June 9-12, Boulder, Colorado. (1991)

MODELLING VERTICAL GROUND MOVEMENTS USING SURFACE CLIMATIC FLUX

Pamela J. Sattler[1] and Delwyn G. Fredlund[2]

ABSTRACT:

An integrated numerical model has been developed which illustrates the relationship between one-dimensional vertical ground movements and meteorological observations. The model consists of four components, namely: 1) a ground movement component, 2) a seepage component, 3) a surface flux boundary condition component, and 4) a Thornthwaite evapotranspiration component. All aspects of the model are presented with an emphasis placed on the surface flux boundary component.

The Thornthwaite potential evapotranspiration computations proved adequate for a first approximation analysis. Based upon several long-term averaging procedures, the actual evapotranspiration was computed to be approximately 70% of the potential evapotranspiration.

The numerical model illustrated the relationship between vertical ground movement and changes in matric suction. The first approximation analysis revealed that it was necessary to consider the infiltration and exfiltration processes separately.

INTRODUCTION

A numerical model has been developed which quantifies the relationship between one-dimensional vertical ground movements (i.e., heave and shrinkage) and meteorological observations. The model is divided into four components as shown in Figure 1. The ground movement model computes the change in the thickness of a soil layer in accordance with a constitutive function relating void ratio and matric suction. The time dependency of ground movements are in accordance with a Darcian type seepage model.

The surface flux boundary condition required for the seepage model is computed from the difference between precipitation and actual

---

[1] Research Engineer, Dept. of Civil Engineering, Univ. of Saskatchewan, Saskatoon, SK, CANADA S7N 0W0

[2] Prof., Dept. of Civil Engineering, Univ. of Saskatchewan, Saskatoon, SK, CANADA S7N 0W0

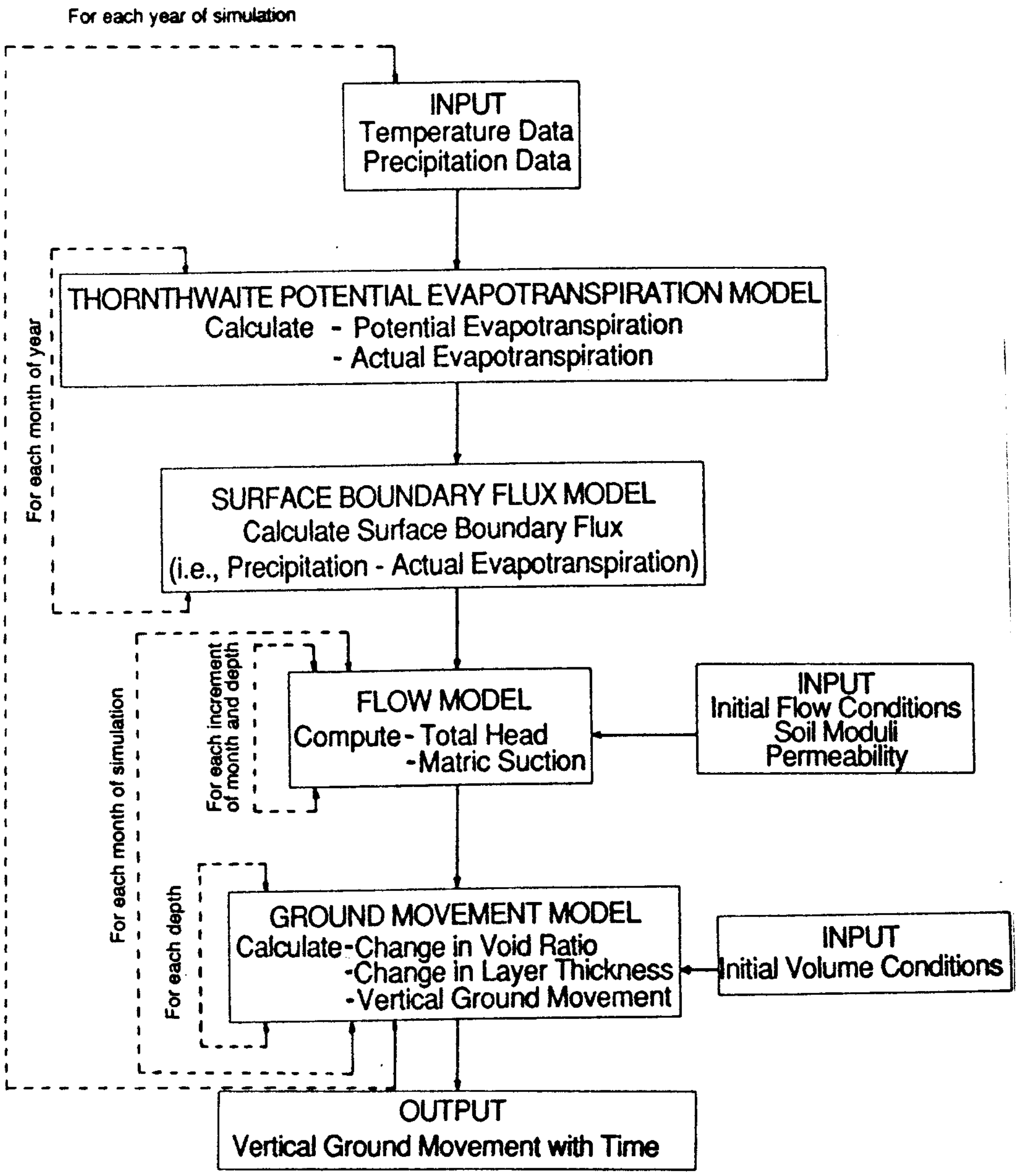

Figure 1. Flowchart for the computer model used to predict vertical ground movements with depth and time

evapotranspiration. Input to the influx portion of the surface flux boundary model is computed from the Thornthwaite (1948) computations for potential evapotranspiration. Based upon several long-term averaging procedures, the actual evapotranspiration was computed to be approximately 70% of the potential evapotranspiration.

This paper describes the one-dimensional numerical model with emphasis placed on the Thornthwaite model and the surface flux boundary model. Further research into quantification of the surface flux boundary condition has recently been initiated (Wilson, 1990).

BACKGROUND

The relationship between meteorological observations and vertical ground movements was illustrated by Hamilton (1965). He summarized the results of two decades of research performed by the Division of Building Research (DBR) of the National Research Council (NRC) of Canada. Vertical ground movements in open fields and beneath building foundations located on expansive soils were measured at various sites. Ground movement measurements were recorded about once a month and water content changes were measured periodically over the span of several years (Hamilton, 1963).

Building slabs were observed to exhibit a gradual heave over long periods of time whereas open-field test plots exhibited more of an upward and downward seasonal movement (Hamilton, 1968). Figure 2 illustrates one typical record of vertical ground movements for an open-field test plot at Regina, Saskatchewan. Increases in water content were observed to accompany movement measurements. During wet periods, both open-field plots and building slabs exhibited upward movement. During dry periods in which vegetation demands become predominant, concrete slab elevations remained unchanged whereas open-field gauges indicated downward movement. It was concluded that there was a relationship between meteorological observations and vertical ground movements.

THE DEVELOPMENT OF A MODEL

The observations recorded by Hamilton (1965) were not placed within the context of a theoretical model. Advances from the study of unsaturated soil behavior during the 1970's have made a mathematical model a logical extension to the field measurements.

Fredlund and Morgenstern (1977) suggested the use of two independent stress state variables to describe the engineering behavior of an unsaturated soil; namely, i) net normal stress $(\sigma - u_a)$ and ii) matric suction, $(u_a - u_w)$ where σ = total stress, u_a = pore-air pressure and u_w = pore-water pressure. In other words, the matric suction is the negative pore-water pressure referenced to pore-air pressure. Changes in matric suction occur due to changes in the microclimate.

The constitutive equation used to describe shrinking and swelling is written as follows:

$$de = a_m \, d(u_a - u_w) \qquad (1)$$

where:

de = change in void ratio

a_m = coefficient of compressibility with respect to matric suction

Vertical ground movements (i.e., one-dimensional volume changes) are computed by summing the changes in layer thicknesses as follows:

$$\Delta H = \sum_{i=1}^{i=n} \Delta H_i \qquad (2)$$

where:

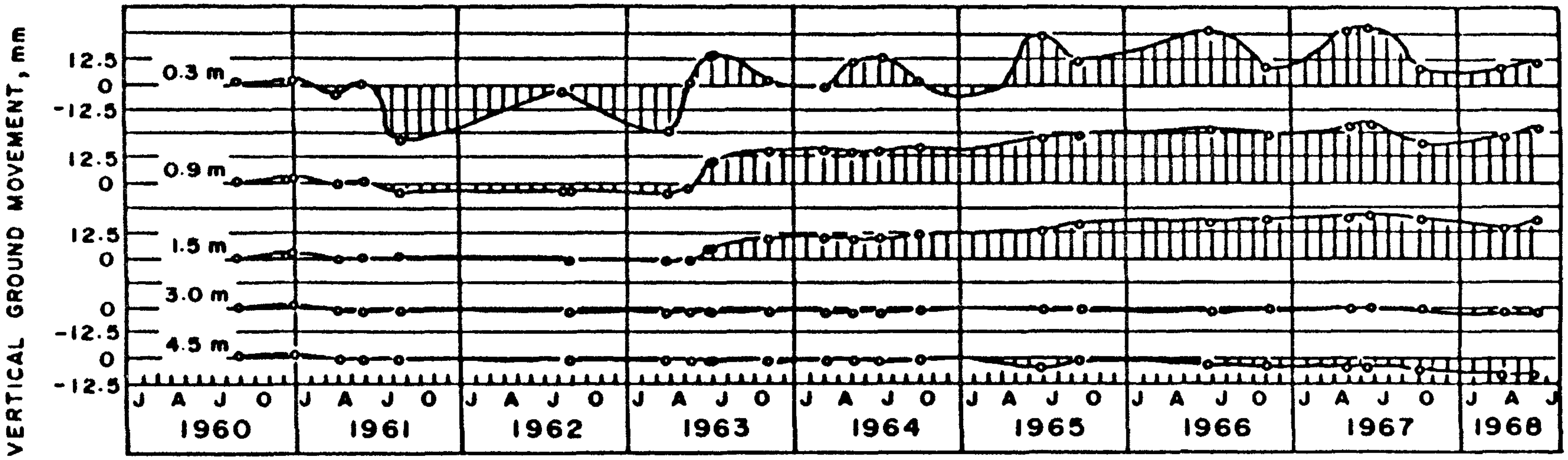

Figure 2. Measured vertical ground movements for an open-field test plot at Regina, Saskatchewan (Hamilton, 1968)

$$\Delta H_i = H_{io}\frac{(\Delta e_i)}{1 + e_{io}} \tag{3}$$

ΔH_i = change in thickness of an individual soil layer
i = number of the soil layer where the soil layers range from 1 to the total number of layers, n
H_{io} = initial layer thickness
Δe_i = change in void ratio for each layer
e_{io} = initial void ratio for each layer

Equations (1), (2), and (3) can be used to compute the vertical ground movement at any depth in the soil. However, the ground movements occur as water becomes available to the soil. The water causes the matric suctions to vary with time as flow occurs from a higher to a lower hydraulic head. The seepage equation is used to describe the flow of water through the soil and thereby solve for the matric suction at any point in space and time. As a first approximation, the transient flow equation can be written:

$$\frac{\partial h}{\partial t} = \frac{k_y}{\rho_w g\, m_2^w}\frac{\partial^2 h}{\partial y^2} \tag{4}$$

where:

h = total hydraulic head, equal to $(Y + \frac{u_w}{\rho_w g})$ where Y is the elevation head above the chosen datum
$\frac{\partial h}{\partial t}$ = the derivative of total hydraulic head with respect to time
k_y = hydraulic conductivity of the soil in the vertical direction
ρ_w = density of water
g = acceleration due to gravity
m_2^w = coefficient of water volume change
$\frac{\partial^2 h}{\partial y^2}$ = the second derivative of total hydraulic head with respect to depth above the datum

The hydraulic conductivity, k_y, for an unsaturated soil is a function of matric suction which in turn is a function of hydraulic head. A more rigorous formulation of the unsaturated soil seepage equation is presented by Lam and Fredlund (1987). However, for the purposes of developing a first approximation model, a linear form of the seepage equation (i.e., Equation (4) above) is proposed. The marching forward finite difference technique was used to solve the transient flow equation.

The volume change modulus, m_2^w, was estimated based upon one-dimensional consolidation data. Values which were used were in the order of 1 x 10^{-2} to 1 x 10^{-5} per kilopascal.

Initial boundary conditions required for the model include the negative pore-water pressure, void ratio, water content, and degree of saturation. Profiles of initial negative pore-water pressures were estimated based upon typical swelling pressure profiles for Regina clay (Yoshida, Fredlund, and Hamilton, 1983) and field matric suction data (van der Raadt, et al., 1987). Profiles of initial void ratio, water content and degree of saturation were established from summaries of statistical properties on Regina clay (Fredlund and Hasan, 1979) and measured water contents at the field locations (Hamilton, 1968).

SURFACE FLUX BOUNDARY CONDITION

For the purpose of first approximation modelling of a flat prairie, it was assumed that all precipitation at the ground surface would contribute to infiltration. Therefore, the surface boundary flux is written as the difference between precipitation and actual evapotranspiration.

Potential Evapotranspiration. Evapotranspiration was defined by Thornthwaite (1948) as "the combined evaporation from the soil surface and transpiration from plants" representing "the transport of water from the earth back to the atmosphere, the reverse of precipitation". Thornthwaite and Mather (1955) defined the term` "potential evapotranspiration" as "the amount of water which will be lost from a surface completely covered with vegetation if there is sufficient water in the soil at all times for the use of the vegetation". A great deal of controversy has surrounded the definition of potential evapotranspiration since in practice actual microclimatic data is used to establish a potential evapotranspiration value (Granger, 1989). However, the main purpose in computing potential evapotranspiration is to provide a means of estimating actual evapotranspiration. Potential evapotranspiration establishes an upper limit for actual evapotranspiration since potential evapotranspiration assumes that there is no limit on the available water and actual evapotranspiration is dependent upon available soil water.

Several methods are available for the estimation of potential evapotranspiration (Thornthwaite and Mather, 1955; Penman, 1963; Priestley and Taylor, 1972). The Thornthwaite method was chosen for its simplicity and applicability to sites where meteorological information may be scarce. The relationship is based on mean monthly temperature and average monthly day length:

$$PE = 0.16\,F\left[\frac{10t}{I}\right]^{a} \qquad (5)$$

where:

PE = the monthly potential evapotranspiration (mm)

F = sunlight duration correction factor based on Thornthwaite and Mather (1955)

t = mean monthly temperature (degrees Celcius)

I = the sum of the twelve monthly heat indices, i

i = $(t/5)^{1.515}$

a = a variable equal to $0.000000675\,I^3 - 0.0000771\,I^2 + 0.01792\,I + 0.49239$

The object of Thornthwaite's research was the classification of climates for geographical purposes. Sanderson (1948) utilized Thornthwaite's method for computing potential evapotranspiration for large regions across Canada (Figure 3). Thornthwaite's method has been used extensively throughout the world for the classification of climates (Sanderson and Rafique, 1979). Classification is based upon a moisture index describing humidity relative to aridity. Figure 4 presents the classification of Canadian climates based upon Thornthwaite's moisture index.

Estimation of Actual Evapotranspiration. Actual evapotranspiration is less than potential evapotranspiration due to a limited amount of water available in the soil. Several methods were investigated in an attempt to establish a relationship between actual and potential evapotranspiration. These are as follows:

i) By Net Groundwater Recharge Observations. Groundwater recharge rates are based upon average conditions over large areas for long periods of time. Zebarth (1988) conducted an extensive literature review of recharge rates for Southern Saskatchewan and concluded that typical recharge rates were in the order of 5% to 10% of annual precipitation. Assuming a surface boundary flux equal to the lower end

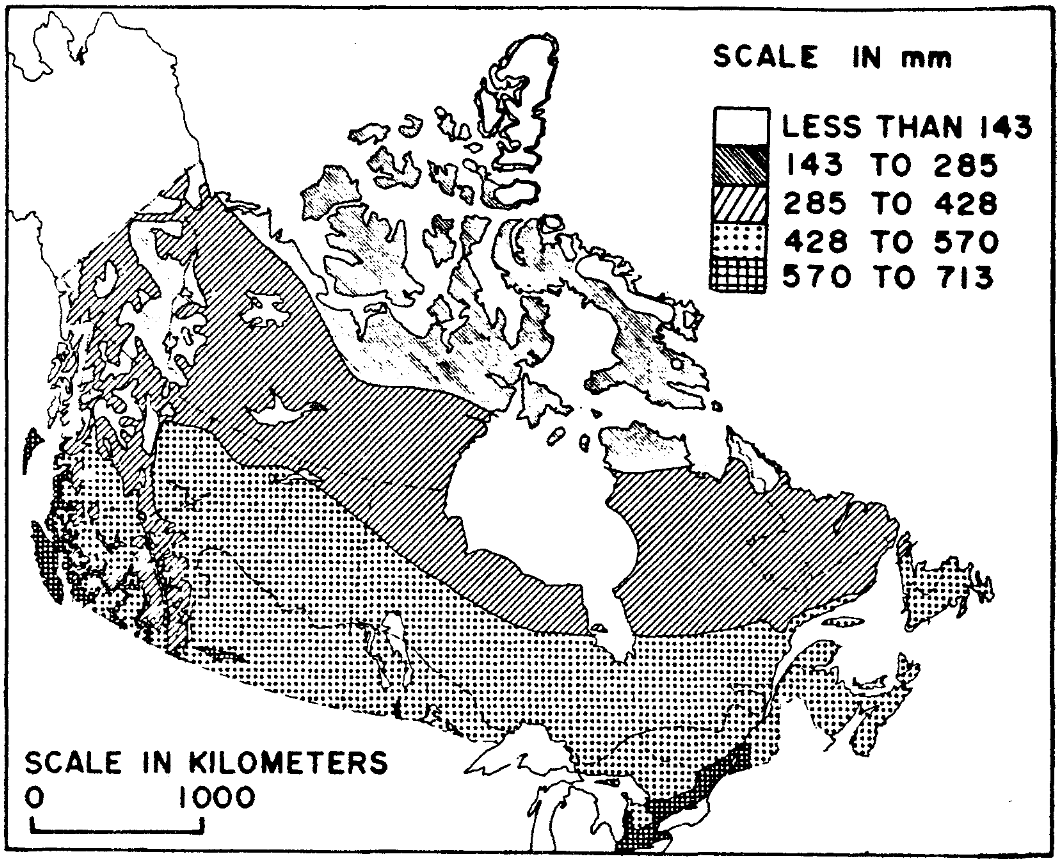

Figure 3. Average annual potential evapotranspiration and climatic types in Canada (Sanderson, 1948)

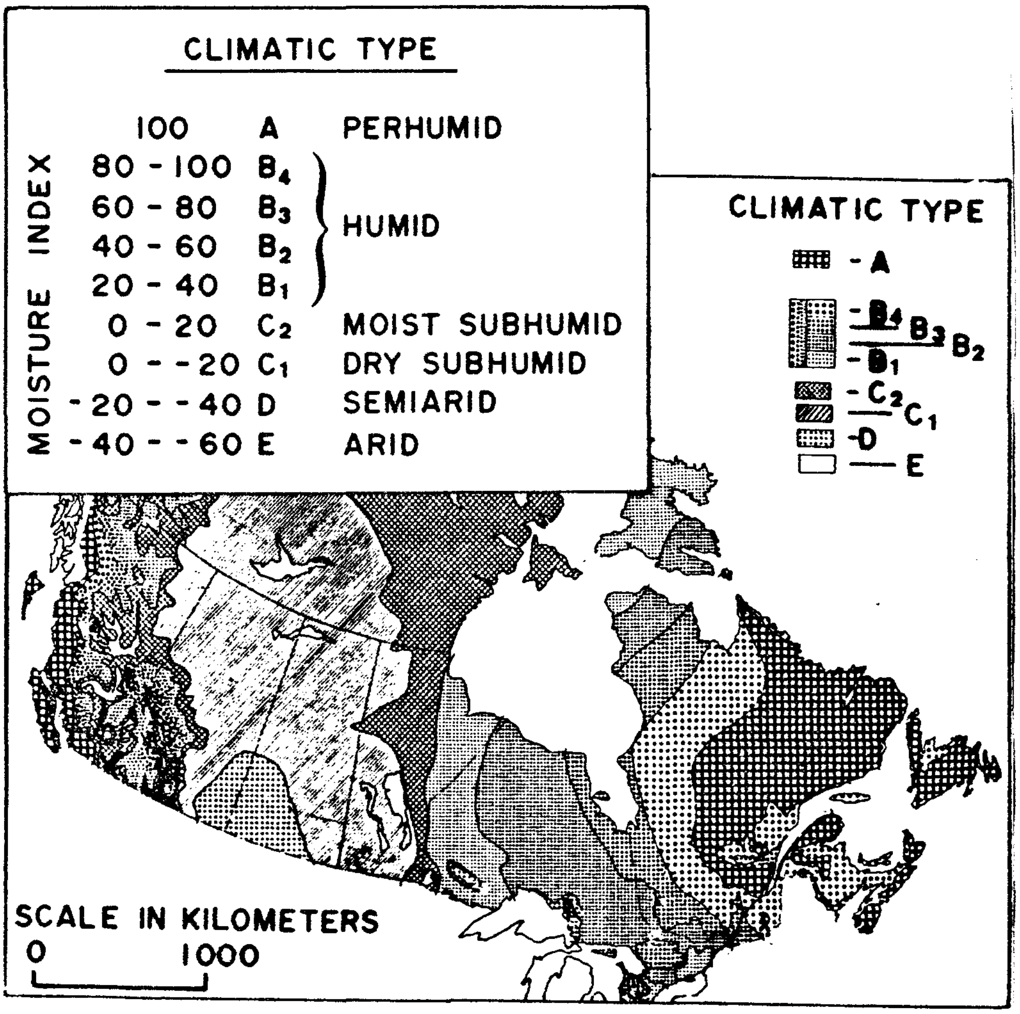

Figure 4. Moisture regions in Canada based on Thornthwaite's classification (Sanderson, 1948)

of the recharge rate (i.e., 5% of precipitation), the surface boundary flux can be written as:

$$F = 0.05\,P = P - AE \tag{6}$$

where:
F = surface boundary flux
P = precipitation
AE = actual evapotranspiration

By rearrangement, actual evapotranspiration is computed as 95% of precipitation. Precipitation data from Environment Canada, for Regina, Saskatchewan for the period of years from 1930 through 1984 is presented in Figure 5. Mean monthly temperature data for the same location along with the Thornthwaite method (1948) were used to compute potential evapotranspiration for the same period of time. Figure 6 illustrates the ratio between actual and potential evapotranspiration for each year between 1930 and 1984. The average ratio between the estimate of actual evapotranspiration and Thornthwaite potential evapotranspiration is 0.71.

ii) By Equating Long-Term Precipitation and Actual Evapotranspiration. Assuming that over the long term, there is neither a net wetting nor a net drying, long-term precipitation must equal long-term actual evapotranspiration. The total precipitation for the years 1930 through 1984 was assumed to equal the total actual evapotranspiration for the same period. The ratio established between actual and potential evapotranspiration was 0.75.

iii) By Equating Long-Term Mean Monthly Values. The long-term mean monthly precipitation for Regina, Saskatchewan is presented in Figure 7. The mean monthly values are based upon 100 years of record collected by Environment Canada. The assumption is made that the mean monthly precipitation is equal to the mean monthly actual evapotranspiration. The corresponding ratio established between actual and potential evapotranspiration was 0.69.

The above discussion illustrates that a reasonable ratio between actual and potential evapotranspiration is in the order of 0.70. It could also be that 55 years of data may be insufficient to estimate long-term values for the ratio between actual and potential evapotranspiration.

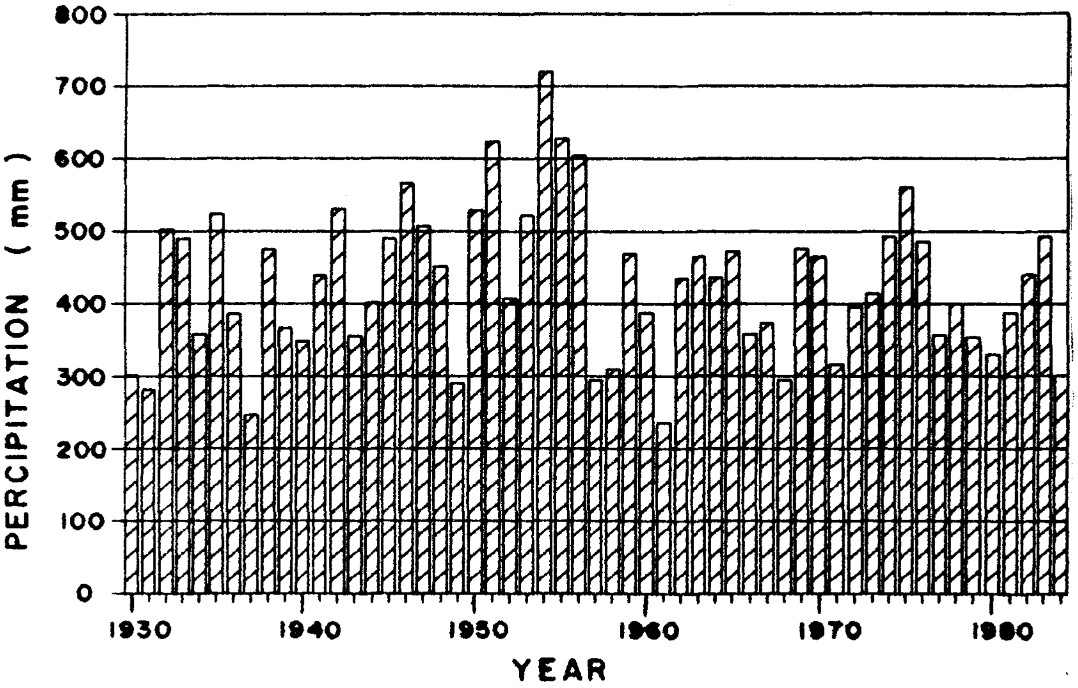

Figure 5. Annual precipitation for 1930 through 1984 at Regina

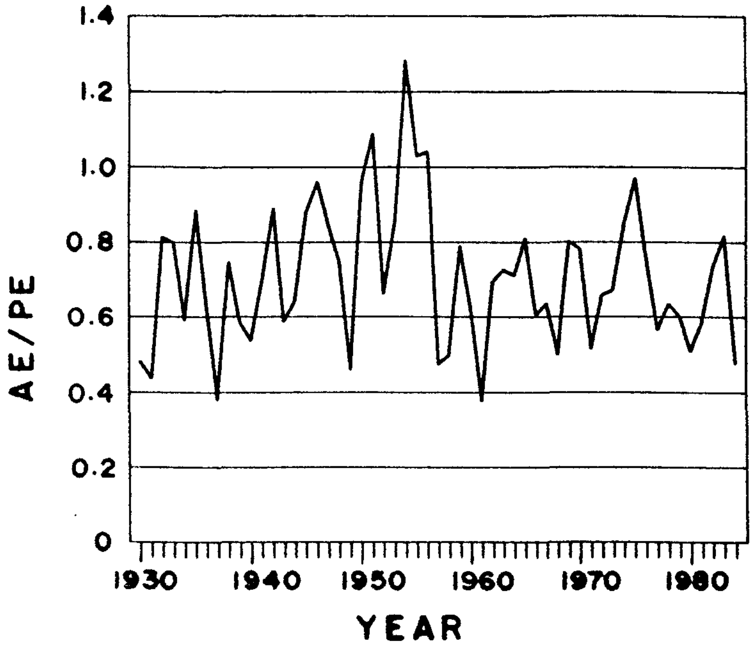

Figure 6. Ratio between actual and potential evapotranspiration based upon the net groundwater recharge method

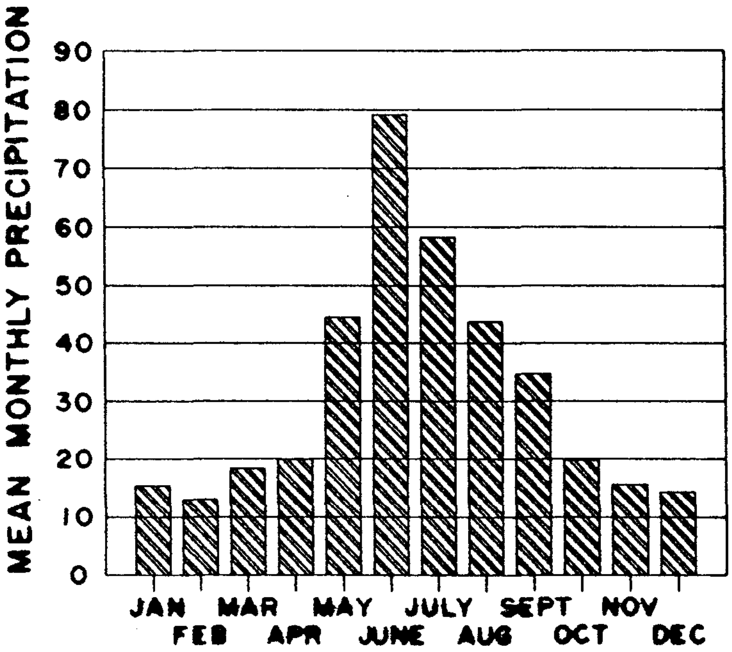

Figure 7. Long-term mean monthly precipitation values for Regina

PRESENTATION AND DISCUSSION OF MODELLING RESULTS

The four components of the model illustrated in Figure 1 illustrate the relationship between matric suction and vertical ground movements. The model shows how increases in matric suction result in shrinkage and decreases in matric suction result in swelling.

Figure 8 illustrates a comparison of simulated vertical ground movements with the recorded ground movements for the Regina location. Movements are in the order of 50 mm at a depth of 0.3 m. Measured surface ground movements were in the order of 75 mm (Hamilton, 1963).

Two independent ranges for the hydraulic conductivity were required; one for the infiltration process and one for the exfiltration process. Values in the order of 10^{-6} to 10^{-8} meters per second were required for modelling infiltration events. These suggest that the soil behaves as though it were relatively permeable during the infiltration process. This can be explained in physical terms by considering the intake of water to the shrinkage cracks evident in the field (Figure 9). The associated volume change behavior is influenced by the macro-structure of the soil mass.

For the exfiltration process, hydraulic conductivities in the order of 10^{-9} to 10^{-11} meters per second were required to obtain reasonable simulations. The exfiltration process appears to occur primarily as a vapor transport process rather than a liquid flow process (Figure 9). The volume change behavior during the exfiltration process appears to be more closely associated with the micro-structure of the soil.

Figure 10 illustrates typical seasonal ground movements for Regina, Saskatchewan. Corresponding typical seasonal suction values vary between 50 kPa and 1400 kPa near the ground surface. The modelling also implies that thermally induced suctions (i.e., winter freezing conditions) could play a significant role in the seasonal vertical ground movements.

CONCLUSIONS AND RECOMMENDATIONS

The numerical model shows the relationship between vertical ground movements and matric suction changes. Reasonable agreement was obtained between measured and simulated movements using soil parameters that vary with depth.

Due to the formation of shrinkage cracks in the field, two ranges of hydraulic conductivity were required for modelling. It was necessary to utilize one for infiltration and one for exfiltration. These ranges reflect the change from water flow during infiltration to predominately vapor flow during exfiltration.

The Thornthwaite potential evapotranspiration computations showed reasonable modelling results with the use of a ratio of 0.70 between actual and potential evapotranspiration. The Thornthwaite potential evapotranspiration computations are suitable for situations in which meteorological data are limited. However, more rigorous physical models would be advantageous. Further research is required to establish better the surface boundary condition and to enlarge the database representing typical soil properties.

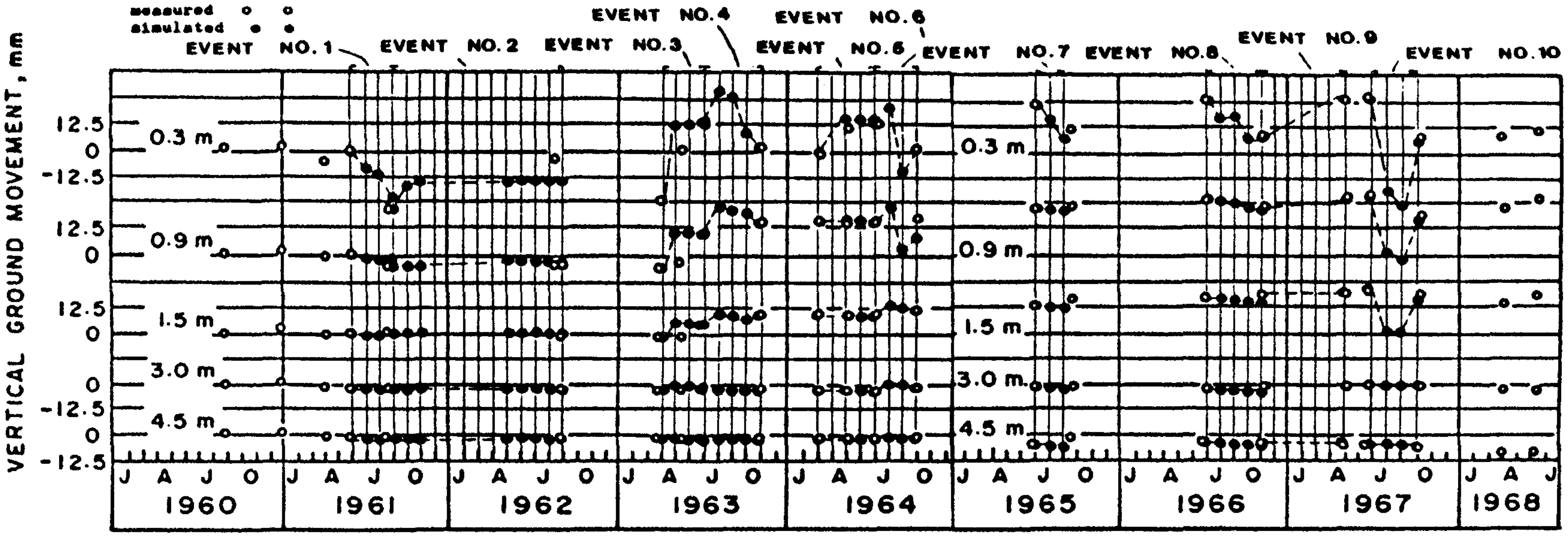

Figure 8. Comparison of predicted and measured vertical ground movements for each of ten events modelled for Regina, Saskatchewan

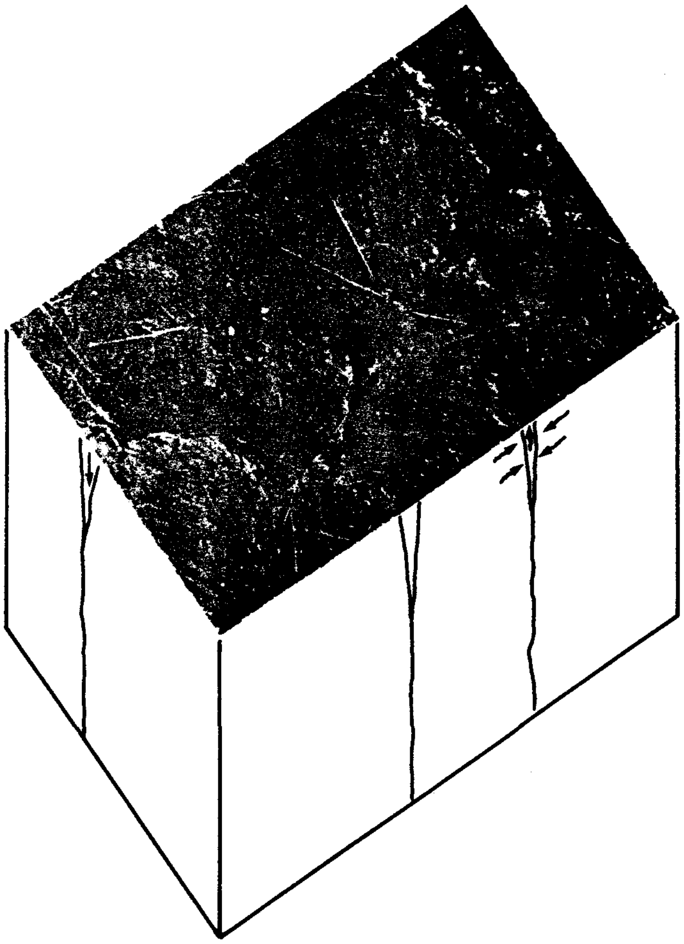

Figure 9. Schematic diagram illustrating infiltration and exfiltration in a cracked clay soil

REFERENCES

1. DG Fredlund & JU Hasan, Statistical geotechnical properties of Lake Regina sediments, University of Saskatchewan, Department of Civil Engineering, Transportation and Geotechnical Group, Saskatoon, Saskatchewan, Internal Report, IR-9, 1979, 102 p.
2. DG Fredlund & NR Morgenstern, Stress state variables for unsaturated soils, Journal of the Geotechnical Engineering Division, American Society of Civil Engineers, Vol. 103 (GT75), 1977, 447 - 446.
3. RJ Granger, An examination of the concept of potential evapotranspiration, Journal of Hydrology, 1989, 9-19.
4. JJ Hamilton, Volume changes in undisturbed clay profiles in Western Canada, Canadian Geotechnical Journal, Vol. 1, 1963, 27 - 41.
5. JJ Hamilton, Shallow foundations on swelling clays in Western Canada, The International Research and Engineering Conference on Expansive Clay Soils, Texas A & M University, College Station, TX, 1965, 183 - 207.

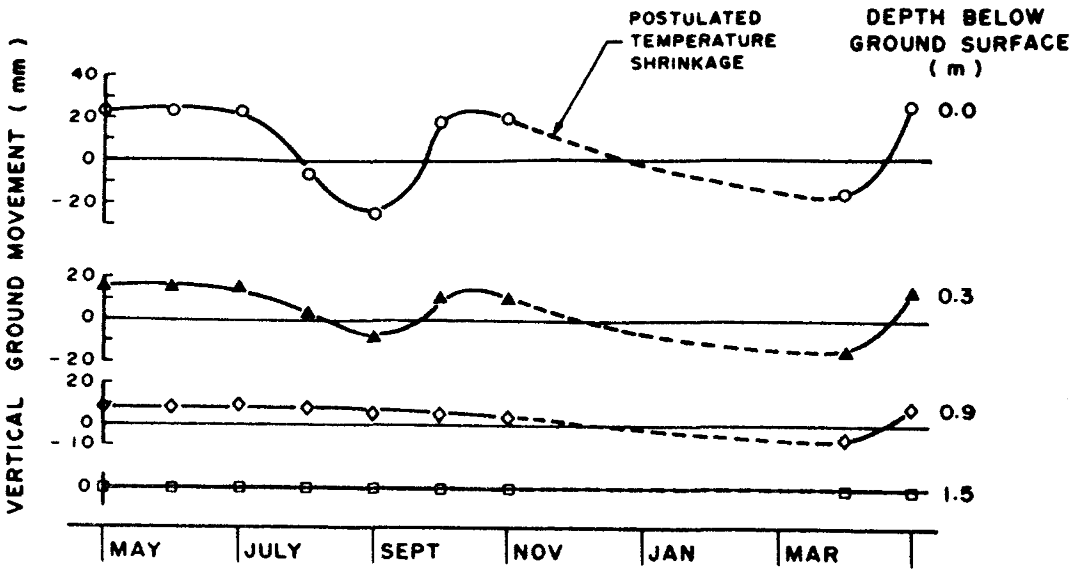

Figure 10. Simulated average vertical ground movements for Regina, Saskatchewan

6. JJ Hamilton, Effects of natural and man-made environments on the performance of shallow foundations, Proceedings of the 21st. Annual Canadian Soil Mechanics Conference, Winnipeg, Manitoba, 1968.
7. L Lam, DG Fredlund & SL Barbour, Transient seepage model for saturated-unsaturated soil systems: A geotechnical engineering approach, Canadian Geotechnical Journal, Vol. 24, 565-580.
8. HL Penman, Vegetation and Hydrology Technical Communication No. 53, Commonwealth Bureau of Soils, Harpenden, Commonwealth Agricultural Bureau, Farnham Royal, Bucks, England, 1963.
9. CHB Priestley, & RJ Taylor, On the assessment of surface heat flux and evaporation using large scale parameters, Monthly Weather Review, Vol. 100, No. 2, 1972, 81 - 92.
10. M Sanderson, The climates of Canada according to the new Thornthwaite classification, Scientific Agriculture, Vol. 28, 1948, 501 - 517.
11. M Sanderson & A Rafique, Potential evapotranspiration and water deficit in Bangladesh using Garnier's modification of the Thornthwaite water balance, Climatological Bulletin No. 25, McGill University, Department of Geography, 1979, 13 - 24.
12. CW Thornthwaite, An approach toward a rational classification of climate, Geographical Review, Vol. 38, 1948, 55 -94.
13. CW Thornthwaite & JR Mather, The Water Balance, (2nd Ed), Publications in Climatology, Vol. 8, No 1, Drexel Institute of Technology, Centerton, New Jersey, 1955.
14. P van der Raadt, DG Fredlund, AW Clifton, MJ Klassen & WE Jubien, Soil suction measurements at several sites in Western Canada, Presented to the Session on Geotechnical Problems in Arid Regions, Transportation Research Board, Committee A2L06TRB, Washington, D.C., 1987.

15. GW Wilson, Soil evaporation fluxes for geotechnical engineering problems", Ph.D. thesis, University of Saskatchewan, Saskatoon, Canada, 1990, 294 p.
16. RT Yoshida, DG Fredlund & JJ Hamilton, The prediction of total heave of a slab-on-grade floor on Regina clay", Canadian Geotechnical Journal, Vol. 20, No. 1, 1983, 69 - 81.
17. BJ Zebarth, Saturated and unsaturated flow in a hummocky landscape in relation to topography and soil morphology, Ph.D. Thesis, Department of Soil Science, University of Saskatchewan, Saskatoon, Saskatchewan, 1988.