Proceedings of the, First Congress on Computing in civil Engineering, American Society of Civil Engineers, Washington, D.C., June 20-22, vol. 1, pp.849-856. (1994)

A Technique to Perform Coupled Consolidation Analysis Using Two Independent Softwares

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Abstract

The consolidation of soils involves two interdependent processes. These are fluid (air and water) flows and the deformation of the soil structure. The two processes tend to be treated separately, especially in the common commercial softwares. This paper illustrates a technique for utilizing these two types of software simultaneously to perform a coupled consolidation analysis. Theoretical formulations and two example problems are presented to illustrate the proposed method. The coupled analysis results exhibit the classic Mandel-Cryer effect as commonly shown in analytical solutions.

Introduction

The application of a load to a soil generates excess pore-water pressures. As the excess pore pressures dissipate, the soil undergoes a time-dependent volume change. This transient process is normally known as consolidation. The first theory for soil consolidation was derived by Terzaghi in the 1930's for saturated soils, and it is still in use. In Terzaghi's consolidation theory, the excess pore-water pressure dissipation is analyzed in a similar manner as in thermal diffusion (Terzaghi, 1943). Biot (1941) developed a more rigorous consolidation theory that takes into account the effect of deformation on the pore-fluid flow. A comparison of Terzaghi's theory and Biot's theory by Cryer (1963) showed that the solutions of the two theories differ considerably with respect to the pore-water pressure changes during

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consolidation. It appears that the pore-water pressure can increase significantly in the early stage of consolidation when Biot's theory is used in the analysis. Mandel (1957) also observed similar behavior when analyzing a brick-shaped body loaded uniaxially under a plain-strain condition. The phenomenon that the pore-water pressures increase before dissipation in certain regions of a consolidating soil has been referred to as the Mandel-Cryer effect. Biot's theory is also applicable to unsaturated soils. Dakshanamurthy, Fredlund and Rahardjo (1984) modified Biot's theory for unsaturated soils by incorporating the appropriate stress state variables.

There have been many numerical solutions developed on the basis of Terzaghi's theory, Biot's theory, and other theories (e.g., Schiffman and Arya, 1977; Christian, 1977; Hsi and Small, 1992). These solutions can be grouped into two categories: coupled and uncoupled solutions. Coupled solutions treat the pore-fluid flow and soil deformation as interdependent processes. The fluid flows cause changes in the pore pressures and consequently in the stress state of the soil. The changes in the stress state result in soil deformations which in turn affect the fluid flows. Computer programs are available to produce coupled numerical solutions. In practice, however, soil consolidation is commonly analyzed by considering the flow and deformation processes independently (i.e., uncoupled solutions). This produces reasonable results as far as the final deformation is of concern. Two types of commercial softwares are commonly available: one for analyzing flow problems only (i.e., seepage analysis) and the other is mainly used to analyze stress/strain problems. It would be useful to find a simple procedure to perform coupled analyses using these two types of independent programs as proposed in this paper. The proposed method is applicable to both saturated and unsaturated soils, but only the procedure associated with saturated soils is presented in this paper due to space limitations.

Deformation and Stress Changes due to Seepage

The finite element equilibrium equation for a continuum is written as follows (Zienkiewicz, 1977):

$$\int_v [B]^T[C][B]dv \, \{a\} = \int_v \langle N\rangle^T\{b\}dv + \int_A \langle N\rangle^T\{s\}dA + \{F\} \qquad (1)$$

where {a} is the vector of nodal displacements, [B]{a} is the strain matrix, [C] is the stress/strain constitutive relationship, dv is the unit volume, [N] is shape functions, {b} is the body forces, {s} is the surface loads, dA is the unit area, {F} is the vector of nodal loads.

During the consolidation process, the external load applied to the soil does not change. The equilibrium equation can then be simplified as

$$\int_v [B]^T[C][B]dv \, \{a\} = 0 \qquad (2)$$

Referring to the following definition,

$$[C][B]\{a\} = \{\Delta\sigma\} \qquad (3)$$

where $\{\Delta\sigma\}$ is the total stress change, equation (2) can be written as

$$\int_v [B]^T\{\Delta\sigma\}dv = 0 \quad (4)$$

Considering that

$$\{\Delta\sigma\} = \{\Delta\sigma'\} + \{m\}\Delta u_w \quad (5)$$

where $\{\Delta\sigma'\}$ is the effective stress change, Δu_w is the change in pore-water pressure, and $\{m\} = [1,1,1,0,0,0]^T$, equation (4) can be transformed as follow:

$$\int_v [B]^T\{\Delta\sigma'\}dv = - \int_v [B]^T\{m\}\Delta u_w dv \quad (6)$$

where

$$\{\Delta\sigma'\} = [C'][B]\{a\} \quad (7)$$

Substituting equation (7) into equation (6) gives

$$\int_v [B]^T[C'][B]dv \; \{a\} = - \int_v [B]^T\{m\}\Delta u_w dv \quad (8)$$

This formulation is equivalent to performing a drained analysis, with nodal forces resulting from the change in pore-water pressures. The $[C']$ constitutive matrix is written in terms of the drained soil properties E' and υ', where E' = elastic modulus and ν' = Poisson's ratio.

For a given Δu_w, $\{a\}$ will be calculated from equation (8). Effective stress change $\{\Delta\sigma'\}$ and total stress change $\{\Delta\sigma\}$ can then be calculated accordingly using equations (7) and (5). The value of $\{\Delta\sigma\}$ may not be zero. This implies that the total stresses in a soil mass can change due to the non-homogeneous dissipation of pore-water pressure. It is a common perception that during consolidation, the effective stress changes by the same amount as the pore-water pressure change. In other words, the total stress remains constant during the consolidation process. However, this is not necessarily the case as revealed by the coupled solutions.

Seepage Analysis

The rate of net water inflow, θ_w, into a soil element can be calculated in accordance with Darcy's law as follows:

$$\theta_w = \frac{\partial}{\partial x}\left(k_{wx}\frac{\partial h_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{wy}\frac{\partial h_w}{\partial y}\right) \quad (9)$$

where k_{wx} and k_{wy} are the coefficients of permeability for the water phase in the x and y directions, respectively; h_w is the hydraulic head with respect to the water phase (i.e., pore-water pressure plus elevation head).

The change in the water content, Δw, of a soil element is equivalent to the volume change of the soil structure caused by a change in the effective stress,

$$\Delta w = -\frac{\Delta\sigma'_x + \Delta\sigma'_y + \Delta\sigma'_z}{3K'} \quad (10)$$

where K' is the bulk modulus with respect to the effective stress, $\Delta\sigma'_x$, $\Delta\sigma'_y$, and $\Delta\sigma'_z$ are the change in effective stresses in the x, y and z directions respectively.

The change in the water content of the soil element can then be linked to the total stress and pore-water pressure changes as follows:

$$\Delta w = -\frac{\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z}{3}\cdot\frac{1}{K'} + \frac{1}{K'}\Delta u_w \quad (11)$$

By combining equations (9) and (11), the governing equation for the water flow can be formulated as follows:

$$\left[\frac{\partial}{\partial x}\left(k_{wx}\frac{\partial h_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{wy}\frac{\partial h_w}{\partial y}\right)\right]\Delta t = -\frac{\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z}{3}\cdot\frac{1}{K'} + \frac{1}{K'}\Delta u_w \quad (12)$$

where Δt is an increment of the elapsed time. The governing equation for water flow involves both the total stress, the pore-water pressure and hydraulic head. In the uncoupled solution, the total stresses are assumed to be constant during consolidation (i.e., $\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z = 0$ in equation 12). As a result, the equation only contains the pore-water pressure head and is similar to the heat flow equation. In the coupled solution, the total stress and the pore-water pressure changes are solved simultaneously.

Calculation Procedure

Rearranging and combining equations (9) and (12) gives

$$\Delta u_w = \Delta\bar{\sigma} + \theta w \Delta t \cdot K' \quad (13)$$

where

$$\Delta\bar{\sigma} = \frac{(\Delta\sigma_x + \Delta\sigma_y + \Delta\sigma_z)}{3}. \quad (14)$$

The term $\theta w \Delta t$ can be computed from equation (9). If the mean total stress, $\Delta\bar{\sigma}$, is known, Δu_w can be calculated using equation (12). If Δu_w is known, then $\Delta\bar{\sigma}$ can be computed using equation (8). The solution involves an iterative procedure where Δu_w and $\Delta\bar{\sigma}$ are modified within each time step.

Let the variable, Δu^*_w, be the pore-water pressure change calculated from equation (12) by assuming that the total stress does not change ($\Delta\bar{\sigma} = 0$):

$$\Delta u_w^* = \theta w \Delta t \cdot K' \tag{15}$$

Let $\lfloor(\Delta u_w)$ represent an operation that calculates the average total stress change, ($\Delta\bar{\sigma}$), due to a change in pore-water pressure, Δu_w:

$$\Delta\bar{\sigma} = \lfloor(\Delta u_w) \tag{16}$$

Equation (12) can now be written as

$$\Delta u_w = \lfloor(\Delta u_w) + \Delta u_w^* \tag{17}$$

Equation (17) is the governing equation which must be satisfied during subsequent reiterations. The reiteration procedure can be outlined as follows,

$$\Delta u_{out} = \lfloor(\Delta u_{in}) + \Delta u_w^* \tag{18}$$

and

$$\Delta u_{in/new} = \xi \cdot \Delta u_{out} + (1-\xi)\Delta u_{in/old} \tag{19}$$

where ξ is a factor specified by user to improve convergence. A typical value for ξ is 1. Initially $\Delta u_{in} = \Delta u_w^*$. The reiteration continues until Δu_{in} and Δu_{out} are within the specified tolerance. The analysis then proceeds to the next time step.

For each time increment, Δu_w^* is calculated using a seepage analysis program, and $\lfloor(\Delta u_w)$ is performed using a stress/strain analysis program. $\lfloor(\Delta u_w)$ normally have to be carried out several or many times before convergence is achieved.

Examples

The following examples were analyzed using a seepage analysis software, SEEP/W, and a stress/strain program, SIGMA/W, which are products of GEOSLOPE International. In the first example, an axially constrained saturated soil cylinder is subjected to a confining pressure of 98.07 kPa that results in initial excess pore-water pressure of equal amount (i.e., 98.07 kPa). The radius of the cylinder is 20 cm, the soil permeability is 10^{-9} m/s and the drained Young's Modulus is 6000 kPa.

Both coupled and uncoupled analyses have been performed with drained Poisson's ratios, ν', of 0.01 and 0.33. Figure 1 shows the development of pore-water pressures along the axis of the cylinder for both the coupled and uncoupled analyses. The results are presented in terms of the ratio of excess pore-water pressure at any time to the initial excess pore-water pressure. Figure 2 shows the spatial distribution of the pore-water pressure at several elapsed times for the coupled analysis with $\nu' = 0.01$.

For the second example, a soil sphere subjected to a uniform pressure on the surface is analyzed. An analytical solution to this problem was presented by Cryer (1963). The radius of the sphere is 6 cm and the applied surface pressure is 98.07 kPa. The soil permeability is 10^{-9} m/s and the drained Young's modulus is 6000 kPa. Two coupled analyses have been conducted using the drained Poisson's ratios, ν', of 0.01 and 0.33.

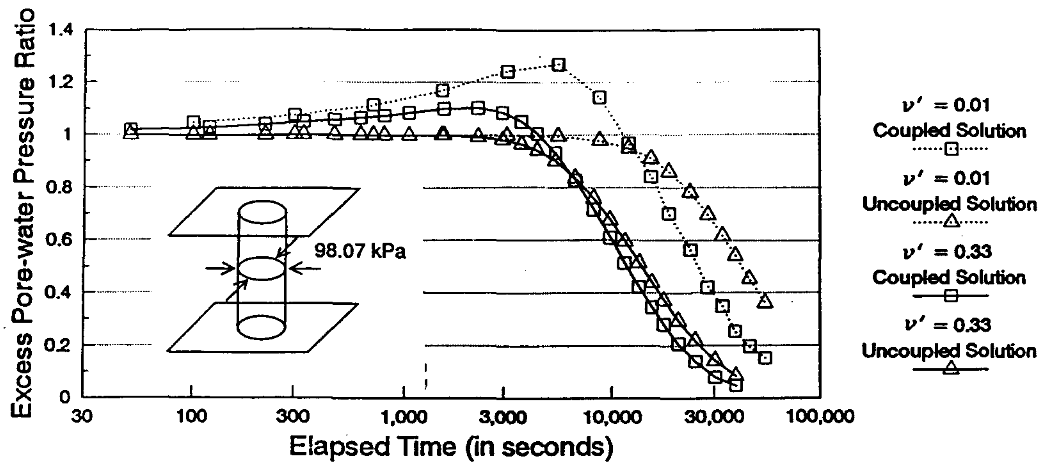

Figure 1. A comparison of coupled and uncoupled numerical solutions with respect to the pore-water pressure at the axis of a cylinder of soil.

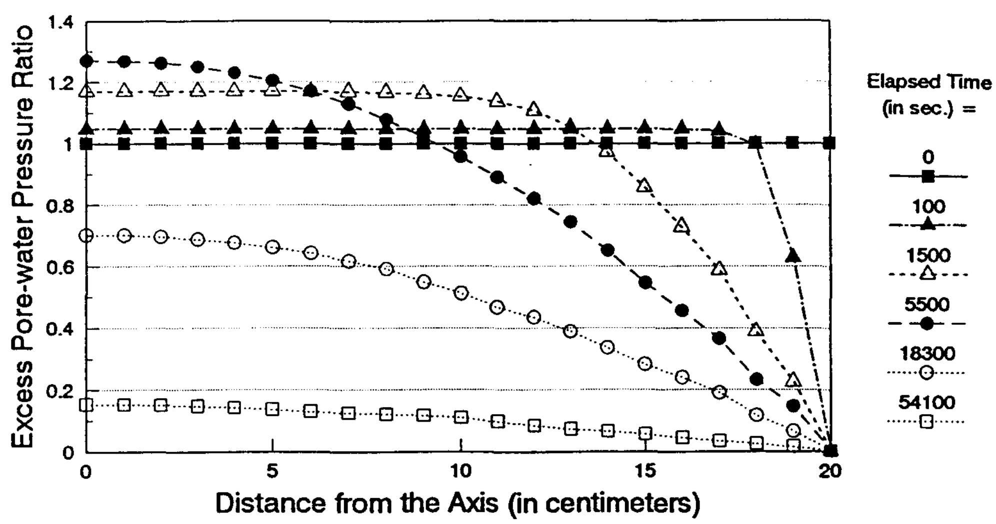

Figure 2. The spatial variations of pore-water pressures at different elapsed times during the consolidation of a cylinder of soil.

The results are compared with the analytical solution with respect to the excess pore-water pressure at the center of the sphere during consolidation (Figure 3). Figure 4 shows the spatial variation of the pore-water pressure at several elapsed times for $\nu'=0.01$.

There is an obvious difference between the results of the coupled and uncoupled analyses as shown in Figure 1. In the coupled analysis, in some parts of the soil the pore-water pressure increases before dissipation (i.e., the Mandel-Cryer effect). The Mandel-Cryer effect is dependent on the Poisson's ratio. It is more significant when the Poisson's ratio is low and it becomes negligible when the Poisson's ratio approaches 0.5.

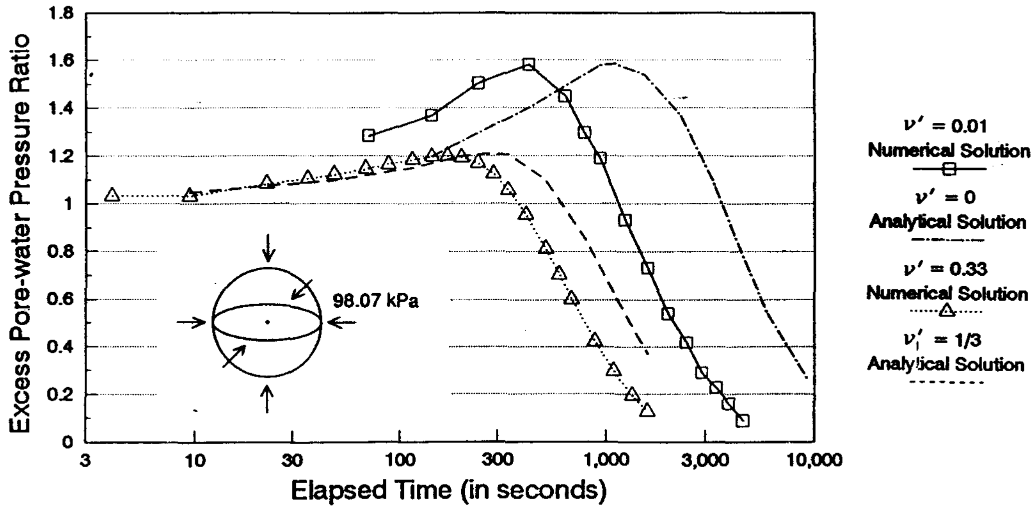

Figure 3. A comparison of numerical and analytical solutions with respect to the pore-water pressure at the center of a sphere of soil.

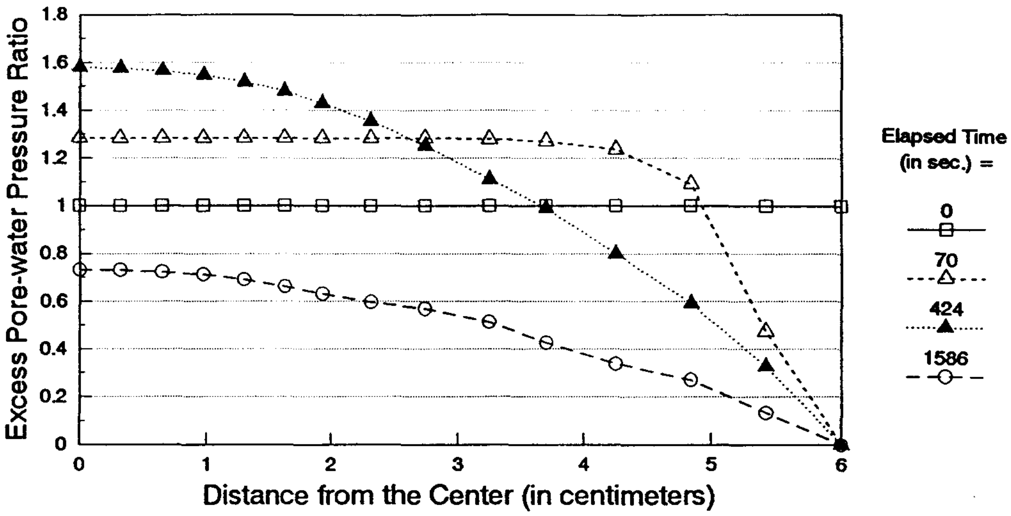

Figure 4. The spatial variations of pore-water pressures at different elapsed times during the consolidation of a sphere of soil.

The magnitude of the pore-water pressure increase obtained using this method agrees closely with that predicted by the analytical solution. However, discrepancies exist in the rate of pore-water pressure changes although both solutions exhibit similar patterns. The results also indicate that the Mandel-Cryer effect is less significant in the case of a cylinder than in the case of a sphere. The localized increase of the excess pore-water pressure in the sphere is almost double the increase in the cylinder.

Conclusion

A coupled consolidation analysis can be performed using two independent seepage and stress/strain finite element programs. The development and dissipation of pore-water pressure during consolidation can be predicted reasonably well following the proposed method.

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