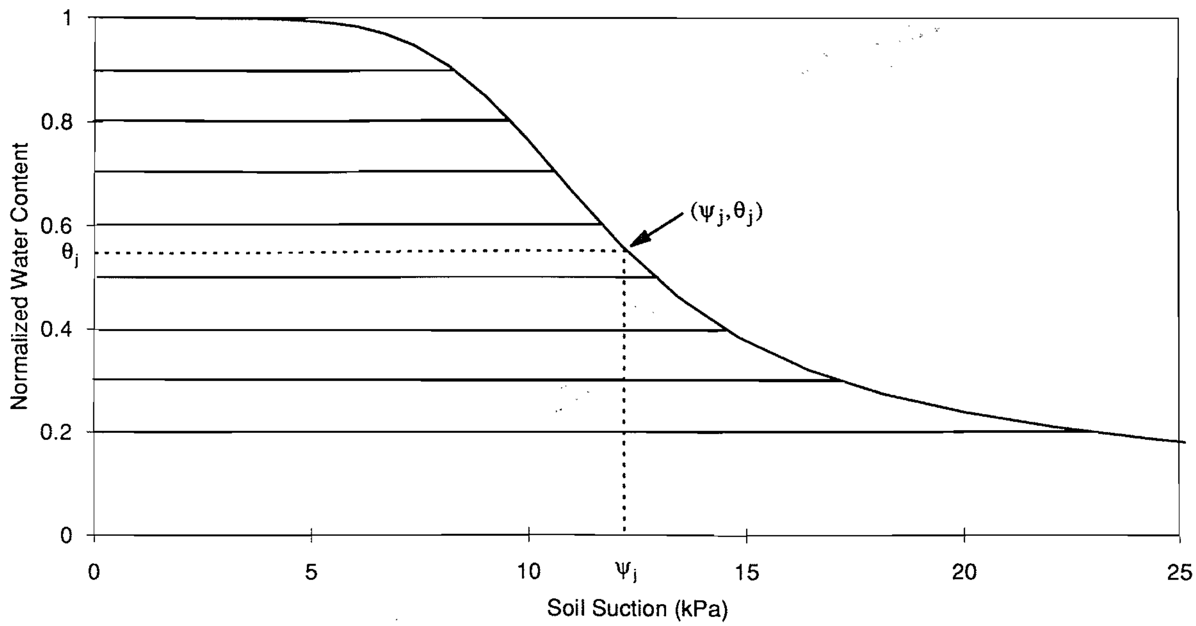

FIG. 3. A typical soil-water characteristic curve for predicting the permeability function. θ_j, midpoint of the jth water-content interval; ψ_j, suction corresponding to θ_j.

ψ_j is the suction (kPa) corresponding to the midpoint of the jth interval (Fig. 3).

The calculation of the coefficient of permeability $k(\theta_i)$ at a specific volumetric water content θ_i involves the summation of the suction values that correspond to the water contents at and below θ_i. The matching factor (k_s/k_{sc}), based on the saturated coefficient of permeability, is necessary to provide a more accurate fit for the unsaturated coefficient of permeability. The shape of the permeability function is determined by the terms inside the summation-sign portion of the equation which, in turn, are obtained from the soil-water characteristic curve.

Mualem (1976*a*) analyzed a conceptual model of a porous medium similar to that of the Childs and Collis-George (1950) model and derived the following equation for predicting the coefficient of permeability:

$$[5] \quad k_r(\theta) = \Theta^q \left(\frac{\int_{\theta_r}^{\theta} \frac{d\theta}{\psi(\theta)}}{\int_{\theta_r}^{\theta_s} \frac{d\theta}{\psi(\theta)}} \right)^2$$