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The character of the shear strength envelope for unsaturated soils.

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Abstract

The shear strength of an unsaturated soil can be written as a linear equation using two independent stress state variables. In this paper, the research literature is reviewed to ascertain the acceptability of the proposed equation. Test results show that in most cases, the linear form adequately defines the shear strength behavior of unsaturated soils. When matric suction varies over a wide range, there appears to be some nonlinearity. The degree on nonlinearity appears to be related to the soil type. The nonlinearity can be satisfactorily accommodated using the linear equation, by giving consideration to the matric suction range of interest.

Introduction

Considerable attention has been given to the study of the shear strength of unsaturated soils during the past three decades. Studies have not been confined to any particular country, but rather there has been interest in various parts of the world. The types of strength tests performed have ranged from unconfined compression tests to modified direct shear tests with the pore-air, pore-water, and total stress measurements. The wide range of test data has often been presented in the absence of a theoretical context to assist in its interpretation. This has resulted in a relatively slow assimilation in understanding the shear strength behavior of unsaturated soils.

The objective of this paper is to both summarize our present understanding of the shear strength of unsaturated soils as well as to address some of the problems associated with the interpretation of shear strength data. For example, the categorization of what is to be considered an "identical" soil (in terms of shear strength parameters) has been clearly defined for *saturated* soils whereas the description of an "identical" soil for *unsaturated* soils has not been well defined. An attempt is made in this paper to provide the necessary definitions and theoretical context for future shear strength studies. Some of the primary needs with respect to future research studies, are suggested. The shear strength of an unsaturated soil is presented in terms of a linear equation while at the same time addressing deviations from the linear form along with suggestions as to how to best accommodate deviations in practice.

When is a Soil not an "Identical" Soil?

In geotechnical engineering practice, it would be expedient to be able to uniquely identify the shear strength (parameters) associated with each geological soil stratum. However, this is not always possible. For example, a particular geological stratum may contain soils with a wide variation in grain size or plasticity. On the other hand, the classification of the soil within a stratum could be consistent while the stress conditions and stress history could vary (e.g., desic-

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cation). In saturated soil mechanics, the shear strength parameters c' (i.e., effective cohesion) and ϕ' (i.e., effective angle of internal friction) have been found to be relatively unique within certain bounds. For example, the plasticity index must be relatively constant and the stress history must be essentially the same. The present insitu confining stress state may also be a variable to consider. If these conditions are *not* met, the soil stratum is simply subdivided and a further set of shear strength parameters must be defined. These are the conditions for an "identical" soil which have been well established in the practice of geotechnical engineering for *saturated* soils.

The testing of *unsaturated* soils has often been undertaken without careful consideration to these conditions. We could ask, "When is an *unsaturated* soil not an "identical" soil?" The conditions necessary for establishing what is an "identical" soil can readily be seen by examining a conventional compaction curve (Fig. 1). It would be convenient to say that the compaction data shown are from the same soil, and therefore, it should be possible to determine unique shear strength parameters for this soil. Unfortunately, this cannot be done. As shown in Fig. 1, the structure of the compacted soil changes along each compaction curve and from one compactive effort to another. If specimens were trimmed from a variety of compaction molds, each specimen would have to be considered to be a "new" soil. In other words, consistent initial soil structure and compaction stress conditions are additional necessary conditions that

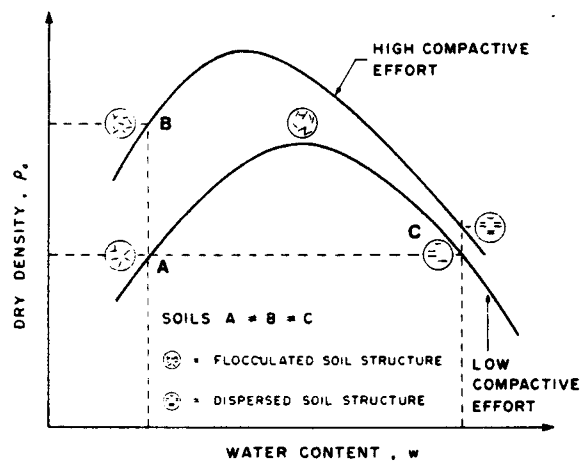


Figure 1 — The particle structure of clay specimens compacted at various dry densities and water contents (from Lambe, 1979).

must be met when identifying an "identical" unsaturated soil. Once an "identical" soil is identified, it should be possible to condition the soil to a variety of further stress conditions for testing. One specimen could be subjected to a matric suction of 50 kPa and tested. Another "identical" could be subjected to a matric suction of 100 kPa and tested and so forth. It should then be possible to develop a shear strength theory which would embrace these data in a unique equation.

On the other hand, it may initially appear to be expedient to change the matric suction of the soil by compacting it at varying initial water contents. Unfortunately, this will not result in an "identical" soil, and it would be highly unlikely that a unique shear strength equation would result. In other words, a procedure that appears to be expedient will violate the necessary conditions for an "identical" soil.

The above conditions for "identical" unsaturated soils have often been violated in research programs. As a result, the associated data must be avoided in formulating an unsaturated shear strength theory. Care has been taken to select data from "identical" soils in justifying the proposed shear strength equation.

Unsaturated Soil Shear Strength Equation

The first shear strength data which met the criteria for "identical" soil was published by Bishop, Alpan, Blight and Donald (1961). Strength data were presented on a compacted shale and a compacted boulder clay. The data were plotted and interpreted in terms of the χ parameter used in Bishop's (1959) effective stress equation. For a detailed discussion on the conceptual and theoretical limitations of such an approach, the reader is referred to the paper entitled, "The Stress State for Expansive Soils", Fredlund (1987) presented to the Sixth International Conference on Expansive Soils.

In 1978 Fredlund, Morgenstern and Widger proposed a linear shear strength equation for an unsaturated soil. The equation was written in terms of two independent stress state variables. The two stress state variables commonly used are the net normal stress, $(\sigma - u_a)$, and the matric suction, $(u_a - u_w)$, where: σ is total normal stress, u_a is pore-air pressure and u_w is pore-water pressure. The proposed shear strength equation has the following form:

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \qquad (1)$$

where:

τ_{ff} = shear stress on the failure plane at failure

c' = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis when the net normal stress and the matric suction at failure are equal to zero. It is also referred to as the "effective cohesion"

$(\sigma_f - u_a)_f$ = net normal stress on the failure plane at failure

ϕ' = angle of internal friction associated with the net normal stress state variable, $(\sigma_f - u_a)_f$

$(u_a - u_w)_f$ = matric suction at failure

ϕ^b = angle indicating the rate of change in shear strength relative to changes in matric suction, $(u_a - u_w)_f$

Equation 1 defines a planar surface which is called the extended Mohr-Coulumb failure envelope (Fig. 2). The envelope is tangent to the Mohr circles representing failure conditions. The shear strength of an unsaturated soil consists of an effective cohesion intercept, c', and the independent contributions to shear strength from net normal stress, $(\sigma - u_a)$, and matric suction, $(u_a - u_w)$. The shear strength contributions from net normal stress and matric suction are characterized by ϕ' and ϕ^b angles, respectively. The failure envelope on the frontal plane (i.e., τ versus $(\sigma - u_a)$ plane) is the same as the Mohr-Coulomb failure envelope for saturated conditions.

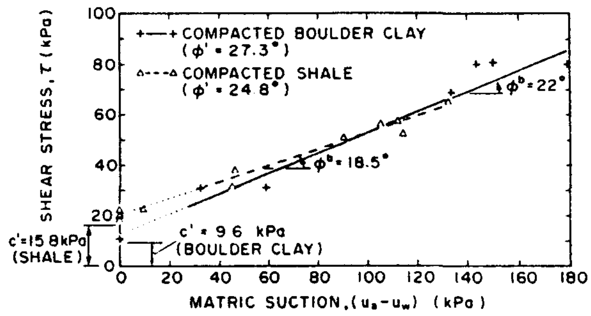


Figure 3 — Planar failure envelopes on the τ versus $(u_a - u_w)$ plane for two compacted soils (data from Bishop, Alpan, Blight and Donald, 1960).

Equation 1 was initially verified by Fredlund, Morgenstern and Widger (1978) by re-analysing the data of Bishop, Alpan, Blight and Donald (1961). The results given in Fig. 3 show essentially a straight line relationship between matric suction and shear strength. Later the re-analysis of other results was presented to support the proposed equation (Ho and Fredlund, 1982; Fredlund and Rahardjo, 1987). The data were accumulated from several authors. Consolidated drained and constant water content tests on unsaturated Dhanauri clay were presented by Satija (1978) and Gulhati and Satija (1981). A series of consolidated drained direct shear and triaxial tests on unsaturated Madrid gray clay were reported by Escario (1980). Most of the data analysed have shown that the failure surface for an unsaturated soil is essentially planar. A summary of best-fit parameters is shown in Table I.

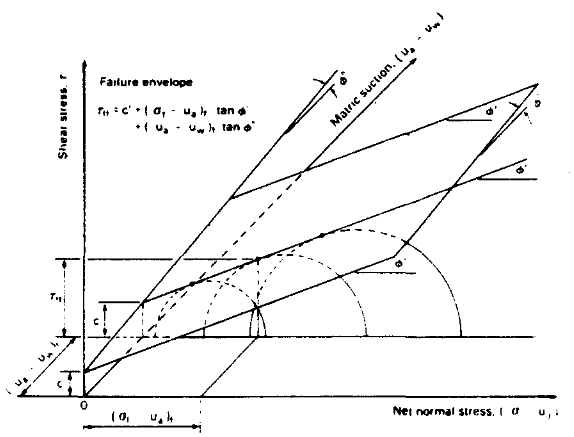


Figure 2 — Extended Mohr-Coulomb failure envelope.

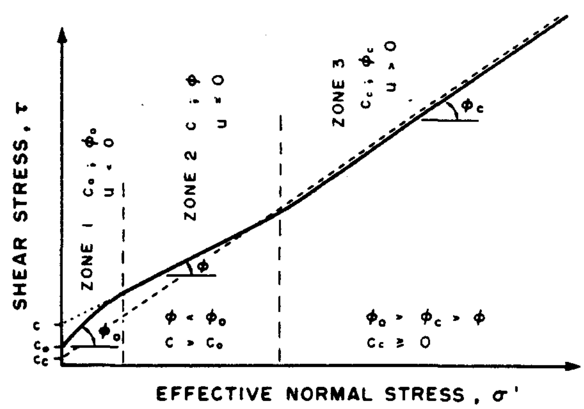


Figure 4 — Mohr envelope for compacted soils (from Mello, 1977).

Table 1 Experimental Values for ϕ^b

Soil Type	c' (kPa)	ϕ' (degrees)	ϕ^b (degrees)	Test Procedure	Reference
Compacted shale; w = 18.6%	15.8	24.6	18.1	Constant water content triaxial	Bishop, Alpan, Blight and Donald (1960)
Boulder clay; w = 11.6%	9.6	27.3	21.7	Constant water content triaxial	Bishop, Alpan, Blight and Donald (1960)
Dhanauri clay; w = 22.2%, ρ_d = 1580 kg/m^3	37.3	28.5	16.2	Consolidated drained triaxial	Satija, (1978)
Dhanauri clay; w = 22.2% ρ_d = 1478 kg/m^3	20.3	29.0	12.6	Consolidated drained triaxial	Satija, (1978)
Dhanauri clay: w = 22.2%, ρ_d = 1580 kg/m^3	15.5	28.5	22.6	Constant water content triaxial	Satija, (1978)
Dhanauri clay; w = 22.2%, ρ_d = 1478 kg/m^3	11.3	29.0	16.5	Constant water content triaxial	Satija, (1978)
Madrid gray clay; w = 29% ρ_d = 131 kg/m^3	23.7	22.5[a]	16.1	Consolidated drained direct shear	Escario, (1980)
Undisturbed decomposed granite; Hong Kong	28.9	33.4	15.3	Consolidated drained multi-stage triaxial	Ho and Fredlund (1982)
Undisturbed decomposed rhyolite; Hong Kong	7.4	35.3	13.8	Consolidated drained multi-stage triaxial	Ho and Fredlund (1982)
Tappen-Notch Hill silt; w = 21.5%, ρ_d = 1590 kg/m^3	0	35.0	16.0	Consolidated drained multi-stage triaxial	Krahn, Fredlund and Klassen (1987)

[a] Average value.

Deviations from a Planar Failure Surface

There is experimental evidence to support nonlinearity associated with the failure surface. It is important, however, not to conclude that a nonlinear shear strength equation is required. One need only consider the evolution of the shear strength theory for saturated soils to realize that a linear theory can adequately serve engineering practice. Let us consider the saturated shear strength envelope for a typical compacted soil as shown in Fig. 4 (de Mello, 1977). The nonlinearity can be adequately handled in practice simply by taking into consideration the effective stress range involved. This type of discretization should be used, if possible, to accommodate nonlinearity associated with the shear strength of unsaturated soils.

In 1956, Donald conducted a series of direct shear tests on fine sands and coarse silts where a negative pore-water pressure could be applied to the base of soil specimens. The specimens were consolidated to 48 kPa, subjected to a negative pore-water pressure and sheared. The results are shown in Fig. 5. The shear strength increased with suction to a peak value and then decreased to a relatively constant shear strength. The commencement of the drop in strength corresponds approximately to the desaturation point of the soil (Fredlund, Rahardjo and Gan, 1987).

Since this behavior is different than that observed by other researchers, it is important to consider this exceptional behavior. The first difference noted is the nature of the soil. Sands which are initially saturated have an exceptionally distinct air-entry value. Correspondingly, the cross-sectional area over which the water acts changes rapidly after a specific suction, with a slight increase in suction. Figure 6 shows typical, generalized water characteristic curves for soils of various grain size gradations. The inference is that a soil with an extremely sharp break in the water characteristic curve could also exhibit the largest nonlinearity (and a possible decrease) in strength with respect to an increase in suction. There may also be some influence due to the confining pressure range used in testing.

Gan (1986) performed direct shear tests on unsaturated glacial till where the pore-air, pore-water, and total stresses were controlled. All specimens were prepared in a similar manner and conditioned to suctions ranging from zero to 500 kPa. A net normal stress of 72.2 kPa was used for all tests. Plots of shear strength versus

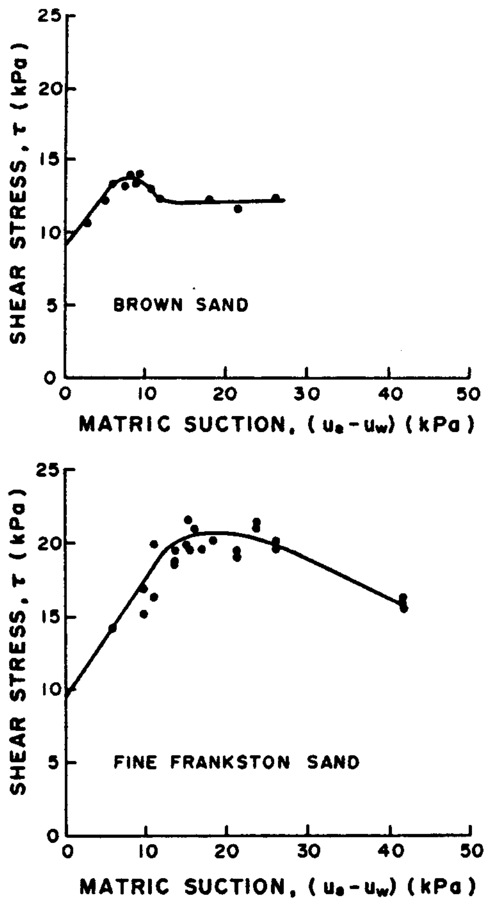


Figure 5 — Results of direct shear tests on sands under low matric suctions (from Donald, 1956).

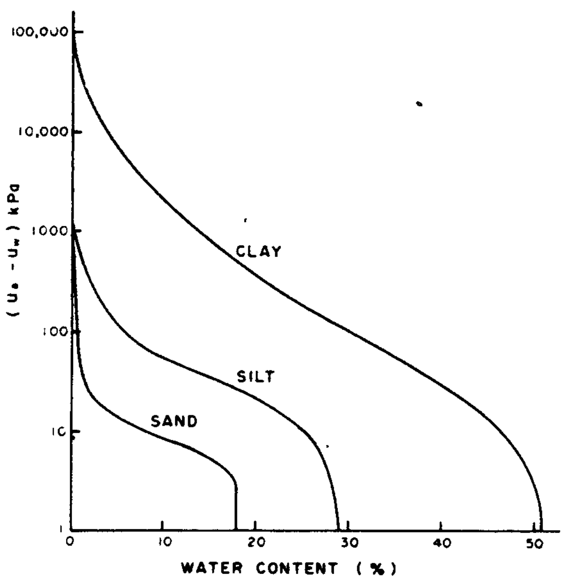


Figure 6 — Typical water characteristic curves for sand, silt and clay.

matric suction showed a nonlinear envelope (Fig. 7). The results fall within a band; the spread being due to differences in the initial void ratios for the specimens.

The slope of the matric suction versus shear strength envelope commences at an angle equal to ϕ' (i.e., 25.5°) near saturation and significantly decreases at matric suctions in the range of 50 kPa to 100 kPa. The ϕ^b angles reach a fairly constant value ranging from 5° to 10° when the matric suction exceeds 250 kPa.

A re-analysis of the triaxial results on compacted Dhanauri clay also reveals some nonlinearity in the failure envelope. The original shear strength parameters along with the parameters computed by Ho and Fredlund, (1982) using a planar failure envelope are summarized in Table 2. The linear interpretation of the failure envelope results in different c' and ϕ^b values when the same soil is tested using two different types of tests (i.e., consolidated drained and constant water content tests). This means that different

Table 2 - Triaxial Tests on Compacted Dhanauri Clay (Data from Satija, 1978)

Soils	CU Tests on Saturated Specimens		Analysis of Test Results on Unsaturated Specimens performed by Ho and Fredlund (1982)		
	c' (kPa)	(degrees)	Types of Test	c' (kPa)	ϕ^b (degrees)
Low Density			CD	20.3	12.6
ϱ_d = 1478 kg/m^3	7.8	29			
w = 22.2%			CW	11.3	16.5
High Density			CD	37.3	16.2
ϱ_d = 1580 kg/m^3	7.8	28.5			
w = 22.2%			CW	15.5	22.6

Note: ϱ_d = dry density; w = water content
CU = consolidated undrained; CD = consolidated drained; CW = constant water content

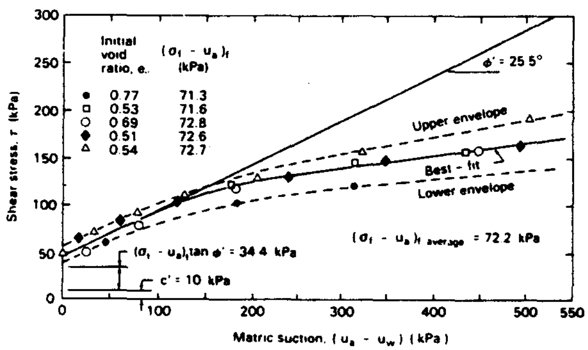


Figure 7 — Curved failure envelopes on the τ versus $(u_a - u_w)$ plane for a glacial till (from Gan, 1986).

types of tests on the same soil yield different shear strength parameters. In other words, the use of a planar failure envelope in analyzing these data causes a problem of non-uniqueness in the resulting shear strength parameters. In addition, the effective cohesion, c', values obtained from the analysis do not agree with values obtained from triaxial tests on saturated specimens.

The re-analysis of this data was performed by assuming a curved failure envelope with respect to the matric suction axis. The results are plotted on the shear strength versus matric suction plane corresponding to a zero net normal stress at failure (i.e., $(\sigma_f - u_a)_f = 0$). Figures 8 and 9 present the results for compacted Dhanauri clay at low and high densities, respectively. The shear strength parameters, c' and ϕ', obtained from the consolidated undrained tests on the saturated specimens (Table 1) were used in this re-analysis. The curved failure envelopes have a cohesion intercept of c' and a slope angle, ϕ^b, equal to ϕ'starting at zero matric suction. The ϕ^b angles start to decrease significantly at matric suctions less than 50 kPa for the low density spec-

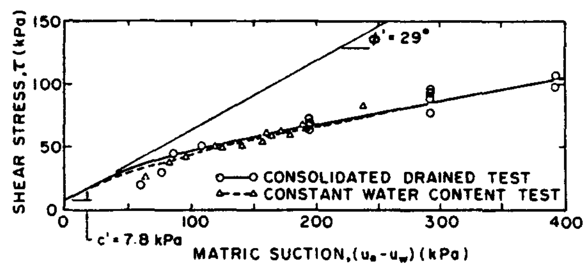


Figure 8 — Curved failure envelopes for compacted Dhanauri clay at low density (data from Satija, 1978).

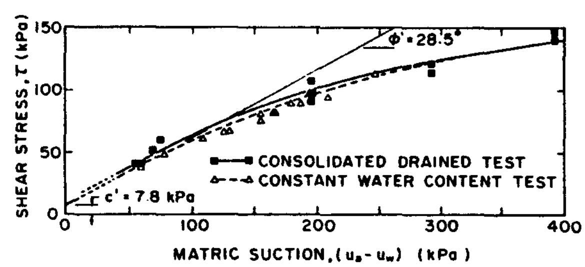
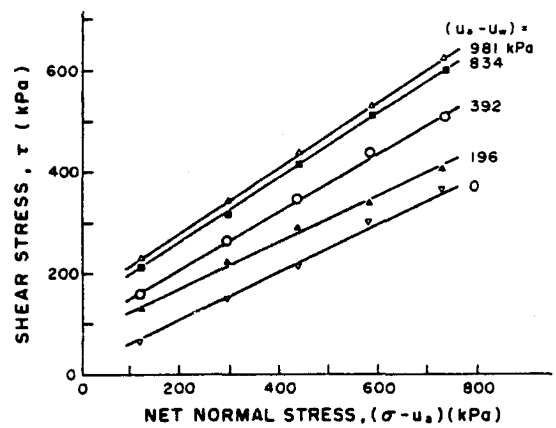
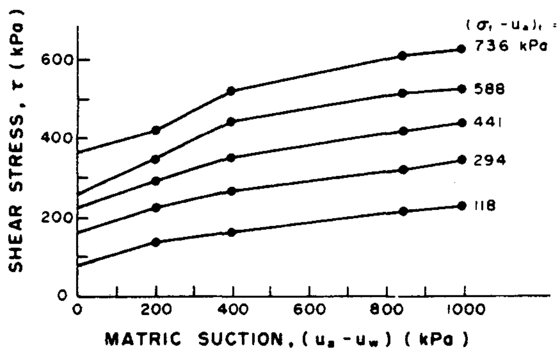


Figure 9 — Curved failure envelopes for compacted Dhanauri clay at high density (data from Satija, 1978).



a) Shear strength versus net confining pressure relationship for various matric suctions.



b) Shear strength versus matric suction relationship.

Figure 10 — Direct shear test results under controlled matric suctions for Madrid grey clay (from Escario and Sáez, 1986).

imens and at matric suction of 75 kPa to 100 kPa for the high density specimens. For the low density specimens, the ϕ^b angles reach a constant value of 11° when the matric suction exceeds 150 kPa. The ϕ^b angles for the high density specimens reach a constant value of 9° when the matric suction exceeds 300 kPa.

There is now good agreement between the failure envelopes through the consolidated drained and constant water content test results when allowing for curved failure envelopes (Figs. 8 and 9). In other words, the application of a curved matric suction, failure envelope to the data leads to a unique failure envelope for the same soil tested using different stress paths or procedures. The uniqueness of the curved failure envelope is also demonstrated at two densities. The specimens prepared at different initial densities should be considered as independent soils.

Escario and Sáez (1986) presented direct shear test data on three soils; namely, Madrid grey clay, Red clay of Guadalix de la Sierra, and Madrid clayey sand. The direct shear box was modified so that the pore-air, pore-water, and total stress could be independently controlled. Figure 10 shows data on the Madrid grey clay. The soil was tested over a wide stress range (i.e., matric suction from zero to 1000 kPa) and the data shows some nonlinearity with respect to matric suction. The low plasticity, Madrid clayey sand showed a distinct levelling off in strength after a matric suction of approximately 100 kPa.

There is need for shear strength tests to be performed on other soil types. The tests should be performed over a wide matric suction and total stress range. In this way, the character of the entire failure surface can be defined for different soils. Regardless of nonlinearities which may be observed, the failure surface can be linearized with respect to specific stress ranges. Details regarding three possible linearization procedures have been presented in the paper entitled, "Non-linearity of strength envelope for unsaturated soils", by Fredlund, Rahardjo and Gan, (1987).

Conclusions

The following conclusions can be made regarding the shear strength envelope for an unsaturated soil:

1) A linear form for the shear strength equation for an unsaturated soil appears to satisfactorily match the experimental data on most soils tested to-date.
2) The relationship between shear strength and matric suction may be somewhat nonlinear, particularly when matric suction is varied over a wide range. The degree of nonlinearity may also be a function of the soil type.
3) When the failure envelope is nonlinear, the following observation has been made: At low matric suctions when the soil remains saturated, the ϕ^b angle is approximately equal to the ϕ' angle. As the matric suction exceeds the air entry value of the soil, desaturation commences and the ϕ^b angle appears to reduce to a relatively constant value. On the basis of existing data, the curved failure envelope can be approximated by a bi-linear envelope.

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