# LABORATORY MEASUREMENT OF THE VOLUMETRIC DEFORMATION MODULI FOR TWO UNSATURATED SOILS

by

David Y.F. Ho, Faculty of Engineering
University of Regina
Regina, Saskatchewan S4S 0A2

and

D.G. Fredlund, Department of Civil Engineering
University of Saskatchewan
Saskatoon, Saskatchewan S7N 0W0

**Proceedings of the 42nd Canadian Geotechnical Conference, October 22-25, Winnipeg, Manitoba. 1989**

## SYNOPSIS

The development of unsaturated soil mechanics consists of several primary areas of study; namely, volume change, moisture movement, and shear strength behavior. Theory and technology developed in these areas form the basis for geotechnical engineering practice involving volume change, seepage, bearing capacity, and slope stability analysis. The ability to describe the volume change behavior of unsaturated soils is important to the solution of these problems. Volume change for an unsaturated soil is described in terms of two independent constitutive relations. The material properties required for these relations can be called volumetric deformation moduli.

A complete understanding of the volume change behavior of an unsaturated soil requires a knowledge of the volumetric deformation moduli on four state planes. The volumetric deformation moduli must be determined either by direct measurement or inferred from empirical relations in order to solve practical problems.

A laboratory test program was conducted to measure the volumetric deformation moduli associated with the soil structure and the water phase with response to changes in the net total stress and matric suction. Two soils, a uniform silt and a glacial till were tested using net total stress loading and matric suction loading. The specimens were also tested in the unloading mode, and in all cases, changes in void ratio and water content were independently measured. Specimens were formed by static compaction at half standard Proctor compaction effort at either "dry of optimum" or "at optimum" initial water contents. The investigation included specimens being loaded and unloaded under $K_0$ and isotropic conditions. Specially equipped modified Anteus consolidometers and stress controlled isotropic cells were used in the test program.

Data obtained from the test program are presented and analysed. The data is used to show how the various moduli are interrelated. As a result of these tests, it is possible to estimate relevant deformation moduli from a few conventional soil tests. The form of the constitutive relations for unsaturated soils were proposed by Fredlund and Morgenstern (1976). However, as far as the authors are aware, this is the first time that all deformation moduli for an unsaturated soil have been measured on "identical" specimens.

## INTRODUCTION

The seepage, volume change, and shear strength behavior are of primary concern in studying unsaturated soils. Theory and technology for these areas form the basis for geotechnical engineering practice involving deformation, seepage, bearing capacity, and slope stability analysis. The ability to describe the volume change behaviour in terms of the stress state of the soil, is important to the solution of these problems. Volume change for an unsaturated soil is described in terms of two independent constitutive relations. The material properties required for these relations can be called volumetric deformation moduli. A knowledge of the volumetric deformation moduli on four state planes is required to completely describe the volume change of an unsaturated soil. These moduli must be determined either by direct measurement or inferred from empirical relations in order to solve practical problems. The measurement of these moduli generally requires modification to conventional laboratory equipment. The tests are complicated and time consuming. An experimental study was undertaken to find the relationships between the various moduli. The objective is to find procedures which can be used to estimate relevant deformation moduli from a few conventional laboratory tests.

A laboratory test program was conducted to measure the deformation moduli associated with the soil structure and the water phase with respect to changes in the net total stress (i.e., $(\sigma - u_a)$) and the matric suction (i.e., $(u_a - u_w)$). Statically compacted specimens with near identical initial conditions were used in the test program. The specimens were tested in both the loading and unloading mode, under $K_o$ and isotropic conditions. The data are used to show how the various moduli are interrelated.

## CONSTITUTIVE RELATIONS THEORY

The continuity requirement justifies the representation of volume change in an unsaturated soil through use of the volume change relation for the soil structure and pore-water (Fredlund, 1973). The mathematical relations between the stress state variables and the volume change of the soil structure or pore-water phase are called constitutive relations. The graphical representations of the constitutive relations are called constitutive surfaces. There is published experimental evidence showing the characteristic form of the soil structure and pore-water phase constitutive surfaces.

The form of the relation between net total stress and void ratio is well established when matric suction is equal to zero. It is the same as the conventional compression curve for a saturated soil. The compression of a saturated soil consists of recompression and virgin compression branches (Terzaghi and Peck, 1967). The virgin compression curve is exponential on an arithmetic scale and can be linearized on a semi-logarithmic plot. The rebound curves are approximately parallel to one another and can be linearized on a semi-logarithmic scale (Schmertmann, 1955) (Holtz and Gibbs, 1956) (Gilchrist, 1963) (Noble, 1966) (Lambe and Whitman, 1962) (Lidgren, 1970) (Chen, 1975). Soils can be overconsolidated by desiccation and rebounded due to suction decrease. The virgin compression branch and rebound curves are essentially linear on a semi-logarithmic scale (Fredlund, 1967) (Aitchison and Woodburn, 1969) (Escario, 1969) (Aitchison and Martin, 1973) (Richards, Peter and Martin, 1984). There are pressure range limits which must be adhered to, to ensure linearity. A constant volume (i.e., constant void ratio) stress path where an unsaturated soil is becoming saturated is essentially a straight line on an arithmetic plot of the stress state variables. The same path becomes a convex asymptotic curve on a logarithmic scale (Escario, 1969).

On an arithmetic plot of void ratio, net total stress and matric suction, the soil structure constitutive surface is a concave, warped surface. The same surface resembles a quarter section of the convex surface of a vertical cone on a void ratio versus the logarithm of net total stress and matric suction plot. A schematic diagram of the soil structure constitutive surfaces for monotonic volume changes is shown (Figure 1). There are four moduli (i.e., $a_t$, $a_{ts}$, $a_m$, $a_{ms}$) associated with the arithmetic representation. These moduli are defined as follows,

$a_t$, $a_{ts}$ = coefficient of compressibility and coefficient of swelling with respect to the net total stress for monotonic volume decrease and increase, respectively

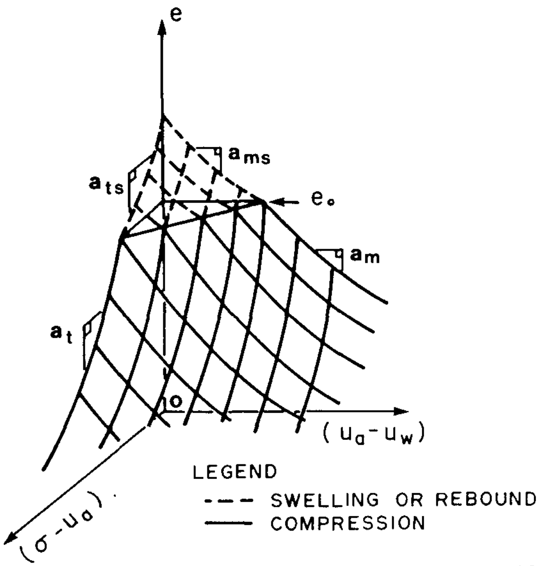
$a_m$, $a_{ms}$ = coefficient of compressibility and coefficient of swelling with respect to matric suction for monotonic volume decrease and increase, respectively.

There are four moduli (i.e., $C_t$, $C_{ts}$, $C_m$, $C_{ms}$) associated with the semi-logarithmic representation. These moduli are defined as follows,

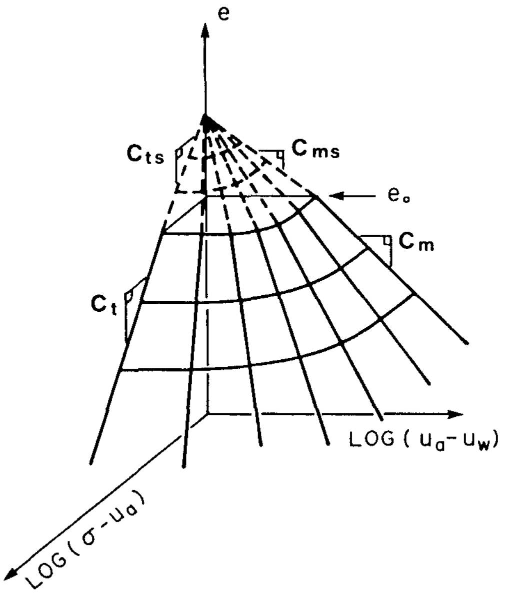
$C_t$, $C_{ts}$ = compressive and swelling index with respect to the net total stress for monotonic volume decrease and increase, respectively

$C_m$, $C_{ms}$ = compressive and swelling index with respect to matric suction for monotonic volume decrease and increase, respectively.

There is insufficient information in the literature to completely define the form of the water phase volume change constitutive surface. The water content and void ratio constitutive relations are interrelated by the relative



a A Schematic Diagram of the Soil Structure Constitutive Surfaces for Monotonic Volume Changes on an Arithmetic Scale



b A Schematic Diagram of the Soil Structure Constitutive Surfaces for Monotonic Volume Changes on a Semi-Logarithmic Scale

Fig. 1 Proposed Soil Structure Constitutive Surfaces for an Unsaturated Soil density, $G_s$, when a soil is saturated. Conclusions arrived at for the void ratio constitutive relation also pertain to the water phase constitutive relation. There is evidence that the water content versus matric suction constitutive curves on a semi-logarithmic scale are essentially linear when the net total stress is equal to zero (Croney and Coleman, 1954) (Fredlund, 1967) (McWhorter and Nelson, 1979) (Mitchell and Avalle, 1984). Once again, there are pressure limits within which there is linearity. The same curves are therefore approximately exponential on arithmetic plots. There is no experimental information available on the shape of a constant water content stress path on the constitutive surface. When a soil approaches saturation, a change in net total stress becomes as effective as a change in matric suction in changing the water content. A constant water content stress path must therefore approach a forty-five degree line when the matric suction tends to zero. This angle forms an upper limit. A constant water content stress path on the water phase constitutive surface will be assumed to be a straight line on an arithmetic plot of water content versus net total stress and matric suction. The resulting constitutive surface is a concave surface when water content is increased or decreased. The same surface resembles a quarter section of the convex surface of a vertical cone on a water content versus the logarithm of net total stress and matric suction plot. A schematic diagram of the water phase constitutive surfaces for monotonic volume charges is presented (Figure 2). There are four moduli (i.e., $b_t$, $b_{ts}$, $b_m$, $b_{ms}$) associated with the arithmetic representation. These moduli are defined as follows,

$b_t$, $b_{ts}$ = coefficient of water content and rebound coefficient of water content with respect to the net total stress for monotonic water content decrease and increase, respectively.

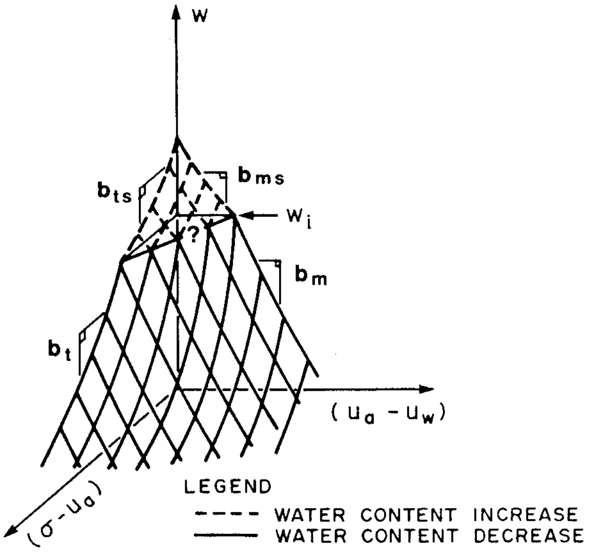
$b_m$, $b_{ms}$ = coefficient of water content and rebound coefficient of water content with respect to matric suction for monotonic water content decrease and increase, respectively.

There are four moduli (i.e., $D_t$, $D_{ts}$, $D_m$, $D_{ms}$) associated with the semi-logarithmic representation. The moduli are defined as follows,

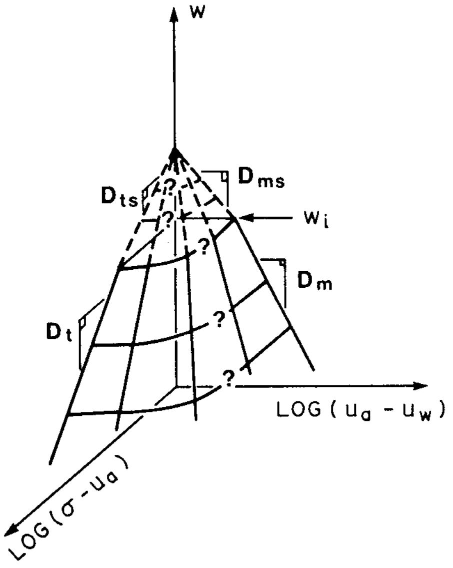
$D_t$, $D_{ts}$ = water content and rebound water content index with respect to the net total stress for monotonic water content decrease and increase, respectively

$D_m$, $D_{ms}$ = water content and rebound water content index with respect to matric suction for monotonic water content decrease and increase, respectively.

The analysis of the laboratory data will make use of moduli associated with the semi-logarithmic representation.



a A Schematic Diagram of the Water Phase Constitutive Surfaces for Monotonic Volume Changes on an Arithmetic Scale



b A Schematic Diagram of the Water Phase Constitutive Surfaces for Monotonic Volume Changes on a Semi-Logarithmic Scale

Fig. 2 Proposed Water Content Constitutive Surfaces for an Unsaturated Soil

## LABORATORY TEST PROGRAM

Two soils, a uniform silt and a glacial till, were tested and their index properties are presented in Table I. An attempt was made to prepare specimens with near identical initial conditions. Each soil was oven-dried and hand mixed with a predetermined quantity of distilled water. The wet soil was placed in a sealed plastic bag and left to cure in a constant humidity and temperature room. The difference in water content between batches of the same soil was controlled to within 0.5%. Specimens were formed by static compaction at one-half standard Proctor compaction effort at either "dry of optimum" or "at optimum" initial water contents. The compaction characteristics of the silt and till are shown in Figure 3 and 4, respectively.

Conventional soil testing equipment was used in the test program whenever possible. However, for some stress paths, it was necessary to develop special equipment. Specially equipped modified Anteus consolidometers (Figure 5) and stress controlled isotropic cells (Figure 6) were used to measure the moduli under one-dimensional and isotropic loading

**Table I**
**Index Properties of the Silt and Glacial Till Used in the Test Program**

| | Silt | Glacial Till |
|---|---|---|
| Liquid Limit | 26.7% | 33.2% |
| Plastic Limit | 14.9% | 13.0% |
| Plasticity Index | 11.8% | 20.2% |
| Percent Sand Sizes | 25.0% | 32.0% |
| Percent Silt Sizes | 52.0% | 39.0% |
| Percent Clay Sizes | 23.0% | 29.0% |
| Relative Density of Solids, $G_s$ | 2.72 | 2.76 |

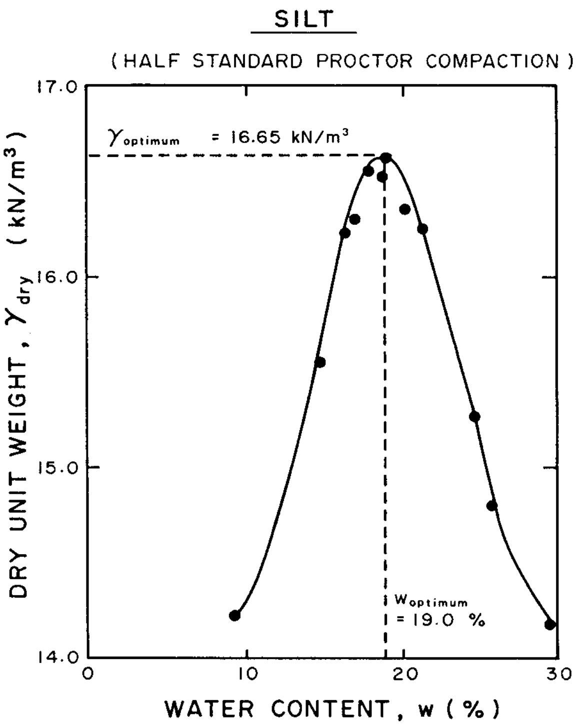


Fig. 3 Compaction Curve for Silt

conditions, respectively. High air entry disks were installed on the base pedestals (i.e., sealed around the edges with epoxy) of both apparatuses. This allowed the axis-translation technique (Hilf, 1956) to control the matric suction in the specimen. The permeability of the high air entry disc is low (e.g., 2 x $10^{-9}$m/s for a 15 bar disc). Therefore, water moving to the specimen through the base pedestal requires a long period of time (Ho and Fredlund, 1982). A newly designed composite loading cap was built consisting of an aluminum top and two layers of porous alundum stones. Three 17-gauge (i.e., 0.147 mm O.D. and 0.104 mm I.D.) hypodermic needles were installed through the loading cap. This allowed water to be added to the specimen under any applied stress condition.

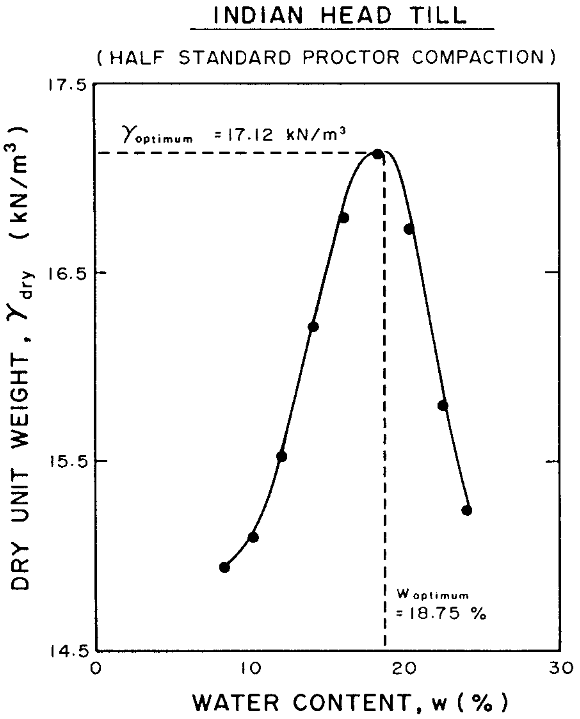
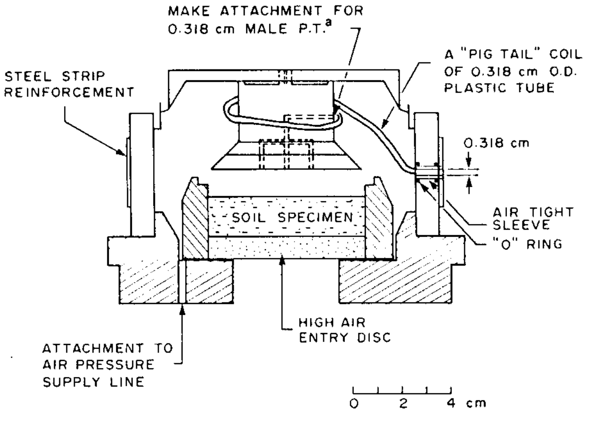


Fig. 4 Compaction Curve for Glacial Till

The main objective of the test program was to experimentally quantify the form of the constitutive surfaces on the net total stress and matric suction planes for various loading conditions. The net total stress and matric suction planes are state planes on which the matric suction and net total stress are at a nominal value, respectively. The form of the constitutive surface will provide an understanding of the relationships between relevant moduli. Two laboratory programs were performed to achieve this purpose. Sub-program I tested soils under stress changes involving no lateral expansion. The term "no lateral expansion", indicates a $K_o$ loading condition as long as there is a tendency for lateral expansion. Sub-program II tested soils under isotropic changes of total stress and matric suction. For both programs, specimens were tested in the loading as well as the unloading mode. In all cases, changes in void ratio and water content were independently measured. Layouts of the sub-programs are shown in Table II and III. The free swell and constant volume loading and unloading tests were performed using the modified Anteus consolidometers and stress controlled isotropic cells for testing under $K_o$ and isotropic conditions, respectively. Stress paths for the free swell and constant volume loading and unloading tests are shown in Figure 7 and 8.

## PRESENTATION OF TEST RESULTS

In the test program, silt and till specimens were tested at both "dry of optimum" and "at optimum" initial water contents. One-dimensional constant volume loading and unloading tests were performed on both silt and till specimens. Results from tests on similar specimens are combined to obtain average void ratio versus net total stress loading and unloading curves. A typical set of data are



NOTE: a) P.T. MEANS PIPE THREAD

Fig. 5 Schematic Layout of the Modified Anteus Consolidometer

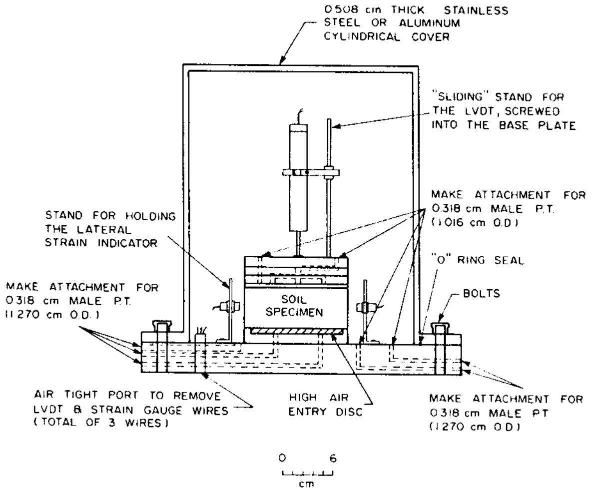
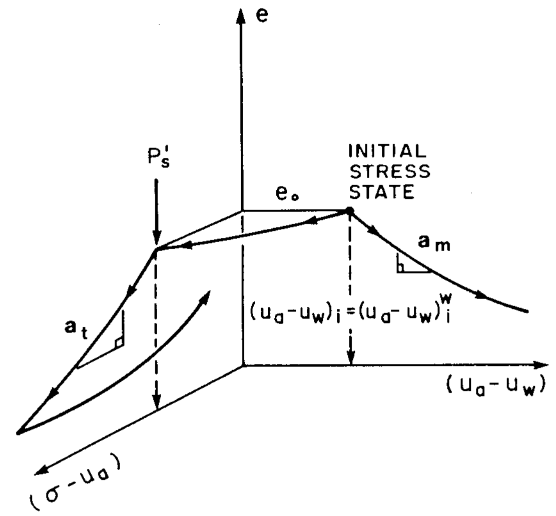
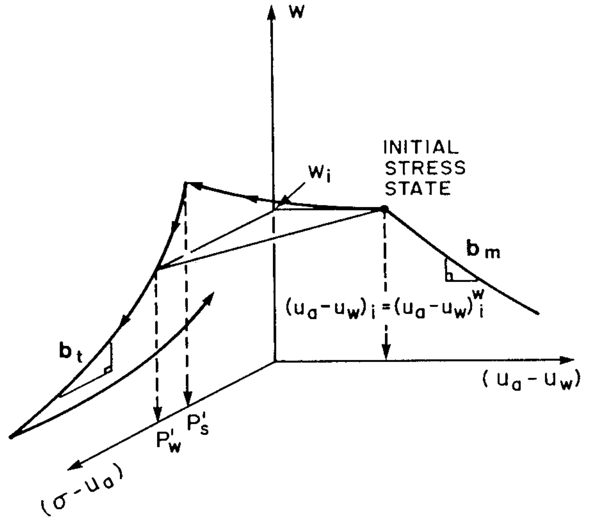


Fig. 6 General Assembly of the Stress Controlled Isotropic Cell

shown in Figure 9. Isotropic constant volume loading and unloading tests were performed only on the silt specimens. The results are shown in Figure 10. A suction test was used to establish the water content versus increasing matric suction relation. Published information on the characteristic shape of the water content versus increasing matric suction curves is incorporated with the test results to define the water phase loading curve. The projected matric suction at zero water content from the linear portion of the water content versus increasing matric suction curve is set to be $3.0 \times 10^5$ kPa (Croney and Coleman, 1954). The matric suction at zero water content is assumed to be at $6.2 \times 10^5$ kPa (Mitchell and Avalle, 1984). A typical set of data are presented in Figure 11. The unconfined shrinkage test was used to establish the water content versus void ratio curve of a soil undergoing increasing matric suction. Combining this information with the water content versus matric suction loading curve gives the loading curve for the soil structure or void ratio versus increasing matric suction relation. A typical set of unconfined shrinkage test results is presented in Figure 12. One-dimensional free swell tests were performed on both silt and till specimens. The test results are used to establish the void ratio and water content versus matric suction unloading curves under $K_o$ conditions. The



a) Void Ratio Constitutive Surface (Loading)



b) Water Content Constitutive Surface (Loading)

Fig. 7 Stress Path of the One-Dimensional and Isotropic, Constant Volume Loading (and Unloading) Tests

results are shown in Figure 13 and 14. Isotropic free swell tests were done only on silt specimens. The resulted void ratio and water content versus matric suction unloading curves are shown in Figure 15.

## DATA ANALYSIS AND CONCLUSION

The void ratio and water content constitutive curves are combined to show the characteristic form of the semi-logarithmic soil structure and water phase constitutive surfaces on the net total stress and matric suction state planes. The combined void ratio and water content loading curves for silt and till compacted dry of optimum and at optimum initial water contents under one-dimensional conditions are presented in Figure 16, 17, 18 and 19. Rieke and Chilincarian (1974) reported that the void ratio of most soils would be reduced to zero at a pressure of approximately $3.5 \times 10^6$ kPa. This information is used as a control to define

Table II Layout of the Sub-Program I

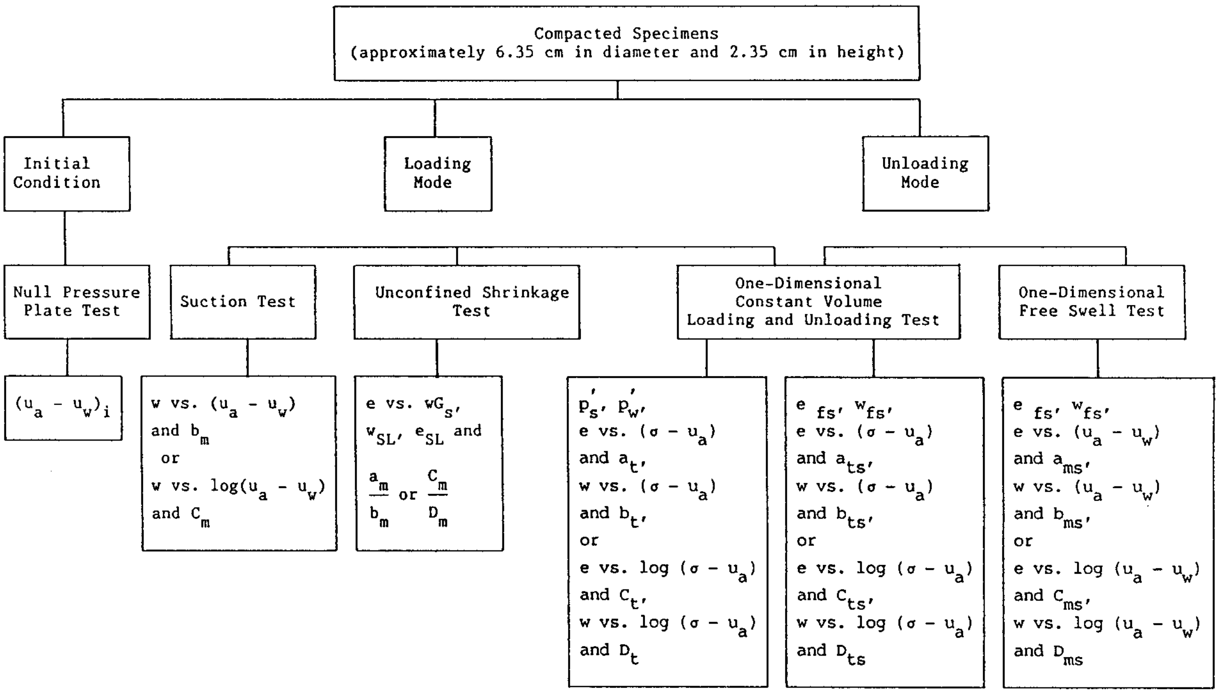
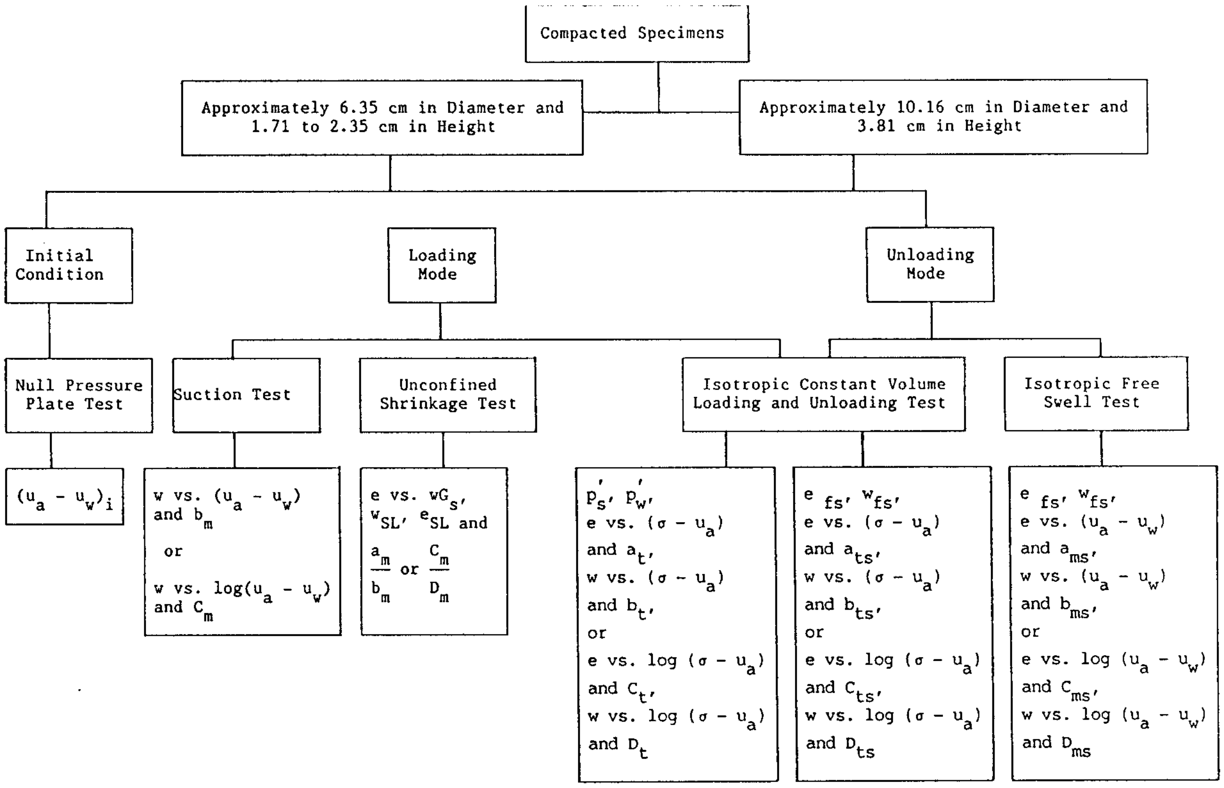
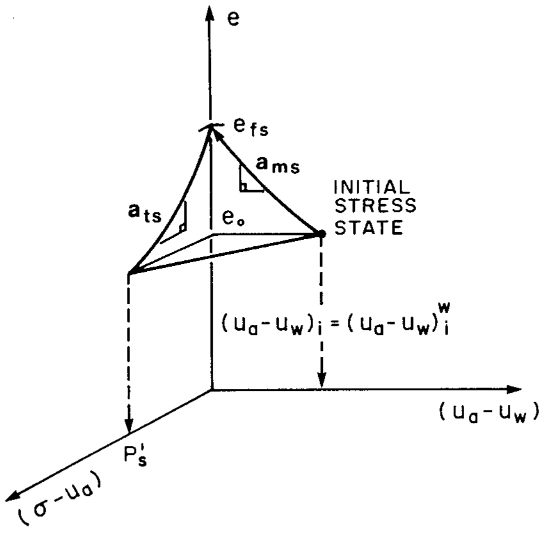
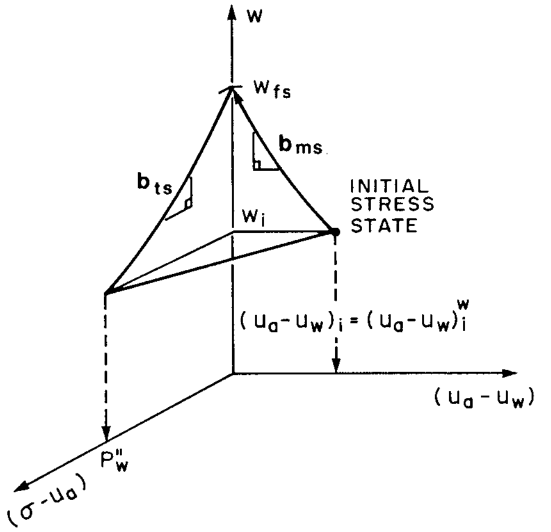


Table III Layout of the Sub-Program II





a) Void Ratio Constitutive Surface (Unloading)



b) Water Content Constitutive Surface (Unloading)

Fig. 8 Stress Path of the One-Dimensional and Isotropic Free Swelling Tests

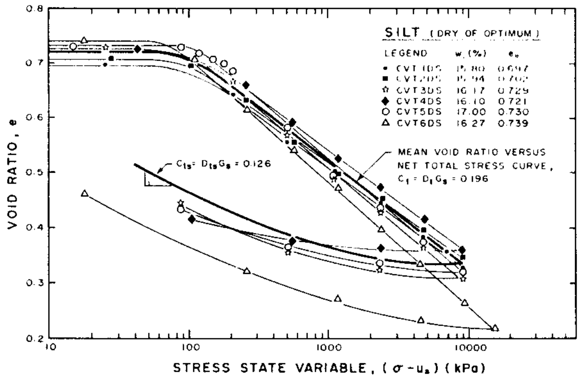


Fig. 9 Results for One-Dimensional Constant Volume Loading and Unloading Tests for Silt with Dry of Optimum Initial Water Content

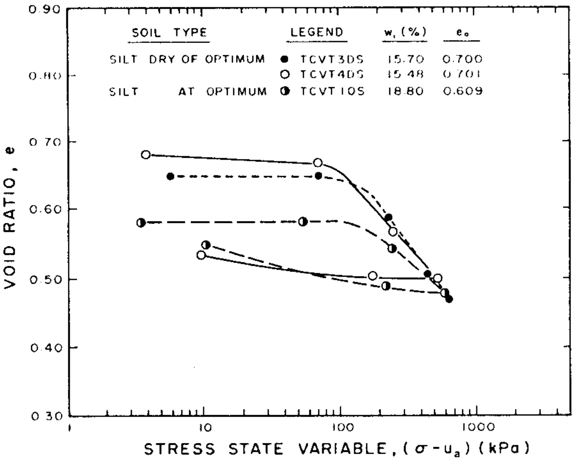


Fig. 10 Results for the Isotropic Constant Volume Loading and Unloading Tests for Silt Specimens

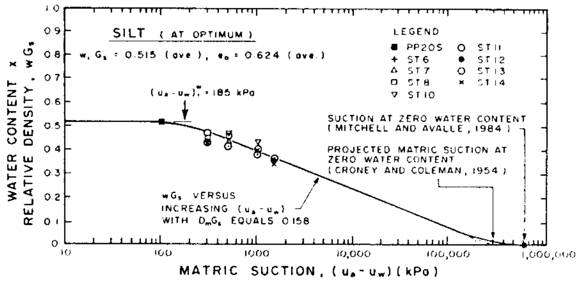


Fig. 11 Average Water Content Versus Increasing Matric Suction Curve for Silt with Optimum Initial Water Content

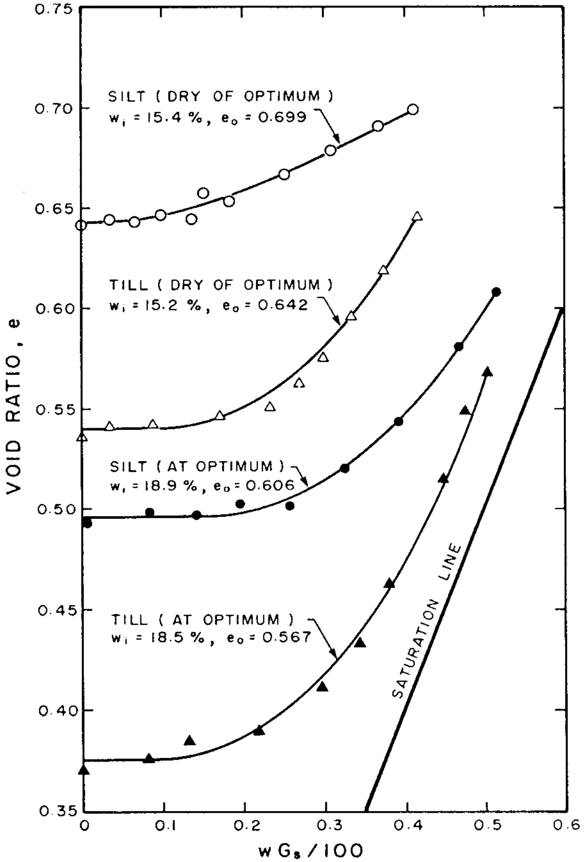


Fig. 12 Typical Set of Unconfined Shrinkage Tests Results

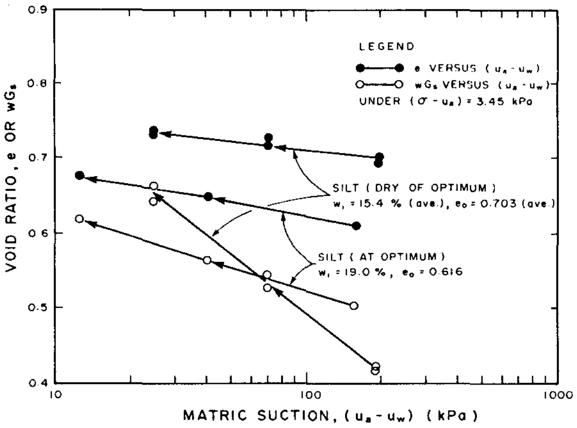


Fig. 13 Results for One-Dimensional Free Swell Tests on Silt Specimens

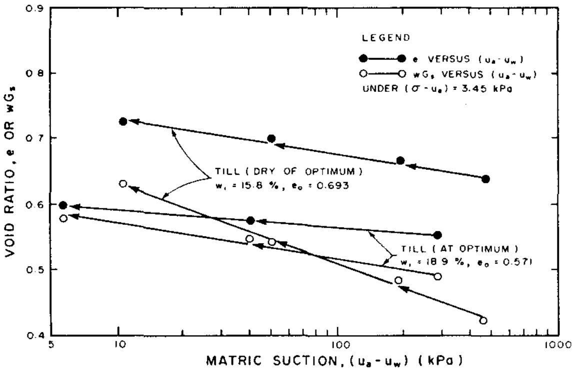


Fig. 14 Results for One-Dimensional Free Swell Tests on Till Specimens

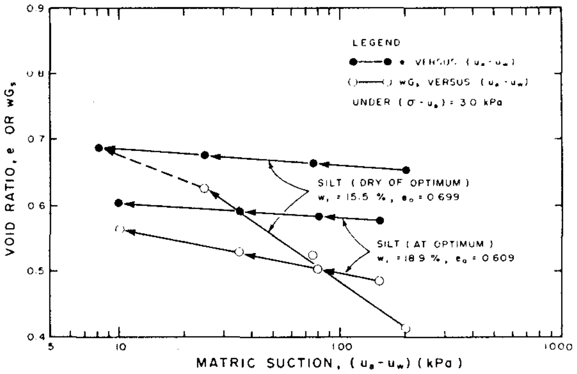


Fig. 15 Results for the Isotropic Free Swell Tests on Silt Specimens

the void ratio versus net total stress loading curve at high pressure range. Similar combined plots can be produced for one-dimensional unloading conditions as well as isotropic loading and unloading conditions. The slopes of the loading and unloading curves are the volumetric deformation moduli. A summary of the moduli is presented in Table IV.

Relationships between the soil structure moduli can be developed from the characteristic form of the soil structure constitutive surfaces. The moduli measurements are used as verification. Relationships between the water phase moduli can be derived from the experimental results. As a result, it is possible to estimate relevant deformation moduli from a few conventional soil tests. The analytical relationships between the moduli will be dealt with in a separate paper.

Table IV A Summary of the Experimentally Measured Volumetric Deformation Moduli

| SOIL TYPE+ | LOADING | | | | | | | | SOIL TYPE+ | UNLOADING | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | One-dimensional | | | Isotropic | | | | | | One-dimensional | | | | Isotropic | | | |
| | $w_iG_s$ | $e_o$ | $C_t$ or DtGs | $C_m$ | $D_mG_s$ | $C_t$ or DtGs | $C_m$ | $D_mG_s$ | | $w_iG_s$ | $e_o$ | $C_{ts}$* | $D^{**}_{tsGs}$ | $C_{ms}$ | $D_{ms}G_s$ | $C_{ts}$* | $D_{ts}G_s$** | $C_{ms}$* | $D_{ms}G_s$ |
| DS | 0.420 | 0.699 | 0.196 | 0.030 | 0.124 | | | | DS | 0.419 | 0.702 | 0.040 | 0.126 | 0.033 | 0.263 | | | | |
| DS | 0.424 | 0.700 | | | | 0.230 | 0.030 | 0.124 | DS | 0.421 | 0.699 | | | | | 0.024 | 0.024 | 0.020 | 0.194 |
| OS | 0.516 | 0.606 | 0.177 | 0.082 | 0.158 | | | | OS | 0.517 | 0.616 | 0.055 | 0.076 | 0.052 | 0.101 | | | | |
| OS | 0.511 | 0.609 | | | | 0.187 | 0.082 | 0.158 | OS | 0.514 | 0.609 | | | | | 0.024 | 0.040 | 0.022 | 0.050 |
| DT | 0.427 | 0.642 | 0.206 | 0.089 | 0.159 | | | | DT | 0.436 | 0.693 | 0.066 | 0.084 | 0.056 | 0.122 | | | | |
| OT | 0.516 | 0.567 | 0.179 | 0.106 | 0.171 | | | | OT | 0.523 | 0.571 | 0.037 | 0.057 | 0.024 | 0.060 | | | | |

NOTES: +) "DS" stands for silt at dry of optimum initial water content
"OS" stands for silt at optimum initial water content
"DT" stands for till at dry of optimum initial water content
"OT" stands for till at optimum initial water content
*) average slope of the unloading curve
**) slope of the linear portion of the unloading curve

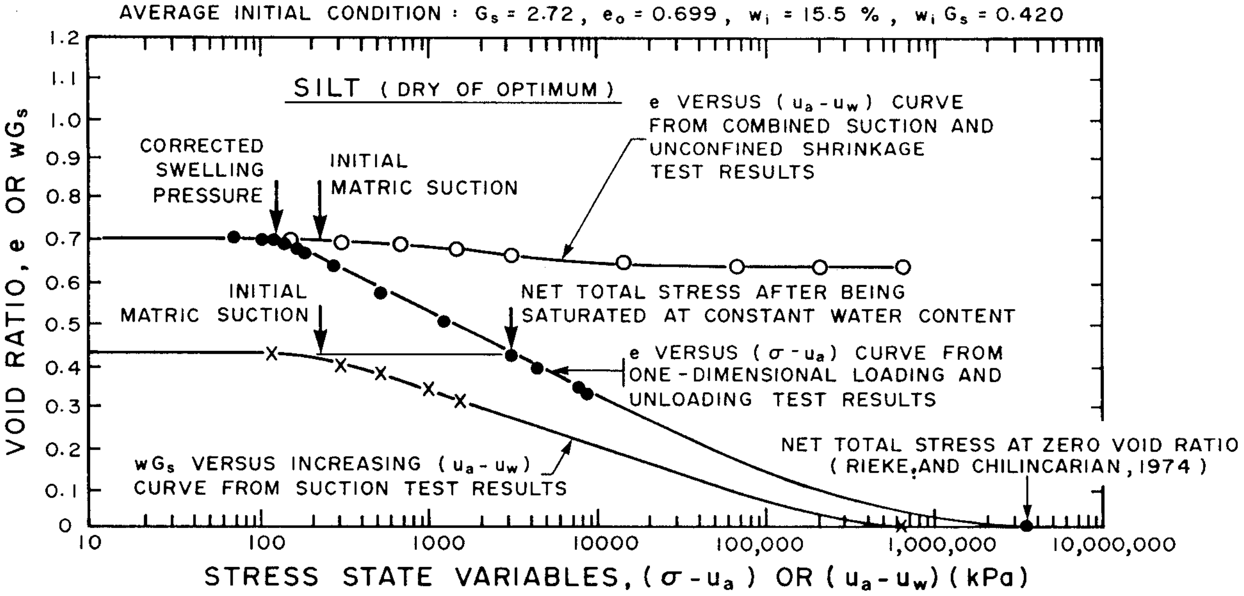


Fig. 16 Constitutive Relations for Silt with Dry of Optimum Water Content Under One-Dimensional Loading Conditions

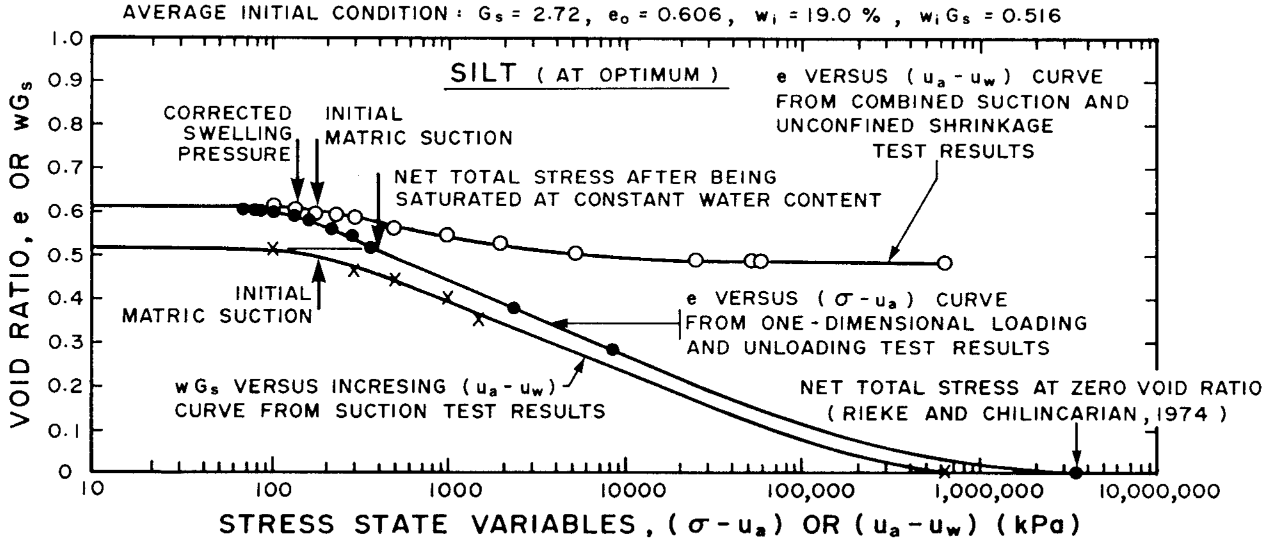


Fig. 17 Constitutive Relations for Silt at Optimum Water Content under One-Dimensional Loading Conditions

## SUMMARY

A two-dimensional combined plot of void ratio and water content versus net total stress and matric suction constitutive curves can be used to illustrate conceptually the relationships between the moduli (Figure 20). The void ratio versus net total stress loading curve can be viewed as the most fundamental constitutive relation. The water content times the relative density, $G_s$, is equal to the void ratio when a soil is saturated. Therefore, a plot of water content times relative density, at any degree of saturation, gives an indication of the degree of saturation in terms of void ratio. In addition, a change in matric suction or net total stress are equally effective in changing the void ratio or water content of a soil when it is saturated. The water content versus matric suction loading curve therefore shifts upward and eventually coincides with the void ratio versus net total stress loading curve when the degree of saturation approaches one hundred percent. Similarly, the void ratio versus matric suction loading curve lowers down towards the void ratio versus net total stress loading curve with an increasing degree of saturation. Such interrelations amongst the soil structure and water phase constitutive curves form the basis for relating the moduli.

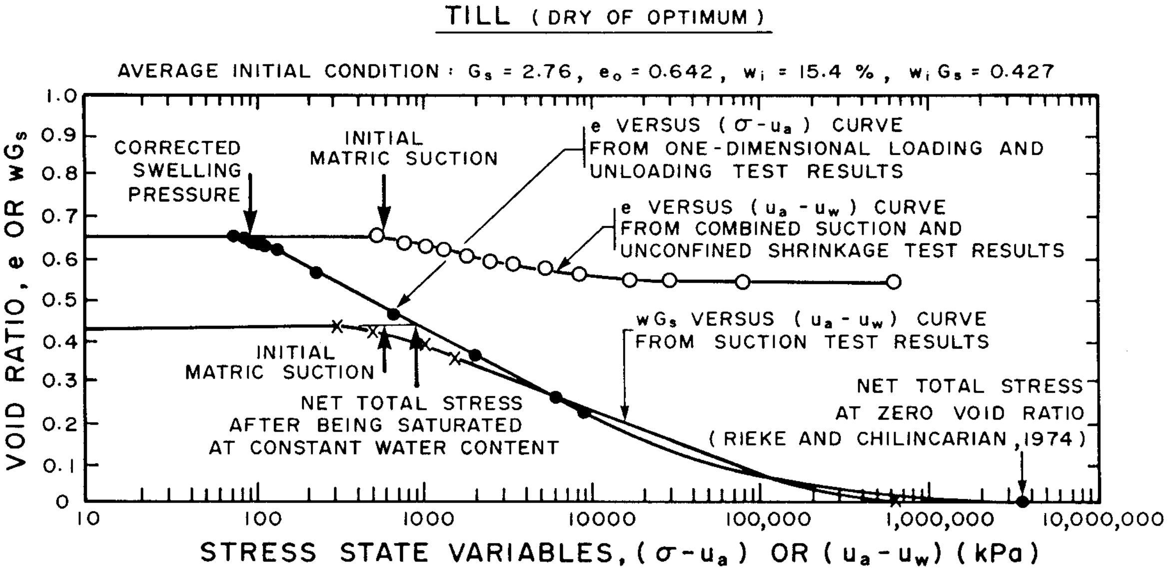


Fig. 18 Constitutive Relations for Till With Dry of Optimum Water Content Under One-Dimensional Loading Conditions

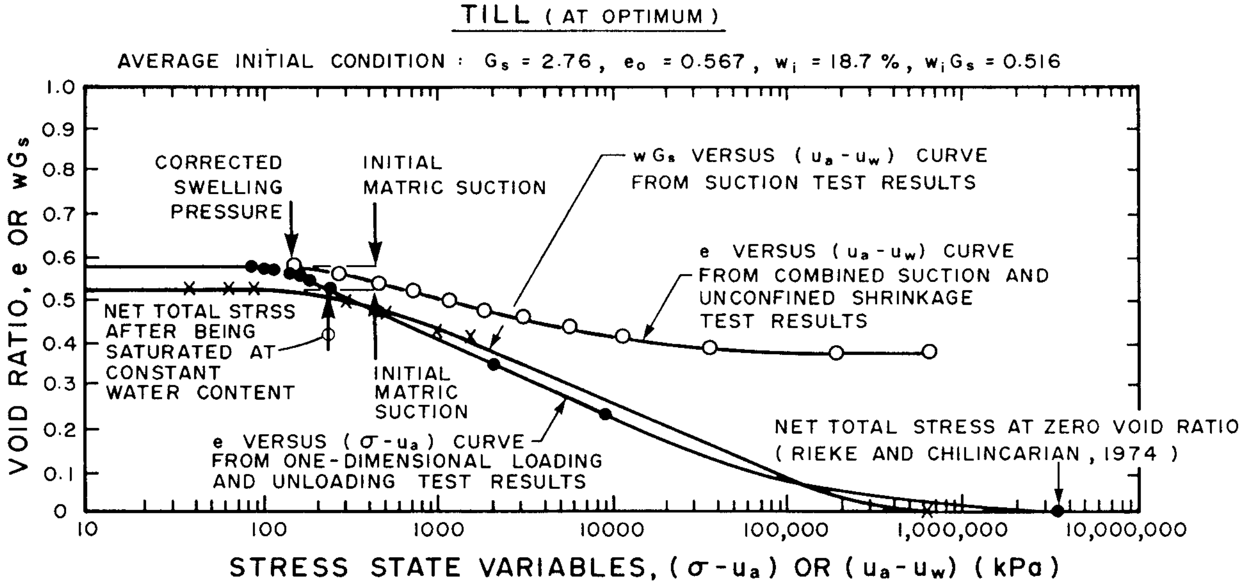


Fig. 19 Constitutive Relations for Till with Optimum Water Content Under One-Dimensional Loading Conditions

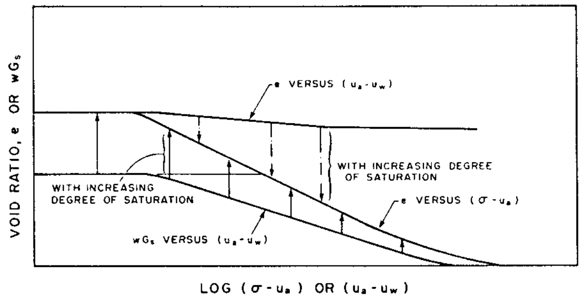


Fig. 20 Schematic Illustration of the Conceptual Relationships Between the Various Moduli of an Unsaturated Soil under Loading Conditions

## REFERENCES

Aitchison, G.D. and R. Martin. 1973. A Membrane Oedometer for Complex Stress-Path Studies in Expansive Clays. Proceedings of the Third International Conference on Expansive Soils, Haifa, Israel, Vol. 2, pp. 83-88.

Aitchison, G.D. and J.A. Woodburn. 1969. Soil Suction in Foundation Design. Proceedings of the Seventh International Conference on Soil Mech. Found. Engg., Mexico, Vol. 2, pp. 1-8.

Chen, F.H. 1975. Foundation on Expansive Soils. First Edition. Elsevier Scientific Publishing Co., New York.

Croney, D. and J.D. Coleman. 1954. Soil Structure in Relation to Soil Suction (pF). Journal of Soil Science, 5(1), pp. 75-84.

Escario, V. 1969. Swelling of Soils in Contact with Water at A Negative Pressure. Proceedings of the Second Int. Research and Engg. Conf. on Expansive Clay Soils, Texas A & M University, College Station, U.S.A., pp. 207-218.

Fredlund, D.G. 1967. Comparison of Soil Suction and One-Dimensional Consolidation Characteristics of a Highly Plastic Clay. National Research Council of Canada, Division of Building Research, Technical Report No. 245.

Fredlund, D.G. 1973. Volume Change Behaviour of Unsaturated Soils. Ph.D. Dissertation, University of Alberta, Edmonton.

Fredlund, D.G. and N.R. Morgenstern. 1976. Constitutive Relations for Volume Change in Unsaturated Soils. Canadian Geotechnical Journal, Vol. 13, No. 3, pp. 261-276.

Gilchrist, H.G. 1963. A Study of Volume Change of a Highly Plastic Clay. M.Sc. Dissertation, University of Saskatchewan, Saskatoon.

Hilf, J.H. 1956. An Investigation of Pore Water Pressure in Compacted Cohesive Soils. Bureau of Reclamation Tech. Mem. 654.

Holtz, H.G. and H.J. Gibbs. 1956. Engineering Properties of Expansive Clays. Transactions of the ASCE, Vol. 121, pp. 641-663.

Lambe, T.H. and R.V. Whitman. 1979. Soil Mechanics. John Wiley & Sons Inc., New York.

Lidgren, R.A. 1970. Volume Change Characteristics of Compacted Till. M.Sc. Dissertation, University of Saskatchewan, Saskatoon.

McWhorter, D.B. and J.D. Nelson. 1979. Unsaturated Flow Beneath Tailings Impoundments. Journal of the Geot. Engg. Div., ASCE, Vol. 105, pp. 1317-1334.

Mitchell, P.H. and D.L. Avalle. 1984. A Technique to Predict Expansive Soil Movements. Proceedings of the Fifth Int. Conf. on Expansive Soils, Adelaide, Australia, pp. 124-130.

Noble, C.A. 1966. Swelling Measurements and Prediction of Heave for a Lacustrine Clay. Can. Geot. Journal, Vol. 3, pp. 32-41.

Richards, B.G., P. Peter and R. Martin. 1984. The Determination of Volume Change Properties in Expansive Soils. Proc. of the Fifth Int. Conf. on Expansive Soils, Adelaide, Australia, pp. 179-186.

Rieke, H.H. and G.V. Chilincarian. 1974. Compaction of Argillaceous Sediments. Development in Sedimentology No. 16, Elsevier Scientific Publishing Co., Amsterdam.

Schmertmann, J.M. 1955. The Undisturbed Consolidation of Clay. ASCE Transactions, Vol. 120, p. 1201.

Terzaghi, K. and R.B. Peck. 1967. Soil Mechanics in Engineering Practice. John Wiley & Sons Inc., New York.