Effect of the axis of moment equilibrium in slope stability analysis

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Some of the methods of slices satisfying moment equilibrium derived for circular slip surfaces have been extended to accommodate noncircular (or composite) type slip surfaces. A question arises regarding the point about which moment equilibrium should be taken and whether varying the center for moment equilibrium has a significant effect upon the computed factor of safety. This paper addresses the question of the effect of the center for moment equilibrium as it pertains to noncircular (or composite) slip surfaces. In particular, extensions of the Ordinary, Bishop's simplified, and the General Limit Equilibrium (GLE) methods are examined. The results show that considerable variations in the factor of safety can occur when using the extended Ordinary method. The extended Bishop's simplified method shows varying factors of safety as the moment axis moves vertically. Variations in the computed factor of safety can generally be expected to be less than 12%. The GLE, Morgerstern–Price, and Spencer methods are independent of the axis for moment equilibrium.

Key words: slope stability, limit equilibrium, moment equilibrium, factor of safety, noncircular slip surface.

Quelques-unes des méthodes des tranches satisfaisant l'équilibre des moments dérivé pour les surfaces de glissement circulaires, ont été élargies pour accommoder les surfaces de glissement de type non-circulaire ou composite. Une question se pose quant au point par rapport auquel l'équilibre des moments devrait être calculé et quant à savoir si le fait de faire varier le centre d'équilibre des moments a un effet significatif sur le coefficient de sécurité. Cet article pose la question concernant l'influence de la position du centre de l'équilibre des moments dans le cas des surfaces de glissement non-circulaires ou composites. L'on examine en particulier les méthodes ordinaire et simplifiée de Bishop, et celle de l'équilibre limite général (GLE). Les résultats montrent que des variations considérables du coefficient de sécurité peuvent se produire lorsque la méthode ordinaire élargie est utilisée. La méthode simplifiée élargie de Bishop montre des coefficients de sécurité qui varient lorsque l'axe des moments bouge verticalement. L'on peut s'attendre à ce que des variations dans le coefficient de sécurité calculé soient généralement inférieures à 12%. Les méthodes GLE, de Morgenstern-Price et de Spencer sont indépendantes de la position de l'axe de l'équilibre des moments.

Mots clés : stabilité de talus, équilibre limite, équilibre des moments, coefficient de sécurité, surface de glissement non-circulaire.

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Introduction

Slope stability analyses play an important role in many civil engineering projects. Various methods of slices have been developed and have the characteristics of being able to solve problems with complex geometries and variable soil and water conditions (Terzaghi and Peck 1976). A comparison of the most commonly used methods of slices has been provided by Fredlund and Krahn (1977). The theoretical and quantitative relationship between the results from various methods has also been defined by Fredlund *et al.* (1981).

The most commonly used methods are the Ordinary or Fellenuis method, Bishop's simplified method, Spencer's method, Janbu's method, the Morgenstern–Price method, Sarma's method, and the General Limit Equilibrium (GLE) method. Each of the methods differs with respect to the statical equilibrium equations used in the derivation and (or) the assumptions made regarding the inter-slice forces. It is possible to classify the above-mentioned method into two categories. (1) Methods that use either force or moment equilibrium alone to solve for the factor of safety. These can be viewed as simplified methods. (2) Methods that satisfy all conditions of statical equilibrium. These can be regarded as rigorous methods.

The differences between the factors of safety computed using these varying conditions of statical equilibrium are as follows. (1) Methods that satisfy all conditions of equilibrium give essentially the same value for factor of safety. Deviations between the methods should not be more than 5% (Duncan and Wright 1980). This statement is contingent upon the solution converging. (2) Methods that use only force or moment equilibrium to solve for factor of safety can be expected to yield quite different values for the factor of safety when compared with rigorous methods. In particular, the use of force equilibrium methods may give substantially different results than those satisfying moment equilibrium (Spencer 1967).

These differences have become common knowledge among geotechnical engineers. However, the significance of another important difference between the two has always been ignored. For all the simplified methods satisfying moment equilibrium, unlike the rigorous methods, the factor of safety is a function of the position of the axis about which the moments for all slices can be summed. From this expression, the factor of safety with respect to moment equilibrium can be derived. For the simplified methods of slices with circular slip surfaces, it has become acceptable to use the center of rotation as the axis of moments. The reasonableness of this choice has unfortunately never been investigated.

The objectives of this paper are (*i*) to study the effect of the position of the moment axis on the factor of safety (three methods of slices, namely the Ordinary, Bishop's simplified, and the GLE methods, have been selected to show the effect of the axis of moments); and (*ii*) to conduct a quantative investigation to ascertain the most reasonable position for the axis of moments when using Bishop's simplified method for composite slip surfaces.

General evaluation

The effect of the axis of moments on the factor of safety is studied for three methods, namely the Ordinary (Fellenuis 1936), Bishop's simplified (Bishop 1955), and GLE (Fredlund *et al.* 1981) methods. All these allow the use of a common axis for the summation of moments.

A single homogeneous slope as shown in Fig. 1*a* is used as the reference case. A case with a composite slip surface (i.e., slip surface consisting of circular arcs and a straight line) illustrated in Fig. 1*b* is used for a comparative study. The study was conducted using the PC-SLOPE software developed by GEO-SLOPE International Ltd. (Fredlund 1985). By using the GLE formulation, this package makes it possible to obtain the factors of safety for all three methods.

Each case was analyzed by systematically changing the position of the axis about which moments were taken. The water table in all cases was assumed to be below the slip surface. More than 900 stability analyses were performed for various centers of moments, and the factors of safety for the various methods were determined. The computed factor of safety at each moment center was compared with the factor of safety obtained by the GLE method. The factor of safety computed using the GLE method should be essentially unaffected by the axis about which moments are taken, since all elements of statical equilibrium are satisfied. As shown in Fig. 2, the percentage difference was essentially 0%, except near the edge where there are numerically unstable zones. The empty space in Fig. 2 represents the numerically unstable zones. When the axis of moments was chosen in this region, the net resistant moment due to the unstabilized shear force was so small that the factor of safety equation becomes unstable and cannot achieve convergence. The effect of the axis of moments for the Ordinary and the Bishop's simplified methods were determined in terms of the percentage difference in the factor of safety between each method and the GLE method at each of the moment centers.

The detailed contours of the percentage difference in the factors of safety are presented in Figs. 3–6. The results verify that the percentage difference for all axis positions is essentially zero for the GLE method (Fig. 2). The results from Bishop's simplified and the Ordinary methods are contoured (Figs. 3–6). The following conclusions can be drawn from these figures.

(1) For the ordinary method, the factor of safety changes widely with changing the location of the axis of moments. The factor of safety is significantly affected when the axis of moments changes in a horizontal direction (Figs. 3 and 4).

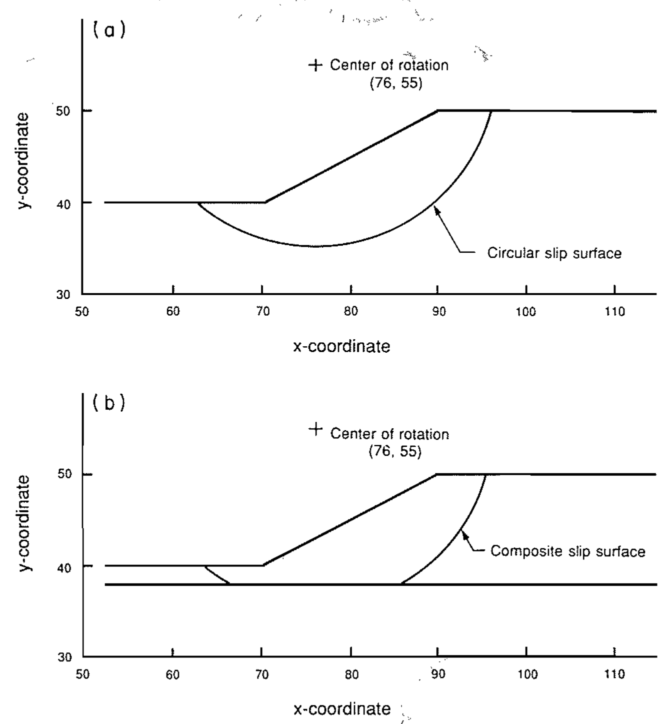

FIG. 1. Geometry of example problems.

The percentage of difference is not a minimum when the axis of moments is assumed to be the center of rotation. Rather, the region of smallest difference is about 5–6 m to the right of the center of rotation. This is true for slopes with a composite slip surface and slopes with a completely circular slip surface.

(2) For Bishop's simplified method, the factor of safety is affected where the axis for moment equilibrium is changed in the vertical direction (Figs. 5 and 6). This finding has been verified by all the cases analyzed. Using Bishop's simplified method for circular slip surfaces shows that the rotation center and the moment center gave the same factor of safety as that of the GLE method. As shown in Fig. 6, the best moment center was not coincidental with the rotation center for the circular portion of a composite slip surface. The results indicate that the Bishop's simplified method can be used for most cases, provided a reasonable axis of moments is selected.

The above findings led to a further study on the effect of the location of the axis of moment equilibrium.

Effect of the location of the axis on moment equilibrium for Bishop's simplified method

Bishop's simplified method is probably the most widely used of all the simplified methods. This is not only because of its easy application to relevant engineering problems but also, more importantly as pointed out above, because it gives a factor of safety similar to that of the rigorous methods, such as Morgenstern–Price, Spencer's, or GLE methods. This is true for all cases where the slip surface is circular. This method was extended to the case of a composite slip surface by Fredlund and Krahn (1977). A derivation of the factor of safety equation for Bishop's simplified method for a composite slip surface is presented in the Appendix. However, a question arises regarding the uniqueness of the solu-

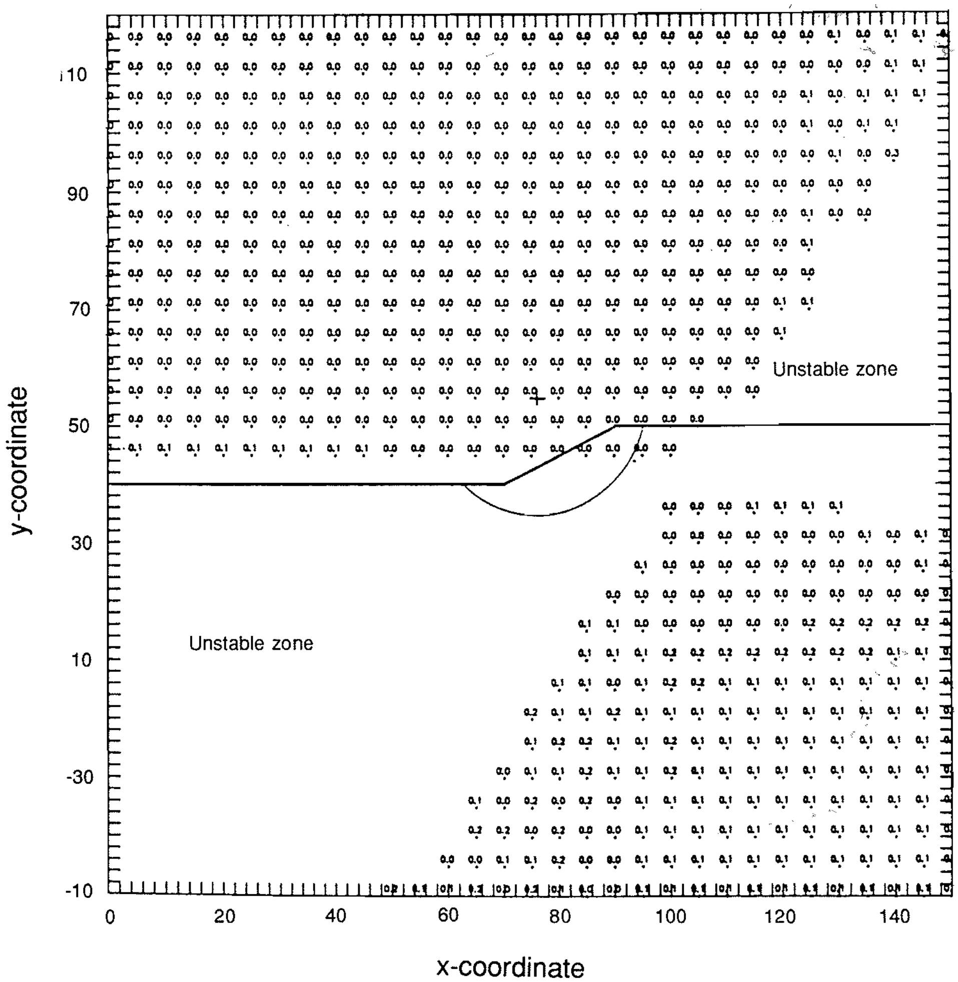

FIG. 2. Percentage difference of the factor of safety at different axes of moments for the reference case using the GLE method. Numbers in the graph indicate percent difference between various axes.

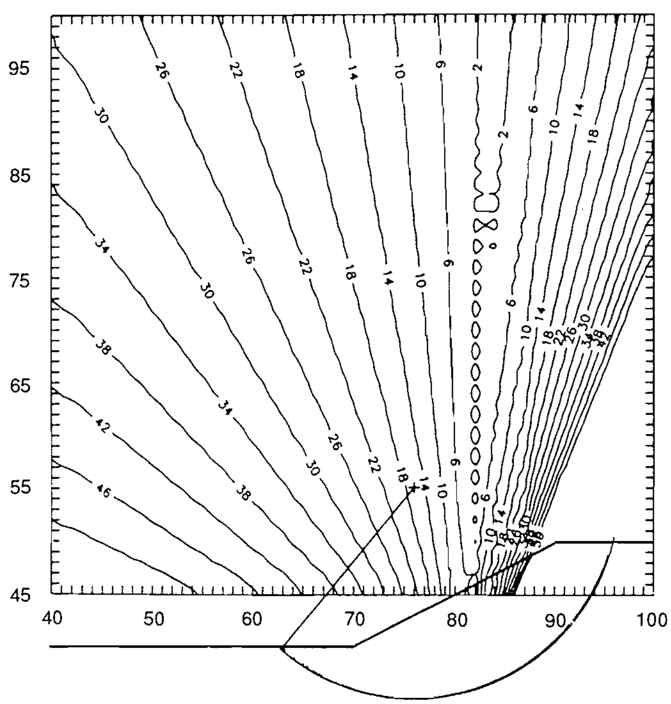

FIG. 3. Contours of the percentage difference in the Ordinary method factor of safety at different axes for moment equilibrium referenced to the GLE method.

tion relative to the axis used for summing moments. In other words, is the solution now a function of the axis of moments, and if so, what is the best axis to use in performing the analysis.

To locate the right axis for moment equilibrium and determine how the axis is affected by the geometry and soil parameters of a slope, a parametric study was undertaken. The homogeneous condition shown in Fig. 7 was selected for this part of the study. Pertinent variables are defined as follows: β = slope angle with respect to the horizontal; α = tilting angle of bedrock (or hard stratum) with respect to horizontal; H = height of the slope; 0 = rotation center for the circular portion of the slip surface; R = radius of the circular portion of the slip surface; D = depth from the top of the slope to the bedrock (or hard stratum); D' = depth from the top of the slope to the bottom of the extended circular slip surface; S = vertical distance from the rotation center to the top of the slope; d = maximum perpendicular distance from the bedrock surface to the extended circular surface; $0'$ = moment center yielding a factor of safety which is equal to that obtained using the GLE method; dy = vertical distance between 0 and $0'$ (a positive dy value means that the moment center is located above the rotation center).

The PC-SLOPE software can define any position for the axis of moments over a grid. It is then possible to locate

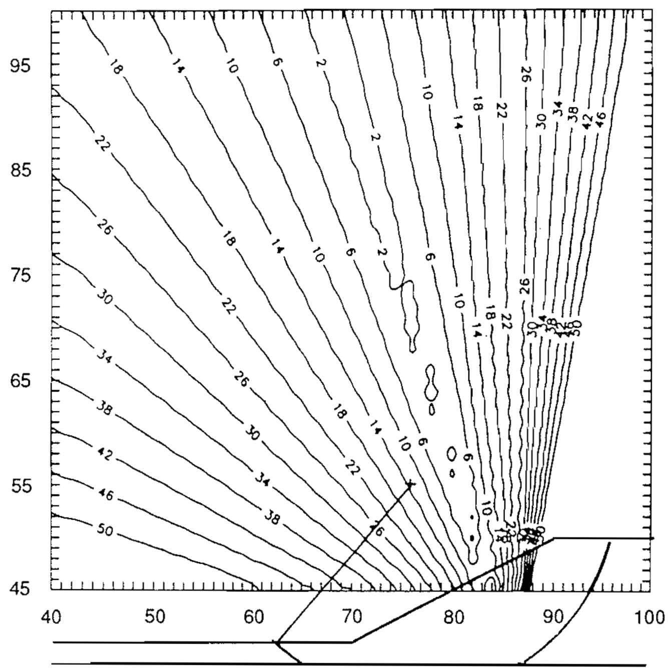

FIG. 4. Contours of the percentage difference in the Ordinary method factor of safety at different axes of moment equilibrium for the non circular case referenced to the GLE method.

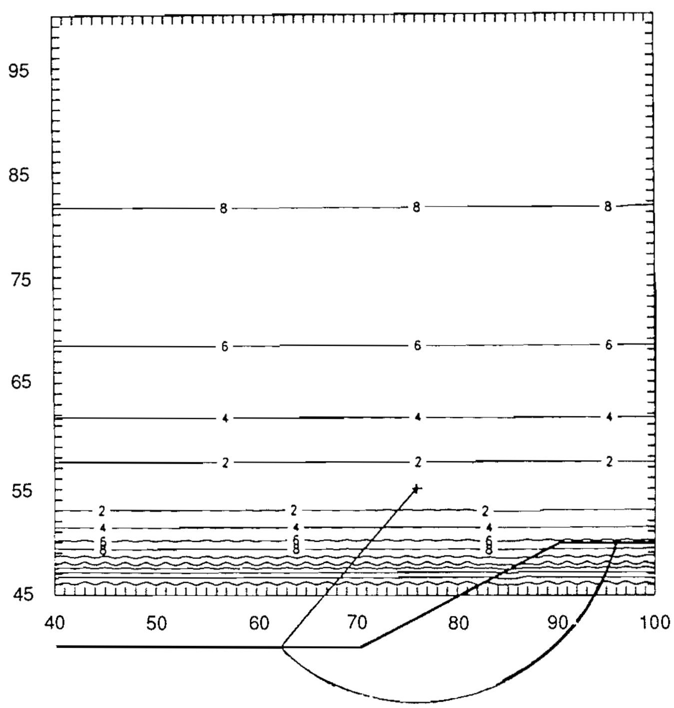

FIG. 5. Contours of the percentage difference of Bishop's simplified method factor of safety at different axes of moment equilibrium referenced to the GLE method.

the axis corresponding to the minimum (or best) factor of safety.

Owing to the multiplication factors that could be incorporated into this problem, the authors did not attempt to develop detailed design charts. Rather, an attempt was made to investigate a series of factors that affect the position of the axis of moments.

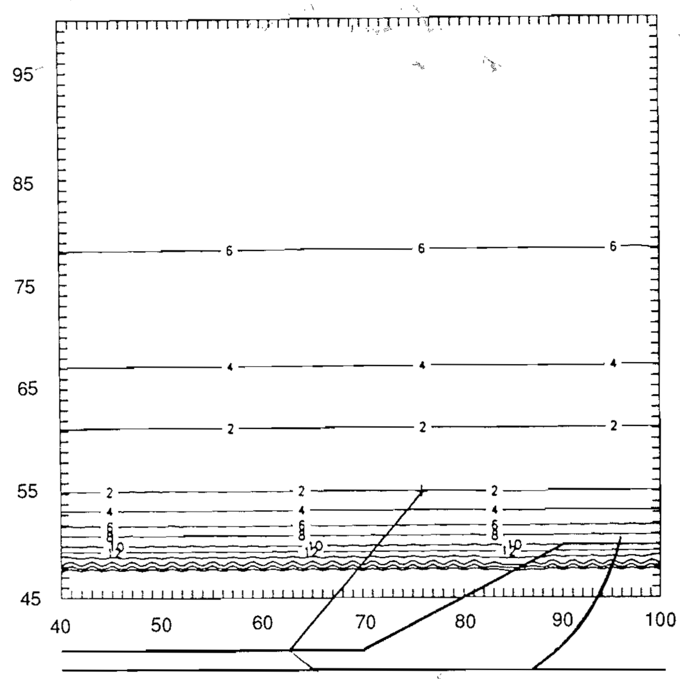

FIG. 6. Contours of the percentage difference of Bishop's simplified method factor of safety at different axes of moment equilibrium referenced to the GLE method.

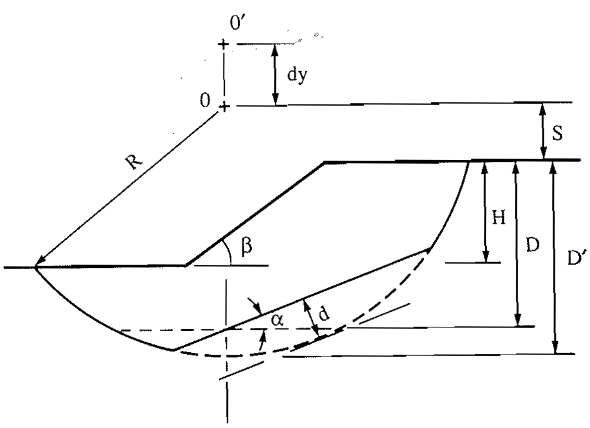

FIG. 7. Definitions of variables on the geometry.

Effect of radius and horizontal bedrock depth

Morgenstern and Price (1965) pointed out that because of material heterogeneity, most slopes fail with a noncircular slip surface. This situation is frequently encountered in nature with a horizontal bedrock layer or a hard stratum intersecting a circular arc. This form can be referred to as a composite slip surface.

The results of the study indicate that for a composite slip surface with a horizontal bedrock line, dy increases with an increase in the ratio d/R. The extent of the composite nature of the slip surface plays an important role (Fig. 8).

The dy value is strongly influenced by the radius of the circular portion, R. A larger dy value occurs when a deep-seated slip surface is analyzed.

In the design charts proposed by Bishop and Morgenstern (1960), the critical depth of a circular surface (D', in this paper) was controlled by the ratio $c'/\gamma H$. When the $c'/\gamma H$ ratio is equal to 0.025, the D'/H ratio must be less than 1.25. When the $c'/\gamma H$ ratio is equal to 0.050, the D'/H ratio

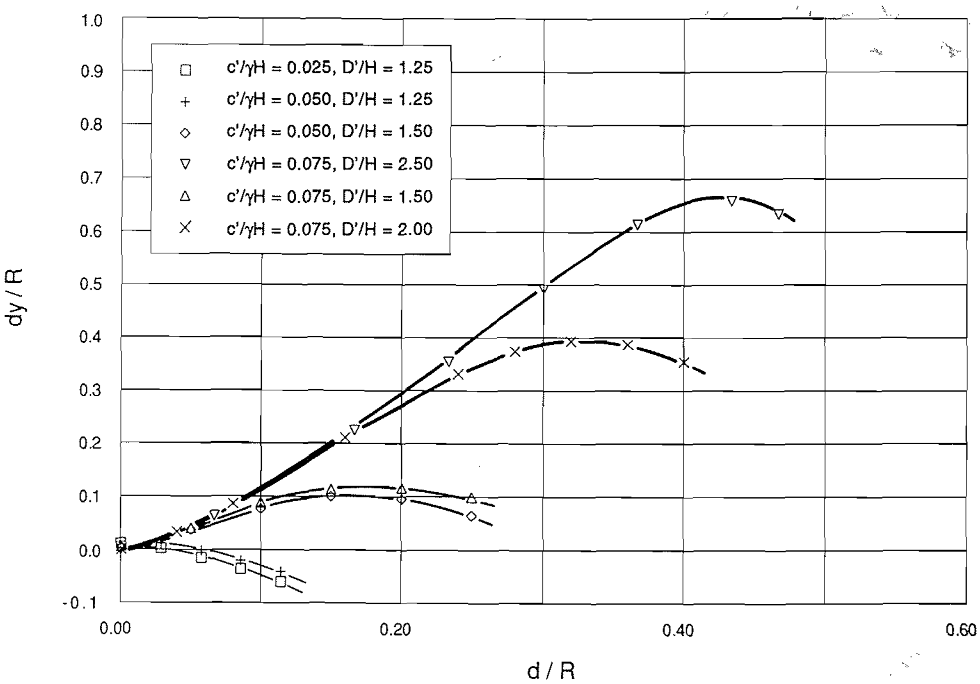

FIG. 8. Effect of the radius R and the parameter d for the case of horizontal bedrock.

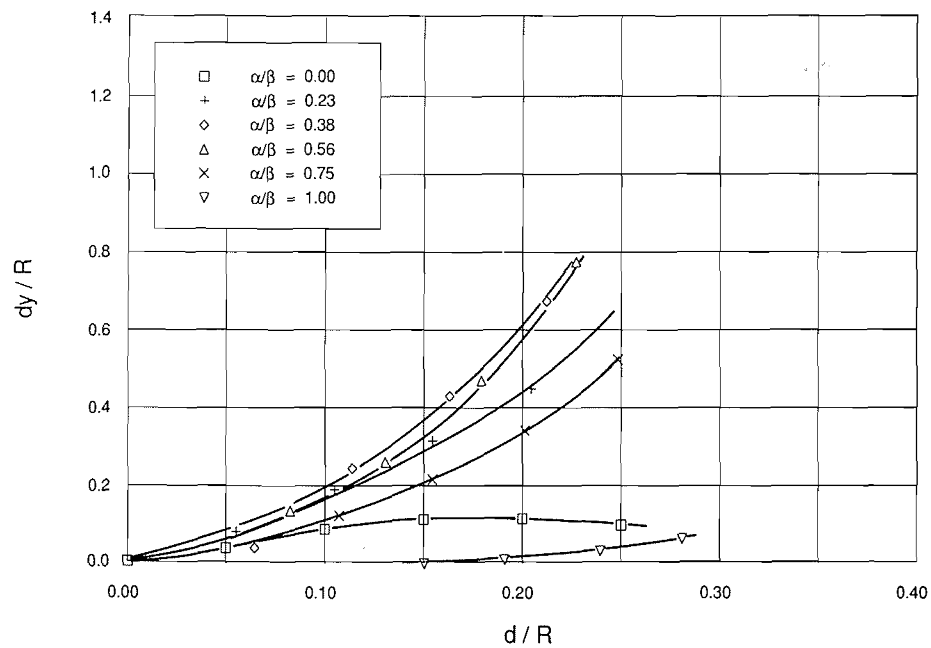

FIG. 9. Effect of bedrock tilt for $D'/H = 1.5$.

must be less than 1.5. Correspondingly, the effects of R and d/R on $\mathrm{d}y/R$ for $c'/\gamma H$ of 0.025, 0.050, and 0.075 are shown in Fig. 8.

Smaller $c'/\gamma H$ values yield shallower critical slip surfaces (i.e., lower D'/H values), and therefore low $\mathrm{d}y/R$ values can be expected. For all cases shown in Fig. 8 where the D'/H ratio is less than 1.5 (or $R < 20$ in the example), $\mathrm{d}y/R$ is always less than 0.15. Where $D'/H = 1.25$, even smaller $\mathrm{d}y$ values were observed. If the rotation center is selected as the moment center, the difference in the factor of safety when compared with the GLE method will be less than about 2–3% (refer to Fig. 16). This is obviously negligible.

When the $c'/\gamma H$ ratio increases to a larger value (i.e., up to 0.075), a larger radius of the slip surface will likely be used. The radius can become an important factor affecting the $\mathrm{d}y$ value. For example, the following observations can be made from Fig. 8: (*i*) when $R = 20$ m, $D'/H = 1.5$ and $\mathrm{d}y_{max} = 2.32$ m at $d/R = 0.25$; (*ii*) when $R = 25$ m, $D'/H = 2.0$ and $\mathrm{d}y_{max} = 9.85$ m at $d/R = 0.32$; and (*iii*) $R = 30$ m, $D'/H = 2.5$ and $\mathrm{d}y_{max} = 19.76$ m at $d/R = 0.44$.

However, even for these cases, the larger $\mathrm{d}y$ values only appear for highly composite conditions. When the slip surface is slightly composite, for example, when $d/R < 0.15$,

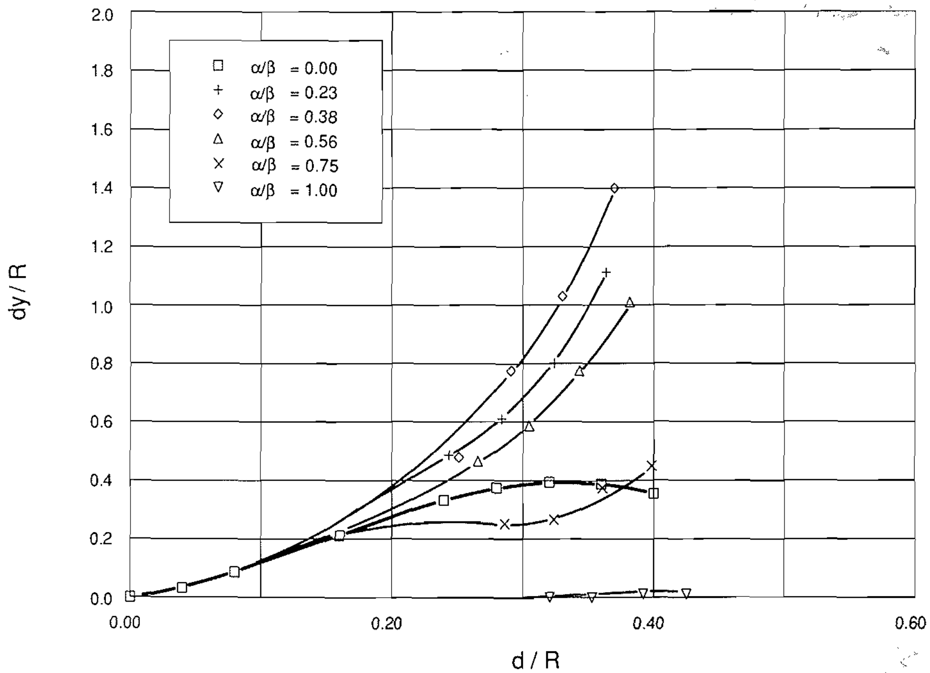

FIG. 10. Effect of bedrock tilt for $D'/H = 2.0$.

dy is still negligible. If the rotation center is selected as the moment center, the error will be less than about 2% (refer to Fig. 16).

Effect of bedrock tilting

A 1:2 slope with $c' = 15$ kPa and $\phi' = 20°$ was selected to study the effect of the bedrock tilting. Some 55 cases, including a variety of intersections d and tilting angles α, were analyzed for conditions of $D'/H = 1.5$ and 20.

The results in Figs. 9–11 show that the best position of the axis for moment equilibrium was strongly influenced by the tilting of the bedrock. Variation in the results also occurred for different D'/H and d/R values.

Quantitatively, the major factor that influences dy/R is the ratio of the bedrock tilting angle α to the slope angle β (Fig. 11). The dy distance increases as the bedrock tilting angle α increases. When α/β is equal to about 0.4, dy is at its maximum value. When α continues to increase, dy decreases. When $\alpha = \beta$ (i.e., the bedrock is parallel to the slope surface), $dy = 0$. In other words, for all semi-infinite slopes, Bishop's simplified method can be applied by selecting the rotation center as the moment center.

Also, for all slightly composite slip surfaces, even when the bedrock is not horizontal, there will not be a significant error if the rotation center is selected as the moment center. For example, from Fig. 11, when $d/R < 0.2$, for any α value, dy/R is less than 0.4 and the error in this factor of safety is less than about 8%.

Effect of slope steepness

A series of geometrics with slopes of 1.5:1, 2:1, 3:1, and 4:1 was studied for the purpose of evaluating the effect of the steepness of the slope on the moment equilibrium axis. The bedrock tilt angle α was assumed to be zero for all cases. The slope length, $H/(\tan \beta)$, which is a projection of the slope surface in the horizontal direction, appears to be a good means of normalizing the effect of slope steepness (Figs. 12 and 13).

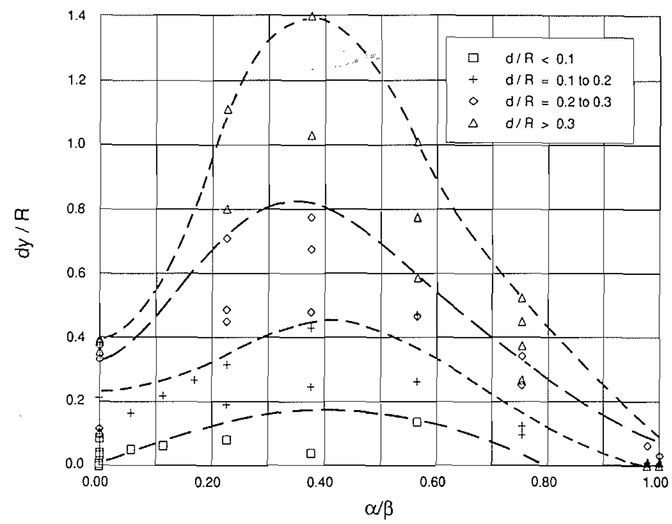

FIG. 11. Effect of bedrock tilt for various d/R values.

Data shown in Figs. 12 and 13 and Table 1 indicate that, although flatter slopes tend to have a larger dy value to get the same factor of safety as that of the GLE method, the change of dy value is not particularly sensitive to the slope steepness when compared with the influence of other factors such as R, α, and d.

The data shown in Figs. 12 and 13 are for the cases with $D'/H = 1.5$ and 2.0. If the rotation center is used as the moment center, most percentage differences in factor of safety will be less than 2% and no larger than 5% for extreme cases.

Effect of soil parameters

Different combinations of shear strength parameters (i.e., c' and ϕ') were used to show the influence of shear strength on the axis of moment equilibrium. The results indicate that the effective cohesion parameter has a stronger influence

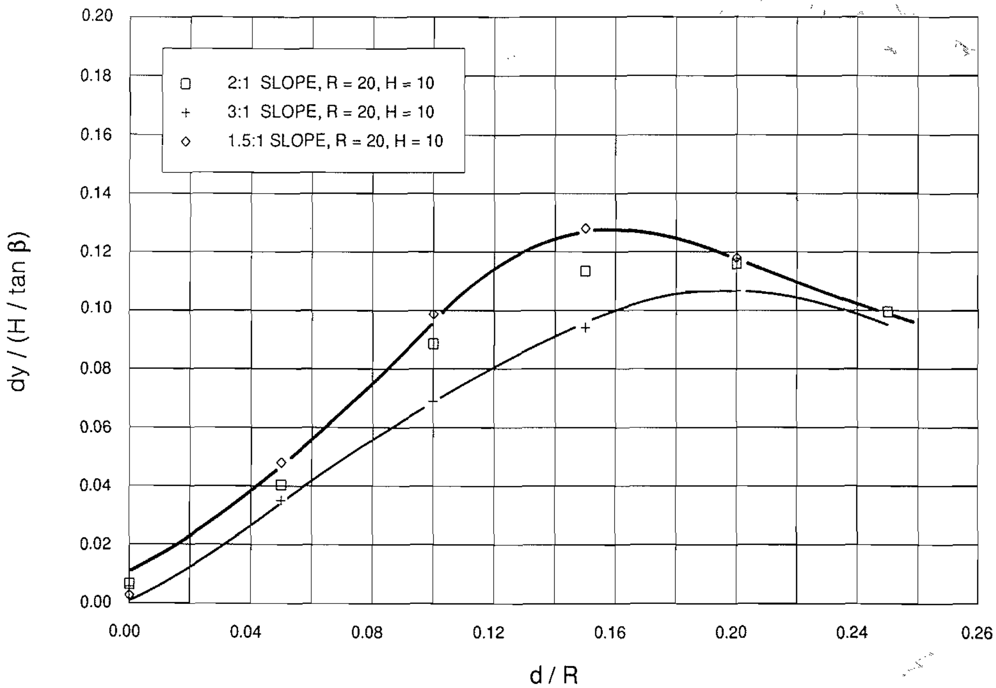

FIG. 12. Effect of slope steepness for D'/H = 1.5.

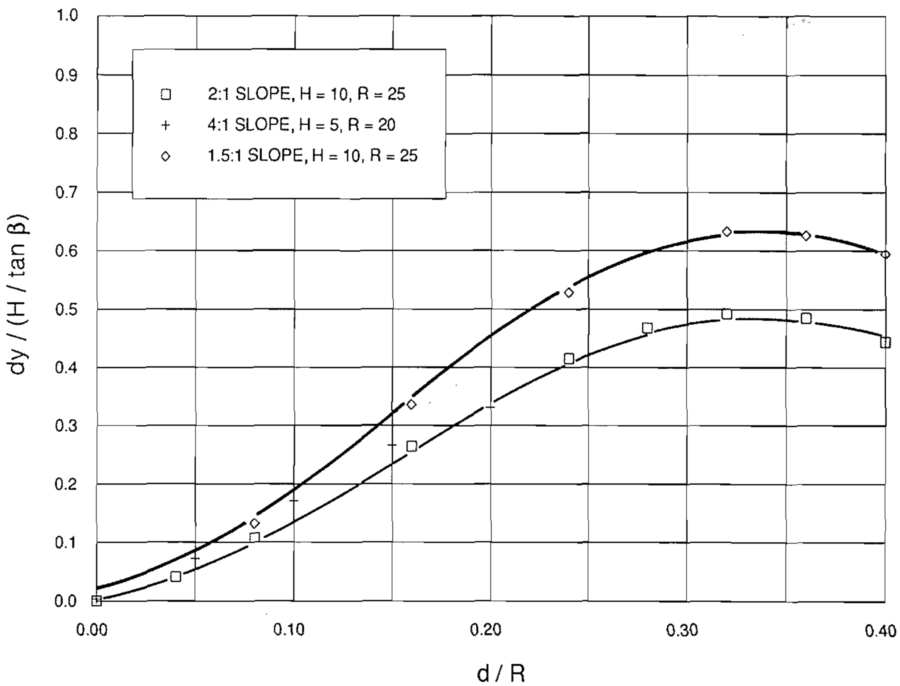

FIG. 13. Effect of slope steepness for D'/H = 2.0.

on dy than does the friction angle ϕ' (Fig. 14). The dy value increases with an increase in cohesion (Fig. 15).

For the example shown in Fig. 15, when cohesion changed from 0 to 15 kPa, dy increased only from 1.2 to 2.4 m. This is of no significance in terms of the error in the factor of safety. The same conclusion can also be drawn by comparing the curves shown in Fig. 8. The curves were close together when D'/H was equal, even though the values of $c'/\gamma H$ were quite different.

Discussion of the results

In total, 133 slopes geometries ranging from 1.5:1 to 4:1 were studied. A wide range of geometries and soil parameters was selected for various aspects of the study. These variables varied as follows: R = 17.5, 20, 25, and 30 m; S = 5–10 m; α = 0–26.7°; β = 18–34°; c' = 0–15 kPa; ϕ' = 10–20°; and $c'/\gamma H$ = 0.025, 0.050, and 0.075.

An attempt was made to assimilate all the data in terms of dy/R versus the percentage difference in the factor of

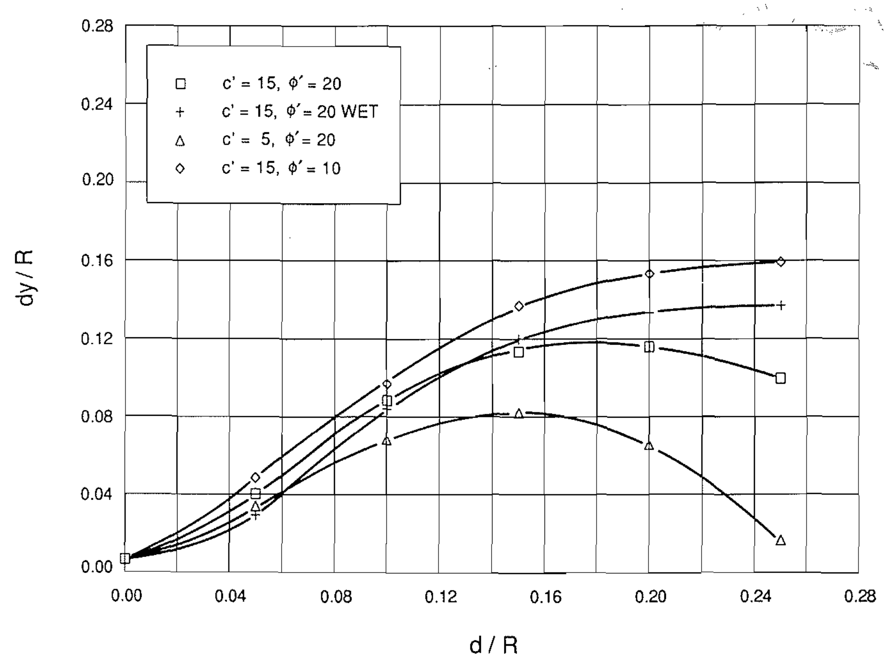

FIG. 14. Effects of c' and ϕ' values on variable dy for a 2:1 slope with D'/H = 1.5 and R = 20.

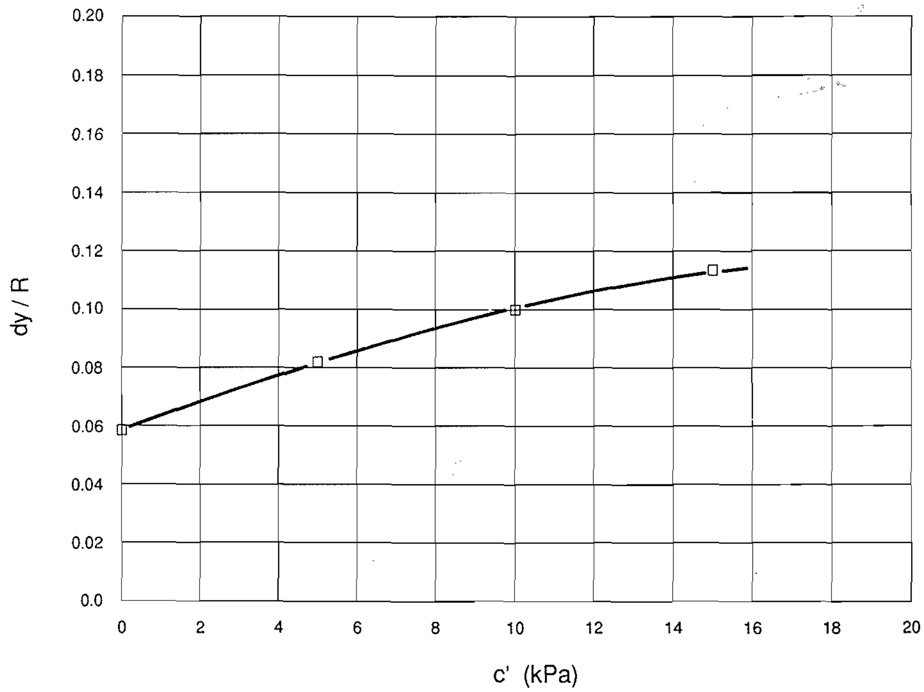

FIG. 15. Effect of c' value on the variable dy.

safety (i.e., the difference in the factor of safety if the rotation center for the circular portion is used as the moment center). The results show the overall influence of all factors mentioned (Fig. 16). The percentage difference in the factor of safety increases with an increase in dy. The percentage approaches a limit at larger dy values. For all the 133 slope geometries analyzed, the percentage difference in the factor of safety was no larger than about 12%; the highest dy value exceeded 20 m. The higher percentage difference in the factor of safety appeared when the radius was large. When $R \leq$ 20 m; $D'/H \leq 1.5$; with the other factor varying, the percentage difference in the factor of safety was never larger than 8%.

Conclusions

(1) Methods of slices satisfying complete statical equilibrium (e.g., the Morgenstern-Price, Spencer's and GLE methods) are not affected by the position of axis of moments. On the contrary, all simplified methods satisfying moment equilibrium are affected by the position of the axis of moments. This study shows that for the Ordinary method,

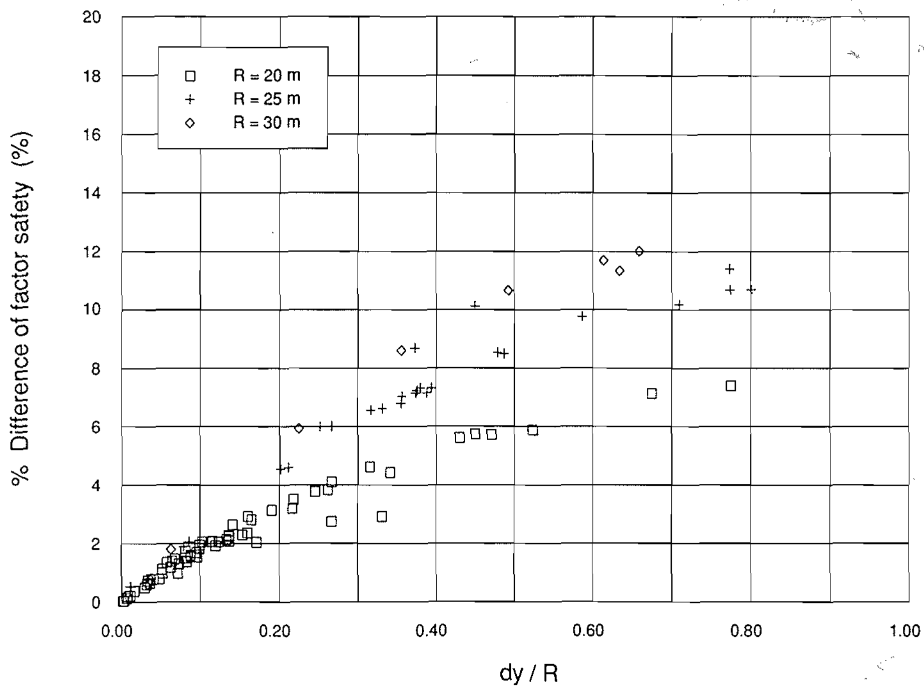

FIG. 16. Percent difference in the factor of safety using the rotation center as center for moment equilibrium.

TABLE 1. Effect of slope steepness on dy values

	dy (m) at the following slopes			
d (m)	1.5:1	2:1	3:1	4:1
1	0.72	0.81	1.05	1.44
2	1.48	1.77	2.70	3.42
3	1.91	2.27	2.82	5.33
4	1.77	2.32	3.20	6.62

NOTE: The dy were obtained from the slope with the variables S = 5–10 m, H = 5–10 m, R = 20 m, c' = 15 kPa, and ϕ' = 20°.

the best position of the axis for moment equilibrium is not the same as the center of rotation for the slip surface. This is also true for the simplest slope geometry with a circular slip surface.

(2) When Bishop's simplified method is used, the same factor of safety can be obtained as with the GLE method by using the center of rotation as the axis moment equilibrium, when (*i*) the slip surfaces are circular; and (*ii*) the geometry is that of a semi-infinite slope (e.g., bedrock parallels the slope surface).

(3) For other conditions, the best axis for moment equilibrium is different than the center of rotation for the circular position. For all the cases analyzed using Bishop's simplified method, the results verified that the factor of safety was only affected by moving the axis of moments in a vertical direction. Therefore, dy alone can be used to define the position of the moment axis corresponding to the difference between the center of rotation for the circular portion and the center of moment equilibrium. The analyses also showed that for all cases of a composite slip surface, positive dy values were obtained. In other words, the best center of moment equilibrium is always above the center of rotation.

(4) It is possible to extend the utilization of Bishop's simplified method to slopes with a composite slip surface. The results showed that the center of rotation of the circular portion can be used as the center for moment equilibrium, with the extreme error in the factor of safety being less than 12%. Except in the cases with D'/H = 1.25, negligible dy values were observed. All the factors of safety obtained by using the center of rotation are on the conservative side.

(5) For a shallow slip surface (for example, R < 20 m or $D'/H \leq 1.5$), the error was negligible (less than 2–3%) for cases with horizontal bedrock. The differences were never larger than 8% for any combination of the geometry of slope and the slip surface. For deep-seated, slightly composite slip surfaces ($R \leq 30$ m, $d/R \leq 0.4$), the error in the factor of safety was less than 8%.

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Appendix. Formulation of the factor of safety equation based on moment equilibrium

Definition of problem

Figure A1 shows the forces that must be defined for a general slope stability problem. The variables associated with each slice are defined as follows: W = total weight of the slice of width b and height h; N = total normal force on the base of the slice over a length β; S_m = shear force mobilized on the base of the slice, it is a percentage of the shear strength as defined by the Mohr–Coulomb equation, i.e., $S_m = \beta[c' + (N/\beta - u) \tan \phi']/F_s$, where c' = effective cohesion parameter, ϕ' = effective angle of internal friction, F_s = factor of safety, and u = pore-water pressure; R = radius or the moment arm associated with the mobilized strength force S_m; f = perpendicular offset of the normal force from the center of rotation; x = horizontal distance from the slice to the center of rotation; α = angle between the tangent to the center of the base of each slice and the horizontal; E = horizontal interslice forces; L = subscript designating left side; R = subscript designating right side, and X = vertical interslice forces.

Derivations for factor of safety

The elements of statics that can be used to derive the factor of safety are summations of forces in two directions and the summation of moments. These, along with the failure criterion, are insufficient to make the problem determinate. More information must be known about either the normal force distribution or interslice force distribution. Either additional elements of physics or an assumption must be invoked to render the problem determinate. Most methods, such as the Ordinary, Bishop's simplified, Spencer's, the Janbu's simplified, and General Limit Equilibrium methods, use the latter assumptions.

Bishop's simplified method, for instance, neglects the interslice shear forces. It thus assumes that a normal or horizontal force adequately defines the interslice forces (Bishop 1955). The normal force on the base of each slice is derived by summing forces in a vertical direction:

$$\text{[A1]} \quad \Sigma F_v = 0$$

$$W - N \cos \alpha - S_m \sin \alpha = 0$$

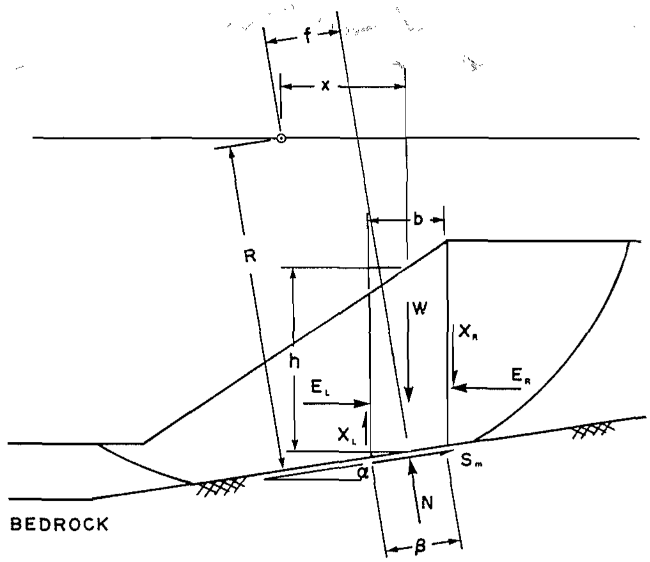

FIG. A1. Forces acting for the method of slices applied to a composite sliding surface.

Substituting the failure criterion for S_m and solving for the normal force gives

$$\text{[A2]} \quad N = \left(W - \frac{c'\beta \sin \alpha}{F_s} + \frac{u\beta \tan \phi' \sin \alpha}{F_s} \right) \Big/ m_\alpha$$

where $m_\alpha = \cos \alpha + (\sin \alpha \tan \phi')/F_s$.

The factor of safety F_s is derived from the summation of moments about a common point. The common point could be any arbitrary point or it could be the real center of rotation for the entire mass:

$$\text{[A3]} \quad \Sigma M_0 = 0$$

$$\Sigma Wx - \Sigma S_m R - \Sigma Nf = 0$$

substituting the normal force N from [A2] and the failure criterion for S_m gives the factor of safety equation:

$$\text{[A4]} \quad F_s = \frac{\Sigma[c'\beta R + (N - u\beta)\tan \phi']}{\Sigma Wx - \Sigma Nf}$$

The factor of safety equation (eq. [A4]) is applicable for all the methods of slices satisfying overall moment equilibrium. However, the definition of the normal force is different in each method.