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The Role of Unsaturated Soil Behaviour in Geotechnical Engineering Practice

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SYNOPSIS

Traditional soil mechanics practice has experienced significant changes during the past few decades. In part, this has been due to increased environmental concerns. The computational capability available to the geotechnical engineer has also strongly influenced practice.

In keeping with the above factors, there has been a need to better understand the engineering behaviour of the soil near to ground surface. This zone, known as the vadose zone, is subjected to a flux type boundary condition for many of the problems faced by geotechnical engineers. As a result, unsaturated soil mechanics has become a necessary tool for analysing common geotechnical and geo-environmental problems.

This paper presents some of the primary aspects associated with unsaturated soil mechanics. It also shows the importance of developing simplified formulations which communicate the primary aspects involved with the vadose zone. The integration of unsaturated seepage and slope stability is used to illustrate a simplified approximation of a common geotechnical problem.

INTRODUCTION

The role of civil engineers and the scope of civil engineering practice are undergoing continual change. One of the most dramatic changes in civil engineering within the past few years has been the area of geo-environmental engineering. It was the increasing awareness of our environment that brought about careful evaluations of the environmental impact of engineering developments.

Geotechnical engineering has traditionally been viewed as an engineering field which is strongly rooted in engineering mechanics, directed at solving problems related to strength, strain and seepage. The effective stress principle is the key concept that has led to the rapid transfer of geotechnology around the world. However, within the past few years there has been an emerging need in society to broaden the scope of geotechnology to include environmental concerns.

In a matter of only about two decades, world attention shifted from the analysis of engineered structures to limiting the impacts of technology and developments on the natural world. Stewardship of the physical and biological environment became an increasing concern. Geotechnical engineers found themselves well positioned, by virtue of their training and experience, to study the impact of a wide range of developments. And geotechnical engineers have, in general, quickly broadened the scope of their domain of practice. In North America and other parts of the world, many geotechnical consulting firms now find that more than 75% of their work involves geo-environmental type projects.

As part of geo-environmental type projects, it has become necessary to study the mass flux of contaminants transported to the groundwater system. Most of these problems occur near ground surface and as such a knowledge of unsaturated soil behaviour is valuable.

The purpose of this paper is to illustrate the development and role of unsaturated soil mechanics in solving geotechnical problems. The application of the theories will be considered not only in the light of geo-environmental needs but also with respect to the broader needs in geotechnical engineering. The paper illustrates how an understanding of unsaturated soil mechanics assists the geotechnical engineer in handling numerous problems encountered in practice.

TRADITIONAL (OR CLASSICAL) SOIL MECHANICS

The effective stress variable, $(\sigma - u_w)$, became pivotal in the

1930's as a means of communicating the behaviour of saturated soils (Terzaghi, 1936). Soil mechanics moved from an empirical basis to a science basis and enjoyed the implied status. Soil behaviour was related to effective stress which was independent of the soil properties.

The scope of saturated soil mechanics embraced three primary areas:

(1) Seepage analyses where the problems were classified as either confined or unconfined flow analysis (Casagrande, 1936),

(2) Plasticity and limit equilibrium analyses where the problems ranged from slope stability, to bearing capacity, to lateral earth pressures, and

(3) Volume change analyses directed primarily towards the prediction of settlement in soft clays.

The basic formulations in classical soil mechanics were primarily of a static and steady state nature. One exception was the theory of consolidation which illustrated the interaction of deformation and seepage. The theory of consolidation provided an excellent mathematical tool to assist the engineer in visualizing saturated soil behaviour. It allowed the prediction of pore-water pressures in time and space, and became the symbol of classical soil mechanics.

With time, there has been a need for the geotechnical engineer to expand the scope of the problems being addressed. The emphasis has been increasingly directed towards the consideration of a wide variety of unsteady state or transient analyses. This has been most visible in the geo-environmental area where the geotechnical engineer is called upon to predict chemical concentrations with respect to time and space. The associated analyses come under the area of contaminant transport modelling. The diffusion of a chemical is superimposed on the conductive movement in the water phase. In other words, an additional unsteady state analysis has been added to the seepage of water through a soil. The soil property, coefficient of permeability, has often been regarded as one of the most difficult property to evaluate. Now, this property is the central focus of many analyses and much research has been directed towards its quantification.

The chemical concentrations of interest may range from inorganic to organic chemicals and their densities may be less than or greater than that of water. The chemicals may be miscible or immiscible with respect to water. The interaction of these chemicals with the soil has made it necessary for the engineer to develop his understanding in the geochemistry area. To complicate matters further, many of the processes of concern occur in the upper portion of the soil profile; in the vadose zone where the pore-water pressures are negative. The groundwater table is no longer the upper boundary of concern. Rather, the ground surface geometry becomes the boundary for the problem being analyzed and movements through the unsaturated zone become the paramount interest.

At the same time as the geotechnical problems have become more complex, the capacity to handle these problems has also improved. The microcomputer has brought a wide range of data management and analytical skills to the desk of engineers. The computer programs used on a routine basis may range from spreadsheet software to powerful coupled, transient application software. The manner in which the application type software is being used in a consulting firm has also taken on new meaning. The software is most often used in a parametric or sensitivity manner in order to embrace possible field conditions and thereby give the geotechnical engineer an indication of the bounds on soil behaviour.

BRIEF HISTORY OF THE DEVELOPMENT OF UNSATURATED SOIL MECHANICS

The development of unsaturated soil mechanics has been relatively slow in comparison to saturated soil mechanics. It is interesting to note that there were a significant number of research papers published on unsaturated soil behaviour at the first International Conference on Soil Mechanics and Foundation Engineering, Harvard, in 1936. Obviously, problems related to unsaturated soils behaviour were considered to be an important part of engineering practice at that time. Most emphasis was placed on the flow of water in the unsaturated, capillary zone.

Subsequent research conferences show a diminishing interest in unsaturated soil behaviour. This was no doubt, influenced by the fact that it was easier and more fruitful to direct research efforts towards saturated soil behaviour. In addition, the empirical handling of many seepage problems as unconfined flow tended to eliminate the need for consideration of the zone where the pore-water pressures were negative (i.e., the vadose zone).

In the 1950's, research at Imperial College, London, was directed towards a fundamental understanding of unsaturated soil behaviour within the classical framework of saturated soils (Bishop, Alpan, Blight and Donald, 1960). The research involved careful laboratory studies, primarily on the shear strength of unsaturated soils. These studies appear to have provided the catalyst for further studies in various countries of the world.

With time, there was increasing awareness that many geotechnical problems could better be analyzed with an understanding of unsaturated soil behaviour (Fredlund and

Rahardjo, 1985 and 1987). As a result, specialty conferences were organized to exchange information on soils labelled as "problematic soils". The first such series of conferences was organized on swelling and shrinking soils and has become known as the expansive soils series of conferences. To-date, a total of 7 conferences have been held. Other "problematic soils" such as residual and collapsing soils, have also been the theme of international conferences. The behaviour of compacted soils has also been studied with the realization that it has not enjoyed the same favourable theoretical context as did saturated soils.

Common to all these "problematic soils" has been the state of stress in the pore-water phase. The pore-water pressures are negative and it is a change in the pore-water pressure that produces behaviour which has been difficult to predict. The difficulty in predicting the behaviour of these soils can, in part, be related to the manner in which the laboratory tests are conducted. It is often considered as acceptable practice to immerse the specimen in water prior to testing. This procedure dramatically alters the properties which need to be measured. In other words, our conventional laboratory equipment is not capable of running laboratory tests under controlled negative pore-water pressure conditions.

The basic theories associated with most of the classic areas of unsaturated soil behaviour were assembled in the 1970's and later (Fredlund, 1979). The primary deterrent to their application was the difficulty associated with measuring negative pore-water pressures insitu (i.e., matric suction) (Fredlund and Rahardjo, 1988). While there is still much research to be done on the measurement of matric suction, the theories of unsaturated soil behaviour are experiencing rapid acceptance. The questions which need to be answered with respect to most unsaturated soils are similar to those which have been addressed for saturated soils.

The greatest impetus for the application of saturated/unsaturated seepage modelling has come about as a result of developments in the geo-environmental area. Procedures for the characterization of the permeability function for unsaturated soils have been established and the use of saturated/unsaturated computer models has become a common practice.

The formulations associated with unsaturated soils are often nonlinear and as a result their solution is computationally intensive (Papagiannakis and Fredlund, 1984). In some applications this has not proven to be a deterrent due to the availability and capability of microcomputers.

NATURE AND ROLE OF THE VADOSE ZONE

The portion of the soil profile above the groundwater table is called the vadose zone (Fig. 1). This zone can be broadly subdivided into a portion immediately above the water table which remains saturated even though the pore-water pressures are negative, and a portion where the soil becomes unsaturated. The desaturation in the upper portion may be due to exceeding the air entry value of the intact soil or due to desaturation in the secondary structure (i.e., the fissures and cracks).

Regardless of the degree of saturation of the soil, the pore-water pressure profile will come to a hydrostatic condition when there is no flux from the ground surface. If moisture

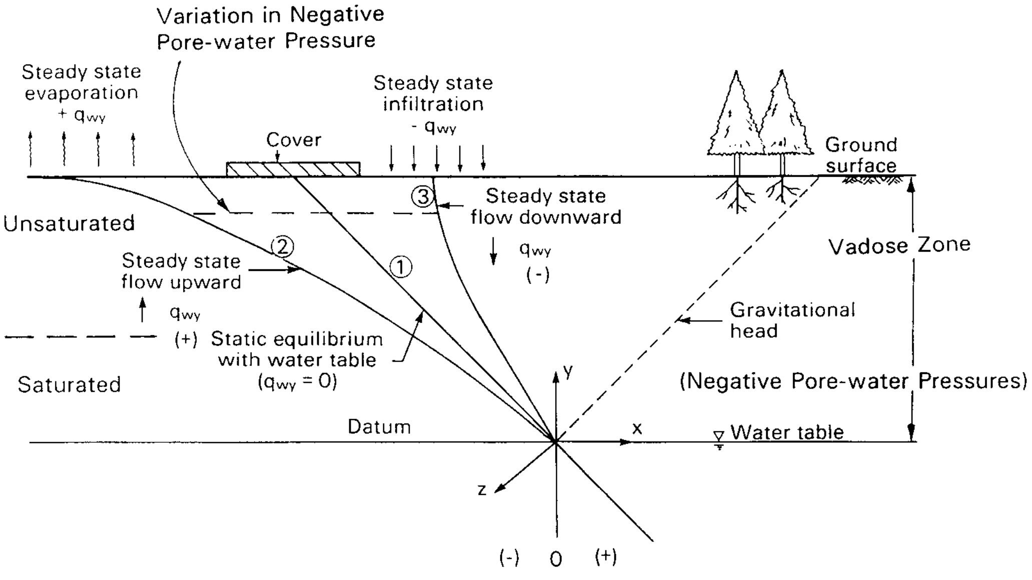

Fig. 1. Definition of negative pore-water profiles in the vadose zone

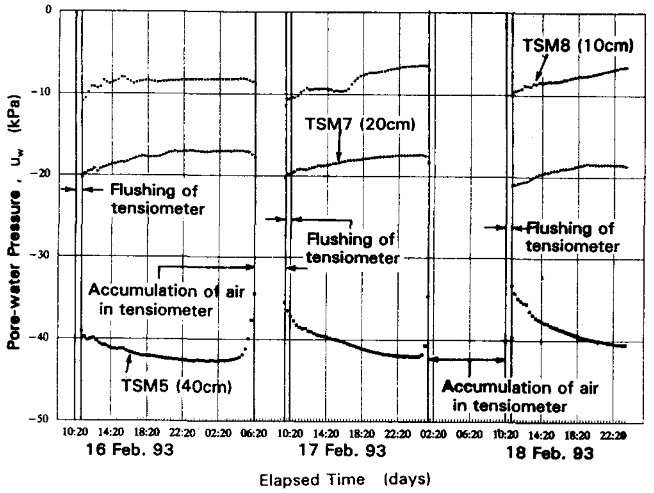

Fig. 2. Tensiometer readings at 3 depths on a slope in residual soil of the Jurong formation at Nanyang Technological University, Singapore.

is extracted from the ground surface (e.g., evaporation), the pore-water pressure profile will be drawn to the left. If moisture enters at the ground surface (e.g., infiltration), the pore-water pressure profile will be drawn to the right. Figure 2 shows the results of tensiometer readings near ground surface in a residual soil of the Jurong formation in Singapore. It can be observed that the pore-water pressures decrease with depth illustrating a net influx to the soil. These principles are basic but important to understanding the continuity which exists when going from positive to negative pore-water pressures.

The precipitation conditions at a site have often been recorded and are often available for design purposes. The evaporative flux must be computed through use of one of several models which have been suggested by various researchers. Some of the most approximate estimates for geotechnical problems have been associated with the assessment of evaporative flux. Only recently have engineers began to include the role of soil suction in computing the rate of evaporation from the ground surface (Fig. 3; Wilson, 1990). This has become a fruitful area of research which is of great value to engineers dealing with geo-environmental problems.

In order for surface water to reach the groundwater table, it must pass through the vadose zone. An understanding of the behaviour of the vadose zone becomes crucial in making long-term predictions on the movement of water and subsequently, the movement of contaminants. Much effort has gone into the numerical modelling of the vadose zone. This research has been done primarily by soil scientists while geotechnical engineers have paid little attention to this topic.

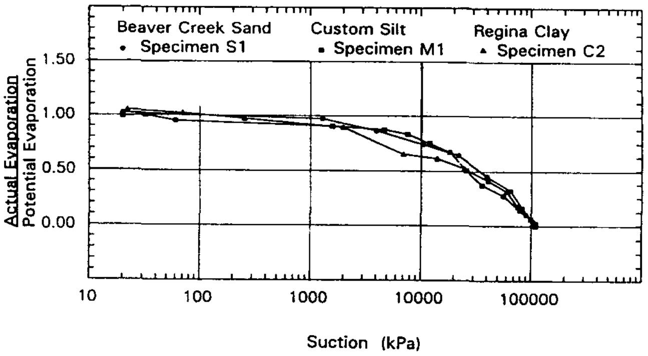

Fig. 3. Relationship between the ratio of actual evaporation to potential evaporation, and soil suction for three soils (from Wilson, 1990).

There are many complexities associated with the vadose zone because of its fissured and fractured nature. The tendency has been to avoid the consideration of this zone. However, in many cases it is an understanding of this zone which holds the key to the performance of an engineered structure.

One of the characteristics of the upper portion of the vadose zone is its ability to slowly release water vapour to the atmosphere at a rate dependent upon the permeability of the intact portions of soil. At the same time, the inflow of water can occur through the fissures under a gradient of unity. There appears to be no impedance to the inflow of water until the soil swells and becomes intact, or until the fissures and cracks are filled with water.

A common misconception is that the water can always easily get into the soil at the ground surface. If the soil is intact, the maximum flux of water at the ground surface is the coefficient of permeability of the soil. This value may be extremely low. If the ground surface is sloping, the surface layer can become saturated and have a higher coefficient of permeability than the underlying soil. As a result, water runs down the top layer of soil on the slope, and never enters the underlying soil.

Long-term predictions related to the closure of a mine, are strongly controlled by the assessment of the surface flux boundary conditions.

TYPICAL QUESTIONS ASKED OF THE GEOTECHNICAL ENGINEER

The types of questions asked of engineers pertaining to the behaviour of unsaturated soils, are similar to questions addressed in saturated soil mechanics. However, questions related to unsaturated soil behaviour have not been

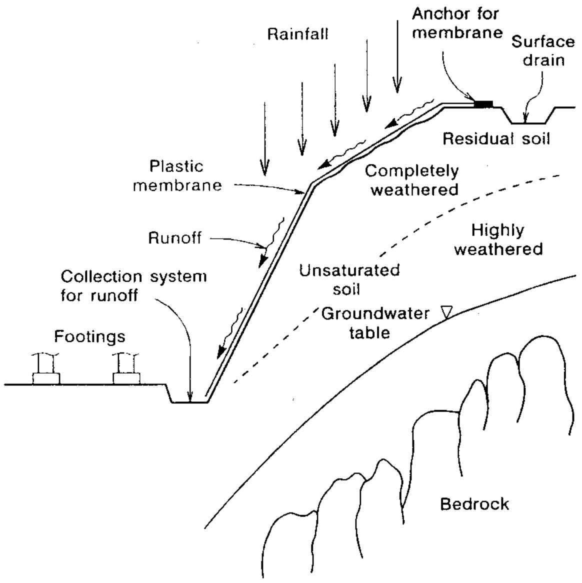

Fig. 4. Control of infiltration through the use of plastic membranes

answered with the same degree of effort. When the soils involved are unsaturated, the engineer has often used the unsaturated condition as an excuse for not getting involved or else not carrying out a significant investigation or analysis. An example is the case of temporary support which must be provided to an open trench or a temporary excavation for the installation of a foundation. In this case, the problem is often left to be solved by the contractor. And one should note that contractors are, in general, quite capable of solving problems in a wise manner. The engineer should be ready to learn from these contractors, and as well add insight by way of theory and analysis in order to come up with a superior, reliable design.

Soil investigation reports often contain statements to the effect that the excavation should be able to stand in an unsupported manner for a certain period of time. In effect, the engineer is "gambling" on there being a minimum amount of rainfall while the excavation is open. The only variable which appears to be changing with time is the negative pore-water pressure in the soil in the backslope. The pore-water pressures will remain essentially constant unless there is an influx of water at the ground surface.

It has been observed in Singapore that it is quite common practice for contractors to place a plastic membrane in the vicinity of a newly excavated slope (Fig. 4). The slope may be part of an excavation for a foundation or part of the site remediation (or landscaping). The plastic membrane ensures that a major portion of rainfall will be shed to the bottom of the slope. In other words, the use of the plastic membrane is an attempt to maintain the negative pore-water pressures in the backslope. The practice of using plastic membranes in this manner has potential usage in other parts of the world.

COMMON PROBLEMS WHERE SURFACE FLUX (AND UNSATURATED SOIL MECHANICS) CAN PLAY AN IMPORTANT ROLE

There are many problems routinely encountered in practice where the surface flux boundary condition has a strong influence on the performance of the structure. Listed below are a few examples.

(1) Stability of natural slopes: Natural slopes generally fail at some time following a higher and more prolonged rainfall than has occurred in the past.

(2) Design of shallow footings: Volume changes in the soil below shallow footings generally take place in response to a moisture flux around the perimeter of the structure. (Fig. 5)

(3) Compacted covers associated with waste management: Compacted clay covers have become a common solution associated with waste management

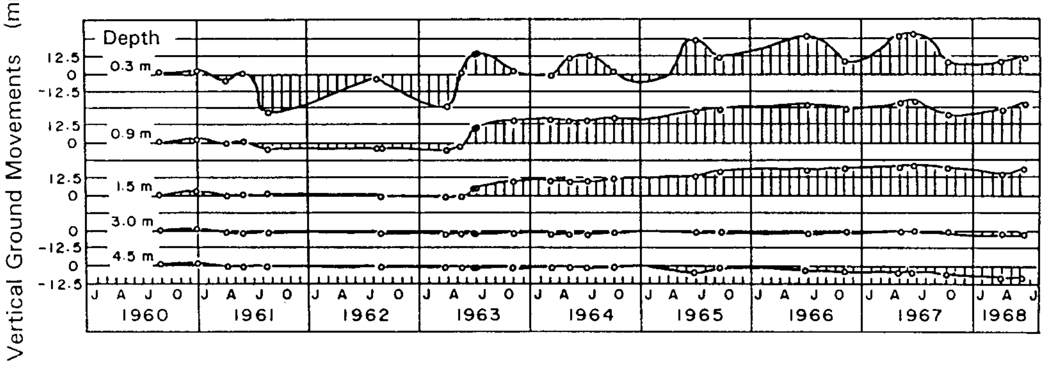

Fig. 5. Measured vertical ground movements in an expansive soil test plot at Regina, Saskatchewan (from Hamilton, 1968)

(Fig. 6). The performance of the cover is highly controlled by volume changes (and thereby permeability changes) associated with the cover in response to the surface flux. The movement of water and contaminants from the waste area is strongly influenced by the surface flux and the permeability characteristics of the cover material.

(4) Lateral earth pressures on retaining structures and walls: Engineers realize that it is preferred design practice to use cohesionless material as backfill for a retaining structure. However, many retaining structures are backfilled with cohesive materials which change volume in response to the intake of water (Fig. 7).

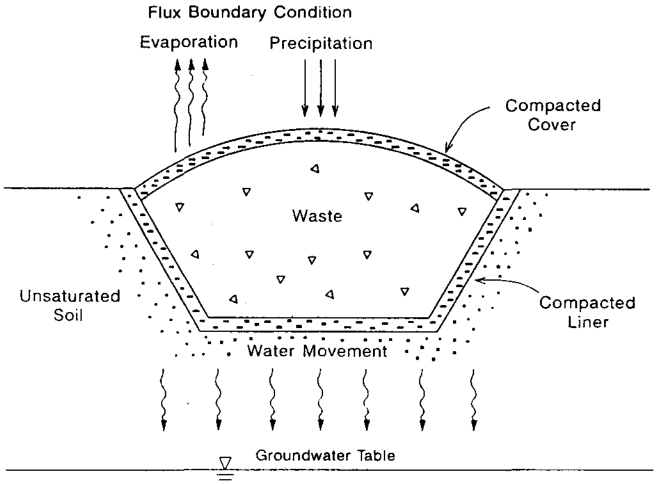

Fig. 6. An example of the movement of water through a cover as well as flow in the unsaturated zone below a liner.

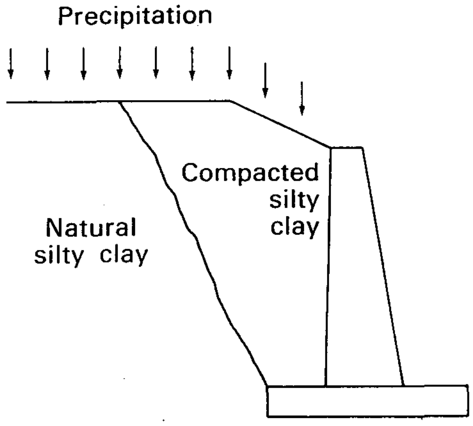

Fig. 7. Lateral earth pressures developed as a result of infiltration into a silty clay compacted against a retaining wall

NEW PROBLEMS WHERE UNSATURATED SOIL BEHAVIOUR PLAYS A DOMINANT ROLE

In recent years there have been a number of new construction techniques that have become common practice. Several of these techniques involve the use of soil in its unsaturated and/or compacted state.

Soil Nailing:
The design of a soil nailing retaining structure assumes that load will come on to the anchors as a result of the influx of water into the soil behind the cover on the slope. It is the change in negative pore-water pressures in response to a ground surface flux which initiates the volume change necessary in order to load the anchors.

Settlement of trench backfill subsequent to construction:
While the settlement of trench materials is by no means a recent phenomenon, it has been a subject of new investigations in recent years. The volume changes (i.e. generally volume decreases) of the backfill material are in response to the influx of water.

Geo-environmental Controls:
The movement of moisture in the entire region surrounding a waste containment area is closely related to surface flux conditions and unsaturated soil behaviour. The mounding of the groundwater table below a waste containment area occurs in response to flow through unsaturated soils (Fig. 8).

SOIL PROPERTIES OF INTEREST FOR UNSATURATED SOILS PROBLEMS

Before identifying relevant unsaturated soil properties, it is important to define the variables which can be used to represent the stress state of the soil. These are the variables which will allow engineering experience from various parts

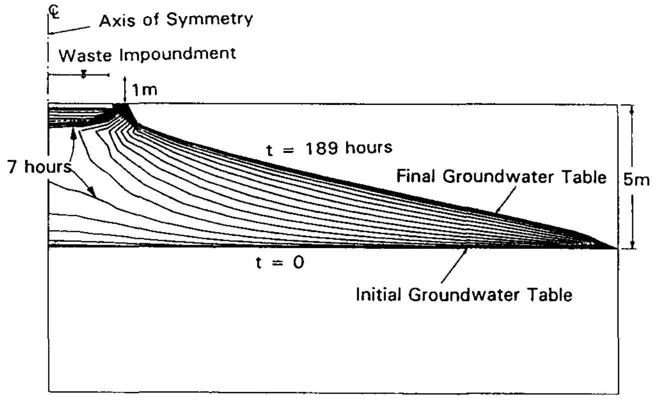

Fig. 8. Positions of the water table below a waste impoundment at various elapsed times.

of the world to be readily compared. It will be the basis on which a transferable science can be built. The stress state is the single element which establishes geotechnical engineering as a science, set apart from an empirical art. As a result, it becomes the basis or the foundation block on which engineers build their engineering formulations.

(1) **Stress State Variables**

There now appears to be a fairly general consensus that the stress state for an unsaturated soil should be described using two independent, normal stress variables (Fredlund and Morgenstern, 1977). These two variables become a logical extension of the effective stress variable used for a saturated soil and are shown in Eq. 1.

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix}$$

$$\text{and} \qquad [1]$$

$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix}$$

The simplest way to visualize the need for two independent stress state variables is to realize that total stress changes and pore-water pressure changes do not produce equivalent responses in an unsaturated soil. This sets their behaviour apart from that of saturated soils. Since the pore-air pressure is constant (i.e., atmospheric) for most practical geotechnical problems, it is logical to independently reference the total stress and the pore-water pressure to the pore-air pressure. All descriptions of soil behaviour (i.e., volume change and shear strength) can now be referenced to the two stress state variables. There is also a smooth transition from the unsaturated case to the saturated case in that the pore-air pressure becomes equal to the pore-water pressure as the soil becomes saturated.

Table 1 : Summary of Classic Saturated and Unsaturated Soil Mechanics Principles and Equations

Principle of equation	Saturated soil*	Unsaturated soil*
Stress state variables	$(\sigma - u_w)$	$(\sigma - u_a)$ and $(u_a - u_w)$
Shear strength	$\tau = c' + (\sigma - u_w) \tan \phi'$	$\tau = c' + (u_a - u_w) \tan \phi^b + (\sigma - u_a) \tan \phi'$ $c = c' + (u_a - u_w) \tan \phi^b$
Constitutive equations (Isotropic loading)	<u>Void ratio and water content</u> $de = G_s\, dw = a_v\, d(\sigma - u_w)$	<u>Void Ratio</u> $de = a_t\, d(\sigma - u_a) + a_m\, d(u_a - u_w)$ <u>Water Content</u> $dw = b_t\, d(\sigma - u_a) + b_m\, d(u_a - u_w)$
Flow law for water (Darcy's law) - Hydraulic head	$v_w = - k_s\, \partial h_w / \partial y$ $h_w = y + u_w/(\rho_w g)$	$v_w = - k_w(u_a - u_w)\, \partial h_w / \partial_y$ $h_w = y + u_w/(\rho_w g)$
Steady state seepage (Isotropic)	<u>One-dimensional:</u> $d^2h_w/dy^2 = 0$ <u>Two-dimensional:</u> $\partial^2 h_w/\partial x^2 + \partial^2 h_w/\partial y^2 = 0$	<u>One-dimensional:</u> $k_w\, d^2h_w/dy^2 + (dk_w/dy)\, dh_w/dy = 0$ <u>Two-dimensional:</u> $k_w\, \partial^2 h_w/\partial x^2 + (\partial k_w/\partial x)\partial h_w/\partial x +$ $k_w\, \partial^2 h_w/\partial y^2 + (\partial k_w/\partial y)\partial h_w/\partial y = 0$

* Variables are defined at the end of text under, "Definition of Soil Terms in Table 1".

(2) **Volume Change Moduli**

The independent stress state variables can be used to formulate the constitutive relations for an unsaturated soil (see Table 1). The volume change constitutive equation can be written in terms of a change in void ratio. However, more than one constitutive equation is required for the complete volume-mass characterization of an unsaturated soil since water content (or degree of saturation) change is independent of the void ratio change.

The water content constitutive equation has generally been used as the second constitutive relationship for an unsaturated soil. When the matric suction of the soil is zero, the change in void ratio and the change in water content are equivalent in terms of the response to a change in total stress. As a result, it is the water content versus matric suction relationship which becomes an important, additional relationship to quantify for an unsaturated soil. Equipment for the quantification of this relationship has been developed in the soil science discipline. The relationship is known as the "Soil Water Characteristic curve" (Fig. 9).

It is the soil water characteristic curve which becomes of great value in quantifying unsaturated soil behaviour. In general, it is possible to estimate most unsaturated soil properties from saturated soil parameters and the Soil Water Characteristic curve. The methods for approximating the unsaturated soil properties are presently an area of fruitful research. In most cases, it appears that approximate estimates of the unsaturated soil properties are satisfactory for geotechnical practice.

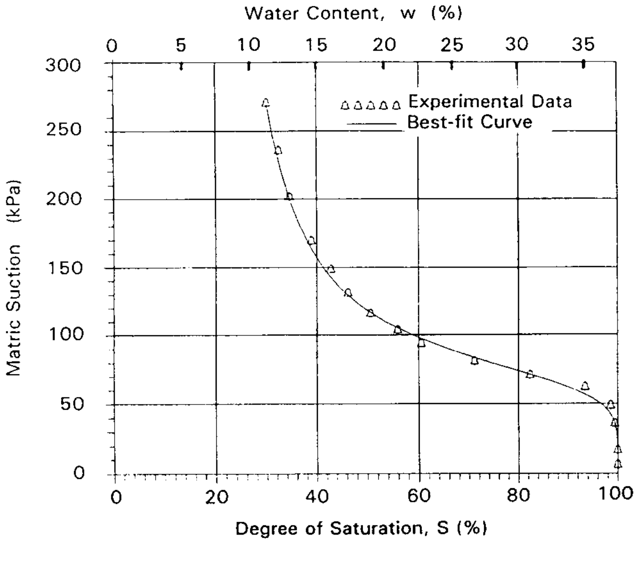

Fig. 9. A typical soil water characteristic curve relating matric suction to the amount of water in the soil

Several mathematical equations have been suggested to best-fit the water content versus matric suction experimental data. All equations have the form of a cummulative frequency distribution curve. It is suggested that one of these forms should be adopted for consistent use in geotechnical engineering.

(3) **Shear Strength Parameters:**

The shear strength equation for unsaturated soils has been formulated as a linear combination of the stress state variables incorporating shear strength parameters. As the testing of unsaturated soils has been extended over an ever increasing suction range, and for many soil types, there has become increasing evidence that the shear strength relationship involving suction can be nonlinear (Fredlund, Rahardjo and Gan, 1987). In general, it is possible to linearize the relationship over a selected suction range. At the same time, the Soil Water Characteristic curve can be used to approximate the shear strength versus suction relationship (Fig. 10).

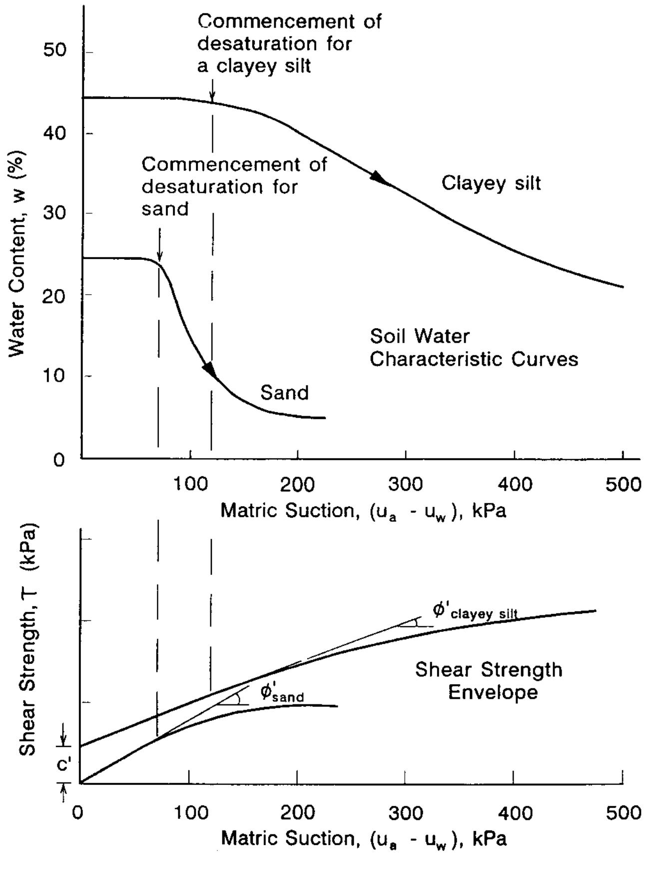

Fig. 10. Relationship between Soil Water Characteristic curve and shear strength for a sand and a clayey silt

Figure 10 illustrates how the ϕ^b angle begins to deviate from the effective angle of internal friction, ϕ', as the matric suction desaturates the soil. As the suction reaches a value corresponding to the residual water content, the ϕ^b angle appears to approach an angle of zero degrees (or it may even be negative).

(4) **Coefficient of Permeability:**

An unsaturated soil no longer has a constant coefficient of permeability and as a result, the engineer must characterize a permeability function for the soil. The function is dependent upon the stress state in the soil. In particular, the coefficient of permeability is a primary function of matric suction.

The permeability function can be empirically computed from a knowledge of the saturated coefficient of permeability and the Soil Water Characteristic curve (Fig. 11). The computations are based on the assumption that water can only flow through the water portion of the soil. Therefore, an integration along the Soil Water Characteristic curve provides a measure of the quantity of water in the soil. For most geotechnical problems, this form of permeability function characterization is satisfactory. Several other functions have also been proposed but at present, Gardner's equation (1958) appears to have quite wide acceptance (Fig. 12).

SIMPLIFICATION OF UNSATURATED SOIL MECHANICS FORMULATIONS

Problems involving unsaturated soils often have the appearance of being extremely complex. This problem is, in part, related to the fact that the engineer is not familiar with unsaturated soil analyses. However, the problem is also related to a lack of simplifications to the formulations. The concepts and theories need to be made as simple as possible in order for engineers to grasp the principles of their application. The formulation of key equations needs to be made simple while still retaining the effect of the primary variables involved.

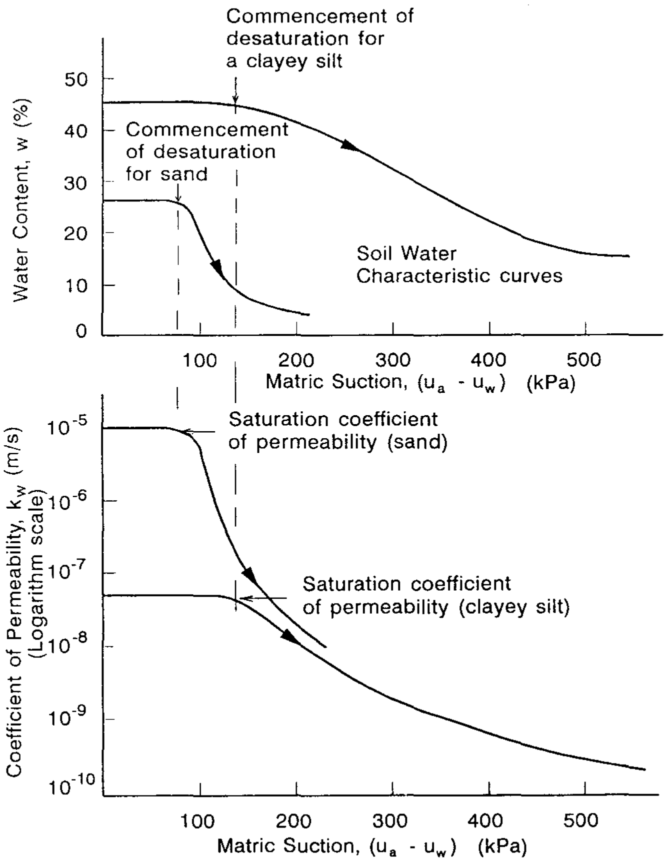

Fig. 11. Relationship between the Soil Water Characteristic curve and the coefficient of permeability for a sand and a clayey silt

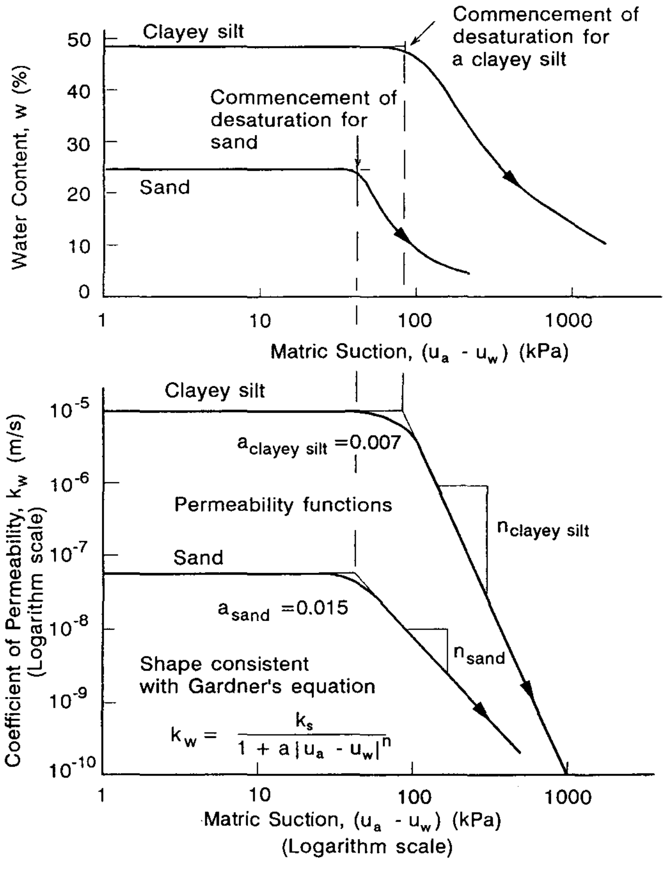

Fig. 12. Typical Gardner's empirical permeability functions shown for a sand and a clayey silt

An examination of the early formulations for saturated soils (and completely dry soils), reveals that early researchers made wise, simplifying assumptions that produced relatively simple formulations. This is readily observed from the classic lateral earth force, bearing capacity, and other formulations.

Our knowledge of unsaturated soil behaviour is presently at the stage where basic research has been completed and all fundamental constitutive relations are known. It has been possible to derive relatively complex formulations for various geotechnical problems. As an example, there are available formulations and calculation procedures for slope stability analyses and other problems involving unsaturated soils (Rahardjo and Fredlund, 1991). However, there is now a need to have simplified models and formulations to assist in geotechnical practice. It is with this need in mind that an example problem involving instability in a residual soils is addressed in the next section.

SIMPLIFIED FORMULATION FOR SLOPE INSTABILITY PROBLEMS IN RESIDUAL SOILS

The following example of a steep slope in residual soils is used to illustrate the type of approximate formulation and analysis which can be used for engineering practice (Fig. 13). The degree of weathering increases towards the ground surface where the effect of physical and chemical weathering is most dominant. Instability often occurs with a slip surface at the interface between the highly weathered and lesser weathered material. The slip surface is often approximately parallel to the ground surface. As a result, a semi-infinite slope analysis can be used to compute the factor of safety of the slope.

The example shown in Fig. 13 has a slope angle, α, and the groundwater table is located at a vertical distance, H, below the ground surface. The seepage of water above and below the groundwater table is assumed to be parallel to the ground surface. A slip surface located above the water table (i.e., in the unsaturated zone) is analysed. The free body diagram for one slice is shown in Fig. 13. The weight of the slice can be written as:

$$W = \gamma\,(H - y) \qquad [2]$$

where:

W = weight of one slice with a unit width,
γ = unit weight of the soil (i.e., ρg),
ρ = density of the soil,
g = gravitational acceleration,
y = height of the slip surface above the groundwater table.

Inter-slice forces do not play a role in a semi-infinite slope analysis and therefore the normal force, N, on the base of the slice can be written,

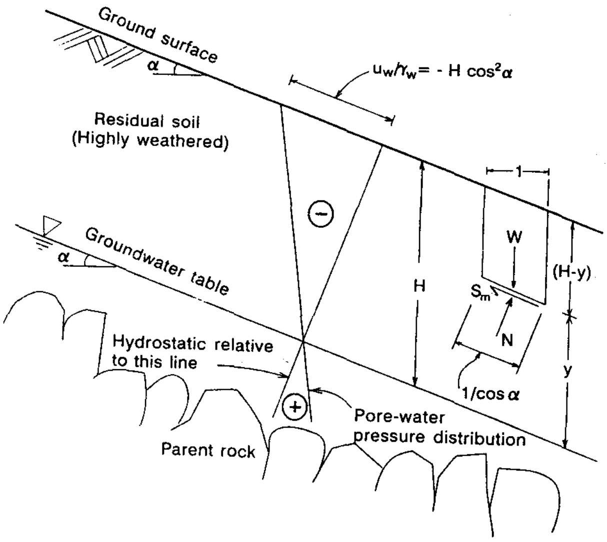

Fig. 13. Geometry and definition of variables associated with a seepage and stability analysis in a residual soil slope

$$N = W\cos\alpha = \gamma\,(H - y)\cos\alpha \qquad [3]$$

The normal stress, σ_n, on the base of the slice is,

$$\sigma_n = N/(1/\cos\alpha) = \gamma\,(H - y)\cos^2\alpha \qquad [4]$$

For hydrostatic conditions relative to a line perpendicular to the groundwater table, the pore-water pressure at the base of the slice, at height y is,

$$u_w = -\,(y/H)\,\gamma_w\,H\cos^2\alpha \qquad [5]$$

where:

γ_w = unit weight of water (i.e., $\rho_w g$)
ρ_w = density of water.

The shear force mobilized, S_m, at the base of the slice can be calculated using the shear strength equation for an unsaturated soil (see Table 1).

$$S_m = \frac{1}{F_s}\,[c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b\,]\,\frac{1}{\cos\alpha} \qquad [6]$$

where:

F_s = factor of safety

At the limiting equilibrium condition, the shear force mobilized must be equal to the actuating shear force.

$$S_m = W \sin\alpha. \qquad [7]$$

or,

$$\frac{1}{F_s}\left[c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b\right]\frac{1}{\cos\alpha}$$

$$= \gamma\,(H - y)\sin\alpha \qquad [8]$$

Rearranging Eq. 8 and assuming that the pore-air pressure is atmospheric (i.e., $u_a = 0$) give the factor of safety for the hydrostatic condition.

$$F_s = \left[\frac{\tan\phi'}{\tan\alpha}\right] + \left[\frac{c'}{\gamma\,(H - y)}\right]\frac{1}{\sin\alpha\cos\alpha} + \left[\frac{y}{H - y}\right]\left[\frac{\gamma_w}{\gamma}\right]\left[\frac{\tan\phi^b}{\tan\alpha}\right] \qquad [9]$$

Figure 14 presents the factors of safety for a slope of 45 degrees when computed in accordance with Eq. 9. The shear strength parameters are assumed to be uniform throughout the slope. This is done for simplicity and comparative purposes. Equation 9 shows that the factor of safety with respect to hydrostatic conditions decreases with depth below the ground surface. However, the slip surface could occur at any depth where the factor of safety decreases below 1.0. The critical depth at which failure occurs will depend upon the rate of increase in the cohesion (and possibly in the friction angle) of the soil with respect to depth below the ground surface.

Let us consider changes in the factor of safety as a result of water infiltration into the slope. As water moves into the slope, the upper portion of the pore-water pressure profile will be affected, down to a wetted depth, y_s. Figure 15 describes three possible pore-water pressure profiles resulting from infiltration and these are labelled as conditions (a), (b), and (c). Condition (a) considers the possibility of zero pore-water pressure at the surface, and a bilinear decrease in pore-water pressures with depth. Condition (b) considers the possibility of zero pore-water pressures throughout the wetted depth. Condition (c) considers the case where positive pore-water pressures are generated from the surface downward through the wetted zone.

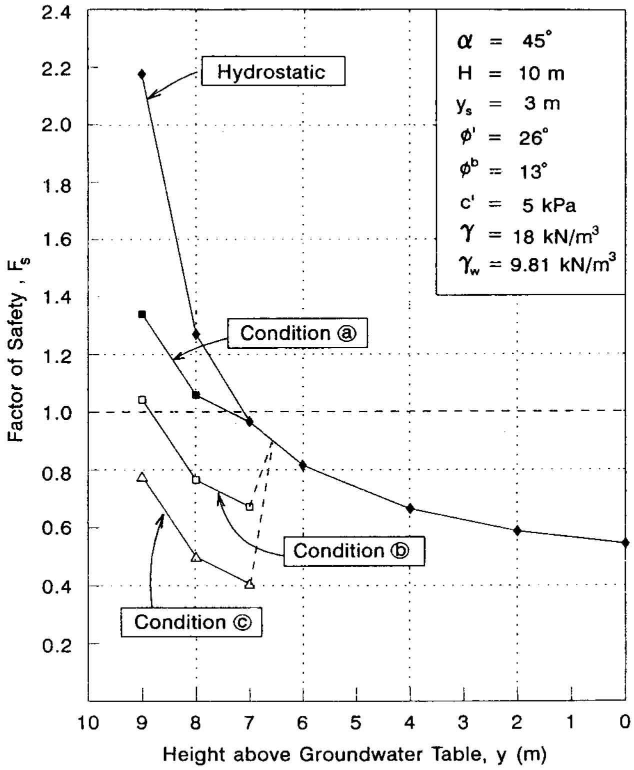

Fig. 14. Computed factors of safety for several pore-water pressure profiles above the groundwater table.

The variation in the pore-water pressure distribution within the wetted zone is the primary difference among the three conditions. The following equations give the pore-water pressure and the factor of safety for slip surfaces within the wetted zone. For condition (a) and depths $(H - y) \leq y_s$, the pore-water pressure can be written,

$$u_w = -\frac{y}{H}\gamma_w H\cos^2\alpha + \frac{y_s - (H - y)}{y_s}\gamma_w H\cos^2\alpha \qquad [10]$$

and the factor of safety as,

$$F_s = \left[\frac{\tan\phi'}{\tan\alpha}\right] + \left[\frac{c'}{\gamma\,(H - y)}\right]\frac{1}{\sin\alpha\cos\alpha} + \left[\frac{H}{y_s - 1}\right]\left[\frac{\gamma_w}{\gamma}\right]\left[\frac{\tan\phi^b}{\tan\alpha}\right] \qquad [11]$$

For condition b and $(H - y) \leq y_s$, the pore-water pressures are zero and the factor of safety can be written,

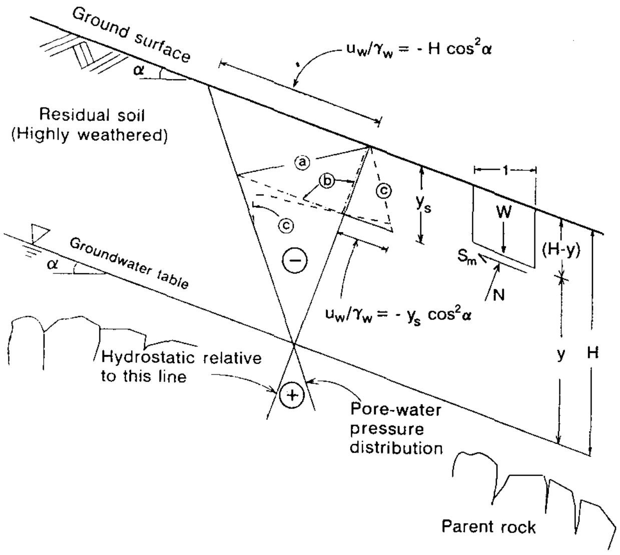

Fig. 15. Three possible pore-water pressure profiles in the zone above the groundwater table.

$$F_s = \left[\frac{\tan \phi'}{\tan \alpha}\right] + \left[\frac{c'}{\gamma\ (H - y)}\right] \frac{1}{\sin\alpha\ \cos\alpha} \qquad [12]$$

For condition (c) and (H - y) $\leq$ y_s, the pore-water pressures can be written,

$$u_w = - (H - y)\ \gamma_w \cos^2\alpha \qquad [13]$$

The factor of safety equation for condition (c) is,

$$F_s = \left[\frac{\tan \phi'}{\tan \alpha}\right] + \left[\frac{c'}{\gamma_{sat}\ (H - y)}\right] \frac{1}{\sin\alpha\ \cos\alpha} + \left[\frac{\gamma_w}{\gamma_{sat}}\right] \left[\frac{\tan \phi'}{\tan \alpha}\right] \qquad [14]$$

where:

γ_{sat} = saturated unit weight. In this example, γ_{sat} is assumed to be equal to γ (i.e., 18 kN/m^3)

The factors of safety corresponding to the three conditions are shown in Fig. 14. It appears that the critical factor of safety for failure within the wetted zone, y_s, decreases to values below 1.0 when moving from the hydrostatic condition to either condition (a), (b) or (c). In other words, the wetted zone becomes the critical zone where failure is likely to occur. In this case, the slip surface will be located at the depth corresponding to the minimum factor of safety. However, changes in the shear strength parameters of the soil may eventually control the location of the slip surface.

The above example illustrates the use of the semi-infinite slope stability analysis to compute the factor of safety in an unsaturated residual soil slope. A knowledge of the extent of water infiltration into the residual soil, and the pore-water profile corresponding to infiltration, is necessary. This process can be studied by simulating the infiltration process using the unsaturated / saturated flow formulation shown in Table 1.

PRIMARY NEEDS AND DIFFICULTIES

It is important that problems associated with unsaturated soil behaviour be put in a simple form. At the same time, it must be realized that there are still research needs and the difficulties associated with problems involving unsaturated soils. Some of these needs are as follows:

(1) Measurement of negative pore-water pressures (i.e., soil suction): The measurement of soil suction may take the form of being either direct or indirect. Both types of suction measurements need to be pursued.

(2) More complete information on the Soil Water Characteristic curves: It is of value to accumulate information on the Soil Water Characteristics curves for various soils. This laboratory test should become a standard test on unsaturated soils.

(3) Simplification of the formulations: There are needs for a series of simplified formulations for various problems involving unsaturated soils.

(4) Complete case histories: There is a need for a complete documentation of stresses, soil properties and behaviour associated with seepage, volume change and shear strength problems involving unsaturated soils.

SUMMARY STATEMENT

(1) Many of the problems that are considered as saturated soil mechanics problems can be analyzed in a superior manner if the engineer has an understanding of unsaturated soil behaviour.

(2) The soil properties and stresses associated with the vadose zone (i.e., unsaturated soil) can often be approximated and thereby provide the engineer with a clearer understanding of various design options.

(3) Many geo-environmental problems involve an interaction with the ground surface such as problems with a flux boundary condition. Precipitation and

evaporation fluxes at the ground surface, along with an understanding of flow through the vadose zone, become crucial in making long term geo-environmental predictions.

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DEFINITION OF SOIL TERMS IN TABLE 1

σ	total normal stress
u_a	pore-air pressure
u_w	pore-water pressure
$\sigma - u_a$	net normal stress
$u_a - u_w$	matric suction
τ	shear stress
c	total cohesion
c'	effective cohesion
ϕ'	angle of internal friction associated with the net normal stress state variable
ϕ^b	angle indicating the rate of change in shear strength relative to changes in matric suction
e	void ratio
G_s	specific gravity
w	gravimetric water content
a_v	coefficient of compressibility
a_t , a_m	coefficients of compressibility with respect to a change in net normal stress and matric suction, respectively
b_t , b_m	coefficients of water content change with respect to a change in net normal stress and matric suction, respectively
v_w	water flow rate
k_s	saturated coefficient of permeability
h_w	hydraulic head of the water phase
k_w	unsaturated coefficient of permeability which is a function of matric suction, $(u_a - u_w)$
ρ_w	water density
g	gravitational acceleration