Calculation Procedures for Slope Stability Analyses Involving Negative Pore-Water Pressures

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Two methods for incorporating the effects of negative pore-water pressures into the slope stability analysis are proposed in this paper. The derivation of equations for the factors of safety with respect to force and moment equilibriums are presented. Several procedures for designating the negative pore-water pressures in the analysis are also outlined. An example is given in order to illustrate the method of calculation.

INTRODUCTION

1. Slope stability analyses have become a common analytical tool for assessing the factor of safety of natural and man-made slopes. Two-dimensional, limit equilibrium methods of slices are commonly used in practice. There is probably no analysis conducted by geotechnical engineers which has received more programming attention than the limit equilibrium methods of slices used to compute a factor of safety (Fredlund, 1980: ref. 1). The main reasons appear to be as follows: first, the limit equilibrium method has proved to be a useful and reasonably reliable tool in assessing the stability of slopes. Its "track-record" is impressive for most cases where the shear strength properties of the soil and the pore-water pressure conditions have been properly assessed (Sevaldson, 1956; Kjaernsli and Simons, 1962; Skempton and Hutchinson, 1969, and Chowdhury, 1980: refs 2-5). Second, the limit equilibrium methods of slices require a limited amount of input information but can quickly perform extensive trial and error searches for the critical slip surface.

2. Saturated, effective shear strength parameters (i.e., c' and ϕ') are commonly used when performing slope stability analyses. The shear strength contribution from the negative pore-water pressures above the groundwater table are usually ignored by setting their magnitudes to zero. The difficulties associated with the measurement of negative pore-water pressures and their incorporation into the slope stability analysis are the primary reason for this practice. This is a reasonable assumption for many situations where the major portion of the slip surface is below the groundwater table. However, for situations where the groundwater table is deep (Fig. 1) or where the concern is over the possibility of a shallow failure surface, negative pore-water pressures can no longer be ignored.

3. In recent years there has developed a better understanding of the role of negative pore-water pressures (or matric suctions) in increasing the shear strength of the soil. Recent developments have led to several devices which can be used to better measure the negative pore-water pressures. Therefore, it is now appropriate to perform slope stability analyses which include the shear strength contribution from the negative pore-water pressures. These types of analyses are an extension of the conventional limit equilibrium analyses.

4. Several aspects of a slope stability study remain the same for soils with positive pore-water pressures (e.g., saturated soils) and soils with negative pore-water pressures (e.g., unsaturated soils). For example, the nature of the site investigation, the identification of the strata and the measurement of the total unit weight remain the same in both situations. On the other hand, extensions to conventional procedures are required with respect to the characterization of the shear strength of the soil. The analytical tools used to incorporate pore-water pressures and calculate the factor of safety also need to be extended.

THEORY OF SLOPE STABILITY

5. The limit equilibrium method of slices are based upon the principles of statics (i.e., static equilibriums of forces and/or moments). The method derived in this paper uses the summation of forces in two directions and the summations of moments about a common point in deriving the factor of safety. These elements of statics, along with the failure criteria, are insufficient to make the slope stability problem determinate (Morgenstern and Price, 1965; Spencer, 1967: refs 6-7). Either additional elements of physics or an assumption regarding the direction or magnitude of some of the forces is required to render the problem determinate. The following method utilizes an assumption regarding the direction of the interslice forces. This approach has been widely adopted in limit equilibrium methods (Fredlund and Krahn, 1977: ref. 8). The various limit equilibrium slope stability methods that follow this approach have been demonstrated to be special cases of this method (Fredlund, Krahn, and Pufahl, 1981: ref. 9).

6. Calculations for the stability of a slope are performed by dividing the soil mass above the slip surface into vertical slices. The forces acting on a slice within the sliding soil mass are shown in Fig. 1 for a composite slip surface, respectively. The forces are designated for a unit width (i.e., perpendicular direction to motion) of the slope. The variables are defined as follows:

W = the total weight of the slice of width 'b' and height 'h'
N = the total normal force on the base of the slice

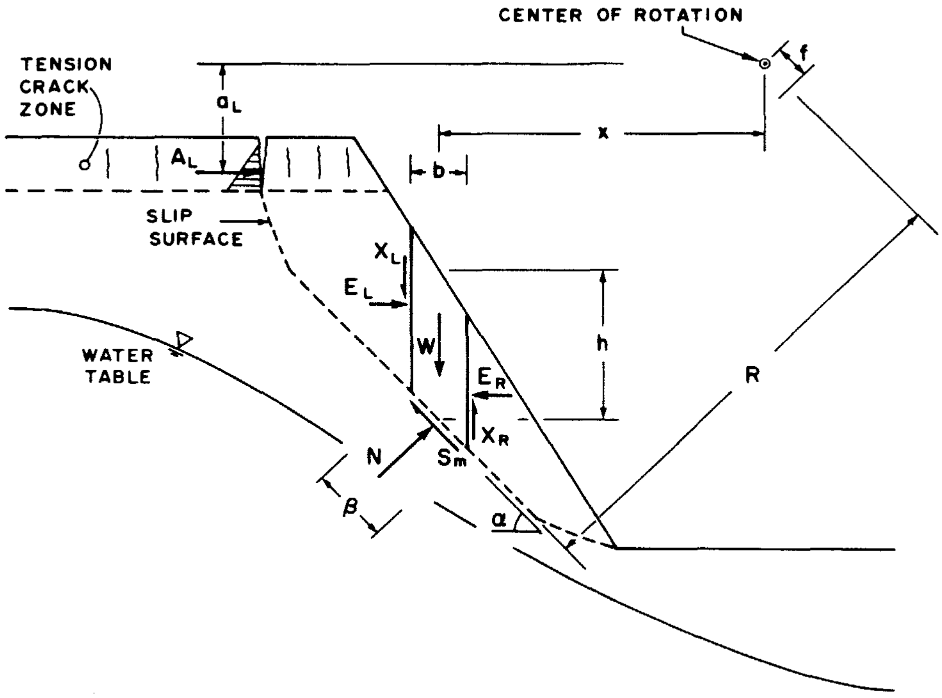


Fig. 1 Forces acting on a slice through a sliding mass with a composite slip surface

S_m= the shear force mobilized on the base of each slice
E = the horizontal interslice normal forces (the 'L' and 'R' subscripts designate the left and right sides of the slice, respectively)
X = the vertical interslice shear forces (the 'L' and 'R' subscripts designate the left and right sides of the slice, respectively)
R = the radius for a circular slip surface or the moment arm associated with the mobilized shear force, S_m for any shape of slip surface
f = the perpendicular offset of the normal force from the centre of rotation or from the centre of moments
x = the horizontal distance from the centre line of each slice to the centre of rotation or to the centre of moments
h = height or the vertical distance from the centre of the base of each slice to the uppermost line in the geometry (i.e., generally ground surface)
a = the perpendicular distance from the resultant external water force to the centre of rotation or to the centre of moments. The 'L' and 'R' subscripts designate the left and right sides of the slope, respectively
A = the resultant external water forces. The 'L' and 'R' subscripts designate the left and right sides of the slope, respectively
α = the angle between the tangent to the centre of the base of each slice and the horizontal. The sign convention is as follows: when the angle slopes in the same direction as the overall slope of the geometry, α is positive, and vice versa
β = sloping distance across the base of a slice

7. The example shown in Fig. 1 is typical of a steep slope with a deep groundwater table. The crest of the slope is highly desiccated and there are tension cracks filled with water. The tension crack zone is assumed to have no shear strength and the presence of water in this zone produces an external water force, A_L. As a result, the assumed slip surface in the tension crack zone is a vertical line. The depth of the tension crack is generally estimated or can be approximated analytically (Spencer, 1968 and 1973: refs 10-11). The weight of the soil in the tension crack zone acts as a surcharge on the crest of the slope. The external water force, A_L, is computed as the hydrostatic force on a vertical plane.

"Total Cohesion" Method

8. The mobilized shear force at the base of a slice can be written using the form of the shear strength equation for an unsaturated soil (Fredlund, Morgenstern and Widger, 1978: ref. 12):

$$S_m = \frac{\beta}{F}\,[c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b] \qquad (1)$$

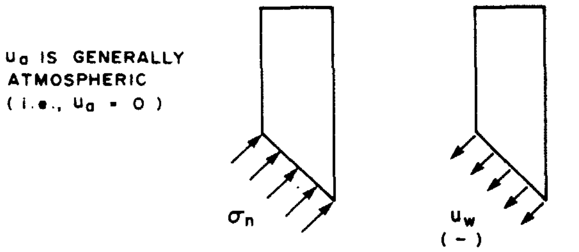
where:
S_m= the mobilized shear force at the base of a slice
F = factor of safety which is defined as a factor by which the shear strength parameters must be reduced in order to bring the soil mass into a state of limiting equilibrium along the assumed slip surface. The factor of safety for the cohesive parameter (i.e., c′) and the frictional parameters (i.e., $\tan\phi'$ and $\tan\phi^b$) are assumed to be equal for all soils involved and for all slices
c′ = effective cohesion intercept
σ_n= total stress normal to the base of a slice
u_a, u_w = pore-air and pore-water pressures on the base of a slice
$(\sigma_n - u_a)$ = net stress normal to the base of a slice

$(u_a - u_w)$ = matric suction

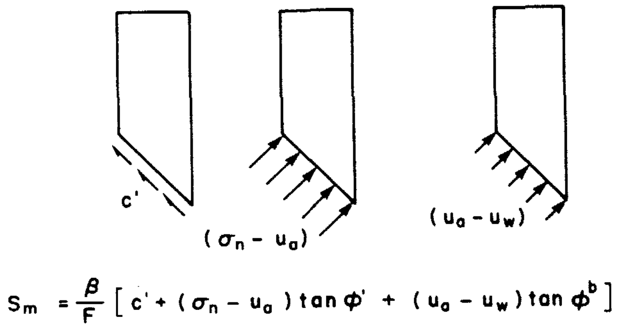
ϕ'= angle of internal friction associated with the net normal stress state variable $(\sigma_n - u_a)$

ϕ^b= angle indicating the rate of increase in shear strength relative to a change in matric suction, $(u_a - u_w)$

9. The components of the mobilized shear force at the base of a slice are illustrated in Fig. 2. The contributions from the total stress and the negative pore-water pressures are separated using the ϕ' and ϕ^b angles, respectively.



(a) Pressure components on the base of a slice



(b) Contributors to the shear resistance

Fig. 2 Pressure and shear resistance components at the base of a slice

10. It is possible to consider the matric suction term as part of the cohesion of the soil. In other words, the matric suction can be visualized as increasing the cohesion of the soil. As a result, the conventional factor of safety equations do not need to be re-derived. The mobilized shear force at the base of a slice, S_m, will have the following form:

$$S_m = \frac{\beta}{F}[c + (\sigma_n - u_w)\tan\phi'] \qquad (2)$$

where:

c = cohesion of the soil that has two components, (i.e., $c' + (u_a - u_w)\tan\phi^b$)

11. This approach has the advantage that the shear strength equation retains its conventional form. It is therefore possible to utilize a computer program written for saturated soils to solve unsaturated soil problems. When this is done, the soil in the negative pore-water pressure region must be subdivided into several discrete layers with each layer having a constant cohesion. The pore-air and pore-water pressures must be set to zero. This approach has the disadvantage that the cohesion is not a continuous function and the appropriate cohesion values must be manually computed. In addition, the soil strata must be discretized in an appropiate manner.

"Extended Shear Strength" Method

12. The following formulations are the revised derivations for the factor of safety equations that directly incorporate the shear strength contribution from the negative pore-water pressures. The mobilized shear force in equation (1) is used throughout the derivation.

13. **Normal force equation**. The normal force at the base of a slice, N, is derived by summing forces in the vertical direction:

$$W - (X_R - X_L) - S_m \sin\alpha - N\cos\alpha = 0 \qquad (3)$$

Substituting equation (1) into equation (3) and replacing the $(\sigma_n\beta)$ term with N results in equation (4). Rearranging equation (4) provides the normal force as written in equation (5).

14. The factor of safety, F, in equation (5) is equal to the moment equilibrium factor of safety, F_m, when solving the moment equilibrium; and is equal to the force equilibrium factor of safety, F_f, when solving the force equilibrium. In most cases the pore-air pressure, u_a, is atmospheric (i.e., $u_a = 0$) and as a result, equation (5) reduces to equation (6).

15. If the base of the slice is located in the saturated soil, the $(\tan\phi^b)$ term in equation (6) becomes equal to $(\tan\phi')$. Equation (6) then reverts to the conventional normal force equation for saturated slope stability analyses. Computer coding for solving equation (5) can be written such that the ϕ^b angle is used whenever the pore-water pressure is negative while the ϕ' angle is used whenever the pore-water pressure is positive. The ϕ^b angle can also be considered to be equal to ϕ' at low matric suction values and lower than ϕ' at high matric suction values (Fredlund, Rahardjo and Gan, 1987: ref. 13).

16. The vertical interslice shear forces, X_L and X_R, in the normal force equation can be computed using an interslice force function.

17. **Factor of safety with respect to moment equilibrium**. Two independent factor of safety equations can be derived; one with respect to moment equilibrium and the other with respect to horizontal force equilibrium. Moment equilibrium can be satisfied with respect to an

$$W - (X_R - X_L) - \left\{\frac{c'\beta}{F} + \frac{N\tan\phi'}{F} - \frac{u_a\tan\phi'\beta}{F} + \frac{(u_a - u_w)\tan\phi^b\beta}{F}\right\}\sin\alpha - N\cos\alpha = 0 \qquad (4)$$

$$N = \frac{W - (X_R - X_L) - \dfrac{c'\beta\sin\alpha}{F} + u_a\dfrac{\beta\sin\alpha}{F}(\tan\phi' - \tan\phi^b) + u_w\dfrac{\beta\sin\alpha}{F}\tan\phi^b}{m_\alpha} \qquad (5)$$

where:

$m_\alpha = \cos\alpha + (\sin\alpha\tan\phi')/F$

$$N = \frac{W - (X_R - X_L) - \frac{c' \beta \sin\alpha}{F} + u_w \frac{\beta \sin\alpha}{F} \tan\phi^b}{m_\alpha} \tag{6}$$

arbitrary point above the central portion of the slip surface. For a circular slip surface, the centre of rotation is an obvious centre for moment equilibrium. The centre of moments is immaterial when both force and moment equilibria are satisfied. When only moment equilibrium is satisfied, the computed factor of safety varies slightly with the point selected for the summation of moments.

18. Consider the moment equilibrium of a composite slip surface (Fig. 1) with respect to the centre of rotation:

$$A_L a_L + \sum W x - \sum N f - \sum S_m R = 0 \tag{7}$$

Substituting equation (1) for the S_m variable into equation (7) and replacing the ($\sigma_n\beta$) term with N yield the factor of safety with respect to moment equilibrium (i.e., equation (8)).

19. In the case where the pore-air pressure is atmospheric (i.e., $u_a = 0$), equation (8) reduces to equation (9).

20. When the pore-water pressure is positive, the ϕ^b value is set equal to the ϕ' value. For a circular slip surface, the radius R, is constant for all slices and the normal force, N, acts through the centre of rotation (i.e., f = 0).

21. <u>Factor of safety with respect to force equilibrium</u>. The factor of safety with respect to force equilibrium is derived from the equilibrium of forces in the horizontal direction for all slices (equation (10)).

22. The horizontal interslice normal forces, E_L and E_R, cancel when summed over the entire sliding mass. Substituting equation (1) for the mobilized shear force, S_m, into equation (10) and replacing the ($\sigma_n\beta$) term with N give the factor of safety with respect to force equilibrium (equation (11)).

23. In the case where the pore-air pressure is atmospheric (i.e., $u_a = 0$), equation (11) reverts to equation (12).

24. When the pore-water pressure is positive, the ϕ^b value is equal to the ϕ' value. Equation (12) remains the same for both circular and composite slip surfaces.

$$F_f = \frac{\sum \{c' \beta \cos\alpha + (N - u_w \beta \frac{\tan\phi^b}{\tan\phi'}) \tan\phi' \cos\alpha\}}{A_L + \sum N \sin\alpha} \tag{12}$$

25. <u>Interslice force function</u>. The interslice normal forces, E_L and E_R, are computed from the summation of horizontal forces in each slice.

$$E_R - E_L + S_m \cos\alpha - N \sin\alpha = 0 \tag{13}$$

Combining equation (3) and equation (13) gives the following equation:

$$E_R = E_L + [W - (X_R - X_L)] \tan\alpha - \frac{S_m}{\cos\alpha} \tag{14}$$

The interslice normal forces are calculated from equation (14) by integrating from left to right. The E_L normal force on the first slice is equal to the external water force, A_L or zero when there is no water present in the tension crack zone.

26. The assumption is made that the interslice shear force, X, can be related to the interslice normal force, E, by a mathematical function (Morgenstern and Price, 1965: ref. 6):

$$X = \lambda f(x) E \tag{15}$$

where:

f(x) = a functional relationship which describes the manner in which the magnitude of X/E varies across the slip surface

λ = a scaling constant which represents the percentage of the function, f(x), used for solving the factor of safety equations

27. The factor of safety equations with respect to moment and force equilibriums (i.e., equations (8) and (11), respectively) are non-linear. The factor of safety, F_m or F_f, appears on both sides of the equations, with the factor of safety being included through the normal force equation (i.e., equation (5)). The non-linear factor of safety equations can be solved simultaneously using an iterative technique.

$$F_m = \frac{\sum [c' \beta R + \{N - u_w \beta \frac{\tan\phi^b}{\tan\phi'} - u_a \beta (1 - \frac{\tan\phi^b}{\tan\phi'})\} R \tan\phi']}{A_L a_L + \sum W x - \sum N f} \tag{8}$$

where: F_m = factor of safety with respect to moment equilibrium.

$$F_m = \frac{\sum \{c' \beta R + (N - u_w \beta \frac{\tan\phi^b}{\tan\phi'}) R \tan\phi'\}}{A_L a_L + \sum W x - \sum N f} \tag{9}$$

$$\sum S_m \cos\alpha - A_L - \sum N \sin\alpha = 0 \tag{10}$$

$$F_f = \frac{\sum [c' \beta \cos\alpha + \{N - u_w \beta \frac{\tan\phi^b}{\tan\phi'} - u_a \beta (1 - \frac{\tan\phi^b}{\tan\phi'})\} \tan\phi' \cos\alpha]}{A_L + \sum N \sin\alpha} \tag{11}$$

where: F_f = factor of safety with respect to force equilibrium

PORE-WATER PRESSURE DESIGNATION

28. Several procedures are commonly used to designate the pore-water pressure conditions in saturated soils. These can be extended to the case of unsaturated soils.

Pore Pressure Coefficients

29. Pore-water pressures are often designated as a pore-pressure coefficient, r_u, for analysis purposes (Bishop and Morgenstern, 1960: ref. 14).

$$r_u = \frac{u_w}{g \sum \rho_i h_i} \tag{16}$$

where: g = gravitational acceleration
h_i = thickness of each soil layer, i
ρ_i = density of each soil layer, i

30. The pore pressure coefficient is generally considered as a positive value. However, it can also be used to represent negative pore-water pressures as well as pore-air pressures. In this case, the water pore pressure coefficient is negative. It is useful to be able to designate pore pressure coefficients for both the water and air phases.

$$r_{uw} = \frac{u_w}{g \sum \rho_i h_i} \tag{17}$$

$$r_{ua} = \frac{u_a}{g \sum \rho_i h_i} \tag{18}$$

where:
r_{uw}= water pore pressure coefficient
r_{ua}= air pore pressure coefficient

31. The water pore pressure coefficient at the phreatic line is equal to zero. A water pore pressure coefficient of +0.5 indicates the verge of artesian pressure conditions since the density of water is approximately one half that of the soil. At points above the phreatic line, the pore-water pressure becomes increasingly negative. At the same time, the overburden pressure is decreasing. As a result, it is possible for the pore pressure coefficient to become highly negative. Let us assume that the pore-water pressure is -200 kPa at a depth of 1 meter (i.e., ρgh = 20 kPa for ρ = 2,000 kg/m^3). This gives rise to a water pore pressure coefficient of -10. The water pore pressure coefficient can often tend to a negative, infinite number as ground surface is approached. In other words, the water pore pressure coefficient becomes a highly variable term as ground surface is approached, thereby creating some difficulty with its usage.

32. Figure 3 shows the cross-section of a dam under steady state seepage conditions. One equipotential line is selected and the water pore pressure coefficients are computed at various depths and plotted in Fig. 4. There is essentially a linear change in the pore pressure coefficient until the ground surface is approached. At this point, the coefficient becomes highly negative.

33. The air pore pressure coefficient in natural soil deposits is always close to zero due to its contact with the atmosphere. In compacted earth fills, the pore-air pressures may become positive due to the weight of the overlying soil layers. In this case,the air pore pressure coefficient will be positive but generally quite small.

Pore Pressure Head Contours

34. Negative pore-water pressures can also be contoured as a series of lines (Fig. 5). An interpolation procedure can be used to obtain the pore-water pressures for points between the contours.

Piezometric Lines

35. Piezometric lines can also be used to designate the pore-water pressures in a slope. The vertical distance from the piezometric line down to a point below the line is equal to the positive pore-water pressure head. On the other hand, the vertical distance from the piezometric line up to a point above the line can be considered as the negative pore-water pressure head.

EXAMPLE USING "TOTAL COHESION" METHOD

36. An example of a steep slope of granitic colluvium is illustrated in Fig. 6. The slope consists of two soil layers of different properties (Fig. 6) which are underlain by a deep bedrock. The water table is assumed to be located at the interface between the bedrock and the soil layer. Several slope-stability analyses were performed in order to illustrate the use of the "total cohesion" method in analyzing a steep slope with negative pore-water pressures. In this case, a specified slip surface (Fig. 6) was used for the entire analyses.

37. The shear strength contribution from matric suction was incorporated into the total cohesion of the soil (i.e., $c = c' + (u_a - u_w) \tan \phi^b$) as shown in equation (2). For this purpose, the soil layers were divided into four subdivisions with each subdivision having a constant cohesion (Fig. 6). The slope was considered to represent an infinite slope condition with each stratum being parallel to the water table. In this case, a hydrostatic condition of the pore-water pressures could be assumed along the perpendicular direction to the water table (Fig. 6). As a result, the negative pore-water pressure distribution above the groundwater table could be established. The matric suction in each subdivision was computed from the pore-water pressure at the center of the subdivision and by assuming an atmospheric pore-air pressure condition. The total cohesion was computed using an equal ϕ^b angle for all subdivisions (e.g., ϕ^b = 10°).

38. Having computed the total cohesion values, the analyses were conducted using various methods of slices with a constant interslice force function (i.e., f(x) = 1.0; Morgenstern - Price's method). Various λ values resulted in different combinations of F_f and F_m. The critical factor of safety is obtained when both F_f and F_m values are equal. When the λ value is equal to zero (interslice forces X = 0, E ≥ 0), the F_m and F_f values represent the factors of safety of Simplified Bishop and Janbu's simplified methods, respectively. The F_m values were found to remain essentially constant at various λ values while the F_f values vary significantly with λ values. Therefore, the stability analyses of a steep slope are commonly performed using the moment equilibrium only (Ching, Sweeney and Fredlund, 1984: ref. 16).

39. The effect of negative pore-water pressures on the factor of safety is illustrated in Fig. 7. The factor of safety is shown to increase with increasing ϕ^b angles. The results

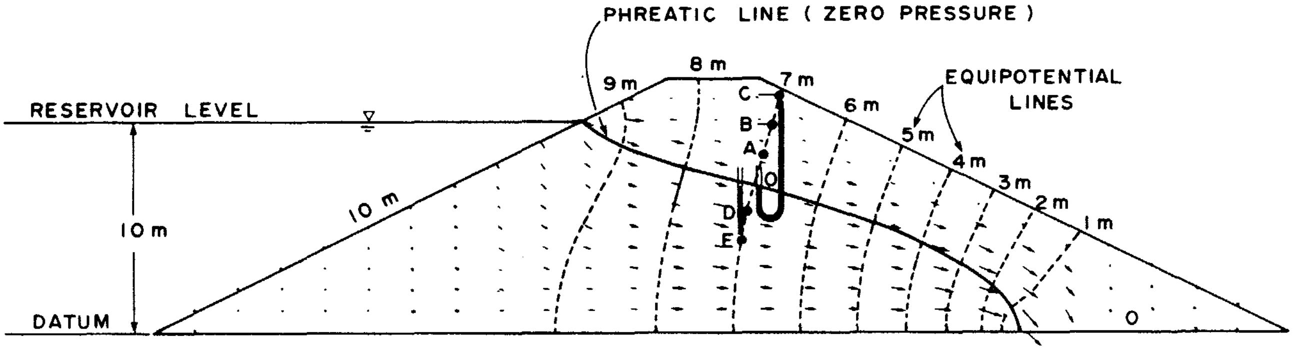


Fig. 3 Steady state seepage conditions in an earth-filled dam

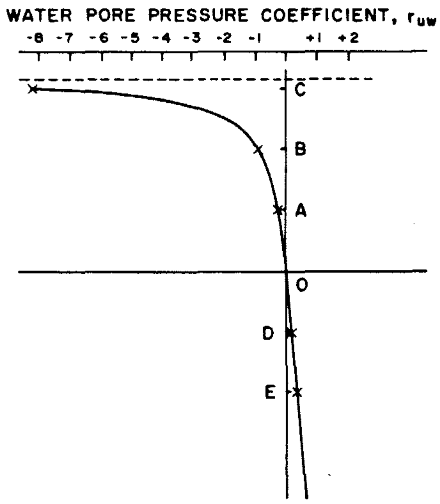


Fig. 4 Water pore-pressure coefficient along the 7m equipotential line

were obtained using the various methods of slices. The ϕ^b angle indicates the matric suction contribution into the shear strength. The increasing ϕ^b angle would in turn increase the cohesive component of the shear strength along the slip surface (Fig. 8). As a result, the factor of safety increases with the increasing total cohesion. In addition, the frictional component of the shear strength will decrease as the slip surface becomes shallower. This gives rise to the more dominant effect of the cohesive component, that includes the matric suction contribution, on the stability of a steep slope.

40. Examples using the "extended shear strength" method will not be presented in this paper due to the limited space available. However, detailed examples on this method have been presented by Fredlund (1989: ref. 17).

CONCLUSIONS

41. The negative pore-water pressures can readily be incorporated into conventional slope stability analysis. The two methods of calculation presented in this paper can be easily used for routine analyses. The conventional slope stability programs require some modifications only when the "extended shear strength" method is used. The "total cohesion" method only requires subdivision of the soil strata of the slope in order to account for the matric suction contribution. Both methods, however, require the measurements of matric suction and the ϕ^b angle in addition to the conventional shear strength parameters.

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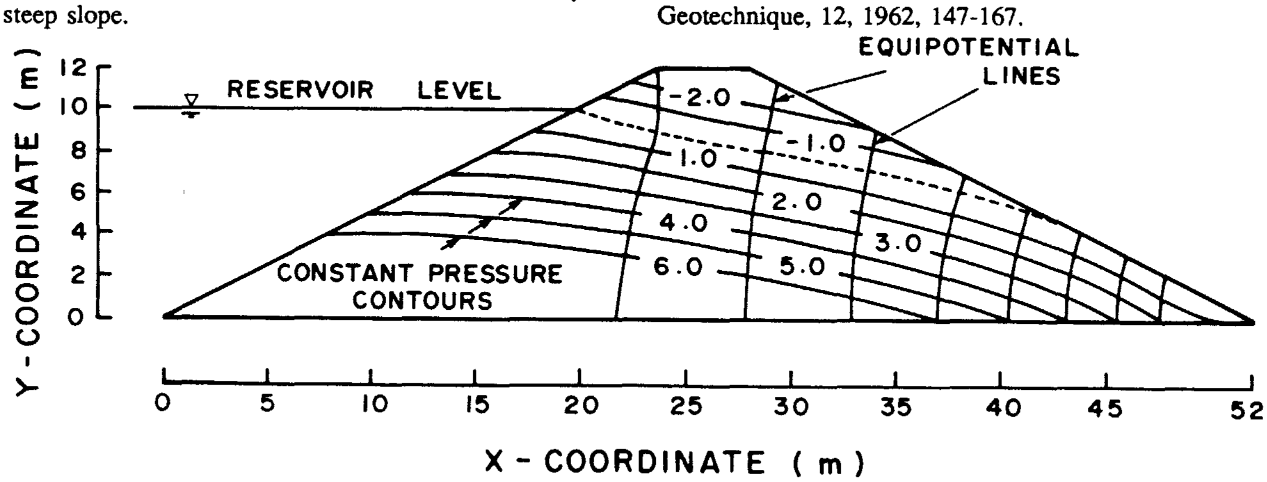

Fig. 5 Contours used to designate pore-water pressure heads

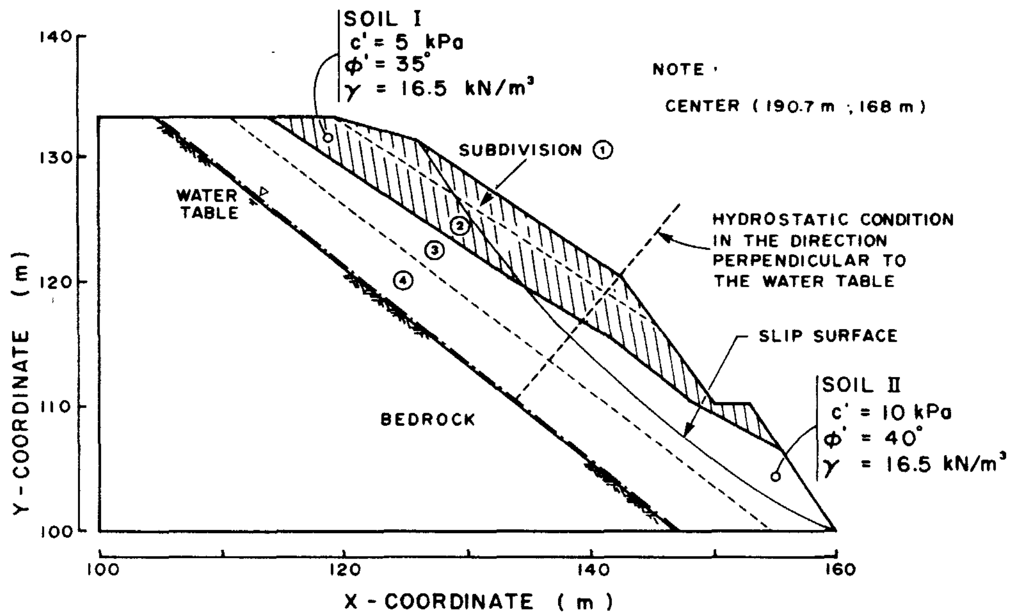

Fig. 6 Example of a typically steep slope (from Sweeney and Robertson, 1979: ref. 15)

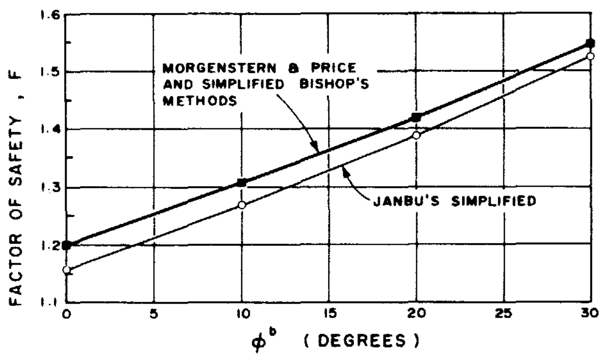

Fig. 7 Increase in factor of safety due to increase in ϕ^b angle

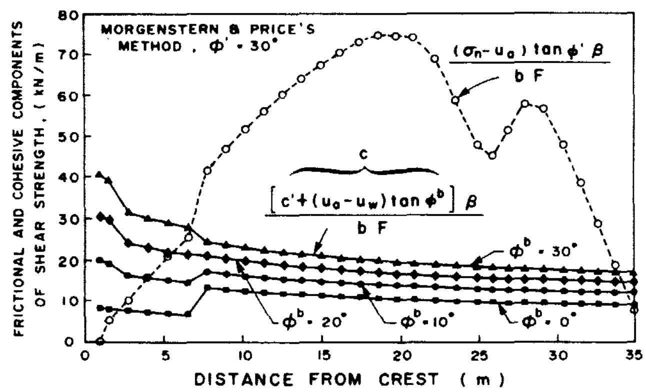

Fig. 8 Frictional and cohesive components of shear strength along the slip surface at various ϕ^b values

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