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Stress Paths for Shear Strength Testing of Unsaturated Soils

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SYNOPSIS : A theoretical framework is proposed for the interpretation of laboratory shear strength tests on unsaturated soils. The framework involves the use of stress paths and is an extension of the system used to interpret saturated soil test results. Some unsaturated soil test results have been obtained for soils from Southeast Asia but there is a need for more test data, particularly on undisturbed and compacted residual soils.

INTRODUCTION

Many geotechnical engineering designs require an assessment of the shear strength of unsaturated soils. The measurement of shear strength of unsaturated soils has gained increasing attention during the past several decades. Triaxial and direct shear tests can be performed on unsaturated soils after modifying conventional testing equipment.

The various shear strength test procedures used for saturated soils can also be used for unsaturated soils. The primary difference lies in the interpretation of the test results. The results for unsaturated soils must be interpreted in the light of developments in unsaturated soil mechanics theories.

A research program on the behavior of unsaturated residual soils is underway at Nanyang Technological University, Singapore. The research involves the study on the shear strength, volume change and permeability characteristics of the residual soils. Triaxial and direct shear testing equipment has been developed and is being used in the program. The shear strength test procedures associated with unsaturated soils are described in this paper.

SHEAR STRENGTH EQUATION

The shear strength for an unsaturated soil is governed by two independent stress state variables; namely, net normal stress, $(\sigma - u_a)$, and matric suction, $(u_a - u_w)$; where σ = total stress, u_a = pore-air pressure and u_w = pore-water pressure.

The shear strength equation for an unsaturated soil can be written in terms of $(\sigma - u_a)$ and $(u_a - u_w)$ (Fredlund, Morgenstern and Widger, 1978) as follows:

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b \qquad (1)$$

where τ_{ff} = shear stress on the failure plane at failure; c' = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis when the net normal stress and the matric suction at failure are equal to zero; it is also referred to as the "effective cohesion"; $(\sigma_f - u_a)_f$ = net normal stress on the failure plane at failure; σ_{ff} = total normal stress on the failure plane at failure; u_{af} = pore-air pressure at failure; ϕ' = angle of internal friction associated with the net normal stress state variable, $(\sigma_f - u_a)_f$; $(u_a - u_w)_f$ = matric suction at failure; u_{wf} = pore-water pressure at failure; ϕ^b = angle indicating the rate of change in shear strength relative to a change in matric suction.

Equation (1) defines a planar surface, which is called the extended Mohr-Coulomb failure envelope (Figure 1). The envelope is tangent to the Mohr circles representing failure conditions.

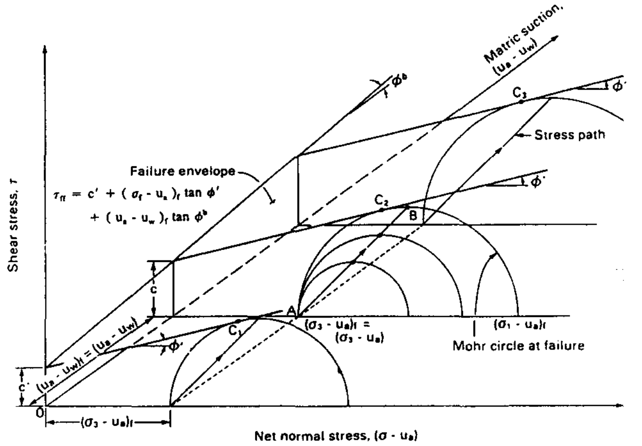

Fig.1 Failure envelope for an unsaturated soil and stress paths followed during consolidated drained tests at various matric suctions under a constant net confining pressure

TRIAXIAL TESTS ON UNSATURATED SOILS

Conventional triaxial tests can be performed on unsaturated soils by modifying triaxial test equipment as shown in Figure 2. The pore-air and pore-water pressures are controlled/measured through the use of a coarse porous disk and a high air entry disk, respectively. The high air entry disk serves as a separator between the pore-air and pore-water pressures.

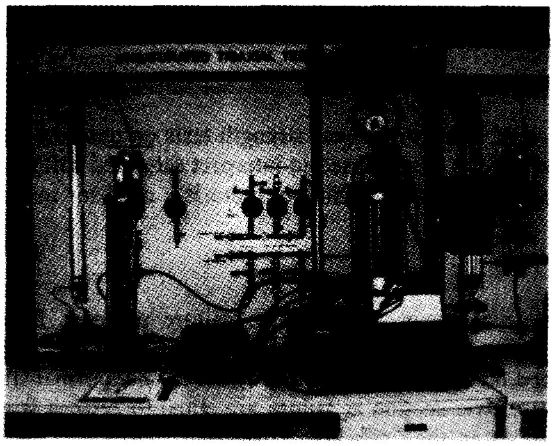

Fig.2 Modified triaxial test apparatus at the School of Civil and Structural Engineering, Nanyang Technological University, Singapore

Triaxial Test Procedures

The test procedures used for unsaturated soils soils can be classified in accordance with the drainage conditions adhered to during the first and second stages of the test. The triaxial test methods can be given a two-word designation or abbreviated to a two-letter symbol. The designations are: 1) consolidated drained or CD test, 2) constant water content or CW test, 3) consolidated undrained or CU test with pore pressure measurements, 4) undrained test, and 5) unconfined compression or UC test. In the case of the CD and CU tests, the first letter refers to the drainage condition prior to shear, while the second letter refers to the drainage condition during shear. The constant water content test is a special case where only the pore-air is kept in a drained mode while the pore-water phase is kept undrained during shear (i.e., constant water content). The pore-air and pore-water are not allowed to drain throughout the test for the undrained triaxial test. The unconfined compression test is a special loading condition of the undrained triaxial test.

The shear strength data obtained from triaxial tests can be analyzed using the stress state variables at failure or using the total stresses at failure when the pore pressures are not known. This concept is similar to the effective stress approach and the total stress approach used in saturated soil mechanics. In a drained test, the pore pressure is controlled at a desired value during shear. Any excess pore pressure caused by the applied load is dissipated by allowing the pore fluids to flow in or out of the soil specimen. In an undrained test, the excess pore pressure due to the applied load can build up because pore fluid flow is prevented during shear. If the changing pore pressures during shear are measured, the pore pressures at failure are known, and the stress state variables can be computed. However, if pore pressure measurements are not made during undrained shear, the stress state variables are unknown. In this case, the shear strength can only be related to the total stress at failure.

Consolidated Drained Test

The test conditions for a consolidated drained or CD test are illustrated in Figure 3. The soil is generally consolidated under an isotropic confining pressure of σ_3, while the pore-air and pore-water pressures are controlled at pressures of u_a and u_w, respectively. The pore-air and pore-water pressures can be controlled at positive values in order to establish a matric suction. This is referred to as the axis-translation technique. At the end of the consolidation process, the soil specimen has a net confining pressure of $(\sigma_3 - u_a)$ and a matric suction of $(u_a - u_w)$ (Figure 3).

During the shearing process, the soil specimen is compressed in the axial direction by applying a deviator stress [i.e., $(\sigma_1 - \sigma_3)$]. During shear, the drainage valves for both pore-air and pore-water remain open (i.e., under drained conditions). The deviator stress is applied slowly in order to prevent the development of excess pore-air or pore-water pressure in the soil. The net confining pressure, $(\sigma_3 - u_a)$, and the matric suction, $(u_a - u_w)$, remain constant throughout the test until failure conditions are reached. Only the deviator stress, $(\sigma_1 - \sigma_3)$, keeps increasing during shear until the net major principal stress reaches a value of $(\sigma_1 - u_a)_f$ at failure.

Typical stress-strain curves for the consolidated drained triaxial test are shown in Figure 4. The stress paths followed during consolidated drained tests under a constant net confining pressure and various matric suctions are shown in Figure 1. The Mohr circles at failure increase in diameter as the matric suction at failure is increased. The Mohr circle at failure is tangent to the failure envelope corresponding to the matric suction used in the test (e.g., at stress points, C_1, C_2, and C_3). However, stress points C_1, C_2 and C_3 do not occur at the same net normal stress. Therefore, a line joining stress points C_1,C_2 and C_3 will not give the angle, ϕ^b. Rather, it is suggested that the failure envelope be extended to intersect the shear strength versus $(u_a - u_w)$ plane to give cohesion intercepts. A line joining the cohesion intercepts at various matric suctions gives the angle, ϕ^b.

Constant Water Content Test

For the constant water content or CW triaxial test, the specimen is first consolidated and then sheared with the

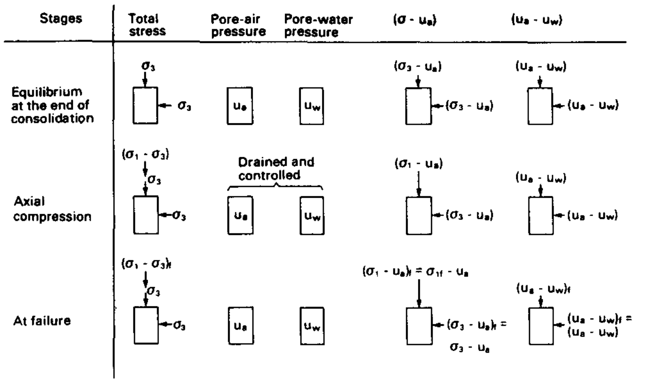

Fig.3 Stress conditions during a consolidated drained triaxial compression test

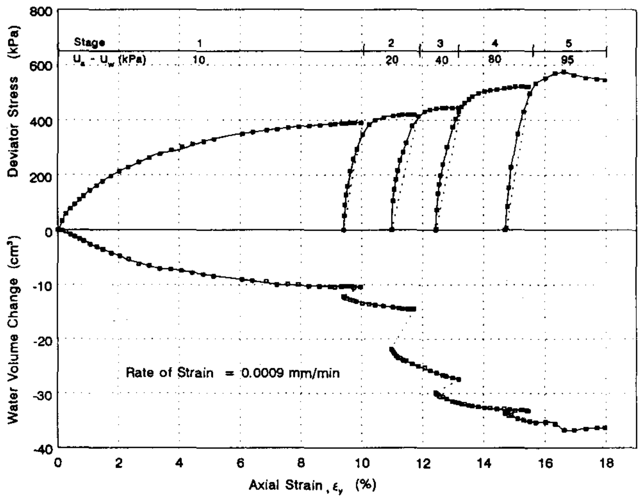

Fig.4 Stress-strain and water volume change-strain curves during a multi-stage, CD triaxial tests on a residual soil of the Jurong formation, Singapore

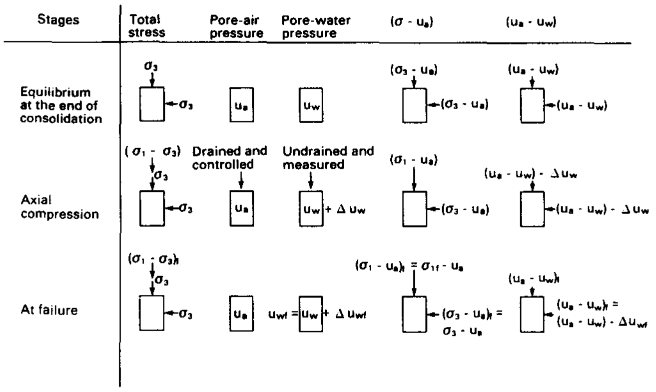

Fig.5 Stress conditions during a constant water content triaxial compression test

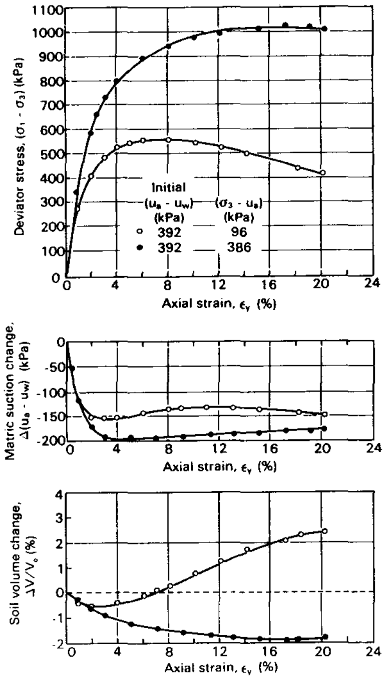

Fig.6 Constant water content triaxial tests on Dhanauri clay (from Satija, 1978)

pore-air phase allowed to drain while the pore-water phase is in an undrained mode. When equilibrium is reached at the end of consolidation, the soil specimen has a net confining pressure of (σ_3 - u_a) and a matric suction of (u_a - u_w). During shear, the drainage valve for the pore-air remains open, while the drainage valve for the pore-water is closed. The pore-air pressure, u_a, is maintained at the pressure applied during consolidation. The pore-water pressure, u_w, changes during shear under undrained loading conditions. The net confining pressure, (σ_3 - u_a), remains constant throughout the test while the matric suction, (u_a - u_w), changes as illustrated in Figure 5.

Typical stress versus strain curves for the constant water content test are shown in Figure 6. The matric suction and the soil volume changes during shear are plotted in Figure 6(b) and (c) respectively. For the results presented, the degree of saturation of the soil specimen increases as the pore voids are compressed as a result of the pore-air being squeezed out of the soil. The water content remains constant. Figure 6(c) indicates that the soil specimens undergo compression until the maximum deviator stress is reached. The soil specimen with the lower net confining pressure dilates after reaching the maximum deviator stress. This is accompanied by a slight increase in matric suction.

A hypothetical stress path that may be followed by a soil specimen during a constant water content test is shown in Figure 7. Stress point A represents the stress state at the end of consolidation. As the soil is compressed during shear, the stress point is assumed to move from point A to

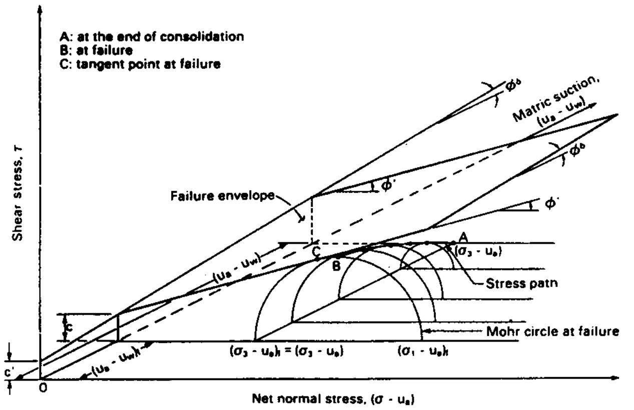

Fig.7 Stress path followed during a constant water content test

point B along stress path $\overline{AB}$. Stress point B represents the stress state at failure. The net confining pressure remains constant at (σ_3 - u_a) along stress path $\overline{AB}$ since the pore-air pressure is maintained at the pressure used for consolidation. The pore-water pressure is assumed to increase continuously during shear. A failure envelope sloping at an angle of ϕ' can be drawn tangent to the Mohr circle at failure. The failure envelope intersects the shear strength versus matric suction plane at a cohesion intercept, c. The cohesion intercepts obtained at various matric suctions can be joined to give the ϕ^b angle.

Consolidated Undrained Test with Pore Pressure Measurements

The consolidated undrained or CU test uses the test condition where the soil specimen is consolidated first and then sheared with both pore-air and pore-water under undrained conditions as shown in Figure 8. The drainage valves for both the pore-air and pore-water pressures are closed (i.e., undrained conditions) during shear. The pore-air and pore-water pressures should be measured during the shear process. At failure, the magnitudes of the net major and minor principal stresses and the matric suction are a function of the pore pressures.

A typical stress path for a consolidated undrained test is illustrated in Figure 9. The stress state at the end of consolidation is represented by stress point A where the net confining pressure is (σ_3 - u_a) and the matric suction is (u_a - u_w). Shear causes the stress state to move from point A to point B along stress path $\overline{AB}$. The stress state at failure is represented by stress point B. In the example shown, the pore-air pressure is assumed to increase continuously during shear. The matric suction is also assumed to decrease continuously. The failure envelope is tangent to the Mohr circle at failure (e.g., at stress point C) and inclined at an angle of ϕ' with respect to the (σ - u_a) axis. The failure envelope intersects the shear strength versus matric suction plane at a cohesion intercept, c. The intersection line joining the cohesion intercepts produced by tests at different matric suctions gives the angle, ϕ^b.

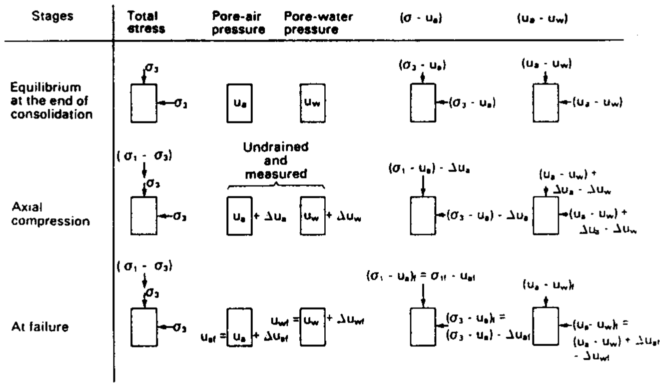

Fig.8 Stress conditions during a consolidated undrained triaxial compression test with pore pressure measurements.

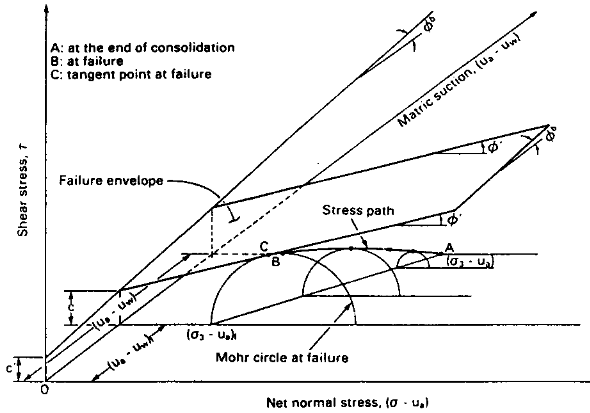

Fig.9 Typical stress path followed during a consolidated undrained test

It should be noted that it is difficult to maintain a fully undrained condition for the pore-air since air diffuses through the pore-water, the rubber membrane and other parts of the triaxial apparatus.

Undrained Test

The pore-air and pore-water are not allowed to drain in the undrained test. This applies both when the confining pressure and the deviator stress are applied to the soil specimen (Figure 10). The excess pore pressures which built up during the application of the confining pressure are not allowed to dissipate. The volume of the soil specimen may change due to compression of pore-air. Undrained loading during shear causes a further development of excess pore-air and pore-water pressures. Generally, the pore pressures are not measured during shear. Therefore, the undrained test results are commonly used in conjunction with a total stress formulation of a problem.

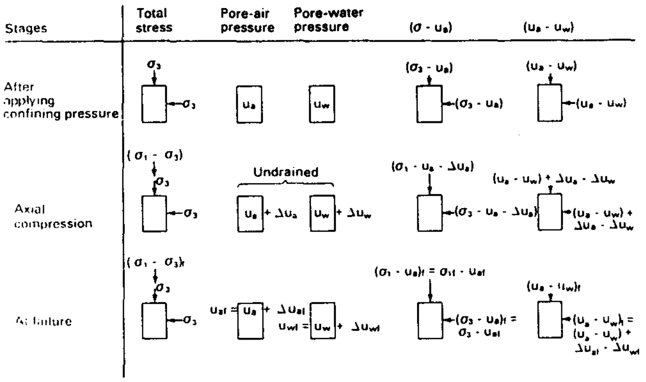

Fig.10 Stress conditions during an undrained triaxial compression test

A typical stress-strain curve for the undrained triaxial test is shown in Figure 11. The stress state variables during shear and at failure are unknown since the pore pressures are not measured. Figure 10 attempts to illustrate how the stress state variables would change, if the pore pressures were measured during an undrained triaxial test.

Hypothetical stress paths that may be followed by a soil specimen during an undrained test are illustrated in Figure 12. Consider four identical specimens that are initially confined at four different total confining pressures. The application of the total confining pressure under undrained conditions results in the compression of the pore fluids and the development of excess pore-air and pore-water pressures. The pore pressure increases in an unsaturated soil are always less than the total stress increment applied. This is in keeping with a B pore pressure parameter which must always be less than 1.0 for an unsaturated soil. Therefore, a higher total confining pressure results in a higher net confining pressure and a lower matric suction. In other words, the four identical soil specimens are brought to four different initial stress states (Figure 12). As the soil is sheared in undrained loading, the pore fluids are further compressed and the pore pressures may further increase. The stress point moves from point A to point B along the stress path $\overline{AB}$. The net confining pressure and the matric suction of the soil specimen decrease when going from stress point A to stress point B.

Figure 12 indicates an increase in the diameter of the Mohr circle at failure as the initial total confining pressure increases. This occurs because the rate of shear strength increase caused by an increase in confining pressure is greater than the reduction in shear strength caused by a decrease in matric suction. This can also be visualized as occurring because the ϕ^b angle is always lower than the friction angle, ϕ'.

The increase in shear strength due to the increase in the initial total confining pressure can also be demonstrated using a shear stress versus total normal stress plot (Figure 13). The Mohr circles at failure are drawn using the total confining pressure at failure and the total major principal stress at failure.

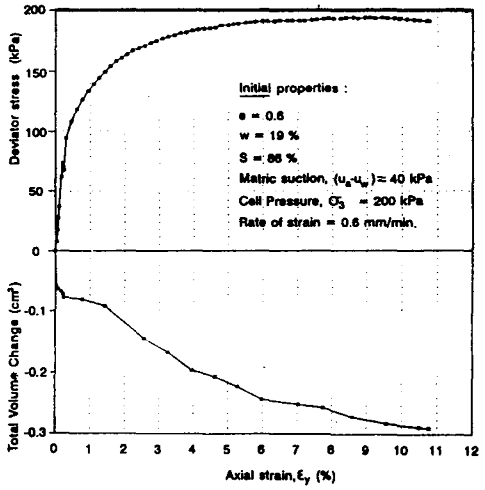

Fig.11 Stress-strain curve during an Undrained triaxial test on a residual soil of the Jurong formation, Singapore

A typical curved envelope can be drawn tangent to the Mohr circles of the unsaturated soil specimens at failure (Figure 13). The envelope defines the curved relationship between shear strength and total normal stress for unsaturated soils tested under undrained conditions. The curve indicates an increase in shear strength as the applied total stress increases. It may be practical in some cases to approximate the curved failure envelope with a straight line. Care should be taken not to exceed the limits for which the envelope is applicable. As the confining pressure increases, the matric suction decreases and the degree of saturation increases. Eventually, a point is reached where the soil approaches saturation. In the saturated condition, an increase in the confining pressure will be equally balanced by a pore-water pressure increase since the B_w pore pressure parameter will equal 1.0. In other words, the effective confining stress, (σ_3 - u_w), remains constant regardless of the total confining pressure, σ_3. A horizontal envelope with a slope of zero (i.e., $\phi = 0$; see Figure 13) can be drawn tangent to the Mohr circles at failure. However, the shear strength versus total normal stress relationship for an unsaturated soil does not produce a horizontal line but is a function of the applied total normal stress.

Unconfined Compression Test

The unconfined compression or UC test is a special case of the undrained test. No confining pressure is applied to the soil specimen throughout the test. Figure 14 illustrates typical changes in the stress state variables that would occur during an unconfined compression if the pore pressures were measured. Figure 15 illustrates two possible stress paths that may be followed in an unsaturated soil specimen during the unconfined compression. The initial stress state is represented by stress point A. During an undrained

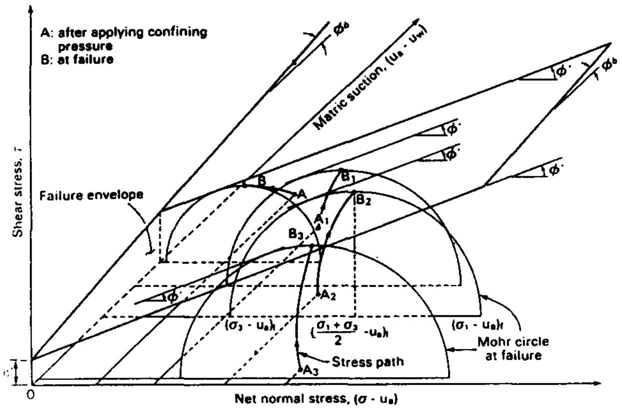

Fig.12 Stress paths followed during an undrained test

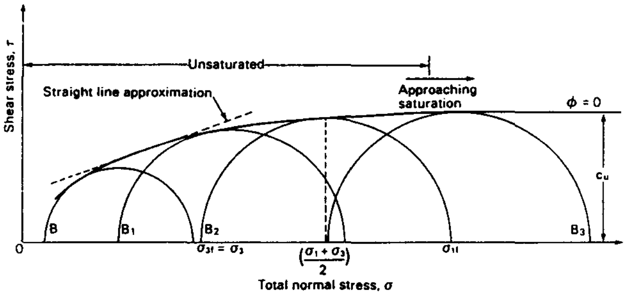

Fig.13 Shear stress versus total normal stress relationship for the undrained test

compression the matric suction can increase, decrease or remain constant depending upon the A pore pressure parameter of the soil. Generally, the matric suction will decrease during an undrained compression test and the stress state in the soil will move forward from point A to point B along the stress path $\overline{AB}$. The pore-air pressure is assumed to increase slightly during compression. This causes the net confining pressure to decrease along stress path $\overline{AB}$ to a negative value. The matric suction at failure will be less than the initial matric suction.

In the case of a constant matric suction during compression, the stress state of the soil could move from point A to point B_1 along the stress path $\overline{AB}_1$. In this case, the stress path lies in a plane of constant matric suction. The pore-air and pore-water pressures are assumed to remain constant during compression. As a result, the net confining pressure remains equal to zero until failure is reached.

For the case of an increasing matric suction during compression (e.g., the soil dilates), the stress state in the soil will move backward from point A to a point (or plane) somewhere behind point A.

Summary

There is a parallel between shear strength testing for saturated and unsaturated soils in that similar test procedures can be used in laboratory testing. One additional test procedure for unsaturated soils is that of the constant water content test. The testing procedures are: 1) Consolidated drained test, 2) Constant water content test, 3) Consolidated undrained test with pore-water and pore-air pressure measurements, 4) Undrained tests in the confined and unconfined modes.

All of the above test procedures have been reported in the research literature and it is now important that the interpretation of the results be done within a consistent theoretical framework. There is also need for more soil types (e.g., undisturbed and compacted residual soils) to be tested and interpreted within the proposed theoretical framework.

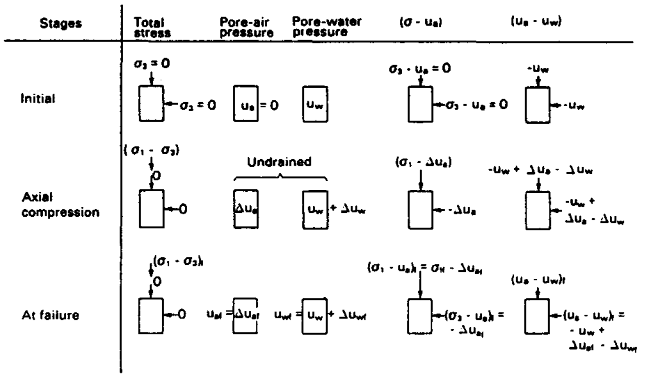

Fig.14 Stress conditions during an unconfined compression test

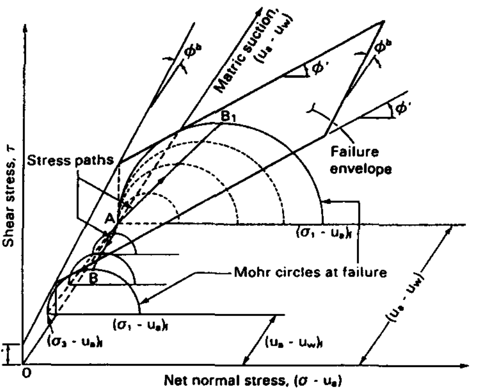

Fig.15 Possible stress paths followed during an unconfined compression test

REFERENCES

Fredlund, D.G., Morgenstern, N.R., and Widger, R.A. (1978). The Shear Strength of Unsaturated Soils, Canadian Geotechnical Journal, Vol. 15, No. 3, pp. 313 - 321.

Satija, B.S. (1978). Shear Behaviour of Partly Saturated Soils, Ph.D. dissertation, Indian Institute of Technology, Delhi, India, 327 pp.