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MECHANICS OF SOILS WITH MATRIC SUCTION

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Abstract: This paper illustrates several geotechnical problems involving soils with negative pore-water pressures or matric suction. The undrained shear strength commonly used in the total stress analyses for bearing capacity and slope stability is a function of the matric suction of the soil. The paper illustrates the role of matric suction in affecting design. Two types of matric suction measuring devices are also outlined in the paper.

Introduction

Soils in many parts of the world are classified as being expansive, collapsible or residual. These soils are commonly unsaturated and the pore-water pressures are negative relative to the pore-air pressure. The pore-water pressures can be highly negative for soils in arid and semi-arid regions or in conditions where the groundwater table is deep. In addition, pore-water pressures in soils can become negative as a result of the process of excavating, remolding or recompacting the soils. This negative pore-water pressure when referenced to the pore-air pressure is referred to as matric suction.

Climate plays an important role in affecting the water content of soils in the proximity of the ground surface. Water is removed from the soil either by evaporation from the ground surface or by evapo-transpiration from a vegetative cover (Fig. 1). These processes produce an upward flux of water out of the soil. On the other hand, rainfall and other forms of precipitation provide a downward flux into the soil. The difference between these two flux conditions on a local scale largely dictates the pore-water pressure conditions in the soil. The negative pore-water pressure distribution with depth can take on a wide variety of shapes as a result of environmental changes (Fig. 1).

The shear strength and volume change behavior of soils with negative pore-water pressures (or matric suctions) are best described in terms of two independent stress state variables, namely $(\sigma - u_a)$ and $(u_a - u_w)$ [3]. The terms $(\sigma - u_a)$ and $(u_a - u_w)$ are referred to as net normal stress and matric suction, respectively, where: σ = total normal stress, u_a = pore-air pressure and u_w = pore-water pressure. The stress state variables can be shown to have a smooth transition when going from an unsaturated soil condition to a saturated soil condition. As the degree of saturation approaches 100 percent, the pore-water pressure approaches the pore-air pressure. The matric suction term goes to zero. The net normal stress reverts to $(\sigma - u_w)$. The term $(\sigma - u_w)$, is commonly referred to as effective stress in saturated soil mechanics.

This paper illustrates the effect of matric suction on the shear strength of unsaturated soils. As an example, changes in the negative pore-water pressures during heavy rainfalls, have been associated with numerous slope failures. Figure 2 shows a typical steep slope with a deep groundwater table. Most or all of a potential slip surface may lie above the groundwater table within the unsaturated soil zone. In other words, the shear strength along the potential slip surface is a function of the matric suction of the soil. Water infiltration into the slope causes a reduction in the matric suction which in turn reduces the shear strength.

Another example is the calculation of the bearing capacity of soils above the water table (Fig. 3). In many cases, the water table is at a considerable depth and the soil below the footing has a negative pore-water pressure. Unconfined compression tests are routinely performed on undisturbed samples which are held intact by negative pore-water pressures. In other words, the matric suction of the soil affects the measured shear strength which consequently influences the computed bearing capacity. Subsequent wetting and drying of the soil produce changes in the matric suction and shear strength, and consequently in the bearing capacity of the soil.

Vertical or near vertical excavations are often used for the installation of a foundation or a pipeline (Fig. 4). It is well known that the backslope in a moist clayey soil will stand at a near vertical slope for some time before failing. The negative pore-water pressures in the soil has an important role in maintaining the stability of the backslope. An increase in the pore-water pressure (or a decrease in the matric suction) is the primary factor contributing to the instability of the excavation.

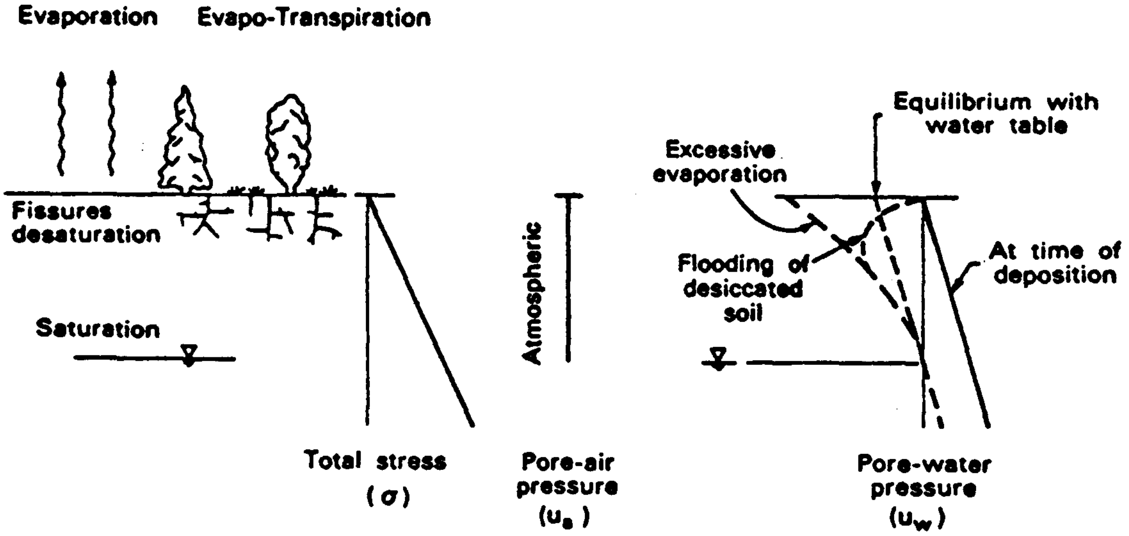

Figure 1 Stress distribution above the water table

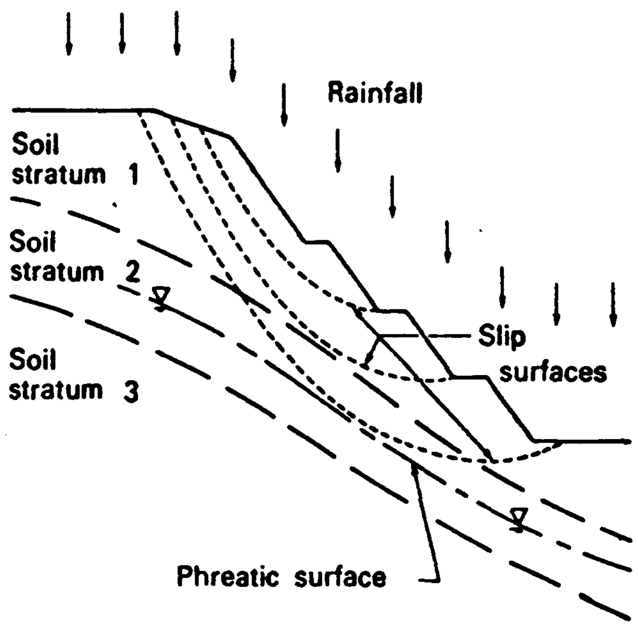

Figure 2 The effect of excavations on a natural slope subjected to environmental changes

The above examples are presented in order to illustrate the importance of understanding the behavior of soils with matric suction. The following sections present the shear strength theory and its application to a total stress analyses for bearing capacity and slope stability problems.

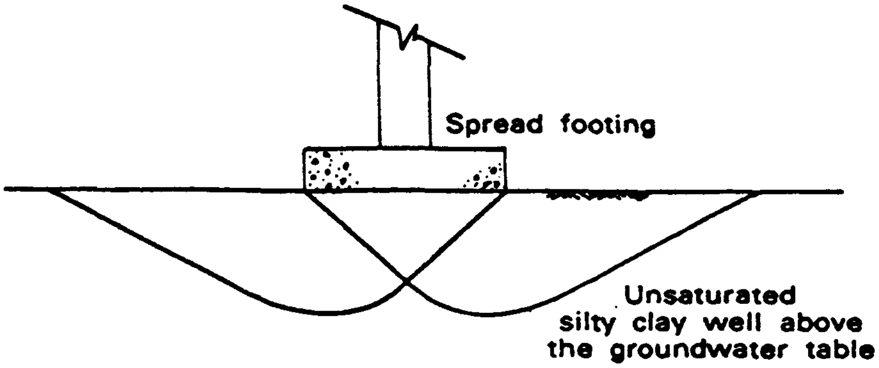

Figure 3 Shallow footing placed on unsaturated soils

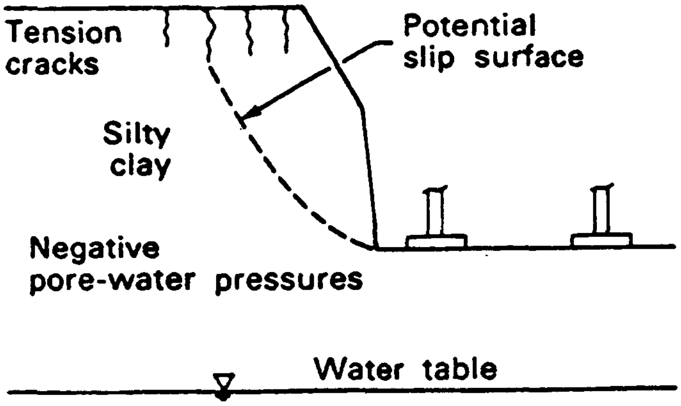

Figure 4 Stability of a near vertical excavation used in the construction of a foundation

Shear Strength Theory

The shear strength equation for an unsaturated soil has the following form [4]:

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b \quad [1]$$

where:

- τ_{ff} = shear stress on the failure plane at failure
- c' = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis when the net normal stress and the matric suction at failure are equal to zero. It is also referred to as the "effective cohesion".
- $(\sigma_f - u_a)_f$ = net normal stress on the failure plane at failure
- ϕ' = angle of internal friction associated with the net normal stress state variable, $(\sigma_f - u_a)_f$
- $(u_a - u_w)_f$ = matric suction at failure
- ϕ^b = angle indicating the rate of change in shear strength relative to changes in matric suction, $(u_a - u_w)_f$

The failure envelope associated with Eq. 1 is referred to as the extended Mohr-Coulomb failure envelope as illustrated in Fig. 5. The shear strength of an unsaturated soil is considered to consist of an effective cohesion, c', and independent contributions from net normal stress, $(\sigma - u_a)$, and matric suction, $(u_a - u_w)$. The shear strength contributions from net normal stress and matric suction are characterized by ϕ' and ϕ^b angles, respectively. Table 1 summarizes various shear strength parameters associated with different types of soil

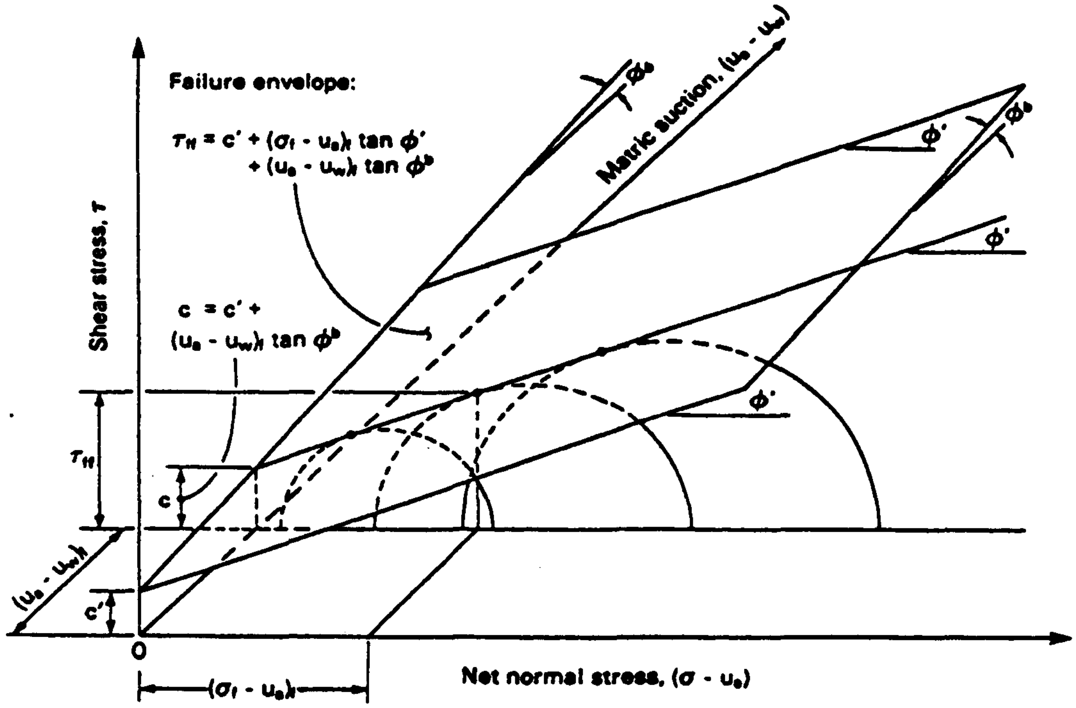

Figure 5 Extended Mohr-Coulomb failure envelope

from various geographic locations. These parameters (i.e., c', ϕ' and ϕ^b) can be obtained by performing drained shear tests on a soil. It appears that the value of the ϕ^b angle is consistently less than or equal to the ϕ' angle.

Undrained shear strength

The undrained shear strength, c_u, is commonly used in geotechnical practice in connection with the total stress design approach. However, the engineer does not normally view his design as one involving the behavior of soils with matric suction. Let us assume that the site under consideration involves a clayey soil with a groundwater table well below the proposed depth for the footings. Typical engineering practice can be described as follows. A field investigation is conducted in which samples are obtained at predetermined depths. The samples are brought to the laboratory where they are extruded and tested for their unconfined compressive strength. The unconfined compressive strength is divided by 2 to give an undrained shear strength for the soil, c_u, while the friction angle is taken as zero.

It is easy for an engineer to lose sight of the fact that the soil has a matric suction (or a negative pore-water pressure) which is holding the soil together. The matric suction in the specimen tested in the laboratory is a function of the insitu negative pore-water pressure and the change in pore-water pressure resulting from unloading the soil during sampling (i.e., releasing $(\sigma - u_a)$ to zero). Therefore, the measured undrained shear strength, c_u, is essentially a function of the matric suction and the net normal stress in the soil. Figure 6 illustrates a possible stress path followed during an unconfined compression test (i.e., stress

path followed during an unconfined compression test (i.e., stress path $\overline{AB}$ where the matric

Table 1 Experimental Values of ϕ^b

Soil Type	c' (kPa)	ϕ' (degrees)	ϕ^b (degrees)	Test Procedure	Reference	
Compacted shale ; w = 18.6%	15.8	24.8	18.1	Constant water content	Bishop, Alpan, Blight and Donald (1960)	[1]
Boulder clay; w = 11.6%	9.6	27.3	21.7	Constant water content triaxial	Bishop, Alpan, Blight and Donald (1960)	[1]
Dhanauri clay; w = 22.2%, ρ_d = 1580 kg/m^3	37.3	28.5	16.2	Consolidated drained triaxial	Satija, (1978)	[9]
Dhanauri clay; w = 22.2%, ρ_d = 1478 kg/m^3	20.3	29.0	12.6	Consolidated drained triaxial	Satija, (1978)	[9]
Dhanauri clay; w = 22.2%, ρ_d = 1580 kg/m^3	15.1	28.5	22.6	Constant water content triaxial	Satija, (1978)	[9]
Dhanauri clay; w = 22.2%, ρ_d = 1478 kg/m^3	11.3	29.0	16.5	Constant water content triaxial	Satija, (1978)	[9]
Madrid grey clay; w = 29%	23.7	22.5[a]	16.1	Consolidated drained direct shear	Escario, (1980)	[2]
Undisturbed decomposed granite; Hong Kong	28.9	33.4	15.3	Consolidated drained multi stage triaxial	Ho and Fredlund (1982a)	[6]
Undisturbed decomposed rhyolite; Hong Kong	7.4	35.3	13.8	Consolidated drained multi stage triaxial	Ho and Fredlund (1982a)	[6]
Tappen-Notch Hill silt; w = 21.5%, ρ_d = 1590 kg/m^3	0.0	35.0	16.0	Consolidated drained multi stage triaxial	Krahn, Fredlund and Klassen, (1987)	[8]
Compacted glacial till; w = 12.2%, ρ_d = 1810 kg/m^3	10	25.3	7 - 25.5	Consolidated drained multi stage direct shear	Gan, Fredlund and Rahardjo, (1988)	[5]

[a] Average value

$$c_u \approx c' + (\sigma_f - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b \qquad [2]$$

where

c_u = undrained shear strength

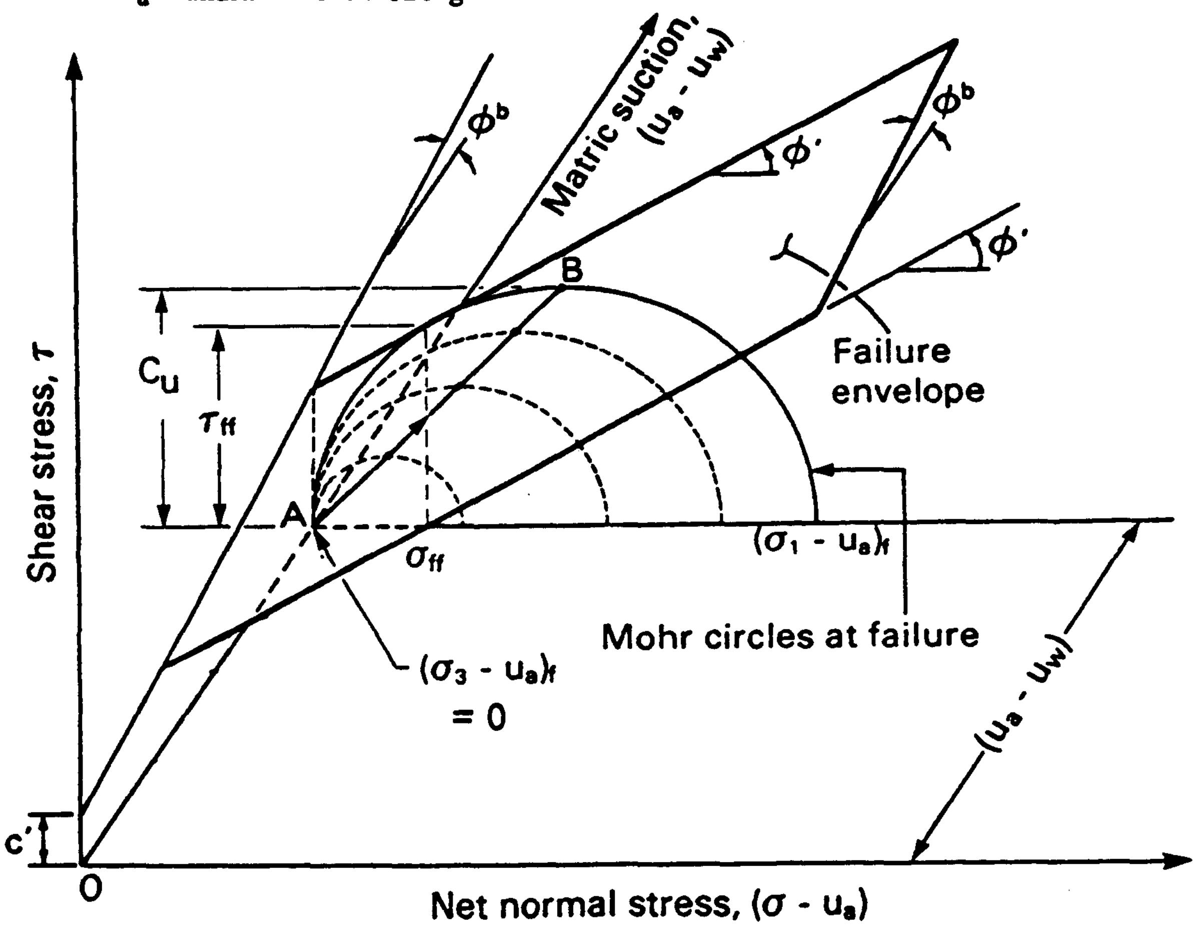

Figure 6 Possible stress paths followed during an unconfined compression test

Example Problems of Application

Bearing capacity and slope stability analyses of clayey soils are sometimes performed using the undrained shear strength, c_u. The following two examples demonstrate the role of matric suction in affecting the values of c_u and consequently the soil bearing capacity and the stability of slopes. The insitu matric suction can increase or decrease in response to changes in the climatic conditions such as evaporation and precipitation. As a result, the undrained shear strength will also change and its change can be expressed as follows:

$$\Delta c_u = \Delta (u_a - u_w) \tan \phi^b \qquad [3]$$

where

Δc_u = change in undrained shear strength due to matric suction change

$\Delta (u_a - u_w)$ = change in matric suction due to drying and wetting

Bearing Capacity

The ultimate bearing capacity of clay can be related to its undrained shear strength as,

$$q_f = c_u N_c \qquad [4]$$

where:

q_f = ultimate bearing capacity
N_c = cohesion bearing capacity factor (i.e., $N_c = 5.14$ for strip footing and $N_c = 6.17$ for square or circular footing [10]

Let us consider a clay with an initially measured undrained shear strength of c_{uo} and an initial ultimate bearing capacity of q_{fo} (i.e., $N_c\ c_{uo}$).

In the field, a change in matric suction, $\Delta\ (u_a - u_w)$ (increase or decrease), will result in a change in the undrained shear strength, Δc_u, as expressed in Eq. 3. As a result, a change in the ultimate bearing capacity, $\Delta\ q_f$ (i.e., $N_c\ \Delta\ c_u$), will occur and the final bearing capacity can be written as,

$$q_f = q_{fo} + \Delta\ q_f \qquad [5]$$

where

q_{fo} = initial ultimate bearing capacity (i.e., $N_c\ c_{uo}$)
$\Delta\ q_f$ = change in the bearing capacity (i.e., $N_c\ \Delta\ c_u$)

Figure 7 illustrates the possible variation in the ultimate bearing capacity of a clay due to matric suction changes.

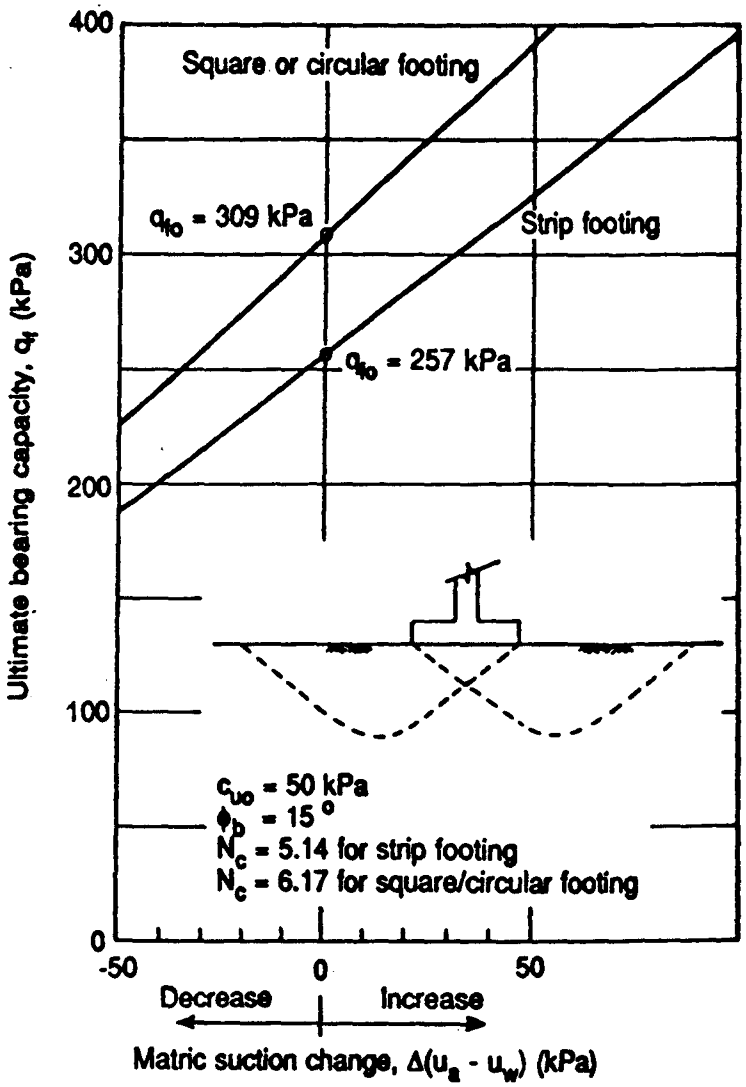

Figure 7 Variation in the ultimate bearing capacity of a clay due to matric suction changes

The clay has an initial measured undrained shear strength, c_{uo}, of 50 kPa and a ϕ^b angle of 15 degrees. The initial computed bearing capacity of the clay, q_{fo}, is equal to 257 kPa for a strip footing or 309 kPa for a square/circular footing. An average change in matric suction is assumed in order to compute the changes in the ultimate bearing capacity of the clay. The ϕ^b angle is assumed to remain constant. Figure 7 shows that an increase in the soil matric suction will increase the bearing capacity while a decrease in the matric suction reduces the bearing capacity.

The percent change in the ultimate bearing capacity can be related to the change in matric suction by combining Eqs. 3 and 5 and assuming a constant ϕ^b angle.

$$\frac{\Delta q_f}{q_{fo}} = \frac{\Delta (u_a - u_w)}{c_{uo}} \tan \phi^b \qquad [6]$$

where

$\Delta q_f / q_{fo}$ = percent change in the ultimate bearing capacity

$\Delta (u_a - u_w) / c_{uo}$ = percent change in the matric suction with respect to the initial undrained shear strength, c_{uo}.

Equation 6 is plotted in Fig. 8 for various ϕ^b values. The relationship is applicable to all shapes of footing since it is not a function of N_c. For a ϕ^b value of 15 degrees, the ultimate bearing capacity will increase or decrease by 27% when the matric suction changes as much as the initial undrained shear strength (i.e., $\Delta (u_a - u_w) = 100\% \; c_{uo}$). The higher the ϕ^b values, the higher will be the percent change in the ultimate bearing capacity.

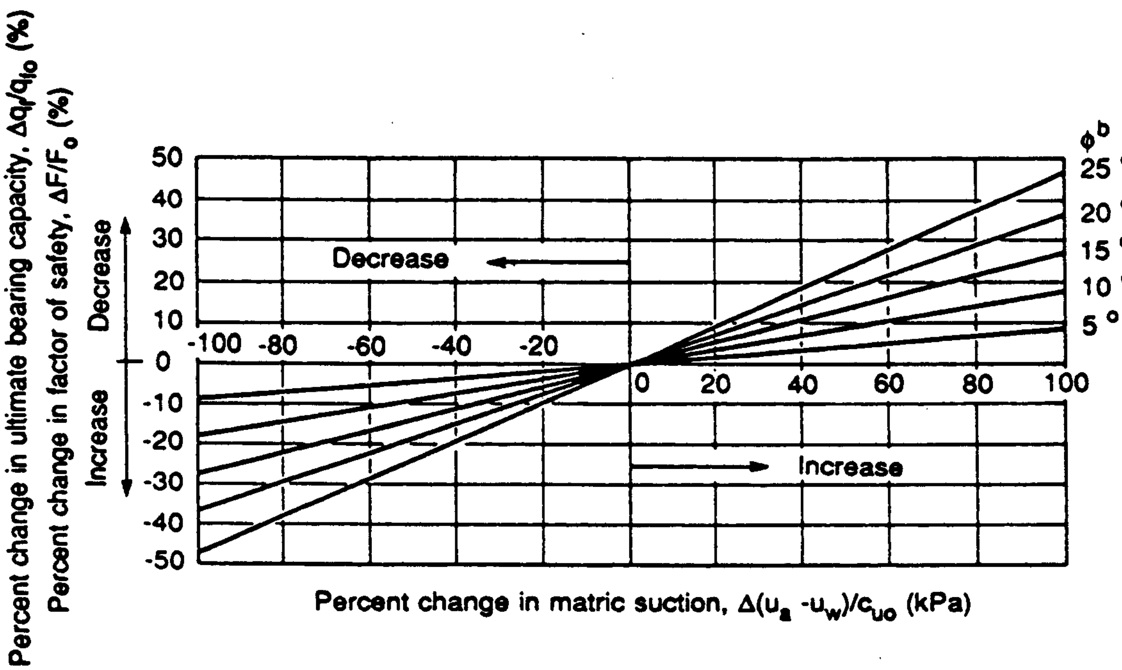

Figure 8 Variation in the ultimate bearing capacity and factor of safety with respect to the variation in matric suction in clay

<u>Slope Stability</u>

The factor of safety of a clay slope can be related to the undrained shear strength of the clay when computed in accordance with the total stress approach (Janbu, 1954)

$$F = N_o \frac{c_u}{\gamma H} \qquad [7]$$

where

- F = factor of safety of a slope
- N_o = stability number which is a function of slope angle and the depth of the impenetrable layer. For example, N_o is equal to 5.53 for slope angles from 0 to 54 degrees when the impenetrable layer is at an infinite depth.
- γ = unit weight of the soil
- H = height of the slope

Let us consider a clay slope with an initially measured undrained shear strength of c_{uo} and an initial factor of safety of F_o (i.e., $N_o c_{uo} / \gamma H$). A change in matric suction, $\Delta (u_a - u_w)$, will result in a change in the undrained shear strength, Δc_u, as given in Eq. 3. Consequently, the factor of safety of the slope will also change by ΔF (i.e., $N_o \Delta c_u / \gamma H$) and the final factor of safety becomes

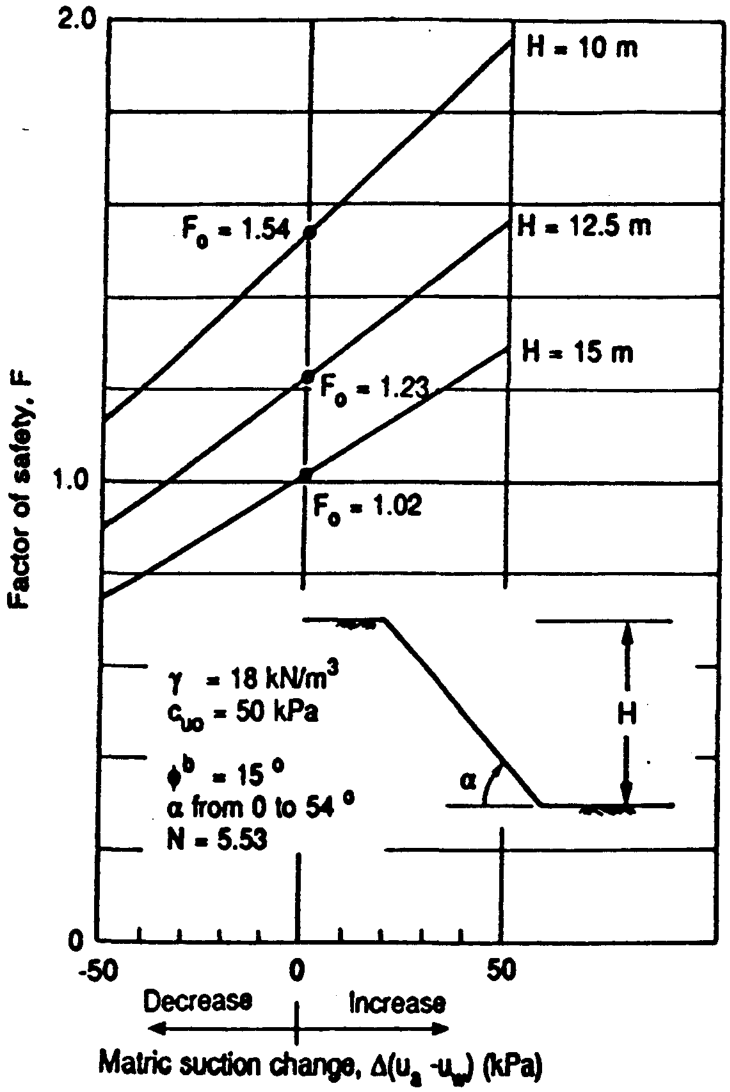

Figure 9 Variation in the factor of safety of a clay slope due to matric suction changes

$$F = Fo + \Delta F \quad [8]$$

where

F_o = initial factor of safety (i.e., $N_o\, c_{uo} / \gamma H$)

ΔF = change in the factor of safety (i.e., $N_o\, \Delta\, c_u / \gamma H$)

Figure 9 illustrates the possible variation in the factor of safety of a clay slope as a result of changes in the matric suction of the soil. The clay has an initially measured undrained shear strength, c_{uo}, of 50 kPa, a ϕ^b angle of 15 degrees and a unit weight of 18 kN/m^3. The initially computed factors of safety of the clay, F_o, are equal to 1.54, 1.23 and 1.02 for slope heights of 10 m, 12.5 m and 15 m, respectively. An average change in matric suction is assumed in order to compute the changes in the factor of safety of the slope. The ϕ^b angle is also assumed to remain constant. Figure 9 shows that an increase in the soil matric suction will increase the factor of safety while a decrease in the matric suction reduces the factor of safety. In some cases, the factor of safety is reduced to 1.0 or lower and this will cause slope instability. In addition, it appears that the factor of safety becomes more sensitive to changes in the matric suction as the height of the slope decreases.

The percent change in the factor of safety can be related to the change in matric suction by combining Eqs. 3 and 8 and assuming a constant ϕ^b angle.

$$\frac{\Delta F}{F_o} = \frac{\Delta\,(u_a - u_w)}{c_{uo}} \tan \phi^b \quad [9]$$

where

$\Delta F/F_o$ = percent change in the factor of safety

Equation 9 is similar to Eq. 6 and therefore Fig. 8 is also applicable to Eq. 9. The relationship given in Fig. 8 is applicable to all angles and heights of slopes since it is not a function of N_o. The higher the ϕ^b values, the higher will be the percent change in the factor of safety.

Measuring Matric Suction

The preceding discussions have illustrated the significance of matric suction (or negative pore-water pressures) in controlling the shear strength of unsaturated soils. Therefore, it is of prime importance to be able to measure matric suction (or negative pore-water pressures) in the soil.

Two devices for the measurement of matric suction; namely, tensiometer and thermal conductivity sensor, will be briefly outlined herein.

A tensiometer measures the negative pore-water pressure in a soil. The tensiometer consists of a porous ceramic, high air-entry cup connected to a pressure measuring device through a small bore tube. The tube is usually made from plastic due to its low heat conduction and non-corrosive nature. The tube and the cup are filled with deaired water. The cup can be inserted into a precored hole until there is good contact with the soil.

Once equilibrium is achieved between the soil and the measuring system, the water in the tensiometer will have the same negative pressure as the pore-water in the soil. The pore-water pressure that can be measured in a tensiometer is limited to approximately negative 90 kPa due to the possibility of cavitation of the water in the tensiometer. The measured negative pore-water pressure is numerically equal to the matric suction when the pore-air pressure is atmospheric (i.e., u_a = zero gauge pressure). When the pore-air pressure is greater than atmospheric pressure the tensiometer reading can be added to the ambient pore-air pressure reading to give the matric suction of the soil. The measured matric suction must not exceed the air entry value of the ceramic cup.

A water reservoir is provided at the top of the tensiometer tube for the purpose of removing the air bubbles. Air bubbles may develop within a tensiometer due to several possible reasons. Dissolved air may come out of solution as the water pressure decreases to a negative value. Pore-air may diffuse through the water in the ceramic cup and come out of solution inside the tube. When the water pressure approaches the vapor pressure of water at the ambient temperature, water molecules can move freely from the liquid to the vapor form (i.e., cavitation occurs). It is important to have an air-free tensiometer tube in order to ensure correct readings and rapid responses.

A thermal conductivity sensor measures the matric suction indirectly and it consists of a porous ceramic block containing a temperature sensing element and a miniature heater (Fig. 10). The thermal conductivity of the porous block varies in accordance with the water content in the block. The water content in the porous block is dependent upon the matric suctions applied to the block by the surrounding soil. Therefore, the thermal conductivity of the porous block can be calibrated with respect to the applied matric suction.

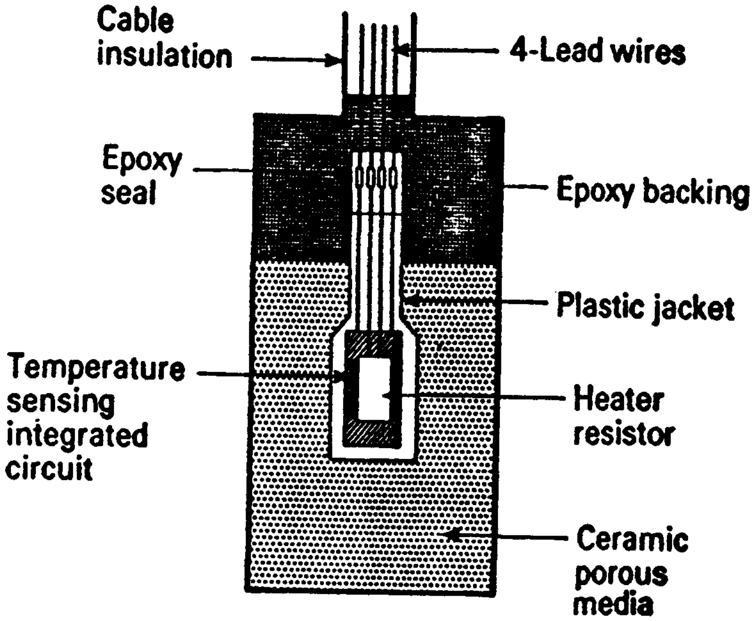

Figure 10 A cross-sectional diagram of the AGWA-II thermal conductivity sensor

A calibrated sensor can then be used to measure the matric suction by placing the sensor in the soil and allowing it to come to equilibrium with the state of stress in the pore-water (i.e., the matric suction of the soil). Thermal conductivity measurements at equilibrium are an indication of the matric suction of the soil.

Thermal conductivity measurements are performed by measuring heat dissipation within the porous block. A controlled amount of heat is generated by the heater at the center of the block. A portion of the generated heat will be dissipated throughout the block. The amount of heat dissipation is controlled by the presence of water within the porous block. The change in the thermal conductivity of the sensor is directly related to the change in water content of the block. In other words, more heat will be dissipated as the water content in the block increases.

The undissipated heat will result in a temperature rise at the center of the block. The temperature rise is measured by the sensing element after a specified time interval and its magnitude is inversely proportional to the water content of the porous block.

It has been found that relatively accurate measurements of matric suctions can be expected from the AGWA-II sensor in the range of 0 to 175 kPa.

Conclusions

Matric suction plays an important role in controlling the shear strength of an unsaturated soil. The reliability of the undrained shear strength of an unsaturated soil depends on the variation of matric suction with respect to changes in the local climate and loading conditions. As a result, the bearing capacity of a soil or its slope stability will also be affected by matric suction variations in the field. Therefore, a knowledge of the possible range of matric suction change in the field will assist engineers in predicting the possible variations in their design factor of safety.

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