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The Role of Kriging in Slope Stability Analysis

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ABSTRACT

The Kriging technique is a numerical procedure which found its original application in the prediction of ore volumes in the mining industry. More recently it has been applied to a wide variety of problems.

This paper examines the role of Kriging in the prediction of pore-water pressures in limit equilibrium slope stability analyses. The pore-water pressures of interest are those at the base of a slice when the method of slices is used.

The results of the study show that the Kriging technique is more accurate than the 4-Way Interpolation technique. However, the Kriging technique requires significant computing time. As a result, the number of points which can be Kriged (on a microcomputer) within a slope stability computer program is somewhat limited.

KEYWORDS: Kriging, 4-Way Interpolation technique, slope stability analyses, computational time, factor of safety

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INTRODUCTION

Kriging is a geostatistical estimation procedure that predicts the value of a parameter at any location based on; a knowledge of the structure of the variability as represented by a set of known data points, and on the minimization of the estimation variance over the entire region for which the prediction is to be made. In geotechnical engineering, geostatistics have been used to study spatial variability of insitu measurements and for quality control in the construction of earth structures. However, the use of geostatistics in prediction of pore pressures or stresses in slopes has not been studied extensively (Soulie, 1983). The technique was first used in the mining industry to predict ore volumes. Presently, it is widely used to contour variables and to produce three-dimensional nets. In the study presented herein, the Kriging technique was used to predict the pore-water pressure at the base of a slice (method of slices) when performing limit equilibrium slope stability analyses.

The Kriging procedure, using a spline function, has been added to the PC-SLOPE code (Fredlund, 1987; Fredlund and Fredlund, 1987). The technique uses the input of pore-water pressures at a limited number of points superimposed on the geometry. Previously a 4-Way Interpolation technique was used to compute pore-water pressures at the base of a slice in PC-SLOPE. The main purpose of this study was to compare the two procedures for computing pore-water pressures; namely, the Kriging technique and the 4-Way Interpolation technique. This paper provides a brief summary of the theory associated with Kriging, comments on possible applications relative to the slope stability problem, and presents data on one application; namely, the prediction of pore-water pressures at a specified point in a soil mass.

EXAMPLES OF KRIGING

Engineers are familiar with the computer programs that are available to produce contour maps and nets showing the variation of a variable in space. The variable may range from the elevation of the ground surface to the concentration of a chemical in the ground. The variable under consideration may have been measured at a few, randomly selected points and the computer program produces a contour map or net which "impressively" presents the variation of the variable over a region. The technique most commonly used to produce these plots is called Kriging. The known data points are used to compute Kriging coefficients which are subsequently used to compute the magnitude of the variable under consideration at all points on a mesh. The points on the mesh are subsequently used for contouring or plotting as a net. Figure 1 shows a typical contour map and Fig. 2 shows the corresponding three-dimensional net. Both are produced as a result of Kriging randomly known values over a two-dimensional region. The plots can become extremely complex (and impressive) depicting minor variations in variables. Such is the net of ground elevations following the eruption of Mt. St. Helens (Fig. 3). Note that one set of lines in the net has been omitted.

The Kriging of data to produce contour maps and nets is by no means

CONTOURS AS A RESULT OF KRIGING

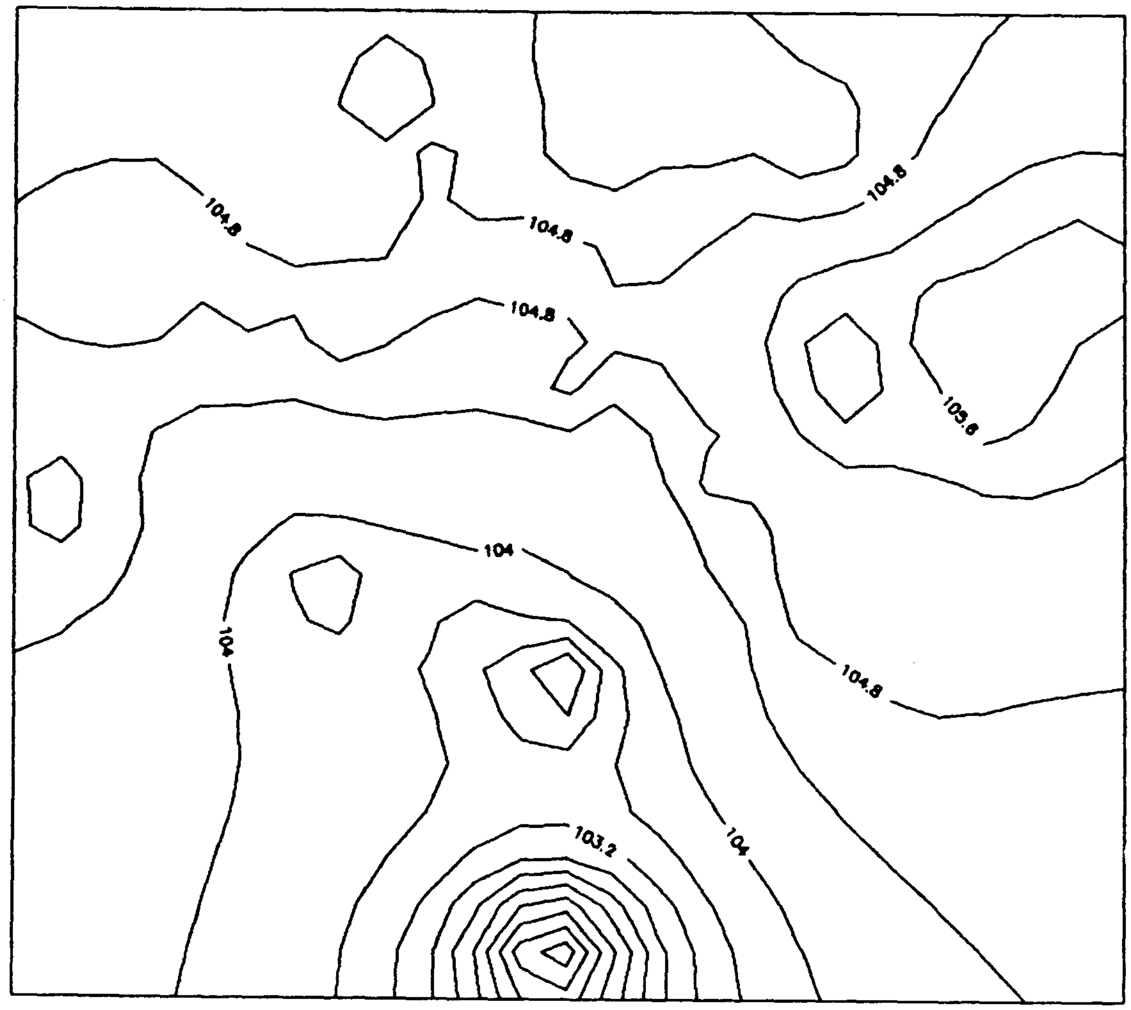


Fig. 1 Typical Contour Map of Elevations over an Area

trivial. The mathematical computations can be compared to those associated with the finite element method. In fact, there is a similarity in the form of the matrix equations used for the finite element method and the Kriging method. Using standard matrix solvers (e.g., Gaussian Elimination) it is possible to krig in the order of 1000 known points using a personal computer with a CPU of 540 Kbytes.

APPLICATIONS OF KRIGING TO SLOPE STABILITY ANALYSIS

There are several possible applications of the Kriging technique to limit equilibrium slope stability analysis.

1) Possibly the most obvious application (and the one considered in this paper) is the use of the Kriging technique to compute the

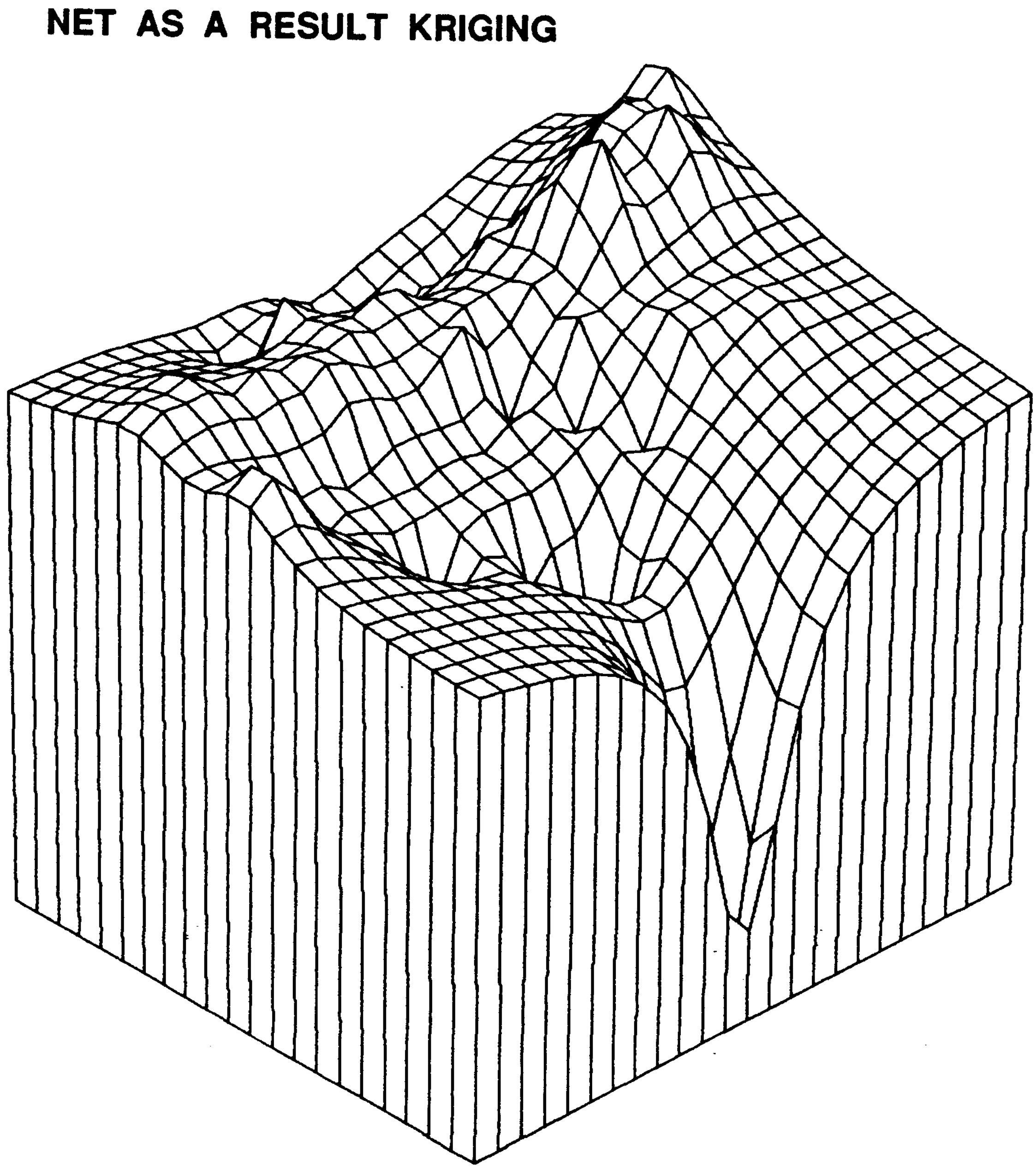


Fig. 2 A Typical Three-Dimensional Net Showing the Variation in Elevation over an Area

pore-water pressure at the base of a slice when using the method of slices. Let us suppose that the pore-water pressure is known at a series of points in the soil mass (Fig. 4). When performing a limit equilibrium analysis, it is necessary to pass a slip surface of some shape through the soil mass. The inscribed sliding mass is then divided into vertical slices and the pore-water pressure must be obtained at the base of each slice.

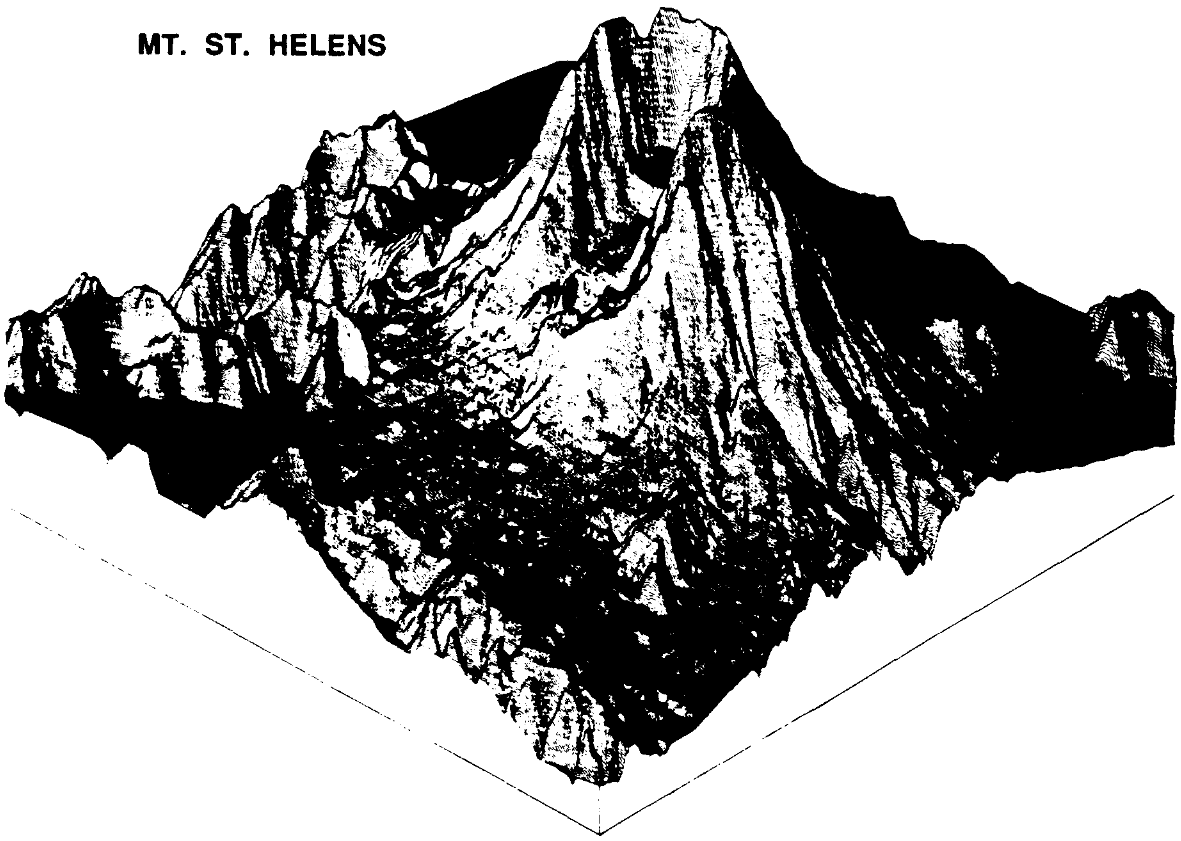


Fig. 3 A Three-Dimensional Net of Mt. St. Helens

The procedure originally used in the PC-SLOPE software for predicting pore-water pressures at the base of a slice was called the 4-Way Interpolation technique (Fig. 5). Using this technique, the nearest point in each of the four quadrants is found and then a 4-Way interpolation is performed to compute the pore-water pressure at the base of a slice. The advantages and disadvantages of this procedure will be later compared to the use of the Kriging technique.

Using the Kriging technique, it is possible to first compute the Kriging coefficients associated with the known points. These coefficients can then be used to compute the pore-water pressure at any other point in the cross-section.

2) The Kriging technique could be used as an interface between a finite element seepage computer program and a slope stability computer program (Krahn et al., 1987). Let us suppose that a transient or steady state seepage analysis has been performed, with the result that the pore-water pressures are known at 1000 points on the cross-section. One thousand points are too many to be kriged within a slope stability computer program when using a microcomputer. Both analyses will not fit within the normal storage limits of the microcomputer (i.e., 540 Kbytes). However, it is possible to use the Kriging technique as an independent program to reduce the 1000 points to a fewer number of points (e.g., 50

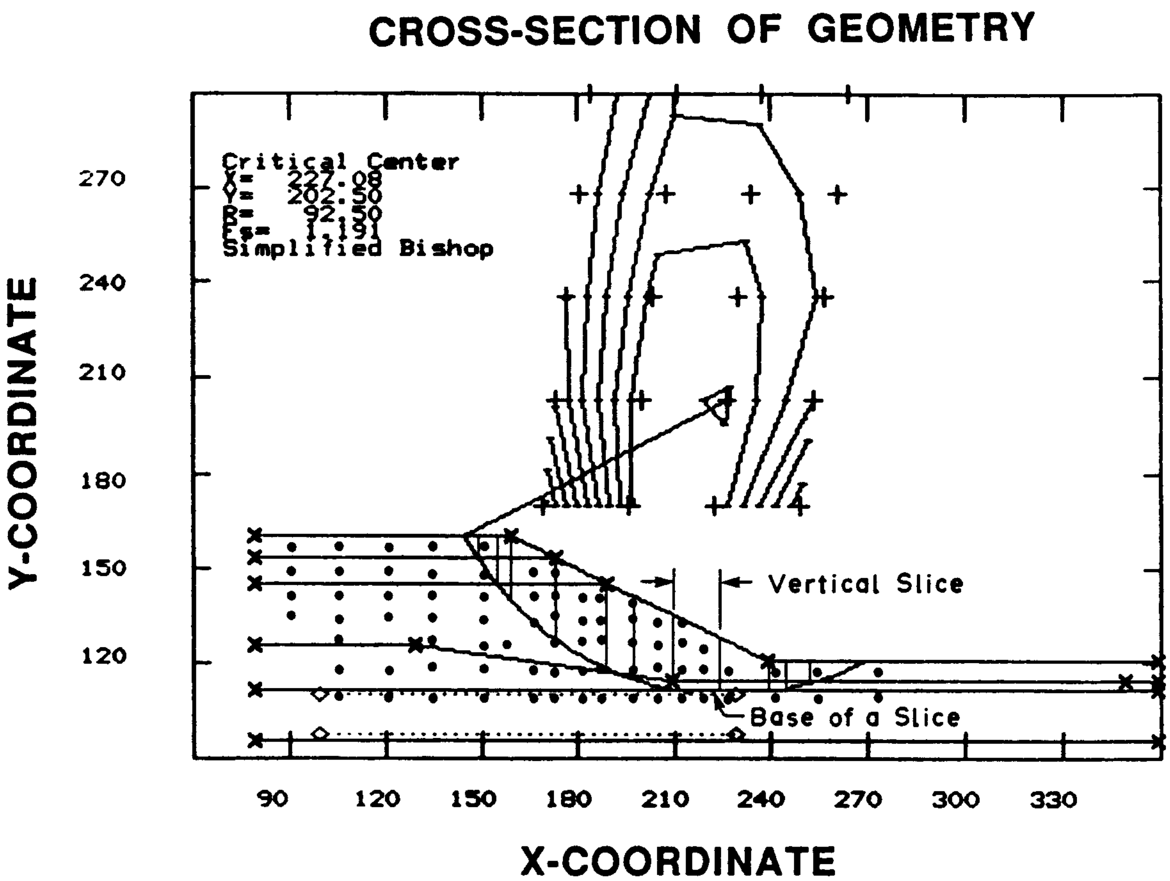


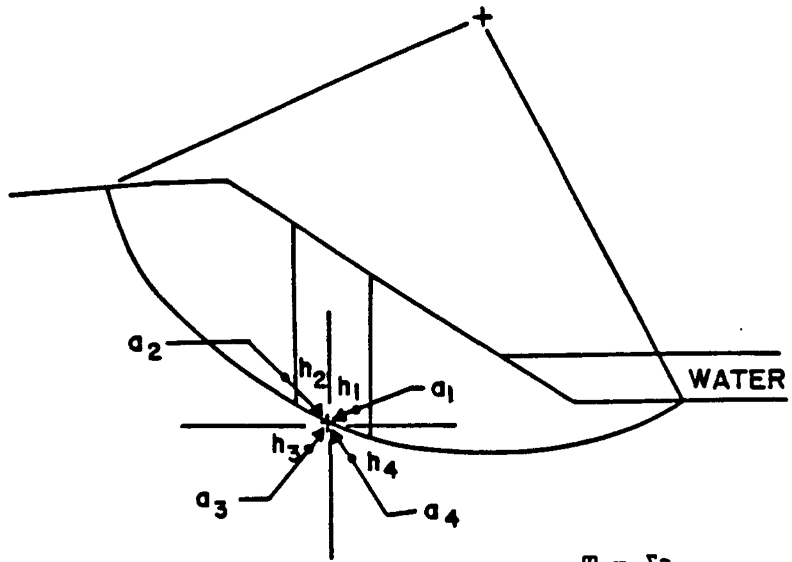
Fig. 4 Cross-Section Through a Soil Mass Showing an Inscribed Slip Surface and Designated Pore-Water Pressure Points

representative points). The 50 points can then be kriged within the slope stability computer program. The above exercise is illustrated in Fig. 6.

3) The Kriging technique could be used to interface a limit equilibrium slope stability analysis with a stress analysis. The stresses of interest could be either the stresses on the base of a slice or the stresses on the slice interfaces.

4) The Kriging technique could be used to depict the variation in the factor of safety. However, this may not be practical since it is as easy to perform a slope stability analysis as it is to krig the factor of safety data.

5) The Kriging technique could be used to advantage to compute the coordinates for the ground surface and the strata interfaces when performing three-dimensional slope stability analysis.



$$T = \Sigma a_i$$

$$u = \frac{\Sigma (T - a_i) h_i \gamma_w}{T(n - 1)}$$

where:

a_i = distance to the point under consideration

h_i = pore-water head at known points

n = number of points used in the interpolation

u = pore-water pressure at the desired point

γ_w = unit weight of water

Fig. 5 Illustration of the 4-Way Interpolation Technique to Compute Pore-Water Pressure at the Base of a Slice

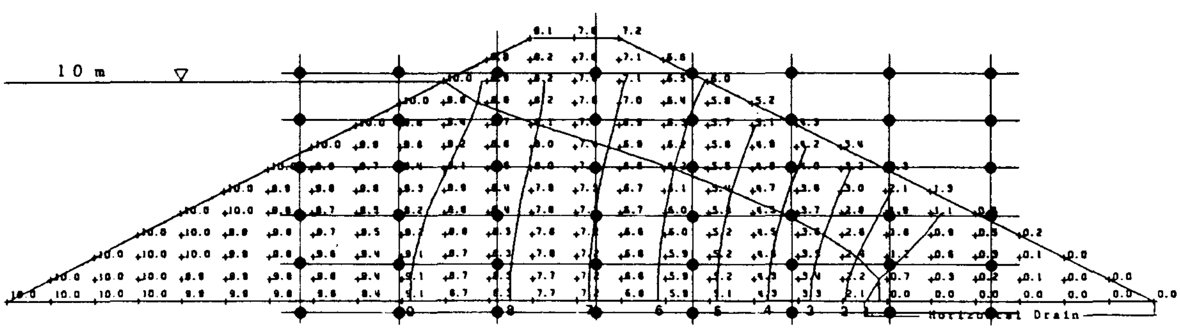


Fig. 6 Illustration Showing the Reduction in Data Points to Represent Pore-Water Pressures

THEORY OF THE KRIGING TECHNIQUE FOR PORE-WATER PRESSURE COMPUTATIONS

The Kriging technique estimates the value of a parameter at a particular locaton based on a linear combination of the value at known data points multiplied by a weighting function. The weighting coefficients are determined subject to the constraints that the estimate be unbiased, and secondly, that the variance of the estimated values from the actual values be minimized. The first criteria is the same criteria as applied by techniques such as inverse distance interpolation (i.e., 4-Way interpolation), or inverse distance squared methods. In these methods, however, the weighting is chosen based on arbitrary criteria which may or may not be the appropriate criteria for the data structure of concern. The second criteria ensures that the weighting coefficients chosen provide the best unbiased linear estimate of the unknown values. (Journel and Huijbregts, 1976).

In order to illustrated the Kriging technique, let us consider the following two-dimensional problem. Suppose a set of values, u_i, at N given points (x_i, y_i with i = 1, ---N) are known, and it is desired to estimate the value of 'u' at some other points. Let:

[1] $$u_m(x,y) = P(x,y) + \sum_{i=1}^{N} \lambda_i K(h_m - h_i)$$

where:
P(x,y) = the chosen trend
K(h) = the chosen interpolation function
h_m-h_i = the distance between the point under consideration and any other point, that is, $(h_m-h_i) = \sqrt{(x_m-x_i)^2 + (y_m-y_i)^2}$
u_m = the value at the point under consideration
λ_i = the computed weighting coefficients referred to as Kriging coefficients

The form of equation used to represent the values u(x,y) is chosen based on the anticipated structure of the variability of the parameter of interest.

The following options for the chosen trend were included in the PC-SLOPE software code (Fredlund and Fredlund, 1987).

[2] $$P(x,y) = a + bx + cy$$

[3] $$K(h) = \delta(0) + bo{*}h^2 \log h$$

where:
$\delta(0)$ = the nugget effect. The nugget effect is explained later; for the present, it is assumed to be zero.

In this case, the first term indicates a linear "drift" of the parameter across the region of interest as represented by the P(x,y) function. The second term expresses the potential for increasing variance between data points at increasing distance without an upper limit or "sill". This would be a typical data structure for values representative of hydraulic head or water pressure across or a slope as a result of seepage.

The coefficients of Kriging are $(a,b,c,\lambda_1,\lambda_2,\ldots\lambda_N)$ and are the solution of the following linear system:

$$[4] \quad \begin{bmatrix} K(0) & K(h_1-h_2)\ldots & K(h_1-h_N) & 1 & x_1 & y_1 \\ & K(0) & & 1 & x_2 & y_2 \\ & & \ddots & \vdots & \vdots & \vdots \\ & & K(0) & 1 & x_N & y_N \\ \text{Symetric} & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \\ a \\ b \\ c \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The above system of linear equations are solved for the Kriging coefficients. The values of u(x,y) can now be computed at any point, x,y using the following equation:

$$[5] \quad u(x,y) = a + bx + cy + \sum_{i=1}^{N} \lambda_i \, [\delta(0) + bo*(h_m-h_i)^2 \log(h_m-h_i)]$$

The following properties can be derived from equation [5]:

- At a point x_1,y_1, $u(x_1,y_1) = u_1$ (if $\delta(0) = 0$ and $K(0) = 0$)
- If for the point x_1,y_1 $\delta(0) = \delta_1 \neq 0$ $(K(0) = \delta_1)$ and $u(x_1,x_1)=u_1$

Therefore, by selecting different nugget values for the initial points, it is possible to assist the estimated values in coinciding with the initial values. At its limit, if $\delta(0)$ is the same for all points and its value becomes large,

$$[6] \quad u(x,y) = P(x,y) = a + bx + cy$$

This is equivalent to the least square solution of fitting. In geostatistics, a large nugget value suggests that no significant structure exists in the variability of the data, that is, it is erratic and consequently, each known data point receives the same weighting in the prediction.

Let us select the function, $K(h) = h^2 \log h$ and let bo = 1.0 in equation [5]. The solution of this problem can be visualized as a thin plate deforming in such a way as to pass through the deflection, u_i, at all points, x_i, y_i.

IMPLEMENTATION OF THE KRIGING TECHNIQUE IN PC-SLOPE

The Kriging technique has been implemented into PC-SLOPE in conjunction with the existing pore pressure techniques utilizing a grid of designated points. These involve procedures using a grid of pore-water pressure heads, a grid of pore-water pressures, or a grid of pore pressure coefficients.

The Kriging technique can be used when 50 or less points are designated. When more than 50 points are designated, the conventional 4-Way Interpolation procedure is used.

An example from the PC-SLOPE S-30 user's manual (i.e., BEN20.SET) is used to illustrate the use of the Kriging technique. This example has a total of 86 points where the pore-water pressure head is designated. The example problem was modified by omitting some of the points on a random basis, in order to reduce the number of designated points to less than 50.

RESULTS USING THE KRIGING TECHNIQUE

Several modifications of the example datafile were used in the study. First, the factors of safety were computed for a single center (i.e., x = 217.5, y = 200.0). Second, a grid of 36 centers was analyzed for the same problem.

SINGLE SLIP SURFACE RESULTS

The original datafile (i.e., BEN20.SET) is shown in Fig. 7. A total of 86 points are superimposed over the geometry. The computed factor of safety using 4-Way Interpolation was 1.238. Figure 8 shows pore-water force per unit length across the slip surface. The pore-water force varies in a somewhat erratic manner across the slip surface. Also shown on this plot is the pore-water force per unit length computed using pore pressure contours. This is considered to be the accurate reference values.

The original datafile was modified to perform further analyses using 43, 29, and 22 points. Figure 9 shows the geometry with 29 randomly selected points superimposed. Figure 10 shows the pore-pressure force per unit length using 43 points and the 4-Way Interpolation procedure. The pore-water force per unit length again appears as a jagged plot when 43 points are used in the analysis. Also shown are the results using pore pressure contours to compute the pore-water force per unit length.

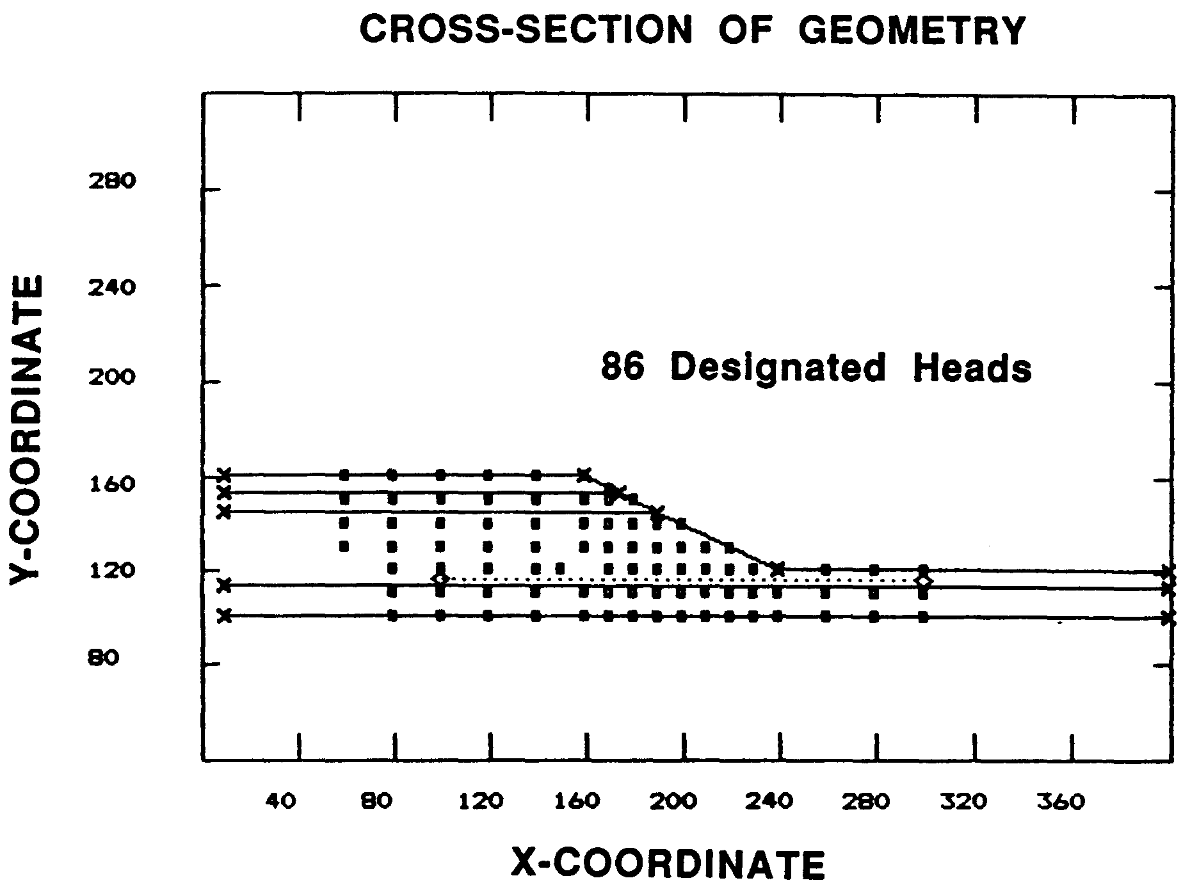


Fig. 7 Original Geometry for the Example Problem Showing 86 Designated Pore-Water Pressure Heads

The results when using both the 4-Way Interpolation technique and the Kriging technique are summarized in Table 1.

Table 1
Summary of Factors of Safety for the 4-Way Interpolation and Kriging Techniques

No. of points	Factor of Safety (Bishop Simplified)	
	4-Way Interpolation	Kriging
86	1.238	
43	1.221	1.285
29		1.287
22	1.236	1.295

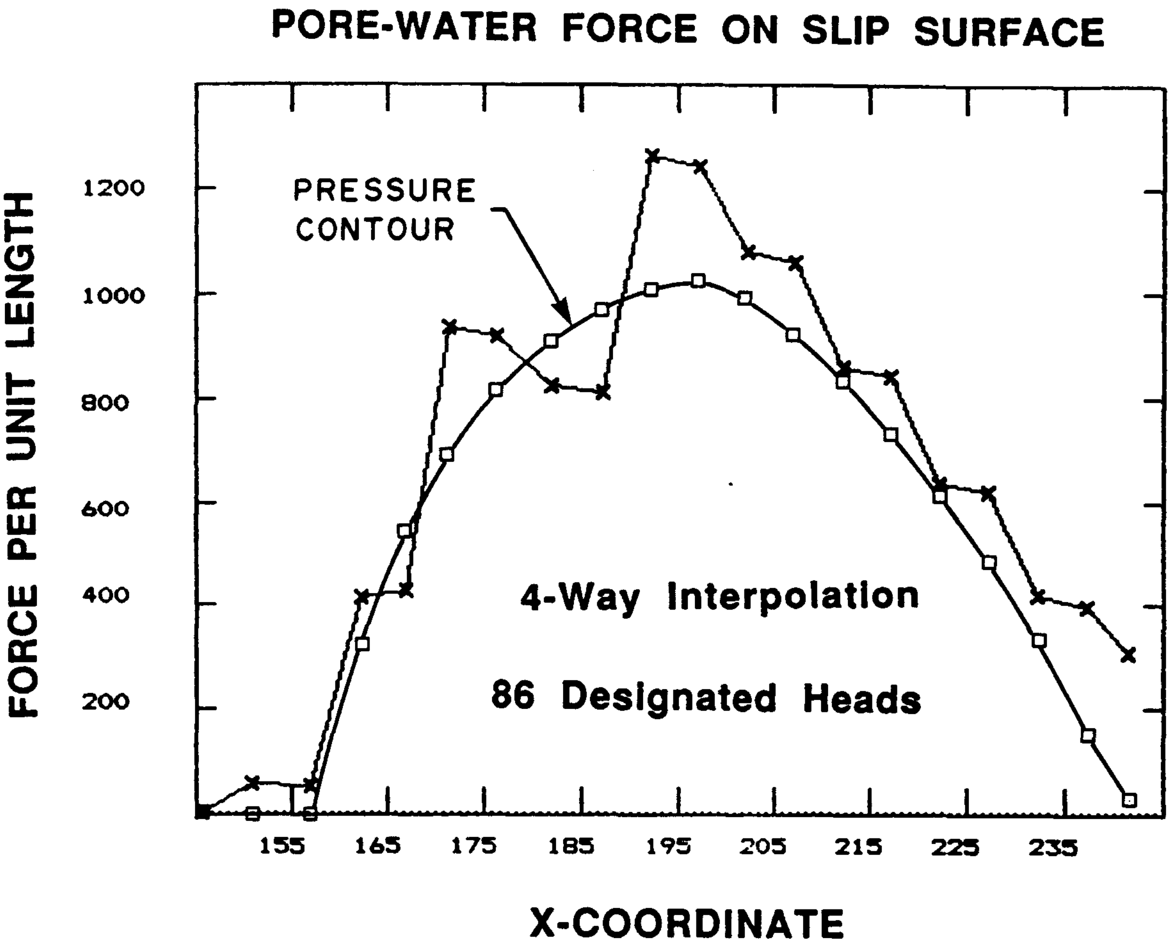


Fig. 8 Pore-Water Force per Unit Length Using 4-Way Interpolation on 86 Designated Heads

The results in Table 1 are shown graphically in Fig. 11. Also shown by a dotted line is the result of an analysis using pore-water pressure contours. The computed factor of safety was 1.285 and this value corresponds to an accurate representation of the pore-pressure conditions. It is noteworthy that the factors of safety computed when using the Kriging technique, are essentially the same as when using the pore pressure contours.

The pore-water force per unit length, when using the Kriging technique on 43 and 22 points, are shown in Fig 12. Both curves are extremely close to the results when using the pore pressure contours.

A grid of centers was selected such that a total of 36 slip surfaces were analyzed. A comparison of the computing times between the 4-Way Interpolation and Kriging techniques are shown in Table 2 and graphically presented in Fig. 13.

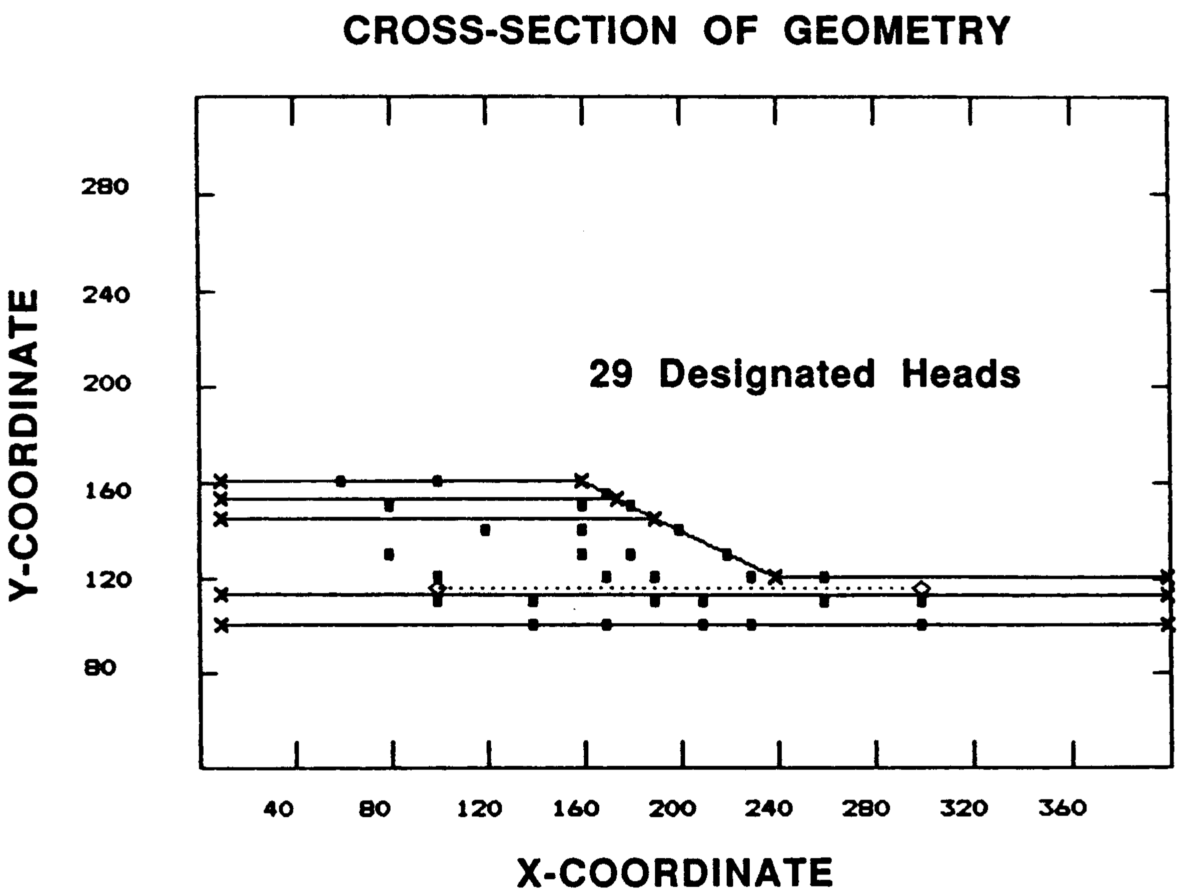


Fig. 9 Geometry Cross-Section with 29 Designated Pore-Water Pressure Heads

Table 2
Comparison of Computing Times

No. of Points	Computing Time per Slip Surface	
	4-Way Interpolation (seconds)	Kriging (seconds)
86	7.0	9.0
43	5.4	9.0
29		6.9
22	4.6	6.1
		5.3

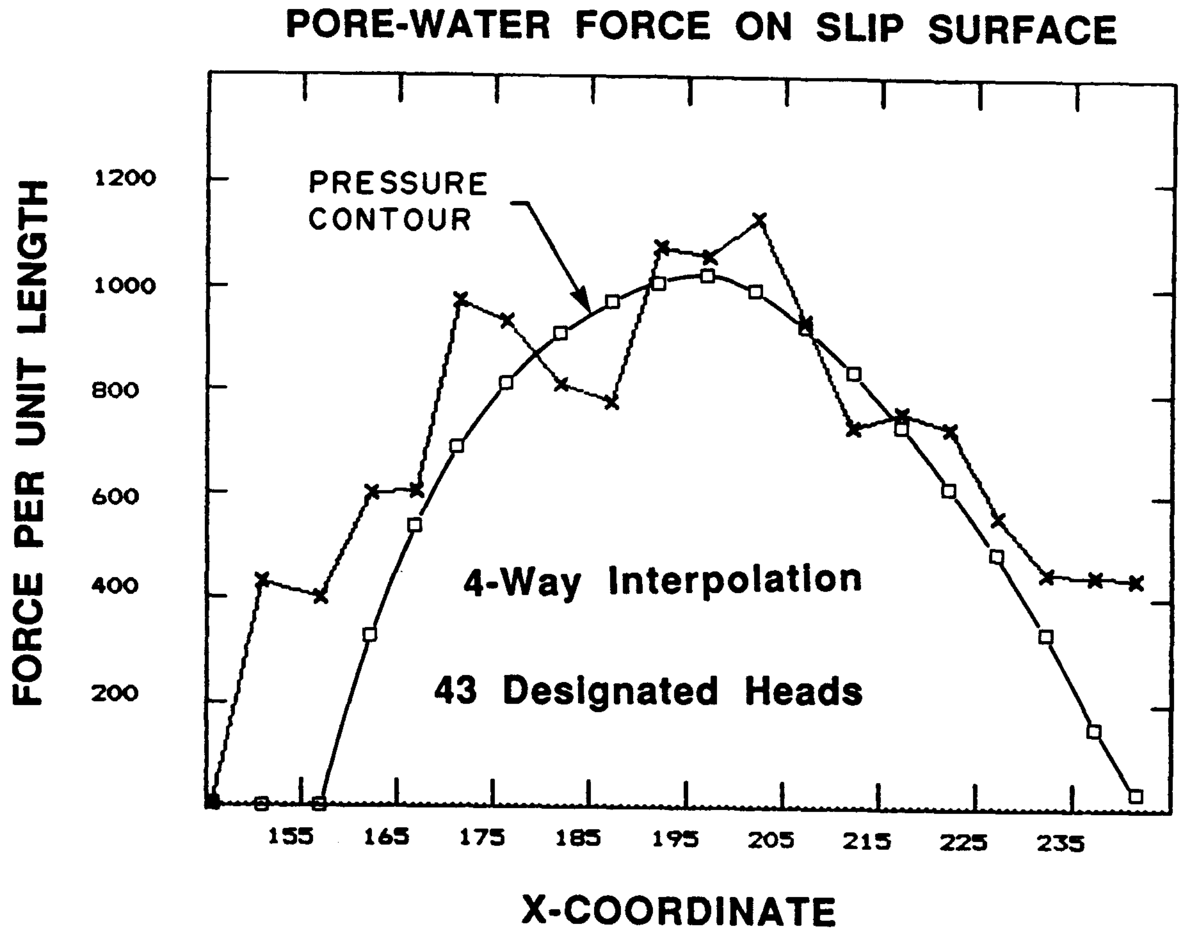


Fig. 10 Pore-Water Force per Unit Length Using 4-Way Interpolation on 43 Designated Heads

The results show the Kriging technique to be more accurate than the 4-Way Interpolation technique. It would appear that even an extremely large number of designated heads (or pore-water pressures) would not produce as reliable pore pressure predictions using 4-Way Interpolation as can be obtained using the Kriging technique. On the other hand, applying the Kriging technique to a limited number of designated heads (or pore-water pressures) appears to produce accurate results. It would be useful to study other examples where the pore pressure changes are abrupt and large.

The Kriging technique requires more computing time. The computing time increases significantly as the number of points used in the Kriging technique increases. As an example, using 40 designated heads resulted in a 70% increase in computing time when using the Kriging technique. However, the accuracy of the pore-water pressure prediction is superior. On the basis of the above study, it is concluded that the Kriging technique can be used to advantage in slope stability calculations.

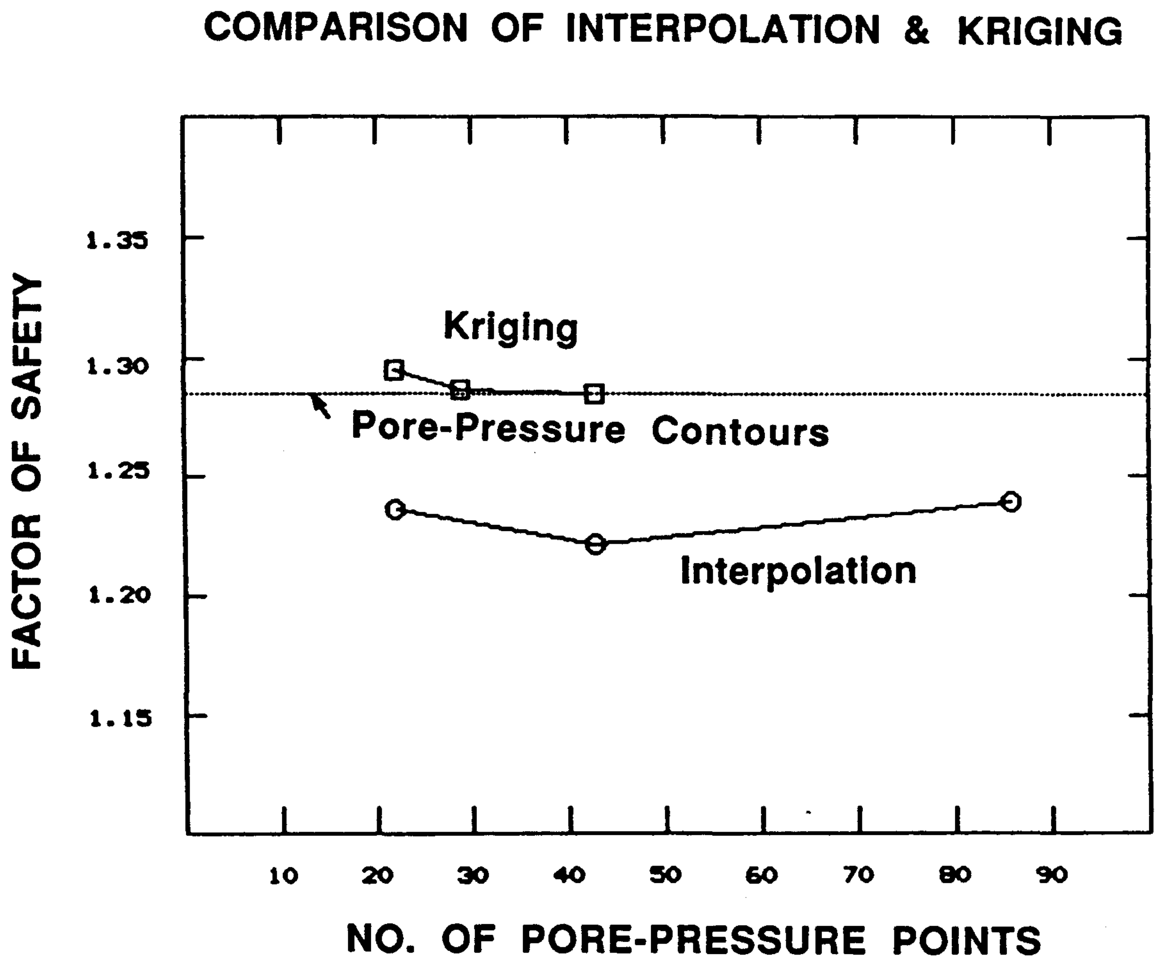


Fig. 11 Comparison of Factors of Safety Computed Using the 4-Way Interpolation and Kriging Technique

It is suggested that the Kriging technique also be given consideration for other applications in geotechnical engineering. Of particular value is the possibility of using the Kriging procedure as an interface between finite element seepage analyses and limit equilibrium slope stability analyses.

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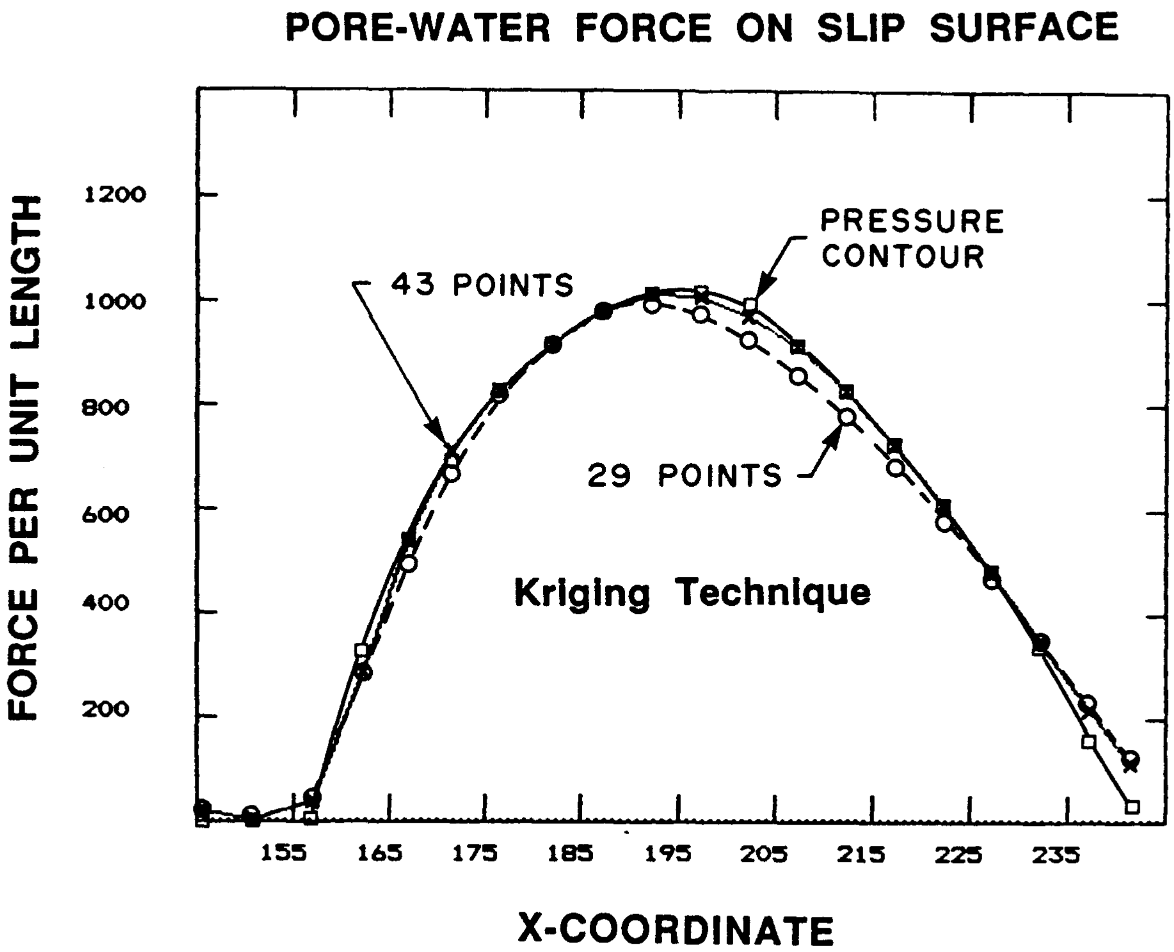

Fig. 12 Pore-Water Force per Unit Length Using the Kriging Technique on 43 and 29 Designated Heads

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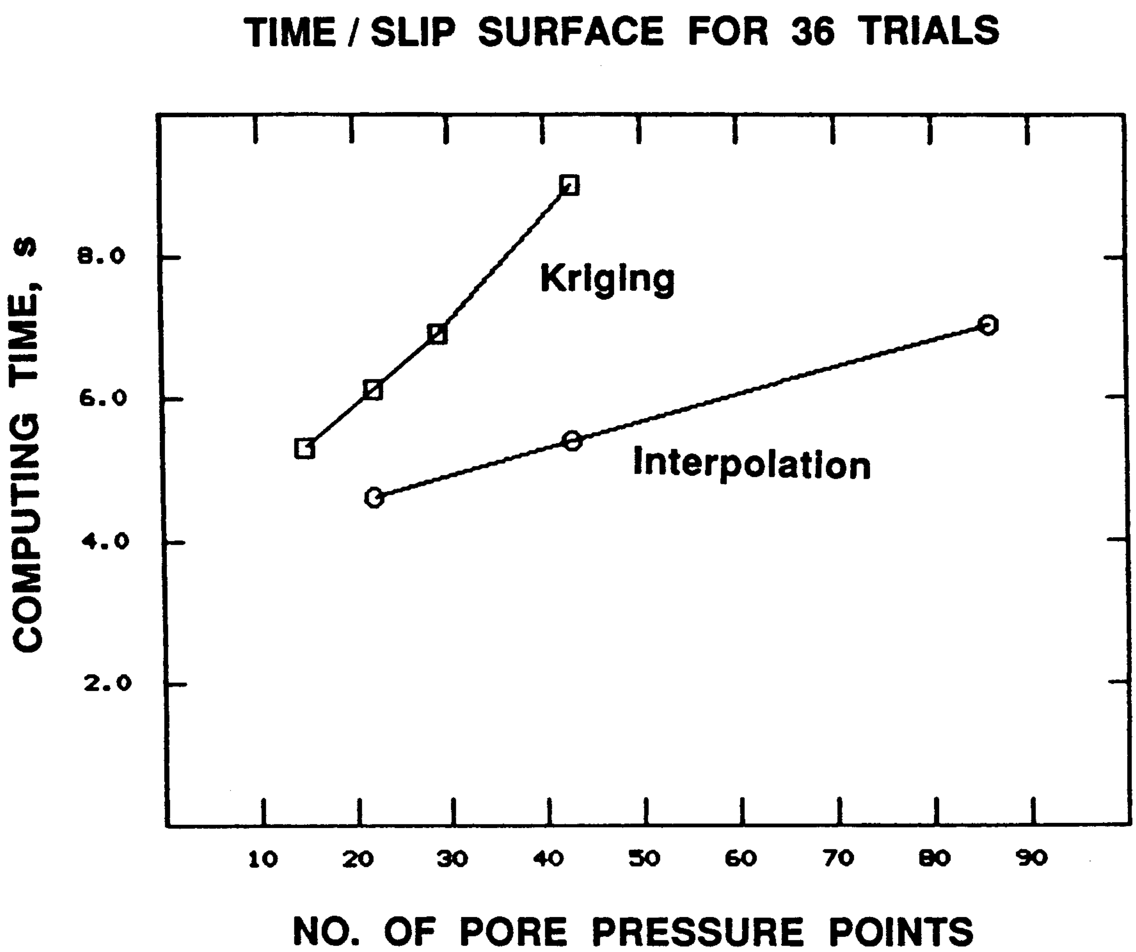

Fig. 13 Time to Compute the Factor of Safety for the 4-Way Interpolation and Kriging Technique