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DESIGN OF ANCHOR SYSTEMS USING LIMIT EQUILIBRIUM

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ABSTRACT

Several procedures are available for the design and analysis of soil anchor systems. This paper compares these methods from theoretical and analytical standpoints. As a result of this study, it is recommended that anchor forces be simulated by one of the following two procedures:
1. Equivalent forces acting along the slip surface,
2. Concentrated load acting on the ground surface.

RÉSUMÉ

Plusieurs méthodes existent pour calculer et analyser des systèmes d'ancrage au sol. Ce papier compare ces differentes méthodes sur les plans théorique et analytique - comme résultat de cette étude, on recommande que les forces agissant sur l'ancrage soient simulées par une des deux méthodes suivantes:
1. Forces equivalentes agissant le long du plan de rupture,
2. Charge concentrée agissant sur la surface du sol.

INTRODUCTION

The stability of earth slopes can often be increased by anchoring the slopes into a suitable stratum below the most probable slip surface. The stabilizing systems require a knowledge of the anchor load, location, orientation, spacing and length. The magnitude of the anchor force is primarily based on the consideration of limit equilibrium forces, although situations occur when limiting deformation governs design.

Methods of analysis which extend limit equilibrium assumptions are commonly used to provide the design tools for anchorage. Nonveiller (1965) included external line loads in Bishop's simplified method. Tenier and Morlier (1982) studied the same method and suggested replacing concentrated anchor loads by statically equivalent forces acting on the failure surface. The various methods of slices were compared by Fredlund, Krahn and Pufahl (1981) who developed a general formulation known as the General Limit Equilibrium (GLE) method. It was concluded that all methods of slices can be written using one formulation, and that methods varied only in the type of interslice force function assumed (or computed) and the elements of statics used. Wilson (1982) and Fan (1983) investigated the use of an interslice force function based on the finite element method.

The main objective of this paper is to provide a computational procedure for the design of an anchor system using a general limit equilibrium approach (i.e., the GLE method). The influence of anchor forces applied to the mass can be simulated in several ways. The paper develops a theoretical context by considering the anchoring effects on the slip surface and the ground surface. Comparisons of these approaches are illustrated using an example problem. There is some discussion regarding the use of interslice force functions.

The scope of this paper is limited to the design of soil anchor systems and does not address issues related to soil nailing systems. The primary difference in these two systems relates to the prestressing force which is applied for anchor systems.

THEORY

The assumed earth-anchor system consists of a structural element which uses a grouted anchor in the ground to secure a tendon which applies a force to the soil mass. The structure can be either a concrete wall or pad, or superficial layers of existing rocks. Figure 1 shows the components of the system. The portion of the tendon between the anchor and the structure is not bonded to the ground and is free to elongate elastically. Force is applied to the anchor by post-tensioning.

Forces on The Slip Surface

The anchoring force provides a component of thrust which directly acts against possible movement of the slope while another component increases the normal stresses on the slip surface. The components of the anchoring forces, _L_n and _L_t, are shown in Fig. 2 and can be evaluated using the theory of elasticity. The soil mass is assumed to be an elastic, isotropic, homogeneous and semi-infinite continuum (i.e.,half-space). For a load of constant intensity acting along a line of infinitesimal width and infinite length, the polar stress acting at any point in the soil, according to Flamant (1886) is given by:

where:

These stresses are integrated along the slip surface to yield the corresponding change in normal force (_L_n) and shear force (_L_t), acting on the plane. The interslice shear forces shown in Fig. 3 are obtained using an assumption regarding the direction of the interslice forces. The expression for the interslice shear forces, X, according to Morgenstern and Price (1965) can be written as follows,
where:
The summation of forces in the vertical and horizontal direction is used to compute the normal force on the base and on the right side of each slice (Fig. 3).
where:

The factor of safety with respect to moment equilibrium of the overall slope is obtained by summing moments about the centre of rotation.

where:
R = radius associated with the mobilized shear force,
x = horizontal distance from centroid of each slice to the centre of rotation,
f = offset of the normal force, N,
a = distance between the centre of rotation and the water force.

The factor of safety with respect to force equilibrium of the overall slope is obtained by summing forces in the horizontal direction.
Both moment and force equilibrium factors of safety can be solved for a range of _ values with a predetermined side force function. These factors of safety are plotted against the lambda values, _, and the point of intersection of these two curves yields an overall factor of safety, F, satisfying both force and moment equilibrium.

Forces at the Ground Surface

Fig. 3 shows all the forces acting on a slice. The anchoring forces are reduced to concentrated line loads acting on the ground surface. The general formulation follows the same procedure as previously illustrated, and the equations derived are expressed as follows:
The anchoring forces, L, are considered as external forces applied to the soil mass increasing the stability of the overall system. The terms L sin_, L cos_ and L are only applied at the ground surface on the slice where the anchor acts.

A functional variation, f(x), relating normal and shear forces is required in the above equation. Wilson (1982) proposed a method of evaluating the side force function for slope stability problems. The procedure involved the use of the finite element numerical method. The entire function is obtained by integrating vertically along each slice and proceeding horizontally across the slope. The function was also developed by Fan (1983) to include lateral earth force problems. It is also possible that the effect of the anchor loads could be taken into account in the finite element stress analysis.

FEASIBILITY STUDIES

The above equations demonstrate the major differences in the manner in which anchor forces can be accommodated. The possibilities for implementing these forces into the factor of safety equilibrium equations need to be examined. A slope stability program available at the University of Saskatchewan was used for this part of study. The program regards anchor loads as external line forces acting on the soil mass in two-dimensional space. The example problem shown in Fig. 4 was used to study the following methods of analysis:

1. Equivalent Forces Acting along the Slip Surface (Method 1)
2. Concentrated Load Acting on the Slip Surface (Method 2)
3. Concentrated Load Acting on the Ground Surface (Method 3)

4. Concentrated Load Acting on the End Slice (Method 4)

The present factor of safety of the slope was 1 and the objective was to increase it to 1.5.

Equivalent Forces Acting along the Slip Surface (Method 1)

The anchor load applied at the ground surface is replaced by equivalent stresses, and subsequently forces, acting on the slip surface. (The equivalent stresses are calculated from equation [1]). These stresses are integrated along the slip surface of each slice to obtain equivalent forces acting on the slip surface. Fig. 5(a) shows the magnitudes of the forces acting on each slice. The forces are assumed to act at the centroid of the base of each slice. Stability analyses were performed using the GLE method and the force system given in Fig. 5(a).

A Concentrated Load Acting at the Slip Surface (Method 2)

The resulting anchor loads applied at the ground surface in this case are translated to the slip surface as a single concentrated load as shown in Fig. 5(b).

A Concentrated Load Acting at the Ground Surface (Method 3)

In this method, the resulting anchor load is assumed to act at the ground surface. The point of application coincides with the location where the actual prestressing operation has taken place. The load is applied only on the slice where the external force acts as shown in Fig. 5(c).

A Concentrated Load Acting on the Last (End) Slice (Method 4)

The resulting anchor load is kinematically translated along its line of action to the edge of the last slice as shown in Fig. 5(d). The edge of the last slice is defined as the intersection between the slip surface and the ground surface. The force system shown in Fig. 5(d) is used in the analysis.

The General Slope Stability Analysis and Overall Lateral Force Computation

The general slope stability problem is statically indeterminate. Assumptions regarding the direction or magnitude of some of the forces are required to render the problem determinate. These assumptions are expressed in the form of an interslice force function. Development of numerical techniques such as the finite element method have facilitated in the evaluation of the interslice force function. A two-dimensional finite element program can be used to accommodate external line loads acting on a slope. Numerical integration is used to compute the interslice shear forces across the entire slope.

The magnitude and direction of the anchor forces used must be selected for the analysis. A reasonable estimate of this force can be determined using the lateral earth force theory. The lateral earth force computer program developed by Rahardjo (1982) was used for this example. Rahardjo (1982) used the summation of horizontal forces and a specified factor of safety to compute the total horizontal force required for equilibrium. The interslice shear force were assumed to be zero in the simplest case. The possibility of using other interslice force functions was investigated and it was concluded that other interslice force functions (e.g., such as half sine, FEM without line load and constant functions) can be used without resulting in substantial difference in the computed factor of safety. However, a general conclusion regarding this assumption requires further study.

Comparison of Methods

The solution obtained using Methods 1 to 4 are plotted in Fig. 6. Methods 2, 3 and 4 yield almost identical results for the computed factors of safety. The solution obtained from Method 1 is about 10% less than that obtained from other methods. This difference may be attributed to the fact that stresses near the ends of the slip surface are ignored. However, it is expected that these forces should not produce a difference of 10%. Carpenter (1988) suggested that due to the geometries of the slope and the slip surface, the sum of the radial forces in the direction of the line load is generally not in equilibrium with the applied load. He introduced a multipler to the radial forces so that the sum of these forces is in balance with the applied load. If a similar adjustment is applied, the factor of safety results are expected to agree more closely with those obtained from Methods 2, 3 and 4.

The values of lambda satisfying both force and moment equilibrium are different from one method to another. This indicates that the resulting forces distributions at the base and on the vertical side of the slices are different for the different methods. Method 2 calculates the normal force on the base of the slice where the load acting can be ten times greater than the average normal force. The distribution of normal forces was found to be negative (indicating tension) near the toe of the slope when using Method 4. Unreasonable normal force distributions may result if the anchor force is simulated using a concentrated line load acting along the the slip surface or on the end (last) slice. Methods 2 and 4 are not therefore recommended for usage in the design of anchors.

Both Methods 1 and 3 appear to be suitable for use in design. A comparsion of the distribution of the normal forces at the base of the slices is shown in Fig. 7. The results calculated using Method 1 gives a more uniform distribution of the normal forces. Tenier and Morlier (1982) compared the force distributions obtained by Flamant's equation (1886) with those obtained from the finite element analysis. Both distributions were in close agreement. Similar conclusions were arrived at by Carpenter (1988). Method 1 appears to yield an acceptable force pattern.

COMPUTATIONAL PROCEDURE FOR THE DESIGN OF ANCHOR SYSTEMS

The design procedure recommended for the design of anchor systems is summarised in the following steps:

1. Calculate the existing factor of safety of the slope assuming no anchors and determine the most critcial slip surface.

2. Calculate the "critical height" on the slope, above which the required factor of safety for stability, is satisfied. The term "critical height" is defined as the height from the lower ground level above which the specified factor of safety is attained.

3. Estimate the required anchor force based on the lateral force corresponding to a selected design factor of safety (Rahardjo, 1982).

4. Distribute the estimated resultant anchor force according to an assumed pressure distribution diagram.

5. The overall factor of safety, including the anchors, is then computed using the GLE method.

6. For the case of slip surfaces for which the specified factor of safety is not achieved, the anchor forces need to be increased.

7. Determine the optimum angle for anchor inclinations and modify the anchor forces if necessary.

8. The required anchor lengths can be determined by extending the anchors to an adequate length beyond the depth of the critical slip surface. The calculated lengths added to the grouted lengths are the total length required for the anchors.

9. The above steps can be repeated until the specified factor of safety is reached. Make a final evaluation of the design anchor forces based on previous local experience.

The above steps may be modified in practice to suit particular design requirements. The design procedure provides a conceptual approach to the problem.

Upon first consideration, it may appear most reasonable to install the anchors normal to a slope. This orientation does not generally provide an optimum angle of inclination for increasing the stability of slope. It is therefore desirous to determine the optimum angle of inclination for the anchors by a trial and error procedure.

Optimization of Anchor Inclinations

The optimum angle for the anchors is defined as the angle which gives the maximum factor of safety with respect to both force and moment equilibrium. The approximate anchor load has to be determined as a known force. For a given anchor load, the factors of safety corresponding to a range of possible angles of inclination are calculated. The factors of safety are plotted against the angles of inclination as shown in Fig. 8. Hryciw (1991) generated similar design charts for anchors (and soil nails) in homogeneous cohesionless soils. The advantage of using the limit equilibrium method over the theoretical approach developed by Hryciw (1991) is the flexibility rendered in solving practical problems.

CONCLUSIONS and RECOMMENDATIONS

Several procedures can be used for the design of soil anchor systems. Some of these procedures have been described and compared in this paper. Based on this study, the preferred procedures are as follows:

1. Method 1 (Equivalent Forces Acting along the Slip Surface).

2. Method 3 (Concentrated Loads Acting at the Ground Surface).

While Method 2 and 4 cannot be proven to be incorrect, it is recommended that Methods 1 and 3 are superior for analysis purposes.

It is recommended that further study be made on, i) the selection of the most appropriate interslice force
function, or ii) the combining of a finite element numerical simulation and the limit equilibrium method.

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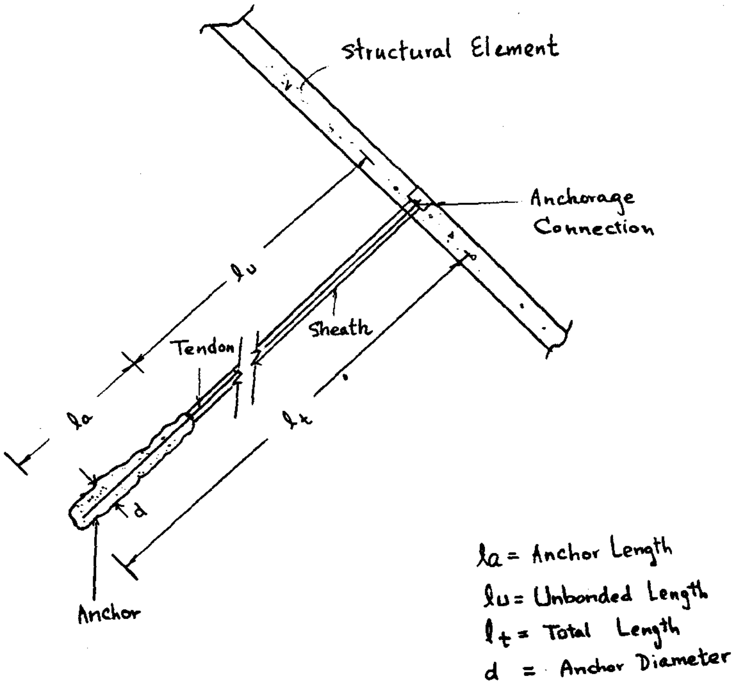
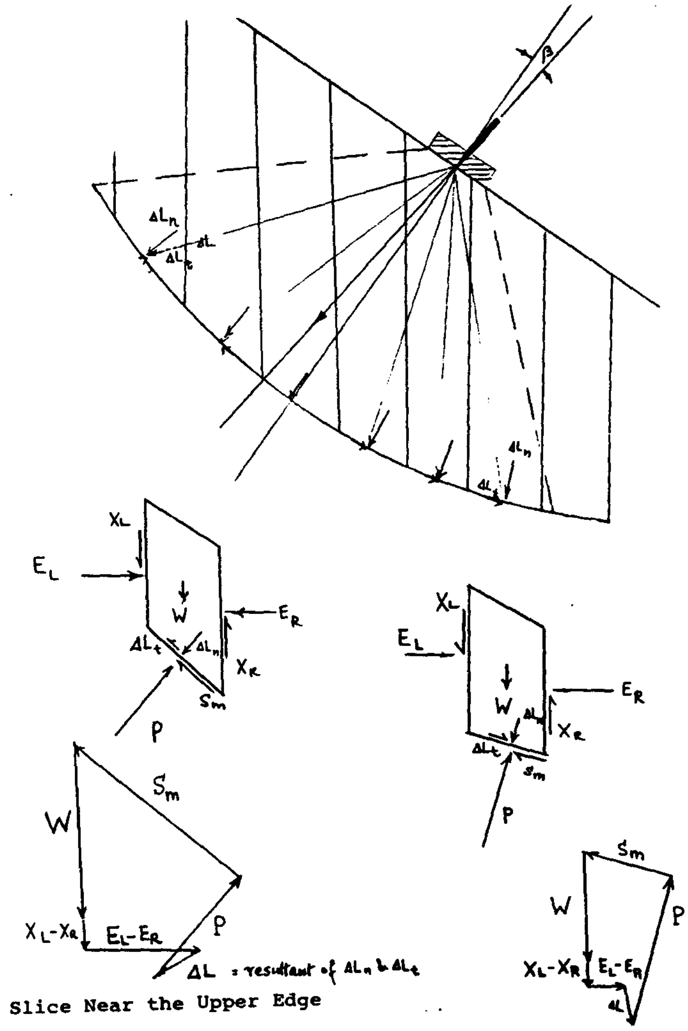


FIGURE 1. An earth-anchor system



Slice Near the Upper Edge

FIGURE 2. Distribution of anchor prestress on a slip surface

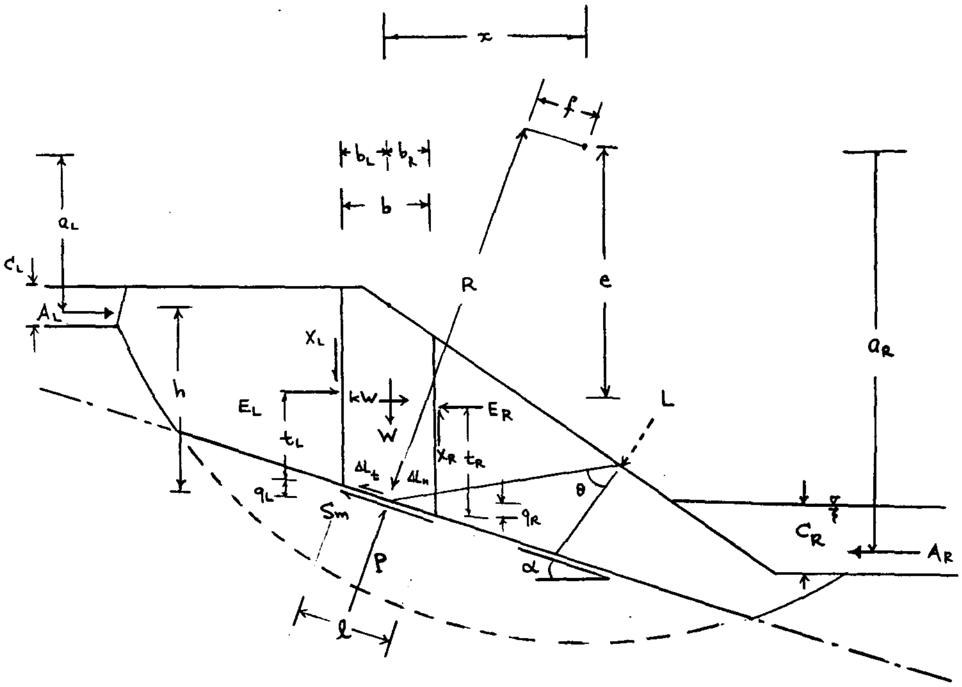
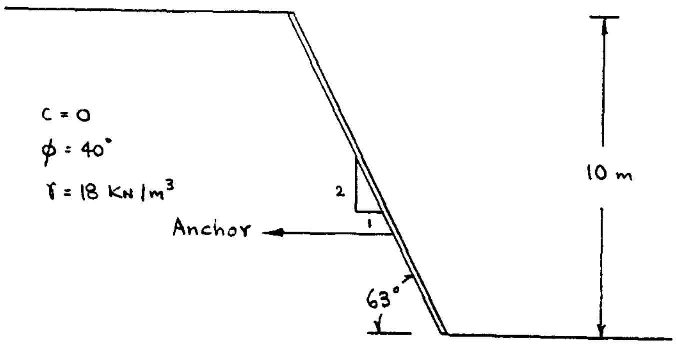


FIGURE 3. Method of analysis: forces acting along the slip surface

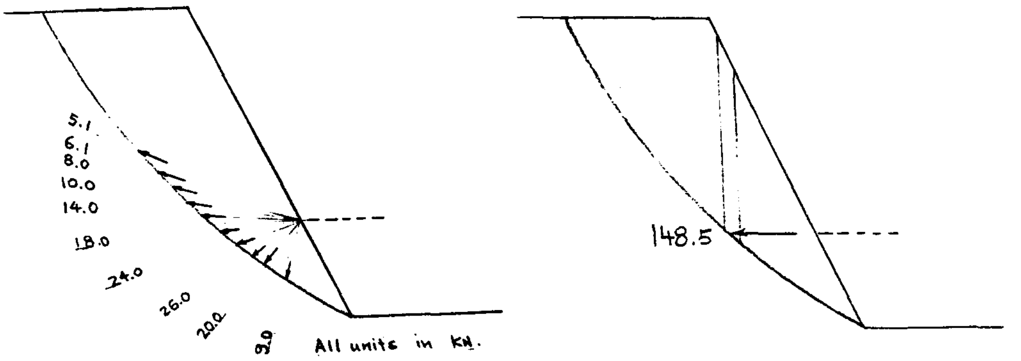
Problem: To increase the factor of safety of the given slope to 1.5

i.e $F_{design} = 1.5$



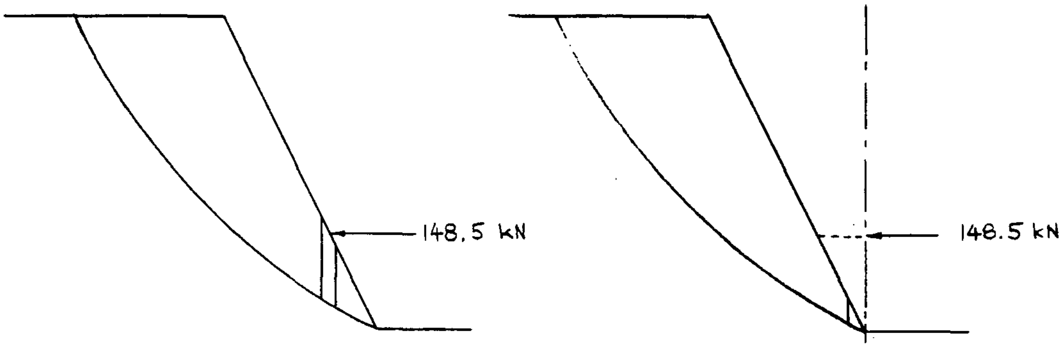
Note: Bedrock and threatic surfaces are well below existing ground surface

FIGURE 4. Example problem used to compare different methods of analysis



(a) Equivalent Stresses Acting on the Slip Surface

(b) Concentrated Load Acting on the Slip Surface



(c) Concentrated Load Acting on the Ground Surface

(d) Concentrated Load Acting on the Last Slice

FIGURE 5. Comparison of concentrated loads acting on the slope using four different methods

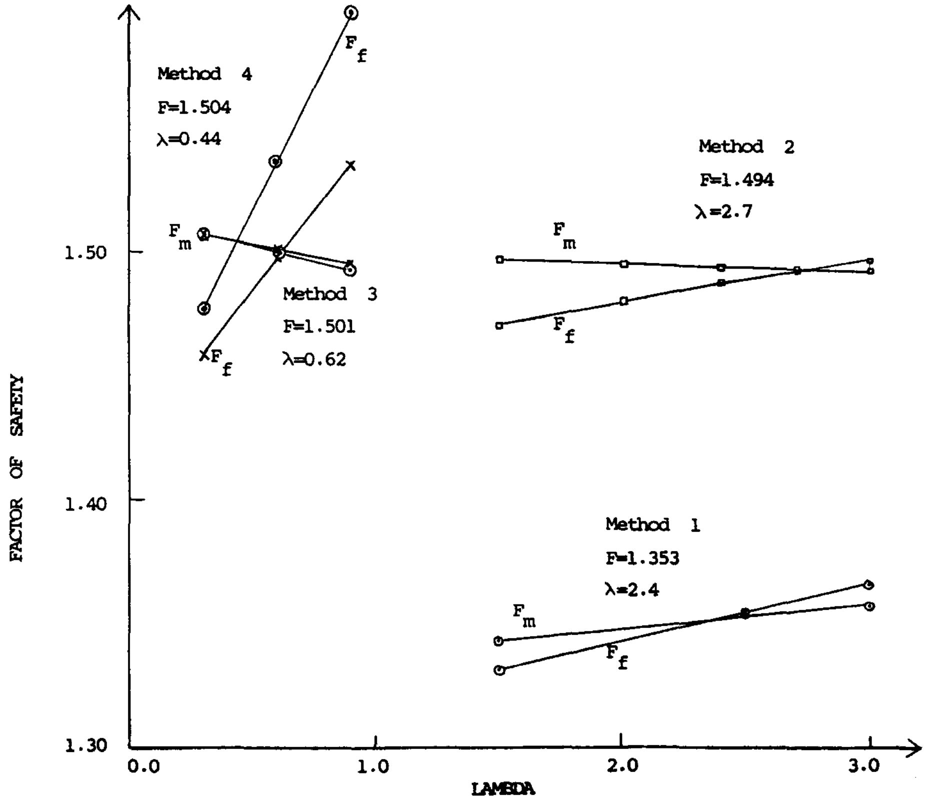


FIGURE 6. Comparison of factors of safety using four different approaches

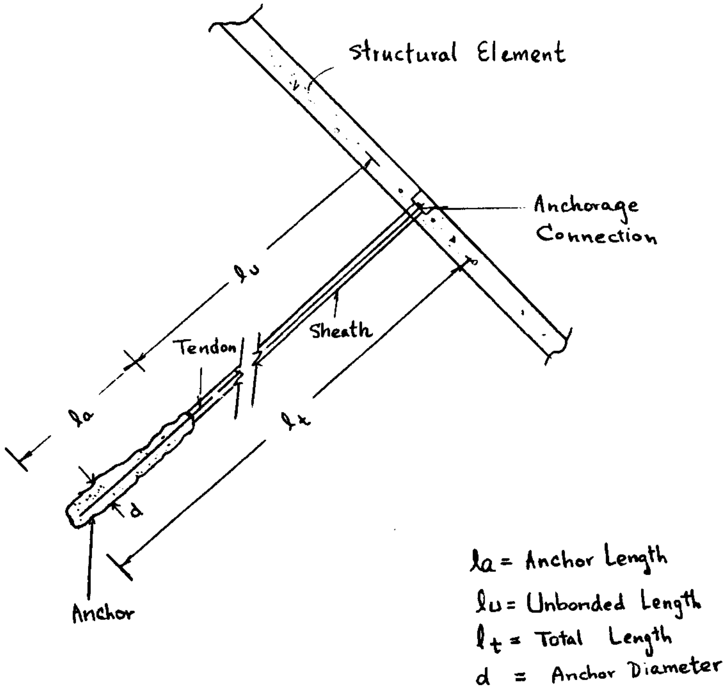
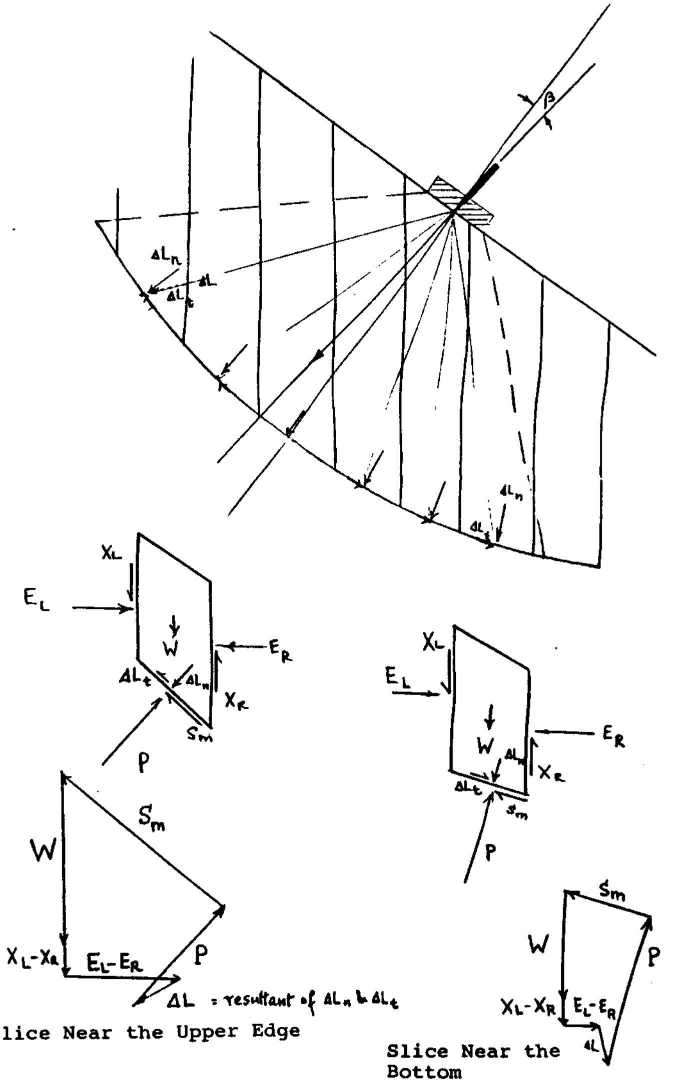


Fig. 1 An Earth-Anchor System



Slice Near the Upper Edge

Slice Near the Bottom

Fig. 2 Distribution of Anchor Prestress on a Slip Surface

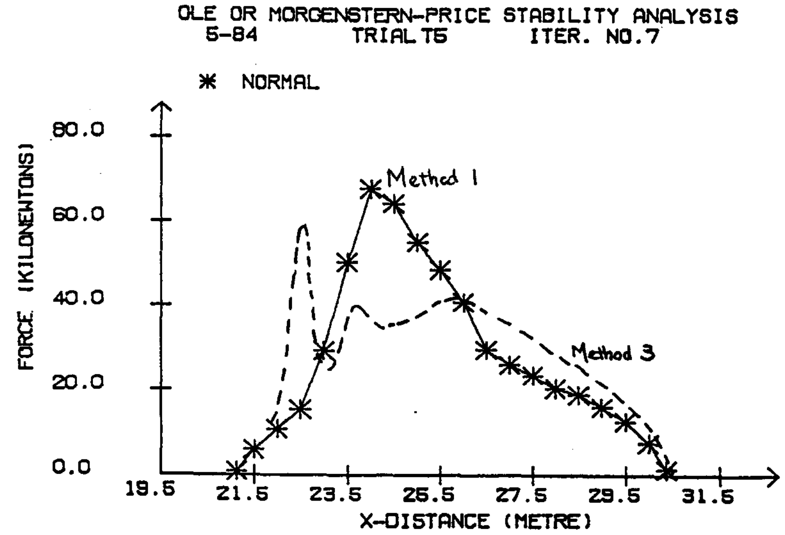


FIGURE 7. Distribution of normal force at the base in methods 1 and 3

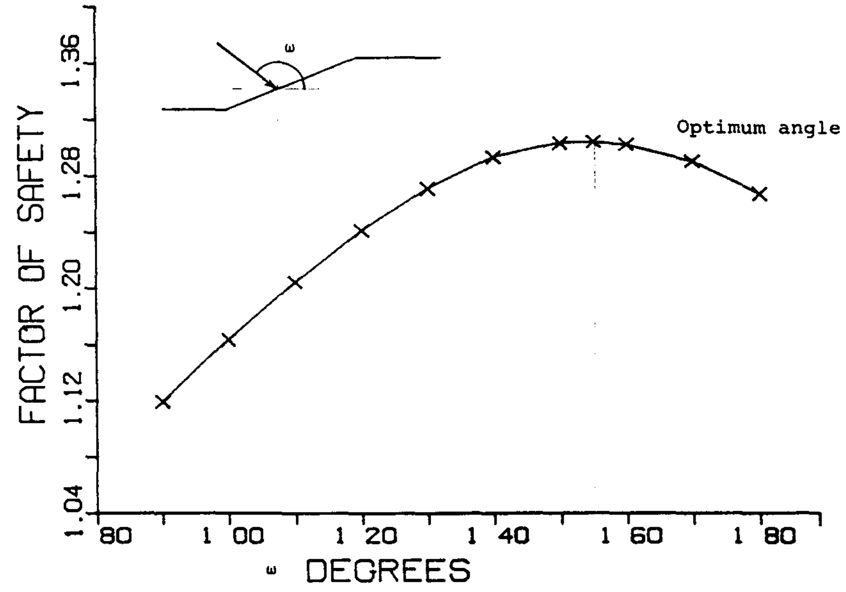


FIGURE 8. Variation of factor of safety with anchor inclination