Effect of pore-air and negative pore-water pressures on stability at the end-of-construction

D.G. Fredlund, H. Rahardjo, and T. Ng

Abstract: An analytical method for predicting the pore pressure change under undrained loading conditions was proposed by the United States Bureau of Reclamation around 1950 (i.e., the USBR method). The same analytical method was later proposed by Bishop (1957). This method was developed by computing pore-air pressure changes during undrained loading using Boyle's and Henry's laws along with the results from one-dimensional consolidation tests. It was then assumed that changes in the pore-air pressure were equal to changes in the pore-water pressure. Subsequently, in practice, the pore-air pressure plot versus total stress has often been assumed to predict the pore-water pressure in a compacted fill. Such an assumption can become quite conservative in terms of the computed factor of safety.

This paper presents an analytical procedure that serves as an extension to the USBR method. In the proposed procedure, the development of the pore-air and pore-water pressures during an undrained loading are not assumed to be equal. The development of pore-water pressure is predicted in an empirical manner, using the initial negative pore-water pressure, (i.e., the matric suction of the soil in an unloaded state), the theoretical saturation pressure and the pore-pressure parameters at saturation. In other words, both the pore-air and pore-water pressures are predicted as independent values for the construction process. The predicted pore-water pressures are consistent with measured results.

A typical example problem is given in the paper to illustrate the use of this method in analyzing the end-of-construction stability for an earth dam. The significance of considering, or omitting the pore-air pressure generation on the stability of the dam is also demonstrated. The difference between including and omitting the pore-air pressure is shown to be significant.

Analytical comparisons are made for various values of the friction angle with respect to matric suction, and for various combinations of pore-air and pore-water pressures at the end-of-construction. The extended method of pore pressure prediction can be used to more accurately compute the factor of safety of a compacted fill at the end-of-construction.

Key words: pore pressure, undrained loading, earthfill dam construction, matric suction, stability ananlysis, factor of safety, computer analysis.

Introduction

The construction of compacted earthfill dams and embankment results in changes in the pore pressures within the compacted soil. In a compacted soil, the pore-air pressure will be either atmospheric or will have a positive magnitude above atmospheric conditions while the pore-water pressure is generally negative with respect to the atmospheric pressure. Pore pressure changes during construction are commonly assumed to take place under undrained conditions during a relatively rapid construction period. As the embankment is constructed, the lower layers are compressed by the placement of the overlying layers. The pore-air and pore-water pressures change within each compacted layer during the period of construction.

The objective of this paper is to present an extended method to more closely simulate the pore-air and pore-water pressures generated in a compacted fill. The effect of various assumptions regarding the magnitude of the pore pressures is studied in terms of factor of safety.

Literature review

Hilf (1948) presented his analytical procedure for predicting the pore-air pressure developed in an unsaturated, compacted fill as a result of total stress changes under undrained conditions. The analysis as used by the United States Bureau of Reclamation and it became known as

D.G. Fredlund. Professor, Department of Civil Engineering, University of Saskatchewan, 57 Campus Drive, Saskatoon, SK, Canada S7N 5A9.
H. Rahardjo. Senior Lecturer, School of Civil and Structural Engineering, Nanyang Technological University, Singapore 2261.
T. Ng. Geotechnical Engineer, Manitoba Highways and Transportation, Government of Manitoba, Winnipeg, MB, Canada.

Reproduced with permission from the *Proceedings International Conference on Dam Engineering*, Johor Bahru, Malaysia, January 12–13, 1993.

the USBR method. The same analytical procedure was again proposed by Bishop (1957).

The pore pressure prediction was based upon Boyle's and Henry's gas laws, along with the results from a one-dimensional oedometer test. The relationship between the change in pore-air pressure and the volume change of the soil under undrained conditions was derived. The change in pore-air pressure was then assumed to be equal to the change in pore-water pressure. This assumption may be reasonable for some low plasticity soils. However, as the plasticity of the soil in the compacted fill increases, likewise the difference between the pore-air and pore-water pressures increases. At the time of compaction, the difference between the pore-air and pore-water pressures can be several atmospheres for a clayey soil.

Skempton (1954) and Bishop (1954) introduced the concept of A and B pore pressure parameters for saturated and unsaturated soils. Their derivations introduced the theory of elasticity, incorporating elastic parameters into the formulation of the pore pressure parameters. Hasan and Fredlund (1980) extended the pore pressure parameter derivations to include the constitutive relations for unsaturated soils (Fredlund and Morgenstern 1976). The A and B pore pressure parameters appear to have experienced limited usage on engineering practice for the prediction of pore pressures at the end-of-construction.

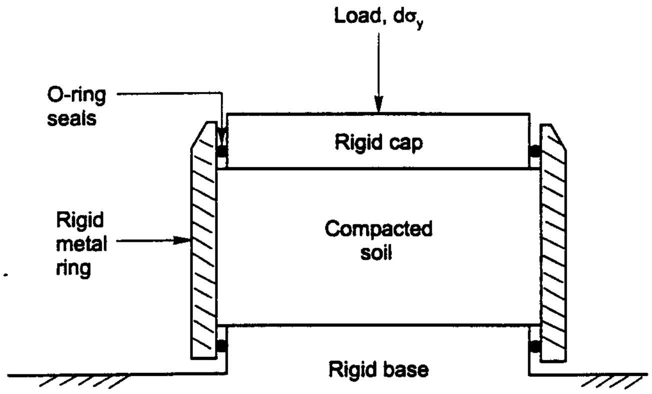
The main reason that predictions of pore pressures at the end-of-construction have not been used in practice, is that until recently there has not been a slope stability formulation which took both pore-air and pore-water pressures into account. The shear strength equation for a compacted soil, which took both pore-air and pore-water pressures into account was proposed by Fredlund et al. (1978). The equation was later applied to a limit equilibrium slope stability analysis (Fredlund 1984). However, research has not been carried out to study the effect of both pore-air and pore-water pressures on the computed factors of safety. The latter is one of the objectives of this study.

Theory related to the analysis by Hilf (1948)

Hilf (1948) simulated the undrained loading of a compacted soil using a one-dimensional loading of the soil with a frictionless piston sealed against a cylinder. Each applied load resulted in a volume change. The volume change was due to the compression of air and the dissolving of air in water (Fig. 1). With known initial and final volumes of air, the final pressure associated with the air phase is computable. The assumptions associated with using the analysis by Hilf (1948) are as follows:

1.) Only vertical strain takes place during loading (i.e., K_0-loading),
2.) the relationship between the effective stress and strain can be measured by performing a conventional, one-dimensional oedometer test on a compacted specimen immersed in water.

Fig. 1. Frictionless piston analogy of the compression of an unsaturated soil.



3.) the initial pore-water pressure can be assumed to be atmospheric (i.e., 101.3 kPa absolute),
4.) strain is due to the compression of air and the solution of air in water,
5.) there is no dissipation of pore-air (or pore-water) pressure with time,
6.) the effects of vapour pressure and temperature are negligible, and
7.) the change in pore-water pressure can be assumed to be equal to the computed change in pore-air pressure.

It is this last assumption which is more fully addressed in an extension to the theory.

The initial volume of free and dissolved air in a compacted soil can be written,

$$[1] \qquad V_{a0} = [(1 - S_0)n_0 + hS_0n_0]V_0$$

where:

V_{a0} = initial volume of free and dissolved air,
S_0 = initial degree of saturation,
n_0 = initial porosity,
V_0 = initial volume of the soil, and
h = Henry's coefficient of solubility by volume which is approximately 0.02 at typical ambient temperatures.

The volume of free air is equal to $(1 - S_0)n_0$ and the volume of dissolved air is equal to (h. The initial pore-air pressure, u_{ao}, is assumed to be atmospheric.

After a load is applied, the final volume of air, V_{af}, can be expressed as:

$$[2] \qquad V_{af} = [(1 - S_0)\,n_0 + hS_0n_0 - \Delta n]V_0$$

where:

V_{af} = final volume of free and dissolved air, and
Δn = change (i.e., decrease) in porosity.

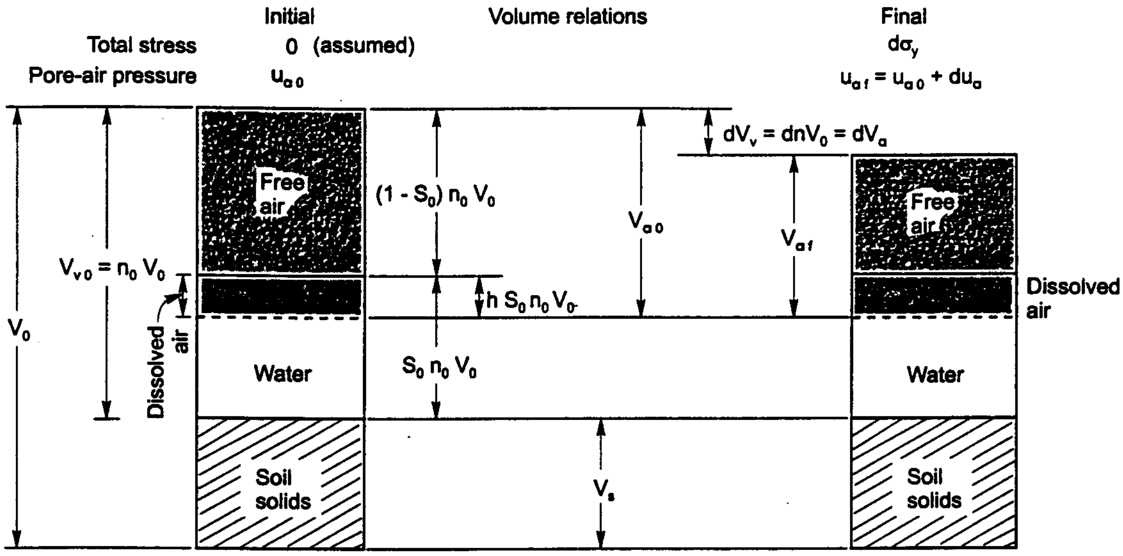
The initial and final volume conditions are shown in the phase diagram in Fig. 2. The final pore-air pressure, u_{af}, can be expressed:

$$[3] \qquad \bar{u}_{af} = \bar{u}_{a0} + \Delta u_a$$

where:

$\bar{u}_{a0}$ = initial pore-air pressure (absolute),

Fig. 2. Initial and final volume relations for before and after loading conditions of the unsaturated soil.



$\bar{u}_{af}$ = final pore-air pressure (absolute), and
Δu_a = change (i.e., increase) in absolute pore-air pressure.

Boyle's law can be applied to the sum of the free and dissolved air volumes.

[4] $\bar{u}_{a0}V_{a0} = \bar{u}_{af}V_{af}$

The initial and final stress and volume conditions can be substituted into eq. [4]:

[5] $\bar{u}_{a0}[(1 - S_0)n_0 + hS_0n_0]V_0 = (\bar{u}_{a0} + \Delta u_a)[(1 - S_0)n_0 + hS_0n_0 - \Delta n]V_0$

The change in pore-air pressure can be solved from eq. [5].

[6] $\Delta u_a = \{\Delta n / [(1 - S_0)n_0 + hS_0n_0 - \Delta n]\}\bar{u}_{a0}$

Equation 5 provides a relationship between the change in volume, Δn, and the change in pore-air pressure. At this point, it has generally been assumed that changes in the pore-air pressure are equal to changes in the pore-water pressure (i.e., $\Delta u_a = \Delta u_w$). The justification and effect of this assumption are later discussed.

The relationship between effective stress and volume change is obtained from the results of a one-dimensional oedometer test. Usually the results are plotted as void ratio versus the logarithms of effective stress. However, for the problem at hand, the void ratio can be converted to a change in volume (i.e., $\Delta n = \Delta e / (1 + e_0)$) and the effective stress is plotted on an arithmetic scale.

The change in volume can he plotted versus effective stress and pore pressure (Figure 3). The principle of effective stress (i.e., $\sigma = \sigma' + u$) is now used to compute the total stress versus the change in volume.

It is possible to plot the total stress and pore-water pressure as shown in Fig. 4. The slope of the resulting curve is quite flat at low total stresses and gradually increases to 45 degrees at saturation. At this point, changes in the total stress are equal to changes in the pore-water pressure. The above analysis appears to be consistent with experimental results associated with the pore-water

Fig. 3. Stress components versus volume change.

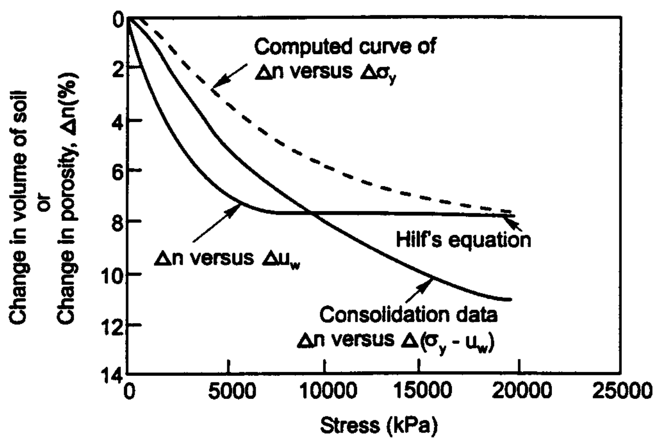


Fig. 4. Non-linear pore pressure versus total stress relationship.

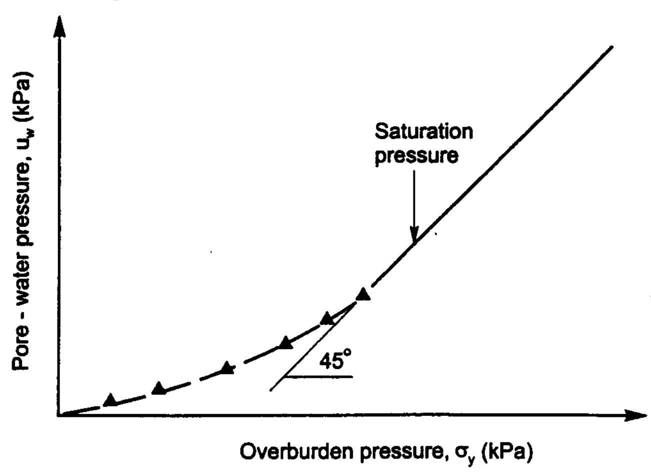
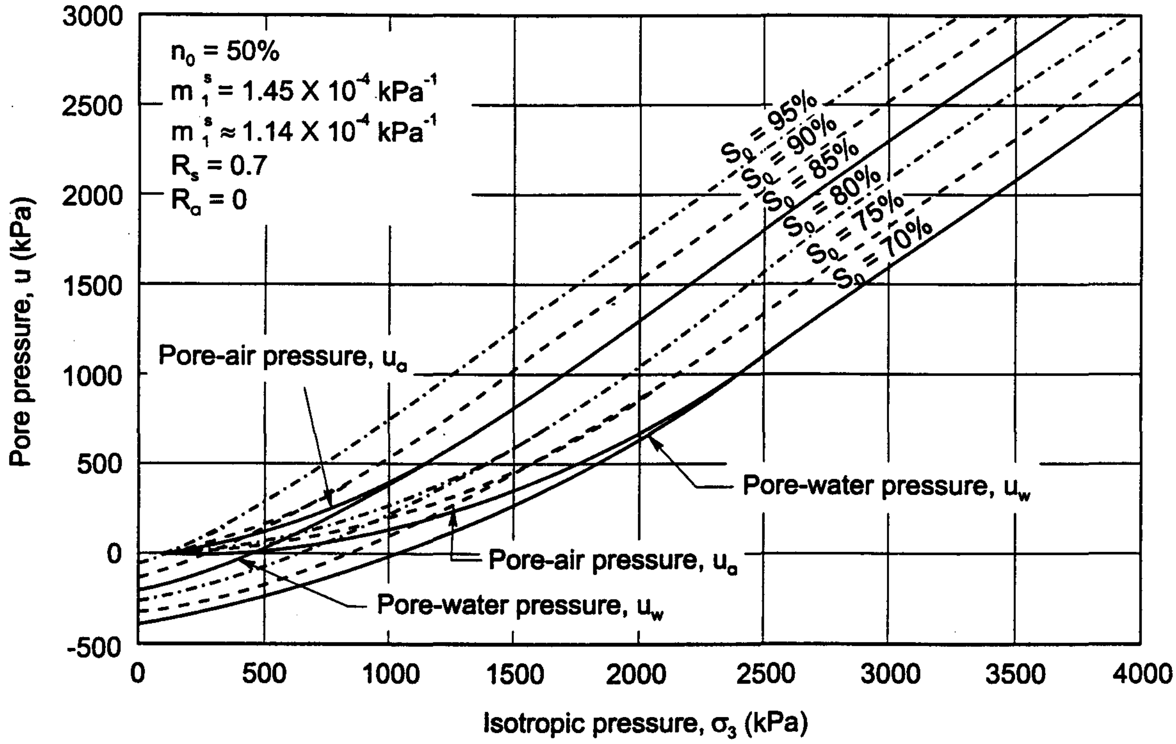


Fig. 5. Shape and character of pore pressure versus total stress.



phase as long as the soil is of low plasticity and has a small initial matric suction (Hasan and Fredlund 1980). The authors feel that more correctly, the analysis by Hilf (1948) applies to the pore-air phase as long as conditions are undrained.

There is a limiting condition to which the equation by Hilf (1948) can be applied. When the soil becomes saturated, the total change in volume will be equal to the initial volume of free air.

[7] $\Delta n = (1 - S_0)n_0$

Beyond this point, no further volume change is possible. Substituting the volume of free air into Eq. [6] gives the pore-air pressure change required for saturation.

[8] $\Delta u_{as} = (1 - S_0)\bar{u}_{a0} / (S_0 h)$

where:

Δu_{as} = pore-air pressure change (i.e., increase) required for saturation.

The relationship between the equation by Hilf (1948) and the pore pressure parameter (i.e., $B_{ah}' = \Delta u_a / \Delta\sigma)_y$ is as follows.

[9] $B_{ah}' = 1 / [1 + (1 - S_0 + hS_0)n_0 / (\bar{u}_{a0} + \Delta u_a)m_v]$

where:

$\Delta\sigma_y$ = change in vertical stress,
m_v = coefficient of volume change, and
B_{ah}' = secant pore-air pressure parameter (i.e., $\Delta u_a / \Delta\sigma_y$) for K_0–undrained loading in accordance with the analysis by Hilf (1948).

Equation [9] applies as long as a single value can be ascribed to the coefficient of volume change.

Extensions to the analysis by Hilf (1948)

Some extensions can be made to the above analysis in order to independently predict the pore-air and pore-water pressure under undrained loading conditions. The analysis by Hilf (1948) is now used to provide the relationship between total stress and pore-air pressure. The shape and character of the pore-water pressure versus total stress relationship have been studied and can be empirically predicted. Typical shape and character of pore pressure and total stress relationships are shown in Figure 5.

The following assumptions can be made regarding the pore-water pressure versus total stress curve:

1.) the pore-water pressure curve is parabolic in shape,
2.) when the soil approaches saturation, changes in total stress, pore-air pressure and pore-water pressure become equal. This means that at saturation the slope of the total stress versus pore pressure becomes 1.0.

If the pore-water pressure versus total stress curve is parabolic, it will adhere to the following general form:

[10] $u_w = A\sigma^2 + B\sigma + C$

where:

A, B, C = constants to be evaluated by considering appropriate boundary conditions

When the total stress is zero, the constant C, is equal to the initial pore-water pressure (or the matric suction when u_a = 0; where u_a = gauge air pressure). This is equivalent to the matric suction of the soil measured in an unconfined condition on a Pressure Plate apparatus.

[11] $C = (u_a - u_w)_0$

When the soil approaches saturation, the applied total stress can be written as σ_3, and the pore-water pressure can be written as u_{ws}. At saturation u_{as} is equal to u_{ws}, and the pore pressure can be designated as u_s. For the point of saturation, eq. [10] can be written:

[12] $u_s = A\sigma_s^2 + B\sigma_3 + (u_a - u_w)_0$

The slope of the pore-water pressure versus total stress curve at saturation is equal to 1 (i.e., $du_w / d\sigma = 1$).

[13] $du_s / d\sigma = 2A\sigma_s + B = 1$

or,

[14] $B = 1 - 2A\sigma_s$

The A constant can be computed after substituting the B constant into eq. [12].

[15] $A = [\sigma_s - u_s + (u_a - u_w)_0] / \sigma_s^2$

Equation [14] can now be used to calculate the B constant:

[16] $B = 1 - 2[(\sigma_s - u_s + (u_a - u_w)_0] / \sigma_s$

The expressions for A, B, C can be substituted into eq. [10] to give a general expression for the pore-water pressure curve.

[17] $u_w = \{[\sigma_s - u_s + (u_a - u_w)_0] / \sigma_s^2\}\sigma^2 + \{1 - 2[(\sigma_s - u_s + (u_a - u_w)_0) / \sigma_s]\}\sigma + (u_a - u_w)_0$

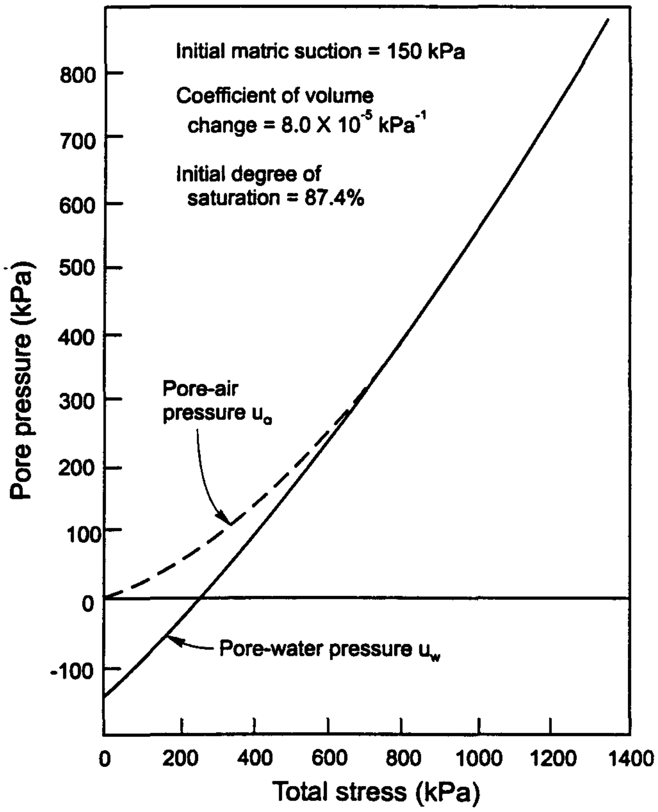
The saturation pore-water pressure, u_s, is computed using eq. [8] (i.e., $u_s = \Delta u_{as} + \bar{u}_{a0}$). The total stress associated with saturation, σ_s, can be obtained when the equation by Hilf (1948) is combined with the results of an oedometer test (Fig. 3).

The relationship between total stress and pore-water pressure can be plotted along with the relationship between total stress and pore-air pressure. The end result is an independent, empirical pore-water pressure curve (Fig. 5) which is consistent in character and magnitude to measured results.

Description of the example problem of a dam

An earthfill dam, 30 meters high and 130 meters wide, with 1:2 side-slopes on the upstream and downstream faces, is used for an example problem. The dam consists of one soil type with a total unit weight of 20.5 kN/m^3. The effective angle of internal friction, ϕ', is assumed to be 30 degrees and the effective cohesion, c', is 20 kPa. The initial degree of saturation of the compacted soil, S, is 87.4% and the initial matric suction, $(u_a - u_w)_0$, is 150 kPa. The coefficient of volume change, m_v, is 8.0×10^{-5} 1/kPa. These properties are used to compute the total stress versus pore-air and pore-water pressures (Fig. 6).

Fig. 6. Relationships between pore-air pressure, pore-water pressure and total stress used for the example problem.



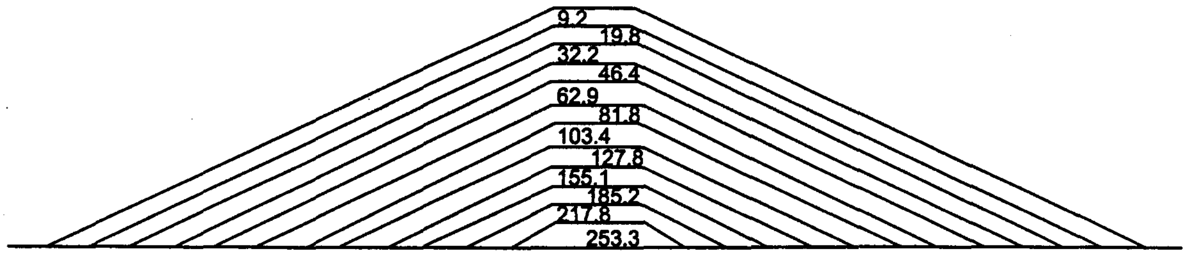
The earthfill dam is divided into 12 layers, with each layer being 2.5 m thick, for the end-of-construction slope stability analyses. The pore-water pressures are input as a series of constant pressure contours while the pore-air pressures are input as independent variables for each soil layer[1] (Fig. 7). Different angles of friction with respect to matric suction, ϕ^b, were selected within the range from 0 degrees to 30 degrees.

Assumptions concerning the pore-air and pore-water pressures

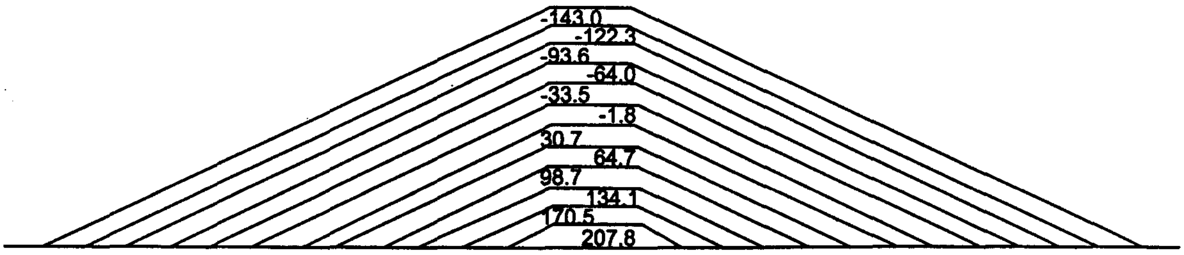
Two separate assumptions were used concerning the pore-air pressures in the analyses. The first assumption was that the pore-air pressures were equal to the values determined using the analysis by Hilf (1948). The second assumption was that the pore-air pressures dissipated and were equal to zero. In both cases, the pore-water pressures were assumed to be equal to those predicted from eq. [17].

[1] The slope stability analyses were performed using PC-SLOPE from Geo-Slope International Ltd., Calgary, Alberta, Canada. The software is able to handle the independent input of pore-air and pore-water pressures.

Fig. 7. Pore-air and pore-water pressures of the dam for the end-of-construction case.



a.) Pore-air pressures at various depths in the dam (kPa)



b.) Pore-water pressures at various depths in the dam (kPa)

These assumptions, together with various ϕ^b values, affect the shear strength equation for unsaturated soils and as a result, affect the computed factors of safety.

Effect of pore pressure assumptions on shear strength

The general shear strength equation for an unsaturated soil can be written as follows (Fredlund et al. 1978):

[18] $\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$

There are several specializations of eq. [18] for various pore-air pressure assumptions and ϕ^b conditions. When the pore-air pressures are equal to values computed using the equation by Hilf (1948), and:

1.) the ϕ^b value is zero, eq. [18] becomes,

[19] $\tau = c' + (\sigma_n - u_a)\tan\phi'$

2.) the ϕ^b angle has a value between zero and ϕ', eq. [18] can be written,

[20] $\tau = c' + \sigma_n \tan\phi' - u_a(\tan\phi' - \tan\phi^b) - u_w \tan\phi^b$

3.) the ϕ^b angle is equal to ϕ', and eq. [18] becomes the same as that used for saturated soils.

[21] $\tau = c' + (\sigma_n - u_w)\tan\phi'$

Let us now assume that the pore-air pressures dissipate rapidly while the pore-water pressure remains the same. When the pore-air pressures are set equal to zero, and,

1.) the ϕ^b value is zero, eq. [18] becomes,

[22] $\tau = c' + \sigma_n \tan\phi'$

2.) the ϕ^b angle, has a value between zero and ϕ', eq. [18] can be written,

[23] $\tau = c' + \sigma_n \tan\phi' - u_w \tan\phi^b$

3.) the ϕ^b angle is equal to ϕ', eq. [18] becomes the same as that used for a saturated soil.

[24] $\tau = c' + (\sigma_n - u)\tan\phi'$

These equations affect the mobilized shear strength computed for the factor of safety calculations.

Results of the stability analyses

The pore-air pressure values computed applying the analysis by Hilf (1948), ranged from 9 kPa at the top layer to 253 kPa near the bottom centre of the dam. The generated pore-water pressures ranged from −143 kPa at the top of the dam to +208 kPa at the bottom centre of the dam.

The computed factors of safety for the above two assumptions regarding the pore-air pressure, are summarized in Table 1. These results are plotted in Fig. 8. The results indicate that with the pore-air pressure introduced into the analysis and the ϕ^b values equal to zero, the factor of safety drops from 1.70 to 1.27 (i.e., 25%). Once the angle of friction with respect to matric suction is increased, the factors of safety increase for both assumptions regarding the pore-air pressure.

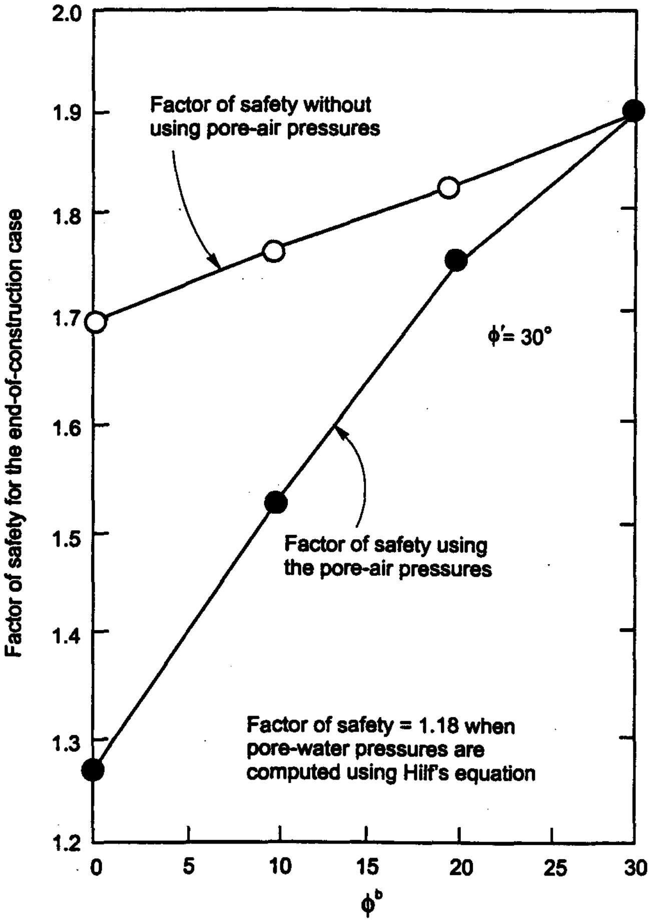
When the angle of friction with respect to matric suction is equal to ϕ', the pore-air pressure no longer affects the shear strength. The computed factor of safety is 1.90. For the case of no pore-air pressures this represents an increase of 12%. For the case with pore-air pressures equal to those computed applying the analysis of Hilf (1948), the factor of safety is increased almost by 50%. These changes in factor of safety are substantial. Ignoring the

Table 1. Results for the end-of-construction case.

ϕ^b	Shear strength equation	Factor of safety
(a) The pore-air pressure values determined applying the analysis of Hilf (1948)	$\tau = c' + (\sigma_n - u) \tan \phi'$	1.90
(b) The pore-air pressure values equal to zero		
0	$\tau = c' + \sigma_n \tan \phi'$	1.70
10	$\tau = c' + \sigma_n \tan \phi' - u_w \tan \phi^b$	1.76
20	$\tau = c' + \sigma_n \tan \phi' - u_w \tan \phi^b$	1.83
30	$\tau = c' + (\sigma_n - u) \tan \phi'$	1.90

Note: The pore-water pressures are the same for both cases.

Fig. 8. Factor of safety with respect to ϕ^b for the end-of-construction case.



influence of pore-air pressures could lead to a too low estimate of the factor of safety.

At first observation, it might appear that the computed factors of safety are the opposite to what one's intuition might suggest. For example, it could be reasoned that if the pore-air pressure is zero, then matric suction would be less and the factor of safety would be less. However, the opposite is true.

In reality, the pore-air pressure influences both the ϕ' and ϕ^b components of shear strength. However, when the ϕ^b value is equal to zero, the pore-air pressure has a significant influence on the factor of safety because it operates on the ϕ' term alone.

The difficulty in understanding the computed results appears to be related to the fact that the engineer usually visualizes shear strength either in terms of the effect of normal stress, σ_n, or in terms of the effect of matric suction, $(u_a - u_w)$. Since the pore-air pressure is involved, it is necessary to visualize shear strength behavior in terms of both $(\sigma_n - u_a)$ and $(u_a - u_w)$. The $(\sigma_n - u_a)$ term operates on $\tan \phi'$. The $(u_a - u_w)$ term operates on $\tan \phi^b$. When the ϕ^b value is equal to zero, the effect of matric suction disappears but the effect of $(\sigma_n - u_a)$ does not disappear.

Another assumptions is made regarding pore pressures at the end of construction. It is sometimes assumed that the pore-air pressure curve obtained with the analysis of Hilf (1948) should be used as a prediction of the pore-water pressure in the fill. At the same time, the pore-air pressures are ignored. The result is the computation of a reduced factor of safety because of the excessively high pore-water pressures used in the calculation. For the example problem, the computed factor of safety would be 1.18 if these pore pressures were used in the analysis.

Suggested procedure for computing factor of safety at end-of-construction

It is suggested that, while numerous assumptions can be used in a stability analysis concerning the pore pressures at the end-of-construction, it is important that the factor of safety also be computed using the most realistic conditions. These conditions would include:

1.) The use of a realistic angle of friction with respect to matric suction for the compacted soil. This can be measured in the laboratory using modified shear strength testing equipment. It can also be estimated as being equal to approximately one half of the ϕ' value.

2.) The use of pore-water pressures in accordance with those predicted using the empirical equation proposed in this paper. The main new information required for such an analysis is a measure of the as-compacted matric suction of the soil.
3.) The use of pore-air pressures set to zero in the analysis could lead to computed factors of safety that are too low relative to an analysis performed using pore-air pressures equal to those predicted using the equation by Hilf (1948).

References

Bishop, A.W. 1954. The use of pore-pressure coefficients in practice. Géotechnique, 4(4): 148–152.

Bishop, A.W. 1957. Some factors controlling the pore pressures set up during the construction of earth dams. Proceedings, 4th International Conference of Soil Mechanics and Foundation Engineering, Paris, France, Vol. 2, pp. 294–300.

Fredlund, D.G. 1984. Slope stability analysis incorporating the effect of soil suction. *In* Geotechnics and Geomorphology of Mass Movement. *Edited by* M.G. Anderson and K.S. Richards, J. Wiley and Son, pp. 113–144.

Fredlund, D.G., and Morgenstern, N.R. 1976. Constitutive relations for volume change in unsaturated soils. Canadian Geotechnical Journal, 13(3): 261–276.

Fredlund, D.G., Morgenstern, N.R., and Widger, R.A. 1978. The shear strength of unsaturated soils. Canadian Geotechnical Journal, 15(3): 313–321.

Hasan, J.U., and Fredlund, D.G. 1980. Pore pressure parameters for unsaturated soils. Canadian Geotechnical Journal, 17(3): 395–404.

Hilf, J. 1948. Estimating construction pore pressures in rolled earth dams. Proceedings, 2nd International Conference on Soil Mechanics and Foundation Engineering, Rotterdam, The Netherlands, Vol. 3, pp. 234–240.

Skempton, A.W. 1954. The pore pressure coefficients *A* and *B*. Géotechnique, Vol. 4, pp. 143–147.