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Use of Spline Interpolation in Slope Stability Analysis

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Abstract: *Spline Interpolation is a numerical procedure which is widely used to contour variables from irregularly spaced data. It has been shown that, under specific criteria, Spline Interpolation is equivalent to a well known estimation technique known as Kriging. Unlike Kriging, Spline Interpolation does not require a preliminary structural analysis. However, as a result, one should expect that the estimates obtained from Spline Interpolation are not as accurate as those obtained from Kriging. This paper examines the role of the Spline Interpolation technique in the prediction of pore-water pressures in limit equilibrium slope stability analyses. The pore-water pressures of interest are those at the base of a slice when the factor of safety is estimated using the method of slices and limit equilibrium techniques.*

The results of the study show that the Spline Interpolation technique is more accurate than the 4-Way Interpolation technique. However, the Spline Interpolation technique requires significant computing time and computer memory. As a result, the number of known data points which can be analyzed (on a microcomputer) within a slope stability computer program is somewhat limited.

INTRODUCTION

Spline Interpolation is a numerical procedure that can, given a set of known data points, estimate the value of a parameter at positions where the variable has not been measured or otherwise defined. The objective of this technique is to produce estimates that honor the data points and, at the same time, produce a 'smooth' surface or curve. The interpolating spline functions, as we know them today, first appeared during the 1940s.[1] With the advent of high-speed digital computers, splines have been used in a wide range of numerical studies. In the study presented herein, the Spline Interpolation technique was used to predict the pore-water pressure at the base of a slice (method of slices) when performing limit equilibrium slope stability analyses.

The Spline Interpolation procedure has been incorporated into the computer program PC-SLOPE.[10] Prior to this, a distance weighting method known as the 4-Way Interpolation technique was used to predict pore-water pressures along a slip surface. The main purpose of this study was to compare the two procedures for computing pore-water pressures. A brief summary of the theory associated with Spline Interpolation is presented along with data on one application; namely, the prediction of pore-water pressures at specified positions in a soil mass.

ESTIMATION OF PORE-WATER PRESSURES IN A SLOPE STABILITY ANALYSIS

Consider a typical slope stability problem like the one shown in Fig. 1. Pore-water pressure conditions within the slope are generally represented by a series of data points obtained from field measurements or from a finite element simulation. When performing a limit equilibrium analysis, it is necessary to pass a slip surface of some shape through the cross-section and divide the inscribed sliding mass into vertical slices. At this point, the pore-water pressure at the base of each slice is interpolated from the other known data points in the soil mass.

The 4-Way Interpolation procedure is a distance weighting method that is a fast and easily implemented means for estimating pore-water pressures within a

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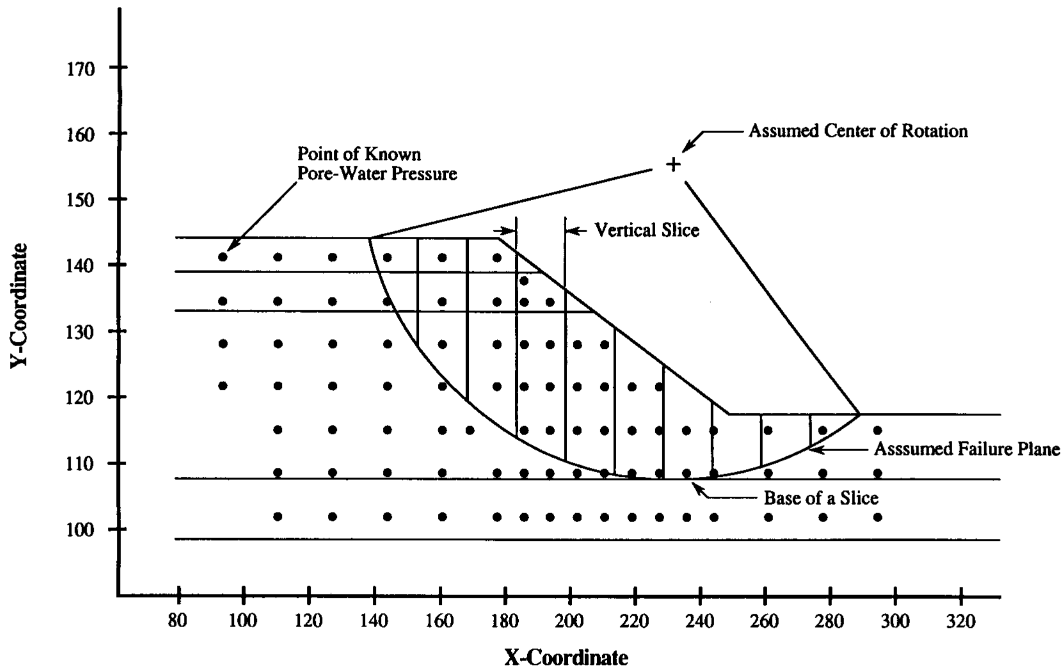
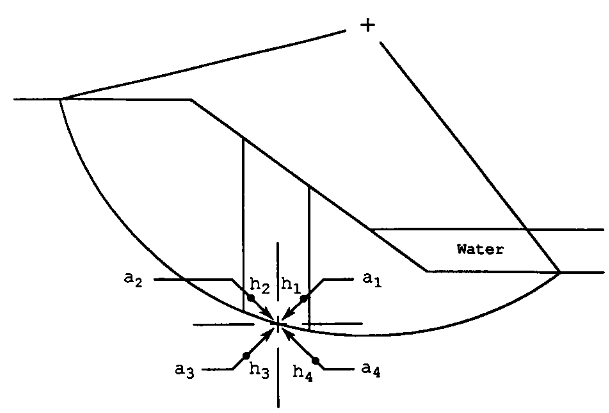


Fig. 1. Cross-section through a soil mass showing an inscribed slip surface and designated pore-water pressure points.

slope stability program (Fig. 2). This technique searches for the nearest point in each of the quadrants surrounding the unknown point. If data points cannot be found in one or more quadrants, the next nearest points are used in their place. This guarantees that no fewer than four data points are used in each interpolation. Estimated values of pore-water pressure are computed using eqn (1) given in Fig. 2. Data points which are used in the interpolation are weighted such that their influence declines linearly with distance from the point being estimated.

The 4-Way Interpolation algorithm is considerably faster than other interpolation methods which require linear equation solvers (e.g. Spline Interpolation). Moreover, its computer memory requirements are minimal; an important factor when one is trying to incorporate an interpolation scheme into an already large application. However, as the results of the study will show, the estimates produced by the 4-Way Interpolation method tend to be somewhat erratic.



$$u = \frac{\sum_{i=1}^{n}\left[\left(\sum_{i=1}^{n} a_i\right) - a_i\right] h_i \gamma_w}{\sum_{i=1}^{n} a_i (n-1)}$$

where:

a_i = distance to the point under consideration;

h_i = pore-water pressure head at known points;

n = number of points used in the interpolation;

u = pore-water pressure at the desired point;

γ_w = unit of weight of water.

Fig. 2. Illustration of the 4-Way Interpolation technique to compute pore-water pressure at the base of a slice.

THEORY

Spline Interpolation

Spline theory has been presented in detail in numerous publications such as Ahlberg *et al.*[1] and Schumaker[17] and will not be presented herein. However, in order to understand the basic principles of Spline Interpolation it is useful to recall the manual interpolation technique used in the past by draftsmen to draw a smooth curve betweeen a set of two-dimensional data points. Using this technique, the draftsman would join the points with a thin flexible strip of plastic or metal. Weights attached to the strip and placed at various positions along the curve were required to hold the strip in place. With the strip in place, the draftsman could then trace a smooth continuous line that passed through all points.

Conceptually, the objective of the Spline Interpolation technique is to determine a function that represents the shape of the strip used by the draftsman. This function $\sigma(x,y)$ is among all the functions f that honor the data points and the one that minimizes a quantity that is analogous to the mechanical energy required to deform a thin strip of infinite extent.[13] The minimized quantity $A(f)$ is equal to:[6]

$$A(f)=\int\int_{R^2}\left[\left(\frac{\partial^2 f}{\partial x^2}\right)^2+\left(\frac{\partial^2 f}{\partial y^2}\right)^2+2\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2\right] \quad (2)$$

The interpolating function $\sigma(x,y)$ relative to N data points is written as:[8]

$$\sigma(x,y)=a+bx+cy+\sum_{i=1}^{N}\lambda_i h_i^2 \log h_i \quad (3)$$

where

$$h_i=\sqrt{(x-x_i)^2+(y-y_i)^2}$$

The coefficients of eqn (3) (i.e., a, b, c and λ_i) are solved by assembling a system of linear equations where

$$\sum_{i=1}^{N}\lambda_i=0$$

$$\sum_{i=1}^{N}\lambda_i x_i=\sum_{i=1}^{N}\lambda_i y_i=0 \quad (4)$$

$$\sigma^*(x_i,y_i)=z(x_i,y_i)(\forall i\in\{1,\ldots N\})$$

The third condition in eqn (4) ensures that the estimated values are equal to the known values at the data points.

Regionalized variables and the intrinsic hypothesis

It has been shown that, under specific criteria, Spline Interpolation is equivalent to another popular interpolation technique known as Kriging.[16] The Kriging method was developed from the theory of regionalized variables; the theoretical backbone of a scientific discipline known as geostatistics. The term 'geostatistics' designates the statistical study of natural phenomena; a field of study that was developed in response to ore-deposit evaluation problems arising in the mining industry.[11] Although the applications of Kriging have been predominate in the mining industry, recent studies have shown this estimation procedure to be well adapted to water resources problems.[5]

Kriging has received considerable attention in the literature in recent years. Details of recognized variable theory and kriging theory has been covered in publications such as Clark,[2] David,[3] Journel and Huijbregts[11] and Matheron.[14,15] Nevertheless, in order to show the relationship between Spline Interpolation and Kriging, a brief outline of the theory behind Kriging is necessary and will be presented herein.

A regionalized variable (i.e., RV) can be described as a phenomenon which is spread out in one, two, or three dimensional space that displays a recognizable spatial distribution or 'structure'.[5] The RV is a function $z(x)$ (where x is the one, two or three dimensional coordinates) that, for many naturally occurring phenomena, has properties that are too complex to be studied easily through common methods of mathematical analysis. However, according to the theory of regionalized variables, a RV $z(x)$ can be interpreted as a realization of a random function $z(x)$. This random function can be seen as a set of random variables $z(x_i)$ defined at each point x_i in the domain of study D, that is:

$$Z(x)=\{Z(x_i),\forall x_i\in D\} \quad (5)$$

These random variables $Z(x_i)$ are correlated but, to describe this correlation it is necessary to introduce the 'intrinsic hypothesis'. This hypothesis is that the expectation of the random function $Z(x)$ is equal to a constant and independent of x (i.e., stationarity):

$$E[Z(x)]=m(x)=m \quad (6)$$

and that the increment $Z(x+h)-Z(x)$ has a zero expectation and a variance which is independent of the point x. This is written as:

$$E[Z(x+h)-Z(h)]=0 \quad (7)$$

$$\mathrm{Var}[Z(x+h)-Z(h)]=2\gamma(h) \quad (8)$$

where h is the separation vector (i.e., h has magnitude and direction). The variogram function $\gamma(h)$ depends only on h and not on the location x.

Characterizing the structure of the data involves estimating the variogram function from the available data. The mean of the squared differences between data points separated by h are computed for a number of different h values. From these values a plot, which is known as the variogram, of $\gamma(h)$ versus $|h|$ is prepared and fitted with a mathematical model. A typical variogram is shown in Fig. 3. Details on the properties, interpretation and fitting of variograms is presented in Clark.[2]

The hypothesis of constant mean is not reasonable for particular types of data; for example in the case of hydraulic heads or pore-water pressures where physical laws govern flow. In these cases the intrinsic hypothesis must be extended so that the random function $Z(x)$ includes a deterministic component called the 'drift' and a random component $Y(x)$ sometimes called 'residual' where:[4]

$$Z(x) = m(x) + Y(x) \quad (9)$$

$$m(x) = E[Z(x)] \quad (10)$$

It should be noted that it is assumed that the drift function $m(x)$ varies slowly in space and that $Y(x)$ is a random function with a structure (i.e., variogram) of its own. In practice, the two most common forms of drift functions are:

$$m(x,y) = a + bx + cy \text{ (linear)} \quad (11)$$

$$m(x,y) = a + bx + cy + dx^2 + ey^2 + fxy \text{ (quadratic)} \quad (12)$$

Universal Kriging

To estimate the value of a parameter, Kriging, like most other interpolation methods, determines a weighting coefficient for each of the known data points used in the interpolation. Given N known data points, $Z(x)$, the Kriging estimator, $Z^*(x)$, is equal to:

$$Z^*(x) = \sum_{i=1}^{N} \lambda_i Z(x_i) \quad (13)$$

The Kriging estimator is the best linear unbiased estimator (BLUE) because the weights λ_i are computed so as to satisfy the two conditions:

$$E[Z^*(x) - Z(x)] = 0 \quad (14)$$

$$\mathrm{Var}[Z^*(x) - Z(x)] \text{ minimum} \quad (15)$$

Equation (14) ensures that there will be no systematic over- or under-estimation (i.e., unbiased) while eqn (15) states that the variance of the estimated values from the actual values must be minimized (i.e., the estimator is optimal).

Universal Kriging is a method that can be used when the intrinsic hypothesis is not appropriate (i.e., drift in the data). In some of these cases one can estimate the form of the drift $m^*(x)$, subtract it from $Z(x)$, and then

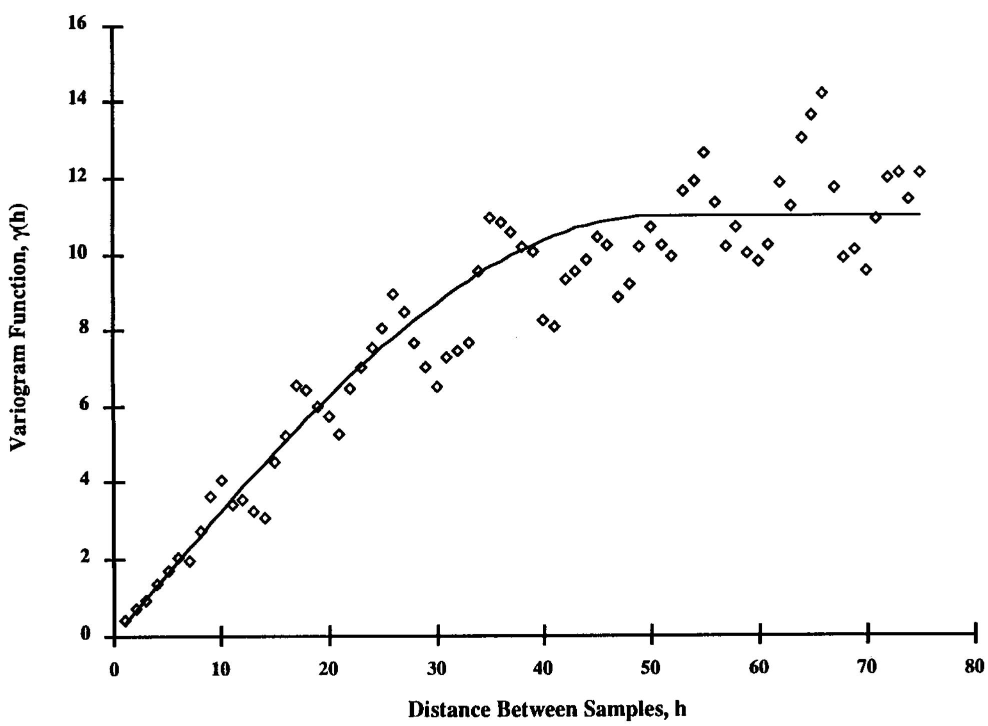


Fig. 3. Fitted spherical model to experimental variogram (after Clark[2]).

fit a model to the variogram of residuals. However, because these variograms are biased, they can be difficult to interpret.[4] Universal Kriging was developed using the Theory of Intrinsic Random Functions (IRF) that was presented by Matheron.[15] This theory made the inference of the spatial structure relatively easy and suitable for automation in spite of the presence of drift.

Under the IRF theory, the Kriging estimator $Z^*(x)$ for N data points can be written as:

$$Z^*(x) = m(x) + \sum_{i=1}^{N} \lambda_i K(x_i - x) \tag{16}$$

where $K(x_i - x)$ is called the 'generalized covariance'. When Kriging with a polynomial drift of order k, the generalized covariance expresses the variance of increments of order k. In other words, the generalized covariance has a role analogous to that of the variogram function when Kriging stationary data.

Not all mathematical functions can be used to represent the generalized convariance. Since the variances of the increments must be positive, certain mathematical restrictions apply. In practice, polynomial models are most frequently used.

Equivalence of Spline Interpolation and Kriging

A two dimensional Kriging system of equations which includes a linear drift function is written as:

$$Z^*(x) = a + bx + cy + \sum_{i=1}^{N} \lambda_i K(h_i) \tag{17}$$

where

$$h_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

and the coefficients are solved subject to

$$\begin{cases} \sum_{i=1}^{N} \lambda_i = 0 \\ \sum_{i=1}^{N} \lambda_i x_i = \sum_{i=1}^{N} \lambda_i y_i = 0 \\ z^*(x_i, y_i) = z(x_i, y_i) (\forall i \in \{1, \dots N\}) \end{cases} \tag{18}$$

Notice that if $h^2 \log h$ is substituted for $K(h)$, the system of Kriging equations in eqns (17) and (18) is the same as the Spline Interpolation system shown in eqns (3) and (4). As it turns out, $h^2 \log h$ is an admissable generalized covariance function if used with a linear drift function.[16] In essence, two dimensional Spline Interpolation is equivalent to Kriging with a linear drift and generalized covariance of the form $h^2 \log h$. However, it is important to remember that the two methods determine the weighting coefficients subject to different criteria. Kriging strives to obtain estimates which are, on the average, as close as possible to the actual values while Spline Interpolation uses the shape of the interpolating function to determine the weighting coefficients which results in estimates that are optimized for aesthetics (i.e., 'smoothness') rather than accuracy.[7]

The most important difference (in a practical sense) between Kriging and Spline Interpolation is that the Kriging method requires preliminary recognition of the spatial structure. In order to minimize the estimation variance, it is necessary to determine the two structural characteristics; first, the order k of the drift, and second, the form of the generalized covariance function $K(h)$ of the variable once its drift has been filtered. However, this procedure can be computationally intensive and requires a significant knowledge of geostatistics. Since Spline Interpolation does not require a structural analysis, it can be more easily implemented for use with applications such as slope stability.

Spline Interpolation was chosen as an alternate method for interpolating pore-water pressures because of its versatility and ease of implementation. Due to the physical laws governing flow, it is reasonable to expect that there will be significant drift in the data. In addition, random fluctuations in the data should be modeled reasonably well with the given generalized covariance. However, without a structural analysis it is impossible to ascertain how well the chosen drift and covariance functions fit the given data.

IMPLEMENTATION OF THE SPLINE INTERPOLATION TECHNIQUE

The Spline Interpolation technique was implemented into PC-SLOPE in conjunction with the existing techniques for representing hydraulic conditions in the soil mass. The user can specify either a set of pore-water pressure heads, a set of pore-water pressures, or a set of pore pressure coefficients. However, due to microcomputer memory (an IBM 286 with a 640 kByte RAM) limitations, the Spline Interpolation technique can be used only when 50 or fewer points are designated. When more than 50 points are designated, the 4-Way Interpolation procedure must be used.

Two benchmark examples were studied to illustrate the use of the Spline Interpolation technique. Datafile

No. 1, a finite element seepage analysis example, was taken from Krahn[12] while Datafile No. 2, a limit equilibrium slope stability example, was taken from Fredlund.[9]

In the first two phases of the study, pore-water pressure data taken from Datafile No. 1 was used to compare the Spline Interpolation and the 4-Way Interpolation methods. The objective of phase one and two was to determine how each interpolation method is affected by the number of known pore-water pressure points that are used in the slope stability analysis. In phase one, the interpolation techniques were compared relative to the computed factor of safety. In order to provide a more direct means of evaluating the two methods, the second phase of the study focused on comparing the accuracy of the actual pore-water pressure estimates.

Datafile No. 2 was used during the third phase of the study to compare the computing time required by the two interpolation methods. The objective of phase three was to determine how the size of the data set, and how the number of slip surfaces affect the computational time.

Phase I: Comparing factors of safety

Pore-water pressure data required for the first phase of the study was obtained from the results of a finite element seepage analysis. Datafile No. 1 illustrates steady state seepage through a homogeneous earth dam with an impervious lower boundary (Fig. 4). The dam cross-section was modeled with a uniform mesh containing 195 nodes and 336 elements. Pore-water pressure heads were computed at each of the nodes.

A new datafile was created to model the slope stability of the downstream face of the dam. The material was assumed to be a sandy clay till with an unit weight of 19·33 kN/m^3, an angle of internal friction, ϕ', of 29° and a cohesion intercept, c', of 24 kPa. Pore-water pressure head data (i.e., 195 data points) from the results of the seepage analysis was placed into the slope stability file. A preliminary slope stability analysis using 30 slices per slip surface was performed to determine the critical slip surface.

Next, a slope stability template file was created. The soil and geometry information specified in the datafile used to determine the critical slip surface was carried over into the template file. However, the template file was designed so that the factor of safety was computed only once; on the critical slip surface determined previously. Pore-water pressure data was excluded from the template file.

Limitations of the slope stability program prevented using more than 50 pore-water pressure points in the slope stability analyses. Therefore, datafiles containing 49, 39, 33, 28, 22 and 18 points were analyzed. In each case, the data points were randomly chosen from the original data set containing 195 points because it was thought randomly selected pore-water pressure points would best simulate measurements taken in the field. In addition, as a means of statistical evaluation, 100 cases of each datafile were produced and analyzed (i.e., 100 files containing 49 points, 100 files containing 39 points, etc.).

A programmed spreadsheet was used to generate the slope stability datafiles analyzed during this phase of the study. First, the slope stability template file and the finite element seepage results were imported into the spreadsheet. Using the programming language of the spreadsheet and its random number generator function, an algorithm was designed such that a predefined number (i.e., either 49, 39, 33, 28, 22 or 18) of

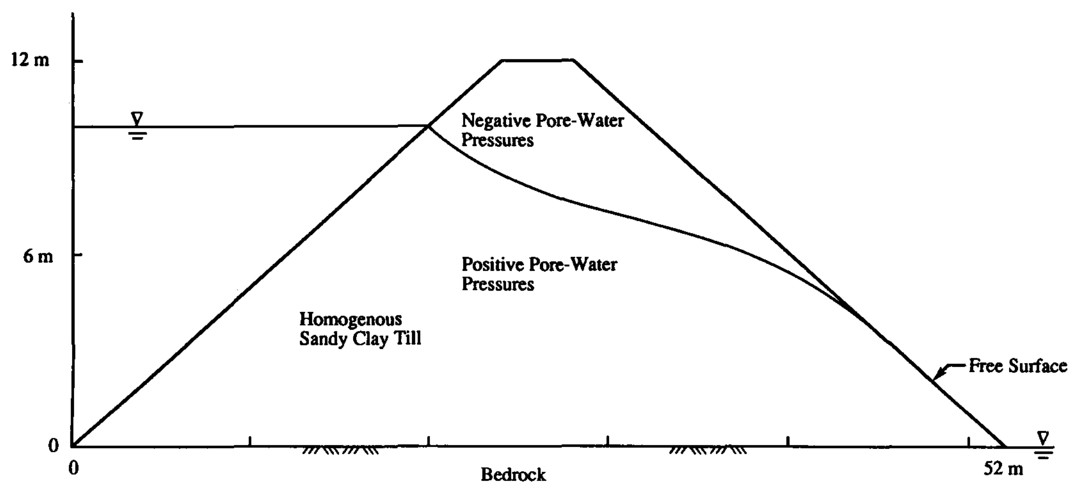


Fig. 4. Geometry and boundary conditions of example problem, seepage through a homogeneous earth dam.

pore-water pressure would be extracted at random from the results of the seepage analysis. The selected points would then be merged with the slope stability template file. Next, the spreadsheet would save the completed slope stability file in the usual PC-SLOPE format. This process would be repeated until 100 slope stability files were generated.

All of the generated slope stability datafiles were analyzed twice; once utilizing the 4-Way Interpolation technique, and once utilizing the Spline Interpolation technique. The mean factors of safety, average from 100 cases, are summarized in Table 1. All factors of safety were computed using the Bishop Simplified method.

The results shown in Table 1 along with the 95% confidence intervals are shown in Figs 5 and 6. Also shown on these plots is the factor of safety computed using a piezometric line to represent the pore-water pressures along the slip surface. This value is considered to be the 'true' factor of safety (i.e., 1·813).

Table 1
Summary of mean factors of safety for the 4-Way Interpolation and Spline Interpolation methods

No. of points	*Mean factor of safety (Bishop Simplified)*	
	4-Way Interpolation	*Spline Interpolation*
49	1·848	1·811
39	1·845	1·812
33	1·850	1·804
28	1·830	1·802
22	1·815	1·799
18	1·839	1·789

The general form of the results shown for the Spline Interpolation method (Fig. 6) appears as would be intuitively expected; that is, the mean factor of safety becomes closer to the true factor of safety as the number of data points used in the analyses increases. Moreover, the confidence interval decreases as the number of data points increases. The mean factor of safety is, for all practical purposes, equal to the true factor of safety when 39 and 49 data points are used.

The 4-Way Interpolation method produces results that are somewhat difficult to interpret (Fig. 5). The mean factor of safety does not change significantly while using either 49, 39 or 33 points. Surprisingly, the mean factor of safety begins to drop closer to the true factor of safety at 28 and 22 points and then loses its accuracy at 18 points. However, the confidence intervals become slightly narrower as the number of pore-water pressure points increases.

The results obtained from the analyses using the 4-Way Interpolation method indicate that mean factor of safety does not appear to converge. However, when a slope stability analysis with 4-Way Interpolation was performed using all 195 pore-water pressure heads, the resulting factor of safety was 1·829. This result indicates that the 4-Way Interpolation method does converge to a value near the true value, albeit very

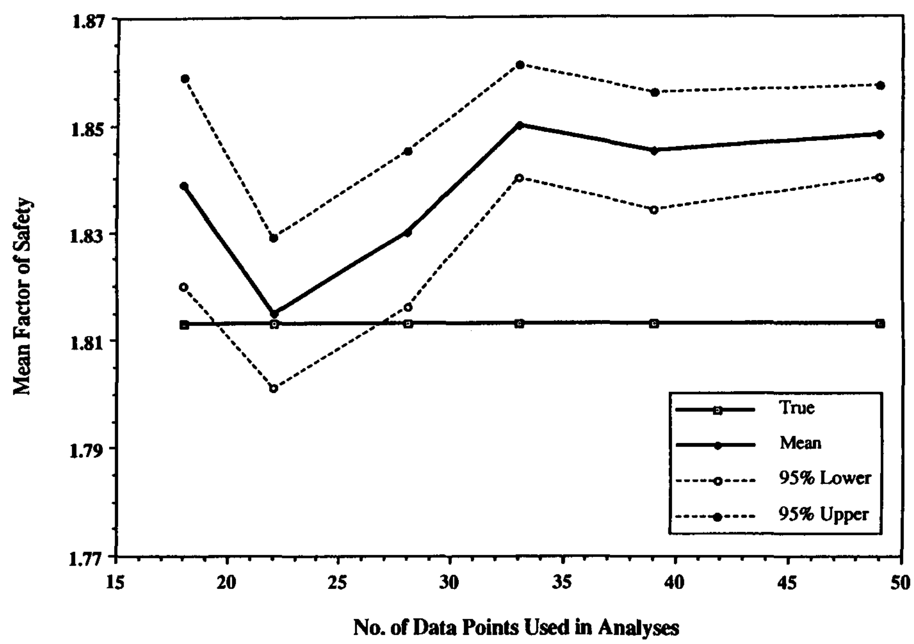


Fig. 5. Mean factor of safety using 4-Way Interpolation.

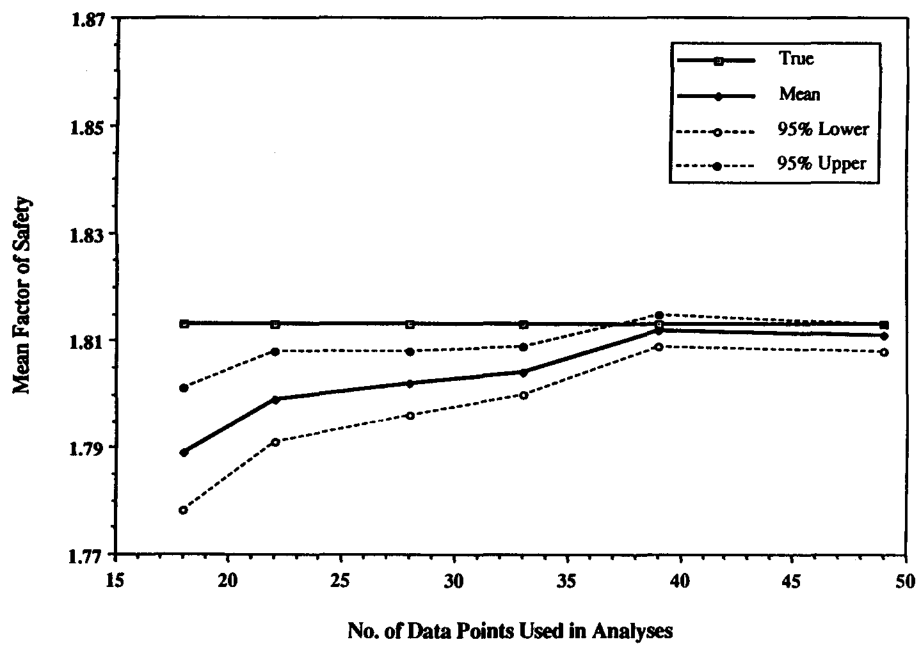


Fig. 6. Mean factor of safety using Spline Interpolation.

slowly, and that over the limited range of data points investigated here, convergence was not significant.

Phase II: Comparing pore-water pressure

Phase II of the study was designed to provide a direct comparison of the pore-water pressure estimates produced by the interpolation procedures. This should avoid any dampening or accentuating effects involved in the factor of safety calculations. The computer program 'INTERP' was developed for direct pore-water pressure interpolation using either the 4-Way Interpolation or the Spline Interpolation methods. For this study INTERP, using a regular input file, was used to perform interpolations using both estimation techniques and, in the process, store the results of the two methods in separate files. This stand-alone program had the capacity to perform interpolations using all 195 pore-water pressure values present in the example problem (i.e., Datafile No. 1).

Pore-water pressures were estimated at 30 positions on the cross-section (Fig. 7). These points correspond to positions on the critical slip circle where the pore-water pressures were being estimated during Phase I. A revised version of the programmed spreadsheet developed during Phase I was used to randomly select a predefined number of pore-water pressure heads from the original finite element seepage results. The spreadsheet would then merge this data with the coordinates of the unknown points (i.e., the 30 positions along the slip surface) and save the information in a format compatible with the program INTERP. This process could be repeated until any user defined number of data files were generated.

Data files containing from 20 to 100 pore-water pressure points were analyzed using the program INTERP. In order to provide a means of statistical evaluation, 150 cases of data files containing 20, 30, 40, 50, 60, 70 and 80 data points were analyzed while only 100 cases of data files containing 90 and 100 points were analyzed.

The program INTERP was designed to analyze all cases of one particular data set size in one run and to write the results into two output files; one file containing pore-water pressure estimates produced by Spline Interpolation, and a second file containing pore-water pressure estimates produced by 4-Way Interpolation. This meant that a typical output file would contain 100 or 150 different pore-water pressure estimates for each of the 30 designated points along the slip surface. With this information, mean pore-water pressures and standard deviations were computed for each position on the slip surface. In addition, the mean of the standard deviations across the slip surface was computed.

A series of graphs showing the variation in the mean pore-water pressure head along the slip surface were plotted. Four of these graphs are shown in Figs 8–11. The correct pore-water pressure head values were also

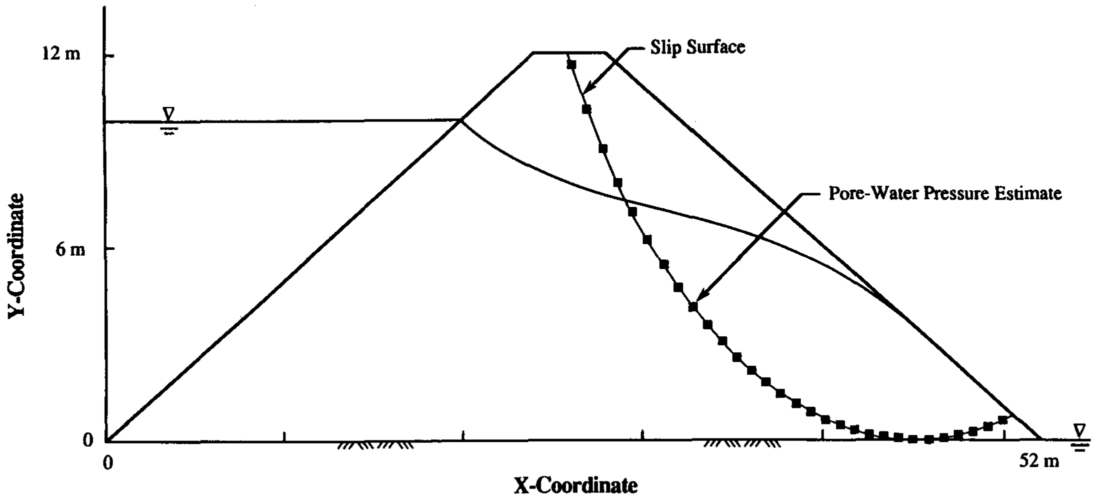


Fig. 7. Cross-section of the dam showing the positions on the slip surface where pore-water pressures are estimated.

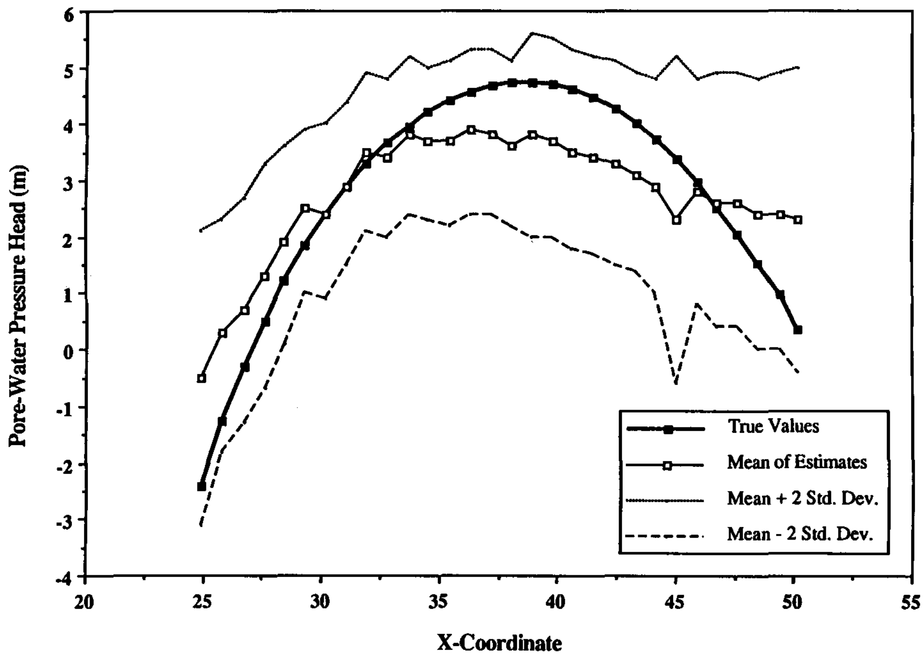


Fig. 8. Pore-water pressure head statistics along the slip surface using 4-Way Interpolation on 20 data points.

included on the plots. These values were obtained by using the Spline Interpolation technique on all of the 195 data points.

The mean pore-water pressure head plots, when using the 4-Way Interpolation technique on 20 and 100 points, are shown in Figs 8 and 10 respectively. The mean pore-water pressure head varies in a somewhat erratic manner across the slip surface. Figures 9 and 11 show the results when using the Spline Interpolation technique on 20 and 60 points. In these cases, the mean pore-water pressure head curve closely follows the plot of true values. Using the Spline Interpolation technique on 60 points produces estimates that are superior to the results of using 4-Way Interpolation on 100 points. Plots illustrating results from analyses using Spline Interpolation on more than 60 points (i.e., 70, 80, 90 and 100 data points) showed insignificant differences between the mean pore-water pressure head values and the true values at all positions on the slip surface. However, these plots did show that the

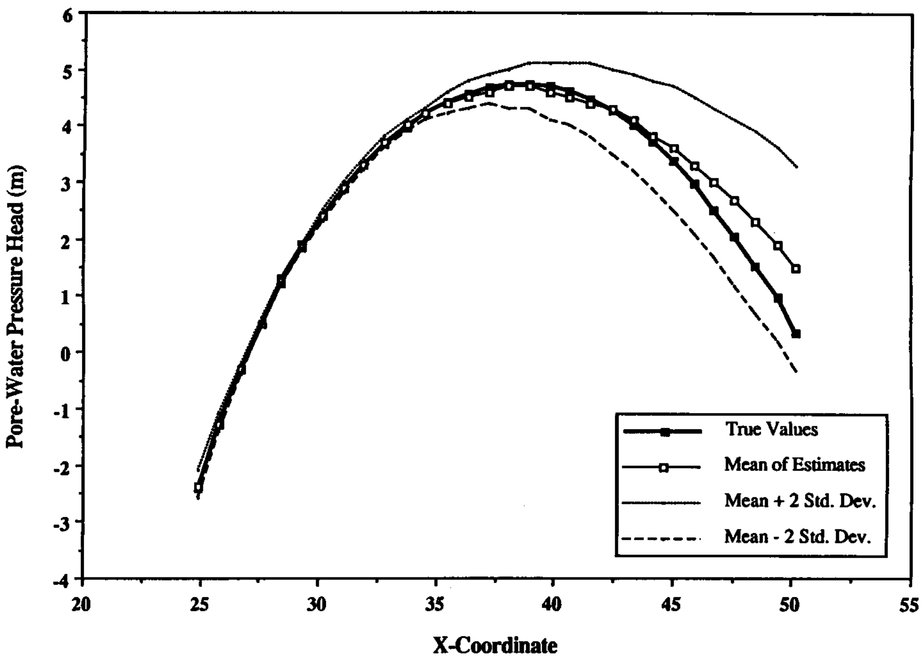


Fig. 9. Pore-water pressure head statistics along the slip surface using Spline Interpolation on 20 data points.

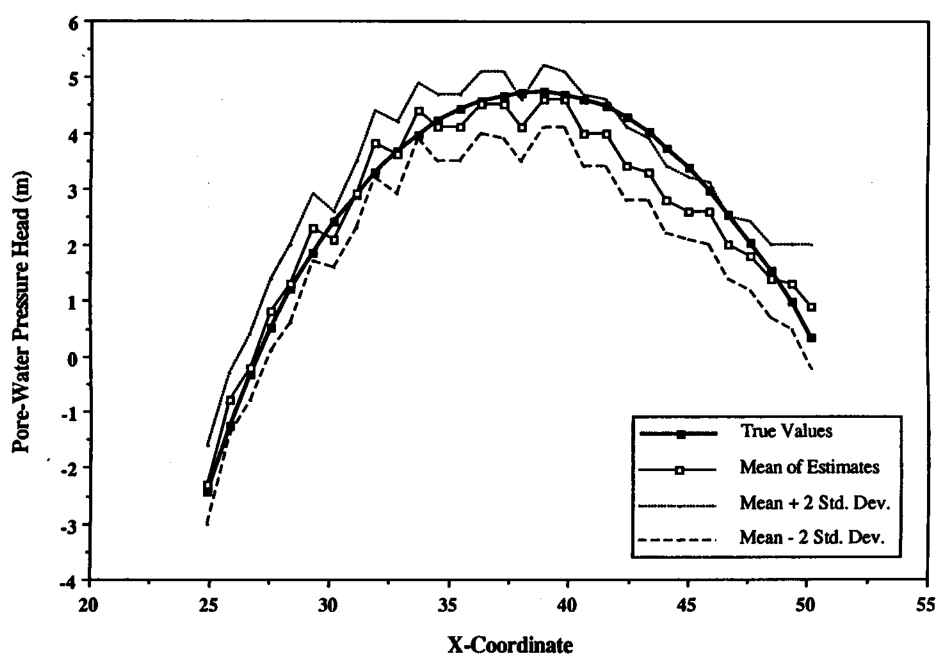


Fig. 10. Pore-water pressure head statistics along the slip surface using 4-Way Interpolation on 100 data points.

variation in the estimates (i.e., standard deviation) decreased as the number of points used in the analyses increased.

Figures 9 and 11 indicate that the estimation error considerably increases with increasing x-coordinate values. This can be attributed to the geometry of the problem. Figure 7 shows that as the x-coordinate along the slip surface increases, the points of estimation located on the slip surface move into the downstream toe of the dam. Recall that the finite element mesh used

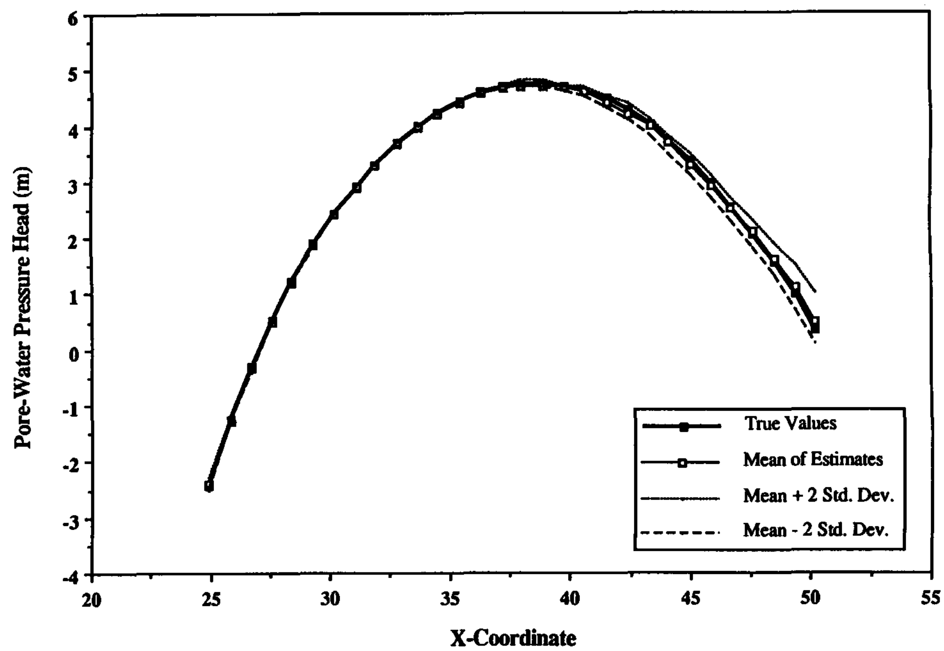


Fig. 11. Pore-water pressure head statistics along the slip surface using Spline Interpolation on 60 data points.

to model the flow was evenly discretized over the cross-section of the dam. Therefore, it one is randomly select, say for example, 40 points from the set of 195 points, it would be reasonable to expect that most of these points would fall within the central portion of the dam. As a result, the estimation accuracy along the slip surface would decrease as one moves toward the downstream toe of the dam simply because the density of known data points becomes rapidly less as the triangular shape of the cross-section 'pinches out'. This effect becomes even more pronounced as the number of data points used in the analysis decreases.

The average of the standard deviations across the slip surface for all data set sizes are listed in Table 2 and plotted in Fig. 12. This plot shows that the average of the standard deviations, in all cases, is significantly lower for Spline Interpolation than it is for the 4-Way Interpolation method. In addition, the average of the standard deviations for the 4-Way Interpolation method increases faster as the number of data points decreases. Using pore-water pressure rather than factor of safety as a basis of comparison produced consistent results, that is, the accuracy of both interpolation methods improved as the number of data points increased.

The results obtained during this phase of the study shed some light on the factor of safety results obtained during the previous phase. The results shown in Fig. 8 indicate that the 4-Way Interpolation method tends to

Table 2
Comparison of the mean of the standard deviations across the slip surface

No. of points	*Mean of the standard deviations (m)*	
	4-Way Interpolation	*Spline Interpolation*
20	0·927	0·287
30	0·746	0·189
40	0·581	0·131
50	0·494	0·082
60	0·423	0·054
70	0·399	0·046
80	0·363	0·035
90	0·340	0·025
100	0·305	0·025

over-estimate pore-water pressure at the outside edges of the slip circle while under-estimating pore-water pressure near the center. As the number of data points used in the interpolation increases (i.e., Fig. 10) the predicted values become closer to the true value but change more frequently from under-estimated values to over-estimated values. It appears that when the number of data points falls between a certain range, (i.e., in this case between 28 and 22 points) the over-estimates and under-estimates tend to offset each other resulting in an improved factor of safety value. However, this trend may not prevail in all slope stability problems.

Phase III: Comparison of computing time

Datafile No. 2 was used during this phase of the study to compare the computing time required by the 4-Way Interpolation and Spline Interpolation techniques. A total of 86 pore-water pressure points were superimposed over the cross-section geometry (Fig. 13). The original datafile was modified to perform analyses using 43, 29 and 22 randomly selected points. The file was modified further so as to allow variation in the

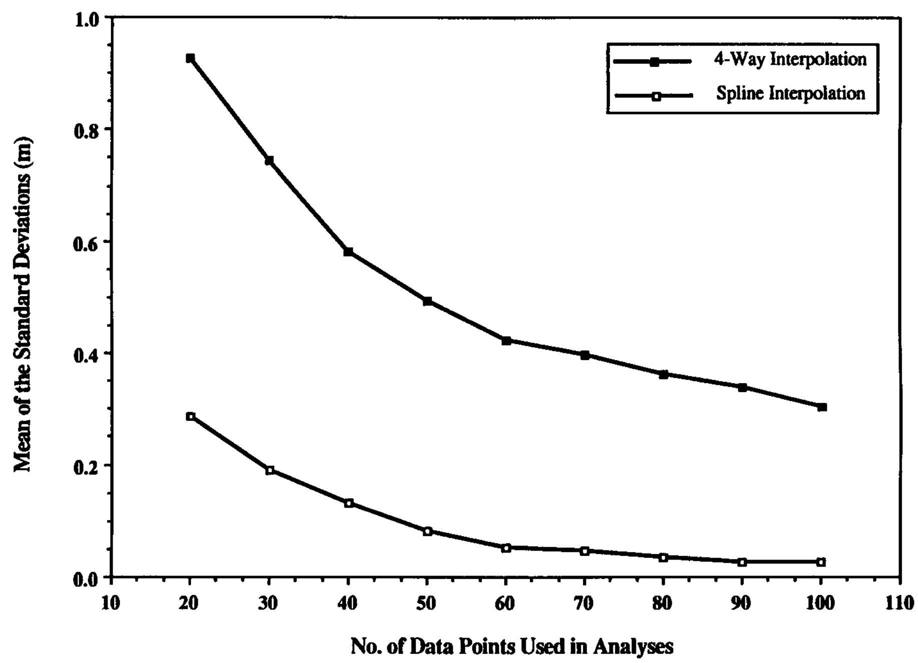


Fig. 12. Mean of the Standard Deviations across the slip surface using the Spline Interpolation and the 4-Way Interpolation method.

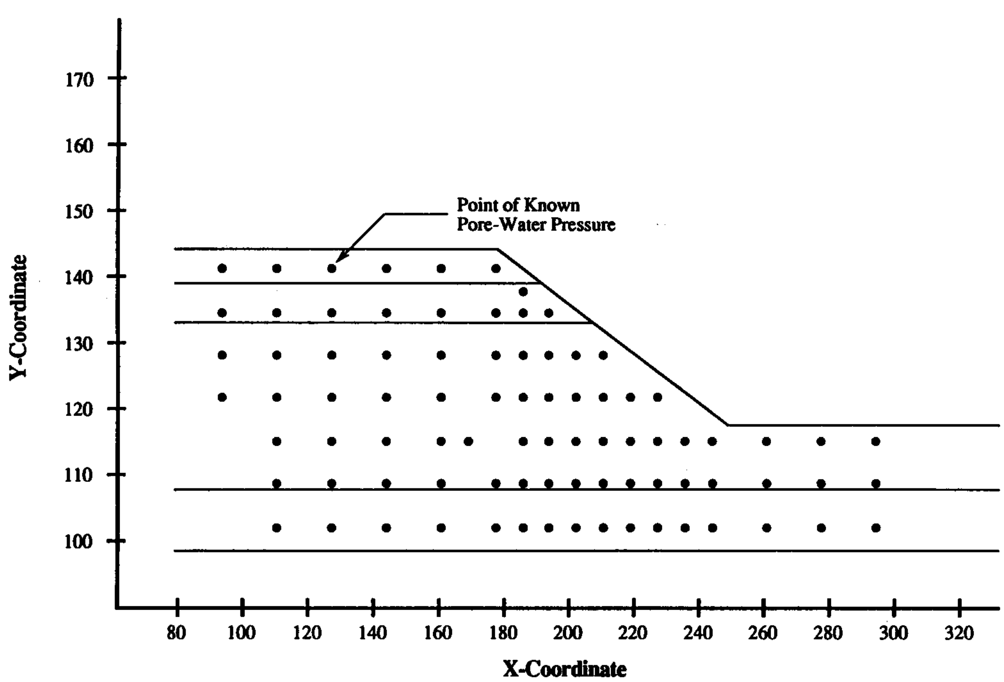


Fig. 13. Original geometry for Datafile No. 2 showing 86 designated pore-water pressure heads.

number of slip surfaces. The results from the analyses are shown in Tables 3 and 4 and presented graphically in Fig. 14. The measured computing time was the time required to perform the complete slope stability analysis on each data file. All analyses were performed using a 10 MHz IBM AT compatible computer with a math co-processor chip.

Table 3
Computing time per slip surface using 4-Way Interpolation

No. of slip surfaces	*Computing time per slip surface (s)*		
	22 Points	*29 Points*	*43 Points*
36	1·11	1·19	1·31
64	1·08	1·20	1·25
100	1·05	1·10	1·23
180	1·02	1·07	1·19

Table 4
Computing time per slip surface using Spline Interpolation

No. of slip surfaces	*Computing time per slip surface (s)*		
	22 Points	*29 Points*	*43 Points*
36	1·42	1·58	1·97
64	1·31	1·48	1·83
100	1·28	1·43	1·75
180	1·24	1·37	1·67

Figure 14 shows the computing time per slip surface versus number of pore-water pressure points for trials using 36, 64, 100 and 180 slip surfaces. The plots show that, as the number of data points increases, the computing time per slip surface increases more rapidly for the Spline Interpolation method than for the 4-Way Interpolation technique. Prior to this study, it was anticipated that perhaps the Spline Interpolation method would show greater efficiency gains that the 4-Way Interpolation method when using a greater number of slip surfaces. However, this trend was not apparent from the results of this example.

CONCLUSIONS

The results show the Spline Interpolation technique to be more accurate that the 4-Way Interpolation technique. The average of the standard deviations across the slip surface was, for all data set sizes, significantly lower for Spline Interpolation than it was for 4-Way Interpolation. It would appear that even an extremely large number of designated pore-water pressures (or pore-water pressure heads) would not produce as reli-

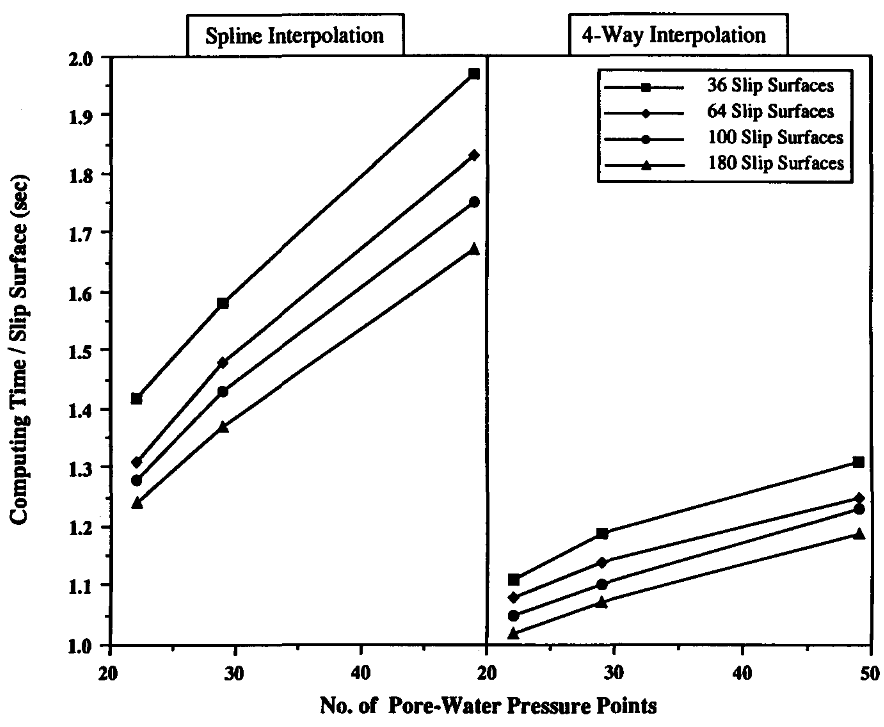


Fig. 14. Comparison of computing time per slip surface required by the 4-Way Interpolation and Spline Interpolation technique.

able pore pressure predictions using 4-Way Interpolation as can be obtained using the Spline Interpolation technique. This is important for factor of safety calculations since large standard deviations in the pore-water pressure estimates make the critical slip surface somewhat loosely defined.

The Spline Interpolation technique requires more computing time than the 4-Way Interpolation method. The computing time increases significantly as the number of points used in the Spline Interpolation increases. However, the accuracy and consistency of its pore-water pressure predictions are superior. On the basis of the above study, it is concluded that the Spline Interpolation technique can be used to advantage in slope stability calculations.

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