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A probabilistic slope stability analysis using deterministic computer software

Yang Dai, D.G. Fredlund & W.J. Stolte
Department of Civil Engineering, University of Saskatchewan, Saskatoon, Sask., Canada

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ABSTRACT: A designation of the probability of failure of a slope can add further understanding to its behaviour, over and above a conventional factor of safety. The purpose of this paper is to illustrate how a deterministic slope stability package can be made into a probabilistic software package or used in a probabilistic manner. The same general procedure could be applied to any deterministic computer program.

A slope stability example was set up to be solved in the conventional manner using a deterministic software package (i.e., PC-SLOPE). Additional, independent code was developed to generate statistical data in order that the deterministic solution could be repeatedly executed to give a probabilistic evaluation of the same example. Normal frequency distributions relative to variations in cohesion, angle of internal friction and pore-water pressure were studied. From the mean values and the standard deviations of the cohesion, angle of internal friction and pore-water pressure, along with the correlation relationship between cohesion and angle of internal friction, sets of random values of the cohesion, angle of internal friction and pore-water pressure were generated using the Monte Carlo method and the Point Estimate method. Each set of these values of cohesion, angle of internal friction and pore-water pressure generated were then used in the deterministic software package to compute a factor of safety. Further code was written to read the output factors of safety files and to analyze the results. In this way, the probability distribution of the factor of safety was obtained for a specific slip surface. The reliability in terms of the probability of factor of safety being greater than or equal to 1.0 was also computed.

INTRODUCTION

Conventional slope stability methods compute the factor of safety of an earth or rock slope based only on a fixed set of conditions or a given data which include shear strength parameters, pore-water pressure and slope geometry. This analysis searches for a minimum factor of safety using a trial and error procedure and is referred to as a deterministic approach. If the factor of safety is greater than unity (i.e., $F > 1$), the slope is assumed to be stable. On the contrary, if the factor of safety is less than unity (i.e., $F < 1$), the slope is assumed to be unstable or susceptible to failure. Uncertainties in any of the input parameters are not taken into consideration in this (deterministic) approach.

The calculated value of the factor of safety is interpreted by the engineer using general design criteria and site conditions. The judgement of the engineer can also influence the interpretation of the factor of safety value based on his experience. It would be preferable to have a knowledge of the statistical variation of the input data. These variations could then be taken into consideration in the analysis.

In this paper, the soil parameters (i.e., cohesion and angle of internal friction) and the pore-water pressure for a slope stability analysis are regarded as uncertainties. The statistical distribution of these uncertainties has been studied by numerous researchers such as Hooper and Butler (1966), Lumb (1966), Schultze (1971), McGuffey, Iori, Kyfor and Grivas (1981) and in general found to follow a normal distribution. Fredlund and Dahlman (1971) and Fredlund and Hasan (1979) found that the logs of the unconfined compressive strength follow a normal distribution more closely than untransformed values do. The correlation between the cohesion and the angle of internal friction has been found to be weakly negative (Lumb, 1970; Holtz and Krizek, 1971; Yucemen, Tang and Ang, 1973; Grivas, 1981; Cherubini, Cotecchia, Renna and Schiraldi, 1983).

There are numerous deterministic software packages which have been coded for solving geotechnical problems such as a slope stability analysis based on single values of the above mentioned parameters. There is an increasing need to have software packages which also provide information on the probability of failure. It would be of value if existing deterministic software packages could be used as part of a probabilistic package. This paper is concerned with developing such an extension of a deterministic, slope stability computer program.

METHODOLOGY

There are various probabilistic approaches available for geotechnical engineering application. Two common methods are the Monte Carlo method and the Point Estimate method and these have been selected for use in this study.

The Monte Carlo method

The Monte Carlo method uses randomly generated values for the component variables to determine the probability distribution of the design variable (i.e., the factor of safety in this case). It requires the statistical distribution of selected input variables to be known. The steps in the Monte Carlo method can be outlined as follows:

1.) Generate random numbers which are independent random variables uniformly distributed over the unit interval between zero and one.

2.) Transform the random numbers from a uniform distribution to the distribution applicable to the component variable.

3.) Calculate values of all component variables based on the appropriate random numbers.

4.) Compute the design variable (i.e., factor of safety) using the generated values of the component variables.

5.) Repeat steps 1.) to 4.) a large number of times. The number of times these steps are repeated depends upon the variability of the input and output parameters and the desired accuracy of the output.

6.) Create a cumulative distribution of the design function using the data obtained from the above simulations.

The details of this approach have been outlined by numerous researchers such as Hammersley and Handscomb (1964), Schreider (1966) and Rubinstein (1981).

The Point Estimate method

The Point Estimate method is an approximate numerical integration approach. It requires a knowledge of the mean and the standard deviation of each variable. This method consists of computing the magnitude of the design variable at values of one standard deviation on either side of the mean values for all component variables (i.e., 2^N terms where N is the number of variables). The mean and the standard deviation of the design function can then be obtained. After assuming a statistical distribution for the design function, the probability distribution of the design function can be obtained. This method was developed by Rosenblueth (1975,1981).

The probabilistic software package

The probabilistic software package is an extension of the deterministic software package, PC-SLOPE. The flow chart of the probabilistic software package is shown in Figure 1.

The computer software is structured as follows. PC-SLOPE, the foundation of the commercial software, requires the associated pre-processor package, PROMSL, to provide the input data file. The base input data file (___.SET) is derived by PROMSL from the means of the component variable, plus other data.

PC-STAT is a part of the probabilistic software package. PC-STAT first reads the data file (___.SET) created by PROMSL. It then allows for the input of the statistical variability for the cohesion, angle of internal friction and pore-water pressures as well as the correlation coefficient between cohesion and angle of internal friction. Either the Monte Carlo method or the Point Estimate method can then be selected and PC-STAT will generate files with random values of component variables (___xxx.SET).

SLOPE is the main-processor in PC-SLOPE and is used to determine the factor of safety based on the files derived from random values of component variables (___xxx.SET) from PC-STAT. An equal number of solution files (___xxx.FAC) are computed using the SLOPE program in a batch processing mode.

PC-PROB is another part of the probabilistic software package. It reads all solution files (___xxx.FAC) and rewrites the factor of safety to a single file (___X.FAC). For the Monte Carlo Method, the relative frequency distribution and the probability distribution of the factor of safety is then computed and saved in another file (___.DET).

For the Point Estimate Method, the normal probability distribution for the factor of safety is also computed.

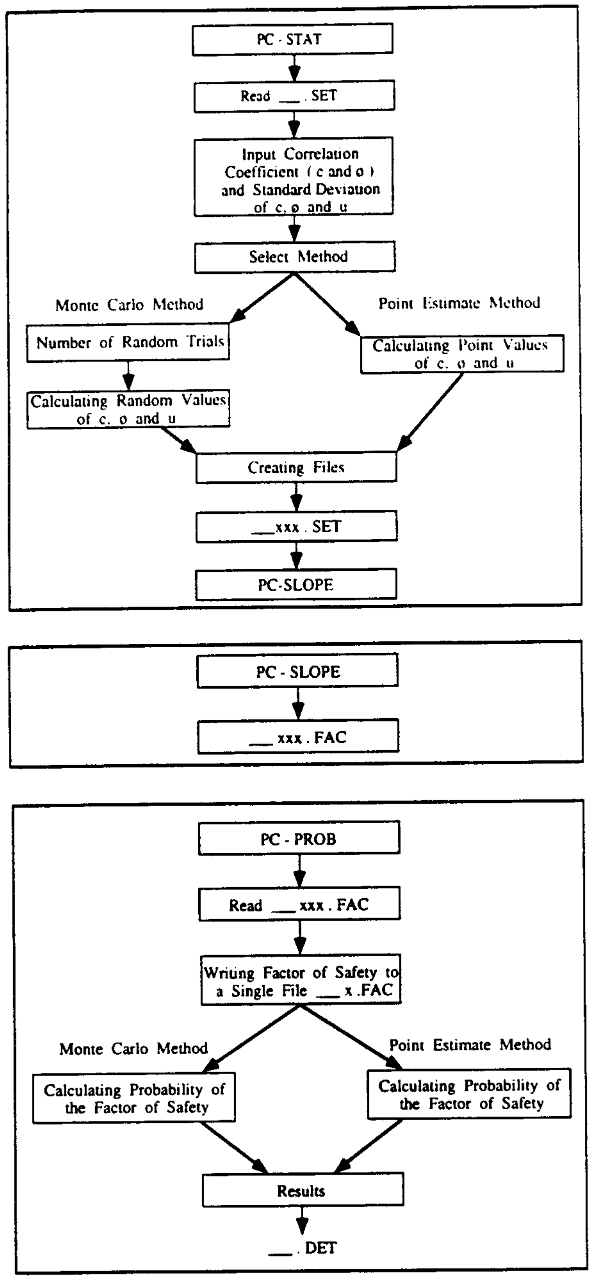


Fig. 1 Flow chart of probabilistic software package.

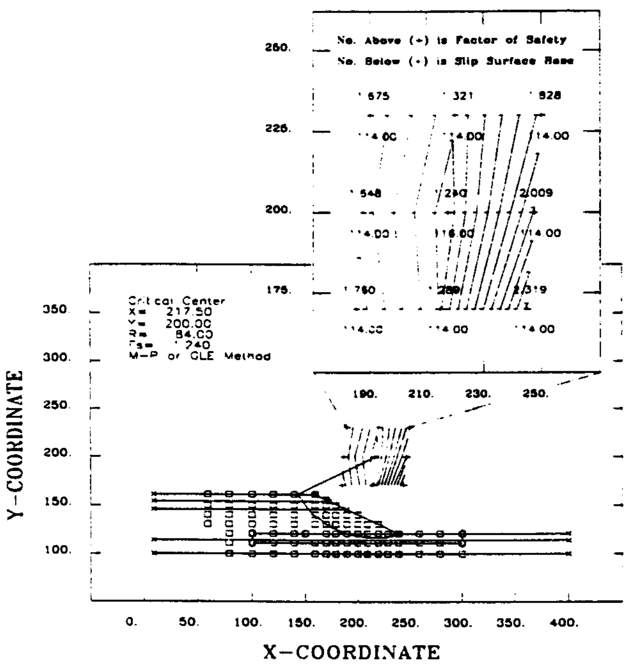
PARAMETRIC STUDY

The probabilistic software package was tested to assess the results relative to the deterministic solution. The deterministic software package has been well studied and only the programs PC-STAT and PC-PROB are considered in this study.

Results of the deterministic solution

A basic slope stability example is selected for both the deterministic and the probabilistic solutions using the Morgenstern-Price method. The geometry, the soil properties and the factor of safety for the basic slope stability example used in the parametric study are shown in Figure 2. The pore-water pressure data were input for a grid.

The Morgenstern-Price Method calculates a factor of safety for each of five radii of the slip circle and nine slip circle centres on a grid, shown expanded in Figure 2. Each grid centre and each radius are combined to locate the slip surface. Each slip surface has a corresponding factor of safety. In total, the deterministic analysis required 45 factor of safety solutions. The minimum factor of safety is the final deterministic solution for the slope.



UNIT WEIGHT	COHESION	PHI	DESCRIPTION
115.00	700.00	22.00	UPPER PORTION OF SILTY CLAY
112.00	600.00	21.00	LOWER PORTION OF SILTY CLAY
110.00	300.00	20.00	SOFT SILTY CLAY
123.00	600.00	27.00	SANDY CLAY TILL
-1.00	.00	.00	BASE OF PROBLEM

Fig. 2 The geometry, soil properties and minimum factor of safety of the basic example used for the parametric study.

The minimum factor of safety for the deterministic solution from the Morgenstern-Price method was 1.240. The coordinates of the slip circle centre for the minimum factor of safety were 217 meters in the x-direction and 200 meters in the y-direction. The y-coordinate of the base of the slip surface for minimum factor of safety was 114 meters.

Results of the probabilistic analysis

The probabilistic analysis had several steps. First of all, with the Monte Carlo Method, the number of simulations to run had to be decided on. Then a check on the consistency of the probabilistic approach was required. Finally, the sensitivity of the probability distribution of a factor of safety to each of the component variables was determined.

1.) Various number of random trials

In the Monte Carlo method, the particular level of reliability depends on the number of simulations. In order to decide on the number, the number of random trials was chosen to be 100, 200, 300, 500, 700 and 900 and the lowest number yielding consistent results chosen. The simulations required the standard deviations of the cohesion, the angle of internal friction and the pore-water pressure head and these were selected to be 50 kPa, 2 degrees and 1.5 meters of head, respectively. In addition, the correlation coefficient between cohesion and angle of internal friction was selected to be - 0.5.

The relative frequency distribution of the factor of safety for various numbers of trials is plotted in Figure 3. The cumulative probability distribution of the factor of safety is plotted in Figure 4. For 100, 200, 300, 500, 700 and 900 random trials, the factors of safety with a 95 percent probability of being exceeded were 1.003, 1.020, 1.024, 1.031, 1.024 and 1.031, respectively. It can be seen that for the example studied, the effect of varying the number of random trials was not significant beyond 200 random trials. Therefore, 200 random trials were used in the Monte Carlo Method.

2.) Consistency check

It was important to verify that the results of the probabilistic analysis were consistent with the deterministic solution. This verification was done first by, holding the circle centre coordinates at the values obtained from the deterministic analysis and then determining if the circle radius producing the maximum probability of failure was the same as that from the deterministic analysis, as would be expected. Then the process was reversed to see if circle centre coordinates producing the maximum

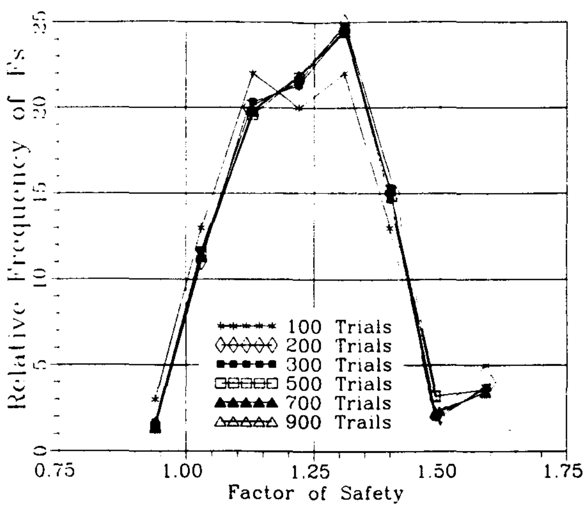


Fig. 3 Relative frequency distribution of the factor of safety for various number of random trials (Monte Carlo method).

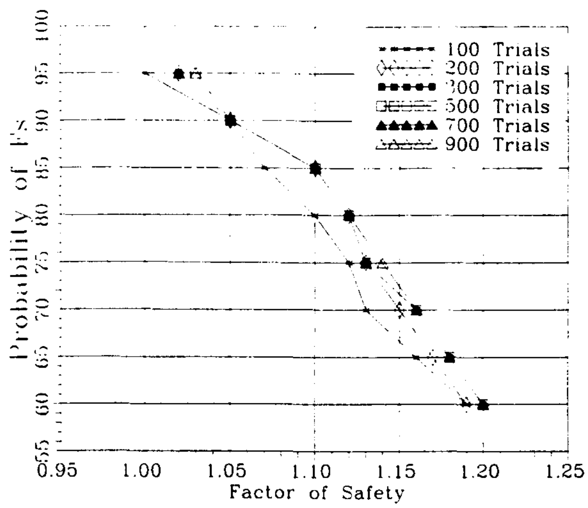


Fig. 4 The probability distribution of the factor of safety for various number of random trials (Monte Carlo method).

probability of failure matched those given by the deterministic analysis.

First of all, the radius was set to the given value and the probability distribution of the factor of safety associated with each circle centre coordinate pair was determined and plotted in Figure 5. The maximum probability of failure is, as expected, associated with the circle centre coordinates given by the deterministic solution. Secondly, Figure 6 shows that the same process used to test the slip circle radius yields maximum probability of

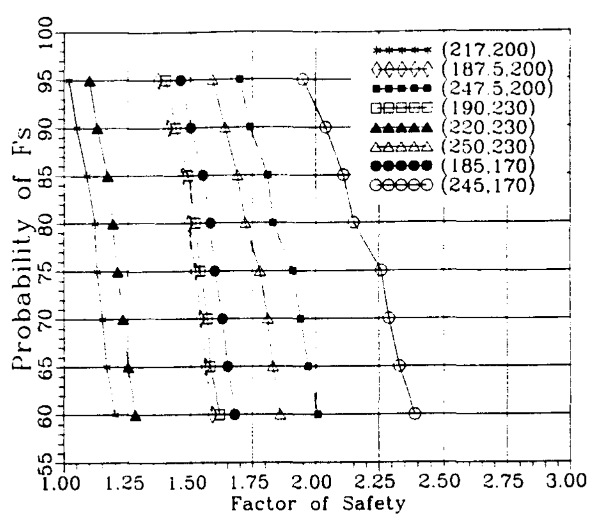


Fig. 5 The probability distribution of the factor of safety for different grid centres (Monte Carlo method).

failure for a radius equal to that given by the deterministic analysis. It can thus be concluded that the results of the probabilistic analysis are consistent with those of the deterministic analysis, thus tending to verify the probabilistic approach.

3.) Parametric analysis

A major benefit of the probabilistic approach is that the sensitivity of the solution to various parameters can be determined. The parameters of most relevance in this study are the standard deviations of the cohesion, angle of internal friction and pore water pressure head, as well as the correlation coefficient between cohesion and angle of internal friction.

a.) Varying the standard deviation of the cohesion

As previously seen, the standard deviations of the cohesion can vary widely. While 50 kPa is a reasonable mean value, values of 10 kPa and 90 kPa are also possible and were thus used for the analysis.

The standard deviation for the angle of internal friction and the pore-water pressure were maintained at the original values of 2 degrees and 1.5 meters of head, respectively. The correlation coefficient between the cohesion and the angle of internal friction was maintained at the original value of - 0.5.

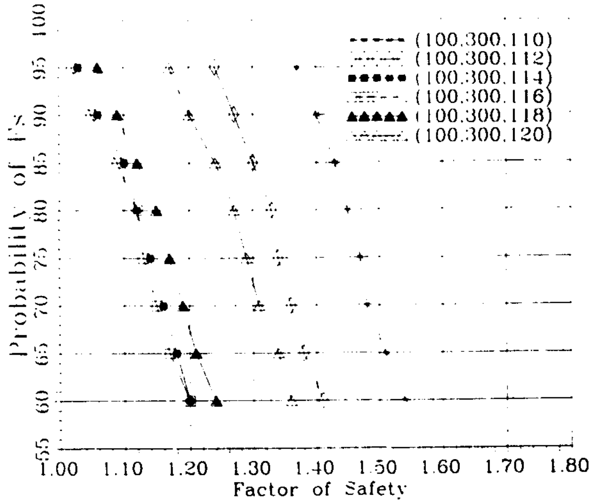


Fig. 6 The probability distribution of the factor of safety for various radii (Monte Carlo method).

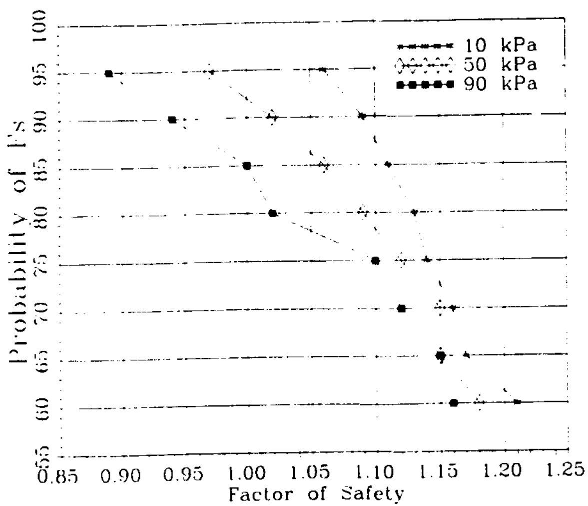


Fig. 7 The probability distribution of the factor of safety for various standard deviations of cohesion (Monte Carlo method).

The cumulative probability distribution of factor of safety is plotted in Figure 7. For standard deviation values for the cohesion of 10 kPa, 50 kPa and 90 kPa, respectively, the factors of safety with a 95 percent probability of being exceeded were equal to 1.059, 0.973 and 0.891, respectively, and the probability of failure ranged from much less than 5 percent to 15 percent.

The plots show that the probability of failure is very sensitive to the standard deviation of the cohesion as would be expected.

b.) Varying the standard deviation of the angle of internal friction

Values of the standard deviation of the angle of internal friction were chosen to be 1.0 degree, 3.0 degrees and 4.5 degrees respectively. The standard deviation for the cohesion and the pore-water pressure were maintained at 50 kPa and 1.5 meters of head, respectively. The correlation coefficient between the cohesion and the angle of internal friction was maintained at - 0.5.

The cumulative probability distribution of the factor of safety is plotted in Figure 8. For standard deviation of the angle of internal friction of 1.0 degree, 3.0 degrees and 4.5 degrees, the factor of safety with a 95 percent probability of being exceeded was equal to 1.071, 0.973 and 0.892, respectively, and the probability of failure ranged from much less than 5 percent to 14 percent.

The plots show that the standard deviation of the angle of internal friction has a great effect on the probability of failure, about the same as that of the standard deviation of the cohesion.

c.) Varying the standard deviation of the pore-water pressure

Values of the standard deviation of the pore-water pressure head were chosen to be 1.0 m, 3.0 m and 5.0 m respectively. The standard deviation for the cohesion and the angle of internal friction were maintained at 50 kPa and 3 degrees, respectively. The correlation coefficient between the cohesion and the angle of internal friction was maintained at -0.5.

The cumulative probability distribution of the factor of safety is plotted in Figure 9. For standard deviations of 1.0 m, 3.0 m and 5.0 m, the factor of safety with a 95 percent probability of being exceeded was equal to 0.973, 0.958 and 0.898, respectively, and the probability of failure ranged from 8 percent to 15 percent.

As expected, the higher the standard deviation for the pore-water pressure head, the higher the probability of failure.

d.) Various magnitudes of the correlation coefficient between cohesion and the angle of internal friction

The correlation coefficient between the cohesion and the angle of internal friction was chosen to range from 0.0 to - 1.0 in intervals of - 0.2.

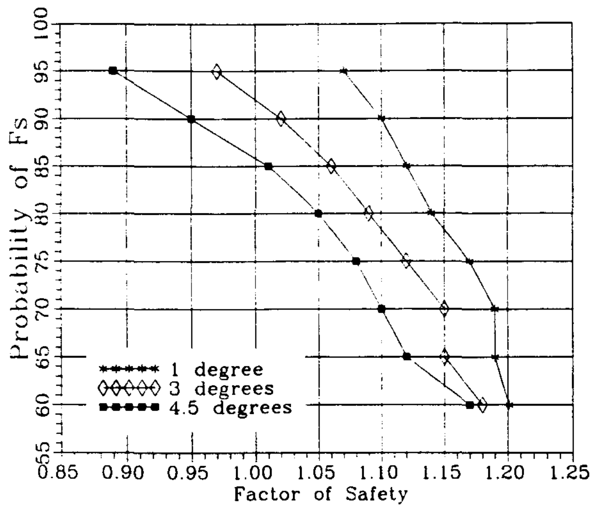


Fig. 8 The probability distribution of the factor of safety for various standard deviation of the angle of internal friction (Monte Carlo method).

The standard deviation for the cohesion, the angle of internal friction and the pore-water pressure head were selected to be 50 kPa, 3 degrees and 5.0 meters.

The cumulative probability distribution of the factor of safety is plotted in Figure 10. For correlation coefficients of 0.0, - 0.8 and - 1.0, respectively, the factor of safety with a 95 percent probability of being exceeded were equal to 0.904, 0.914 and 0.921, respectively.

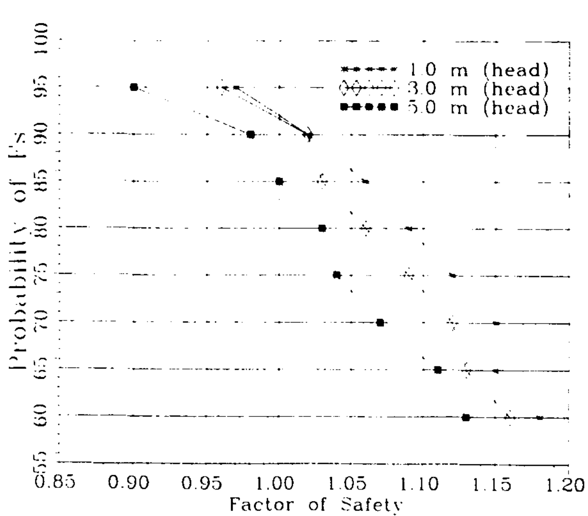


Fig. 9 The probability distribution of the factor of safety for various standard deviations of the pore-water pressure (Monte Carlo method).

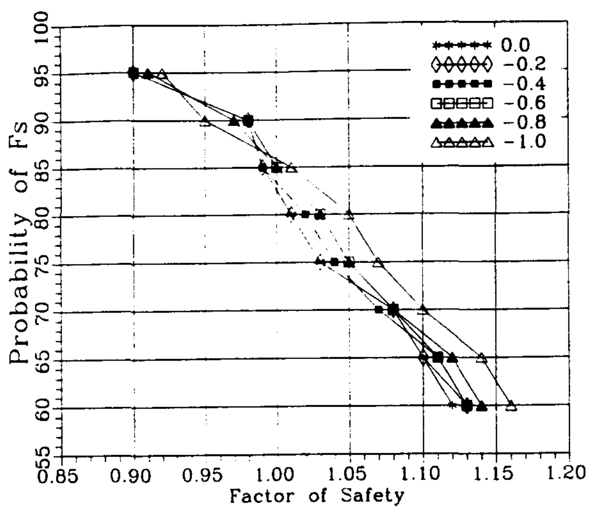


Fig. 10 The probability distribution of the particular factor of safety for various magnitudes of correlation coefficient between cohesion and angle of internal friction (Monte Carlo method).

The change in the probability of a particular factor of safety being exceeded is less than 4 percent for a range from 0.0 to - 1.0 in the correlation coefficient. It is concluded that the correlation coefficient between the cohesion and the angle of internal friction has a relatively small influence on the probability distribution of a particular factor of safety.

SUMMARY

From the parametric study, the following observation can be made.

1.) The effect of varying the number of random trials was not significant beyond 200 random trials for the Monte Carlo method.

2.) The probabilistic analysis can be run on the critical slip surface obtained from a deterministic analysis for both the Monte Carlo method and the Point Estimate method for the example studied.

3.) The results from the probabilistic approach are not significantly affected by the magnitude of correlation coefficient between the cohesion and the angle of internal friction for both the Monte Carlo method and the Point Estimate method.

4.) The higher the standard deviation of the cohesion, the angle of internal friction and the pore-water pressure, the higher the probability of failure as determined bv both the Monte Carlo as compared to using the Monte Carlo method.

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