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Effective stress, limit equilibrium back-analysis of failed slopes: guidelines
(La rétro-analyse de la tension effective et de l'équilibre limit des pentes échouées: des lignes directives)

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ABSTRACT: Guidelines are given to assist the analyst in performing effective stress, limit equilibrium back-analysis of failed slopes. The problem is divided into three categories; namely, site, theoretical, and modelling guidelines. Based on years of study of failed slopes, guidelines are substantiated with data from a number of case studies.

RESUME: Les lignes directives sont présentées pour assister la rétro-analyse de la tension effective et de l'équilibre limit, des pentes échouées. Ces lignes directives sont divisées en trois catégories; celles de terrain, théoriques, et des modèles. Les lignes directives sont basées sur plusieurs années de recherche des pentes échouées. Elles sont appuyées par les données provenant de diverses études.

1 INTRODUCTION

Most geotechnical engineers have the responsibility of performing a back-analysis on a failed slope at some point in their career. The purpose of the analysis is generally to assess reasonable shear strength parameters for the soil with the view of recommending remedial measures. The research literature contains little information on the most suitable procedure to follow, particularly when performing an effective stress back-analysis, even though the analysis has become rather routine in practice.

Common slope stability methods of analysis are indeterminate and it is not obvious how a back-analysis should be performed. The purpose of this paper is to present guidelines to assist the practicing engineer. The suggestions fall under the following categories:

1. site guidelines,
2. theoretical guidelines, and
3. modelling guidelines.

Examples from case histories are presented to illustrate the role of a sensitivity or parametric type analysis.

It is difficult to provide guidelines which are representive of all soil conditions. The authors have performed back-analysis for both soft and stiff soil conditions; however, the major experience has been associated with stiff soil conditions. The authors realize that deviations from the suggested guidelines will be required in certain situations.

2 SITE GUIDELINES

The reliability of back-analysis is sensitive to the quality of information available from the characterization of the site. Essential information includes surface topography, stratigraphy and structural geology; dimensions of the landslide mass; the hydrogeology, including pore-water pressures at the time of failure. These elements are determined from field measurements, core samples, geophysical techniques, and various types of instrumentation. Hopefully, the interpretation of exploration data is representative of the true population. Various degrees of uncertainty, however, will always persist no matter how intensive the exploration program. But there are useful guidelines which can greatly enhance the site information and help minimize uncertainty.

1.1 Stratigraphy and structure

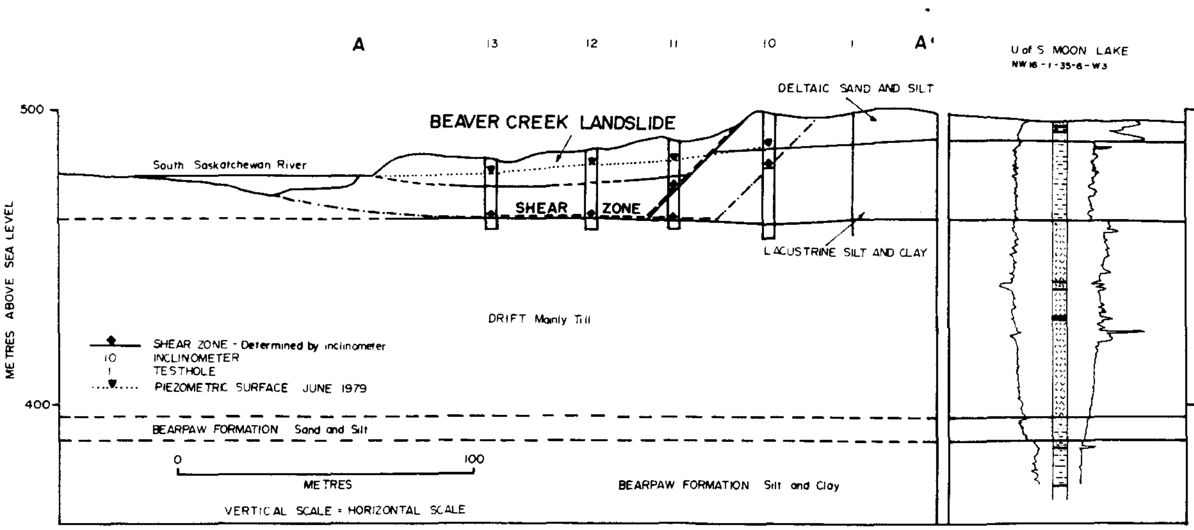
The sequence of beds of sediments of different lithology is determined from stratigraphy. Structure defines the geometric configuration of beds and discontinuities. Together, these provide a framework for the interpretations of the landslide mechanism and history. Stratigraphy establishes meaningful population boundaries for the characterization of materials. For example, in Fig. 1 a glacial till is covered by a regressive offlap of lacustrine clay covered by deltaic sand. In this case, three populations are defined for sampling and assignment of soil properties for analysis.

Structures include stratigraphic contacts, unconformities, and tectonic deformation. For example, in Fig. 1 the lacustrine clay is in abrupt contact with the till. The contact between the lacustrine clay and overlying deltaic sand, however, is gradational finning downward. The separation of these two soils is arbitrary and subject to the interpretation of the analyst. This may or may not present a problem depending on the location of the slip surface. In this case, the slip surface is based on the till and the contact between the clay and sand is less significant.

1.2 Geometry of the failure surface

A landslide is a structural feature so the principles of structural geology apply to defining the mechanism. The failure surface is part of the created structure and is controlled by stratigraphic contacts and structural discontinuities that existed prior to failure. Often, a circular slip surface is assumed during back-analysis. Rarely, however, does this occur in nature except for shallow slips in massive clay deposits or newly constructed embankments. Most failure surfaces are composite, with much of the slip surface planar in shape. The composite shape results primarily because of structural control on the mechanism.

Inclinometer measurements generally provide positive identification of the location of the slip surface. However, site access difficulties and time constraints may preclude the use of the instruments. In addition, the slope may be temporarily inactive. In some cases, shear zones can be identified in core samples. But mobilization difficulties and sampling problems in highly disturbed landslide debris often make this impractical and unreliable. There are some important guidelines, however, that permit the definition of slip surfaces to a degree of certainty acceptable for analysis. Also, a precise definition



Information on landslide from Yoshida (1981)

Figure 1. Typical cross-section through a sliding mass illustrating the stratigraphy.

of the base of the slip surface may not be overly critical when using an effective stress analysis.

The movement of landslides in overconsolidated clays is often translational involving the stretching of sliding blocks along a low angle failure zone (Christiansen 1983, Sauer, 1983). The slip surface will generally seek a stratigraphic change or structural discontinuity and these are commonly planar. The number of possibilities for the failure surface is generally limited by the regional geology. For example, in sedimentary basins the following possibilities exist:

1. Contact of soft clay over stiff clay, till or sand (Sauer, 1984).
2. Contact of hard material such as till over soft clay (Eckel et al, 1987).
3. Interlayered soft materials such as bentonitic gouge (Christiansen and Sauer, 1985).
4. Interlayered clay between sand or tills (Sauer, 1979).

The backslope configuration of the slip surface is difficult to measure. There are few documented examples of measurements of the backslope orientation. Usually it is assumed to be a circular arc. For composite slip surfaces, the backslope should become near planar dipping at an angle close to $45^{\circ} + \phi/2$ from the horizontal, to form the active Rankine state. The effect of the orientation of the backslope is demonstrated analytically in Fig. 2.

1.3 Surface topography

The primary activating forces of a landslide result from the slope of the ground surface. The back-analysis, therefore, is sensitive to the average surface slope angle. The slope angle before failure is steeper than after failure. Limit equilibrium back-analysis requires a known factor of safety and at failure the factor of safety is theoretically equal to unity. As a result, the slope after failure is most emenable to back-analysis. The shear strength mobilized after failure is appropriate for design of works on presently dormant landslides or stabilization of active failures.

The shear strength parameters appropriate for analyzing the point of incipient failure are significantly higher for some clays due to the difference between the peak and residual strength. Smectite clays, in particular, exhibit this property whereas some tills do not show a significance drop-off in strength (Sauer and Christiansen, 1985). In order to obtain strength parameters for incipient failure, it may be necessary to back-analyze existing stable slopes. The prefailure slope angle can often be estimated from natural slopes in similar materials in the vicinity. The factor of safety, however, is actually unknown and the groundwater regime at the time of failure is unknown. This type of

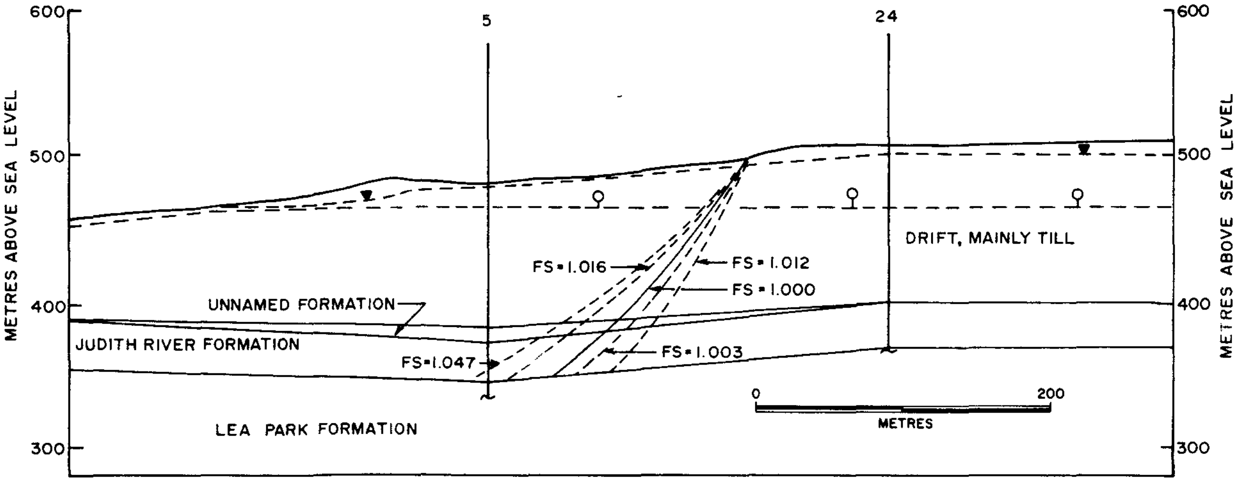


Figure 2. The effects of the backslope slip surface on the computed factor of safety.

back-analysis warrants further investigation. Simulation of the landslide mechanism within a time framework shows promise in dealing with this important problem (Eckel et al, 1987).

The significance of the ground surface slope angle in performing a back-analysis can be illustrated by the stability equation for a long slope in a dry cohesionless material.

$$\tan \phi' = \tan \beta \qquad [1]$$

where:
β = average surface slope angle
ϕ' = effective angle of internal friction

For steady seepage with the water table at the ground surface, the equation becomes,

$$\tan \phi' = 2 \tan \beta \qquad [2]$$

It is obvious that for most field conditions an error of 1° in surface slope results in a nearly 2° error in back-calculated values of ϕ'. Low slope angles (i.e., 5° to 15°) are difficult to measure accurately, even to within 1°. The shear strength parameters are correspondingly low. Thus, an error of 2° in the geometry slope becomes unacceptable where the is only 8° to 10°.

Accurate contour maps are extremely useful for performing reliable back-analysis. The desired contour interval depends on the size of the landslide. It is often difficult to define the surface topography because of the irregular surface characteristic of the landslide. It is sometimes helpful to separate the landslide into blocks and to analyze the blocks separately and in combination. This procedure can be used to establish a higher degree of accuracy in the back-analysed results (Sauer 1983, Eckel et al. 1987).

1.4 Shear strength parameters

When the failure surface is confined to one material the back-analysis is relatively straight forward (Sauer 1984). When the slip surface passes through more than one material, however, the problem becomes more complicated. The properties of some materials are better known and more easily characterized than others. In any case, the problem becomes one of trial and error using different combinations of material properties. For low angle slopes the largest part of the slip plane will pass through one material, usually the one of most concern in the analysis. This layer most often is near planar, forming the base of the sliding mass. The back scarp portion of the slip surface may pass through one or more materials (Eckel et al. 1987). But the strength mobilized along the backslope is of much less significance than the strength along the horizontal plane portion. The problem may become one of judgement in many cases and can be greatly assisted through the use of a parametric study.

1.5 Hydrogeology and pore-water pressures

The pore-water pressure at failure is generally not known for certain. In the field, piezometers are used to measure pressure head. These measurements usually represent average conditions over a period of time and not necessarily the conditions at failure. In addition, it may not be possible to install piezometers in the failure zone because of movement along the sliding surface.

The sensitivity of the factor of safety to the pore-water conditions is apparent from equation [1] and [2]. It can be seen that differences in water table elevation could typically vary from being at the ground surface to being at the failure plane. The upper and lower piezometric conditions can be viewed as two limiting conditions (Sauer 1983). In design, a steady seepage condition somewhere between these two limits will generally be appropriate. Generally, water levels do not vary greatly on long flat slopes.

Artesian conditions must be considered in some cases (Eckel et al, 1987). Artesian pore-water pressures probably vary less than surface groundwater conditions. Even though artesian pressures may be critical, the error in estimating pressure heads may be small. This is because artesian conditions are regional flow phenomenon and tend to maintain hydraulic stability.

2 THEORETICAL GUIDELINES

The elements of statics that can be used to derive the factor of safety are the summation of forces in two directions and the summation of moments. These, along with the failure criteria, are insufficient to make the problem determinate (Fredlund and Krahn, 1975). More theory (or physics) is required to render the problem determinate. Most commonly, an assumption is made regarding the direction of the interslice forces. The analysis is now determinate from a moment and force equilibrium standpoint provided the shear strength parameters for all soils are known. A moment and force equilibrium factor of safety can now be independently computed for various interslice force conditions (Fig. 3).

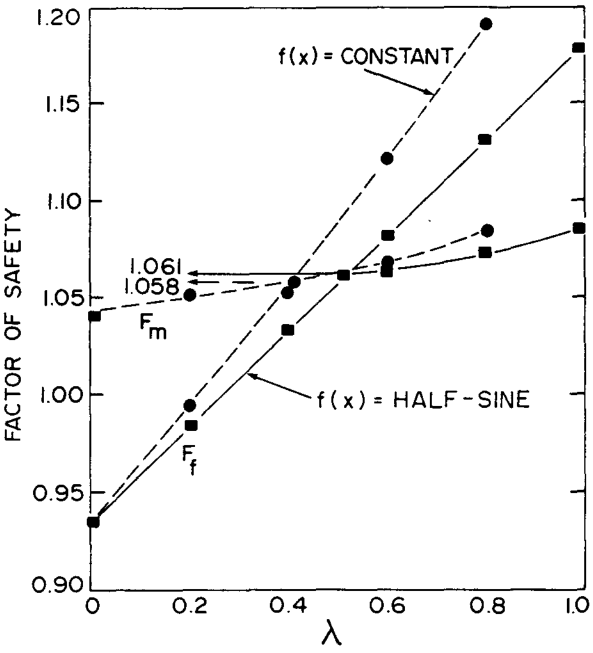


Figure 3. Variation of moment and force equilibrium factors of safety with respect to lambda

In the case of a back-analysis, the factor of safety is set to 1.0 and the shear strength parameters for one soil are solved. However, the shear strength of the soil introduces two unknowns, namely, the effective cohesion intercept and the effective angle of internal friction. Therefore, the problem once again becomes indeterminate. The indeterminancy can be handled in terms of a range in possible parameters or by simply computing an average shear strength for the slip surface. The former procedure is most advantageous as is later shown when considering modelling guidelines.

There is considerable computing effort involved in solving for the factor of safety using complete equilibrium. For this reason, often either the moment or force equilibrium factor of safety is used for analysis purposes. Fig. 3 shows that the force equilibrium factor of safety is more sensitive to the selected interslice force function than is the moment equilibrium factor of safety. All to often, the past experience has been to use a moment equilibrium back-analysis when the slip surface shape is circular. When the shape is noncircular, the force equilibrium factor of safety equation has been used. The authors would like to challenge this practice and suggest that a moment equilibrium factor of safety formulation should be used even for composite shaped slip surface.

The moment equilibrium factor of safety equation has been extended to accommodate noncircular slip surfaces (Fredlund and Krahn, 1977). The forces and moment arms involved in an analysis are shown in Fig. 4. There are two changes from the conventional circular arc analysis. The normal force at the base of each slice need not pass through the center of moments and the moment arm for the shear force mobilized can be variable. Taking these two differences into consideration, the moment equilibrium factor of safety can be written as follows:

$$F_m = \frac{\sum\{c'\beta R + (N-u)R\tan\phi'\}}{\sum Wx - \sum Nf \pm Aa} \qquad [3]$$

where:

F_m = factor of safety with respect to moment equilibrium

The variables in Eq. [3] are defined in Fig. 4. The normal force can be computed based on the summation of forces in a vertical direction. The use of the moment equilibrium equation ensures a factor of safety which is readily computed and which is insensitive to the interslice force conditions. The selected center of moments should be in a location which would permit a circular arc to approximate the slip surface.

If complete equilibrium is to be satisfied in the back-analysis, it is suggested that either a constant interslice force function or the finite element based interslice force function (Fredlund, 1984) be used.

3 MODELLING GUIDELINES

The modelling of failed slopes provides information of value in the selection of reasonable shear strength parameters. In general, soils have a cohesive and frictional component. If the soil is completely frictional in nature, the failure surface would tend to the ground surface. On the other hand, if the soil were purely cohesive, the slip surface would tend to go infinitely deep. In actuality, the depth factor associated with a failed mass provides information on the relative roles of the cohesive and frictional components.

The reliability of a back-analysis is dependent on how accurately the field measurements, assumptions, and theory model the landslide mechanism. The degree of uncertainty will not be the same for all elements of the analysis. A parametric analysis is useful for evaluating the different elements of uncertainty. The influence of the shear strength parameters, surface topography, failure geometry, pore-water pressures, and unit weights within the landslide mass can be evaluated systematically using parametric study. This type of analysis provides information not readily available from a deterministic or even a probabilistic approach.

3.1 Shear strength parameters

It is not possible to back-analyze a landslide and obtain a unique combination of effective cohesion and effective angle of internal friction. Sometimes it can be assumed that the clay in the shear zone has been remolded and softened by excessive strain, and therefore, the soil has no cohesion. This is compatible with Skempton's (1964) concept of residual shear strength.

An alternative approach is to back calculate the effective angle of internal friction assuming the cohesion is zero. This can be followed by an analysis assuming the angle of internal friction is zero and computing the effective cohesion component. A straight line relationship always exists between tan ϕ' and c' as shown in Fig. 5. Any point on this line represents a combination of c' and ϕ' for limiting equilibrium. This diagram is useful for testing other modelling elements as will be shown subsequently.

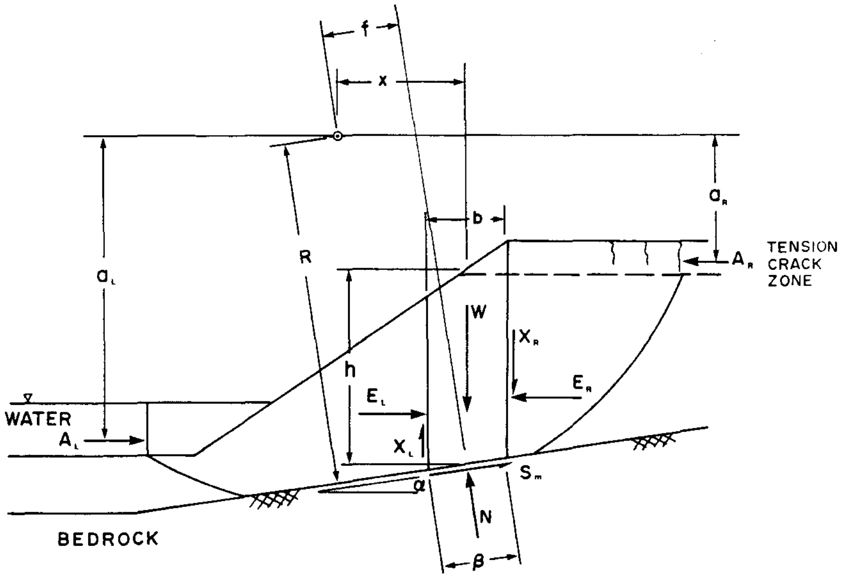


Figure 4. Forces acting on a slice through a sliding mass with a composite slip surface

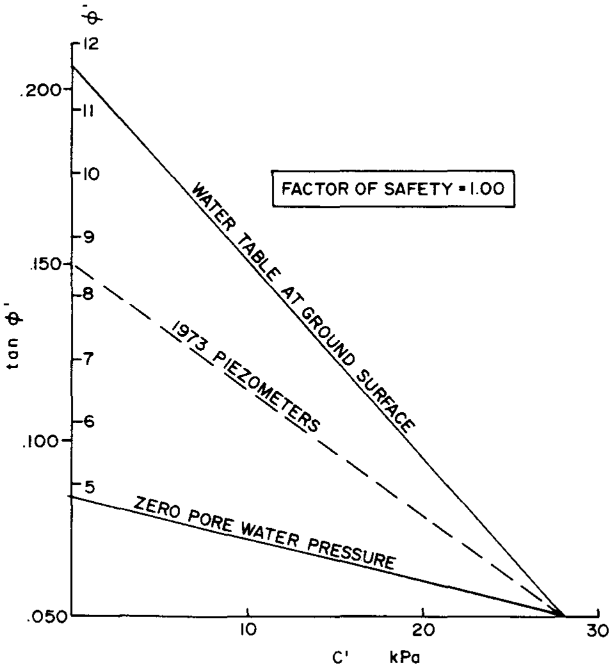


Figure 5. Effective stress parameters required for limiting equilibrium. Two extreme pore-pressure conditions are shown (After Sauer, 1984).

When the slip surface passes through more than one material, the contribution of the second material to the mobilization of shear strength should be evaluated. The influence of both c′ and ϕ' should be evaluated as shown in Figs. 6 and 7. For the after failure condition, the effective cohesion can often be assumed to be zero. This greatly simplifies the analysis.

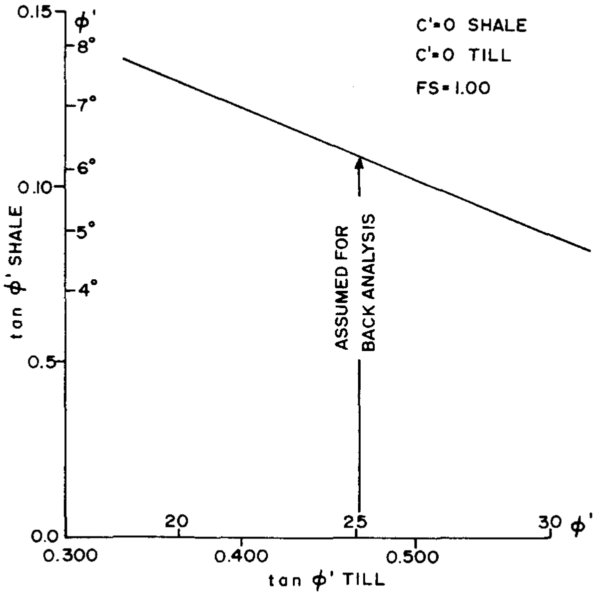


Figure 6. Sensitivity of back-analyzed values of ϕ' for shale for assumed ϕ'values for till. Till forms back slope of slip surface (After Eckel et al., 1987)

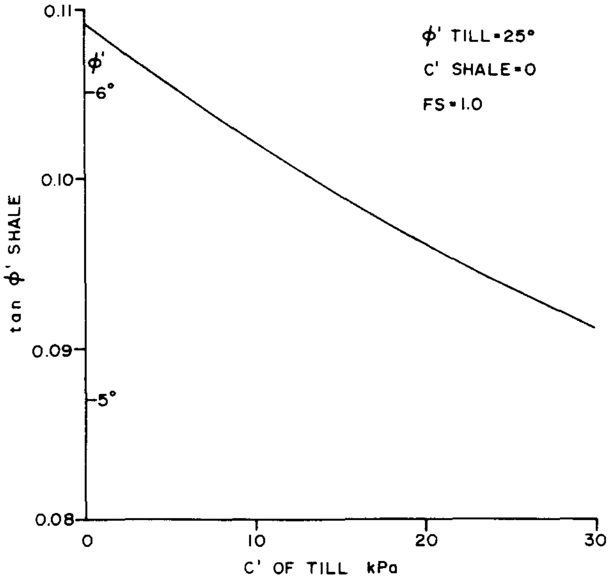


Figure 7. Sensitivity of back-analyzed values of ϕ' for shale for assumed c′ values for till. Till forms back slope of slip surface (After Eckel et al., 1987)

3.2 Surface topographics

Errors introduced by inaccuracy in the measurement of the average slope angle have already been discussed. When landslides have long flat slopes, 500 m or more at angles less than 15^{0}, several blocks of sliding material are usually involved. The surface topography becomes uneven and difficult to measure accurately. In addition, the number of coordinates then can be entered into the computer program is usually restricted. It is often useful, therefore, to model the landslide as a series of individual blocks, and combinations of sliding blocks (Fig. 8). The use of this technique combined with the use of the tan ϕ'versus c′ digagrams, (Fig. 5) has been described by Sauer (1983) and Eckel et al. (1987).

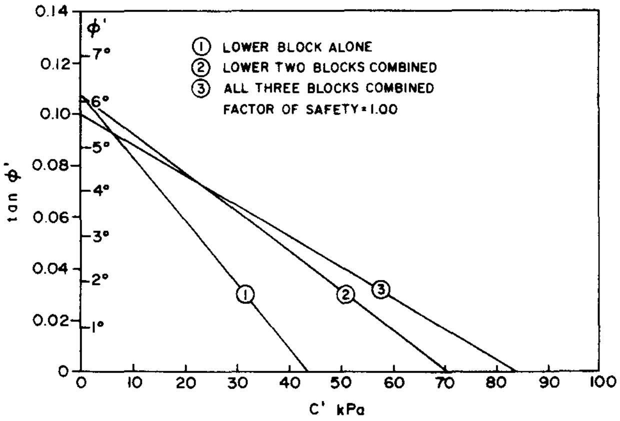


Figure 8. Plot of effective shear strength parameters for three block combinations used in the back-analysis to test modelling accuracy (After Eckel et al, 1987).

3.3 Failure geometry

A precise definition of the slip surface may not be practical for reasons stated previously. The length of the failure surface and its depth below the ground surface can be considered as independent dimensions for composite or wedge-shaped slip surfaces. Uncertainty in these two dimensions affects the results differently.

The depth of the failure surface has an influence on normal stress, and therefore, on the mobilized frictional shearing resistance. Opposing the shear strength mobilized are the forces producing instability. The depth of the failure plane, therefore, has only a minimal effect on the factor of safety for a purely frictional material (Fig. 9). On the other hand, normal stress has no influence on mobilized cohesion. The depth of failure surface, therefore, has a significant effect on the factor of safety calculations of a frictionless or purely cohesive material (Fig. 9).

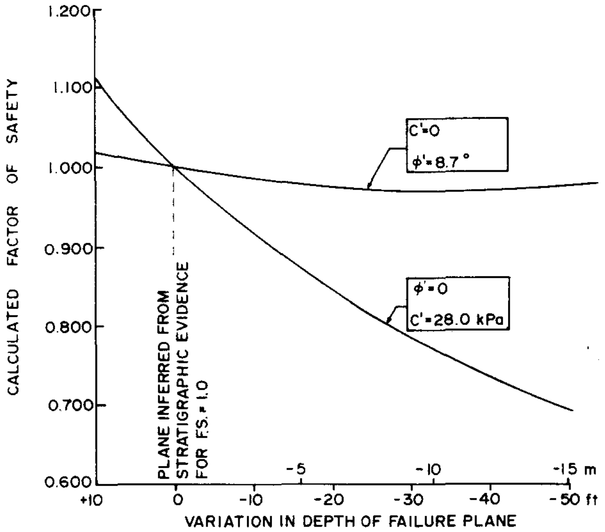


Figure 9 Influence of depth of failure surface for low angle slope on factor of safety for constant values of ϕ' ($c' = 0$) and c' ($\phi' = 0$) (After Sauer, 1984)

The length of failure surface does not affect the back calculated effective angle of internal friction which can be seen from Eq. [1]. For a purely cohesive material, however, the back-calculated value of the parameter c' is directly related to length of failure surface. The influence of the length of failure surface on the back-calculated values of c' and ϕ' can be seen in Fig. 8 where a slope is analyzed as separate blocks. For the same slope angle and piezometric conditions, ϕ' remains constant but c' increases as the size of the landslide blocks increase.

It is obvious that the effective cohesion parameter, cannot be back-calculated unless the dimensions of the failure surface are accurately known. On the other hand, the parameter ϕ' is not greatly affected by the dimensions.

3.4 Pore-water pressure

The sensitivity of back-calculated values of the effective angle of internal friction to changes in pore-water pressure is demonstrated in Fig. 10. The factor of safety is related to the pore-water pressure parameter, r_u, for constant angles of internal friction. It can be seen from this diagram that at a factor of safety of 1.0, a change of r_u from 0.30 to 0.40, results in an increase in friction angle from 7° to 8° required for equilibrium.

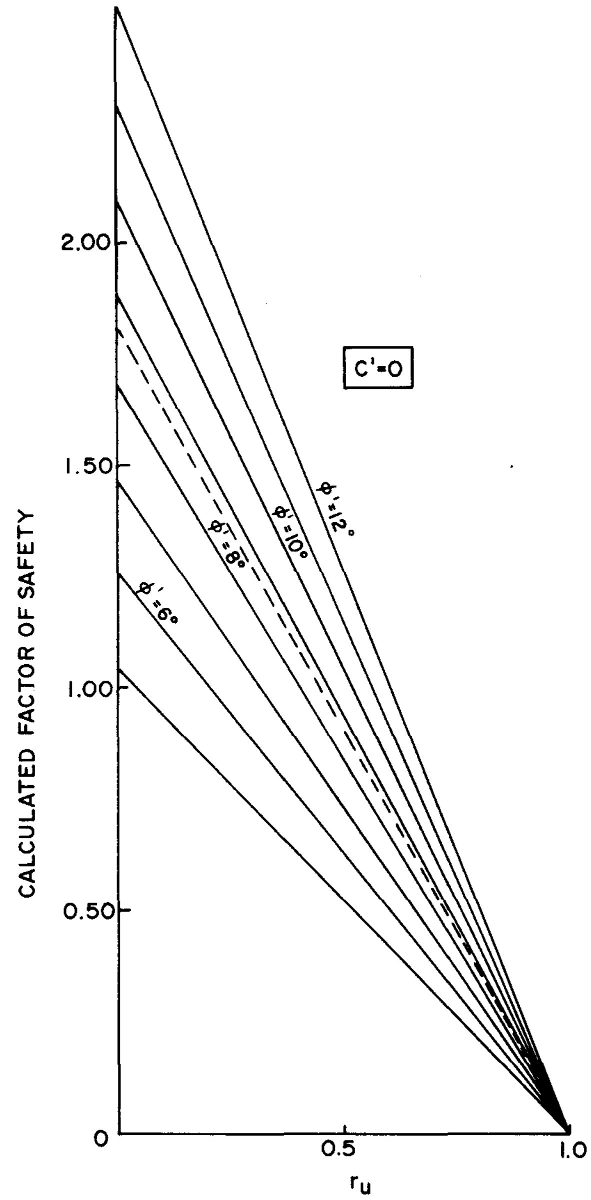


Figure 10. Effect of variation in piezometric levels on the factor of safety as indicated by r_u for constant values of ϕ' (After Sauer, 1984)

Calculated effective friction angles can also be plotted against r_u to study the influence of fluctuations in piezometric levels on back-calculated friction angles (Fig. 11). The case study shown in Fig. 10 shows that a fluctuation of ± 1.0 m in piezometric level results in a variation in the back-calculated friction angle from 6.5° to 6.9°. Changes in pore-water pressure do not have a direct influence on back calculated values of the effective cohesion parameter (Fig. 11).

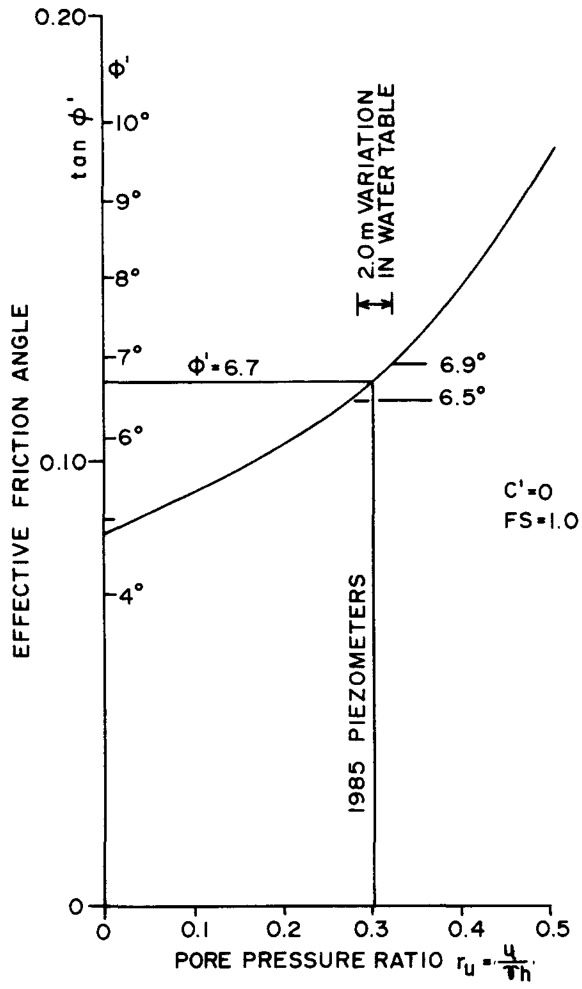


Figure 11. Effect of variations of ± 1.0 m in piezometric level on back-calculation value of ϕ' (After Sauer and Christiansen, 1987)

3.5 Unit weight of landslide debris

The unit weight of the soil affects the mobilized shearing resistance as well as forces causing instability. The influence of unit weight on back-calculated values of ϕ' is significant as illustrated in the case study shown in Fig. 12. There is always a question as to how representative core samples are of the entire sliding block.

Errors in the measurement or assumption of unit weight are apparent from the case study shown in Fig. 13. Back-calculated values of both ϕ' and c' are both affected, but in opposite manner. Typically, the unit weight of a softened clay can be as low as 14.0 kN/m^3, whereas an intact, overconsolidated clay could be as high as 20.0 kN/m^3. The implications of variations in this range are obvious from Fig. 12.

4 SUMMARY

Guidelines for performing back-analysis of landslides can be grouped into three categories; namely, site guidelines, theoretical guidelines, and modelling guidelines.

4.1 Site guidelines

Some site guidelines of value to the analyst can be summarized as follows:

1. The stratigraphy and structure provide valuable input to the analyst.
2. The geometry of failure surface, depth, and length of failure surface, influences the calculated values of c' more than .
3. The back-analysis is highly sensitive to the measured surface slope angles.
4. The shear strength parameters must be assumed and tested parametrically when more than one material is involved.
5. Piezometric measurements provide valuable, necessary information on pore-water pressures.

4.2 Theoretical guidelines

From a theoretical standpoint, a moment equilibrium factor of safety equation is recommended. This is also the recommendation even when the slip surface is composite in shape.

4.3 Modelling guidelines

A parametric analysis is valuable for establishing the reliability of the modelling. The following variables should be tested parametrically:

1. Shear strength parameters.
2. Surface topography.
3. Failure surface geometry.
4. Pore-water pressure conditions.
5. Unit weight of the materials.

REFERENCES

Christiansen, E.A., 1983. The Denholm Landslide: Part I Geology. Canadian Geotechnical Journal, 20, pp. 197-207.

Christiansen, E.A. and Sauer, E.K., 1984. Landslide Styles in Southern Saskatchewan. Proceedings 37th Canadian Geotechnical Conference in Conjunction with IV International Symposium on Landslides.

Eckel, B.F., Sauer, E.K. and Christiansen, E.A., 1987. The Petrofka Landslide, Saskatchewan. Canadian Geotechnical Journal, 24, pp. 81-99.

Fredlund, D.G. and Krahn, J. (1977). "Comparison of Slope Stability Methods of Analysis", Canadian Geotechnical Journal, Vol. 14, No. 3, pp. 429-439.

Fredlund, D.G. (1984). "Analytical Methods for Slope Stability Analysis", State-of-the-Art address, Fourth Int. Sym. on Landslides, Toronto, Canada.

Sauer, E.K., 1979. A Slope Failure in Till at Lebret, Saskatchewan. Canadian Geotechnical Journal, 16, No. 4, pp. 242-250.

Sauer, E.K., 1983. The Denholm Landslide: Part II. Canadian Geotechnical Journal, Vol. 20, pp. 208-220.

Sauer, E.K., 1984. A Landslide in Clay in the North Saskatchewan River Valley: Engineering Geology, Amsterdam, Vol. XX, pp. 279-300.

Sauer, E.K. and E.A. Christiansen, 1985. A Landslide in Till at Varman, Saskatchewan: Canadian Geotechnical Journal, Vol. 22 pp. 195-204.

Sauer, E.K. and Christiansen, E.A., 1987. The Denholm Landslide, Saskatchewan, Canada, an update. Canadian Geotechnical Journal, 24, pp. 163-168.

Skempton, A.W., 1964. Long-term stability of clay slopes, Rankine Lecture, Geotechnique, 14; 77-101.

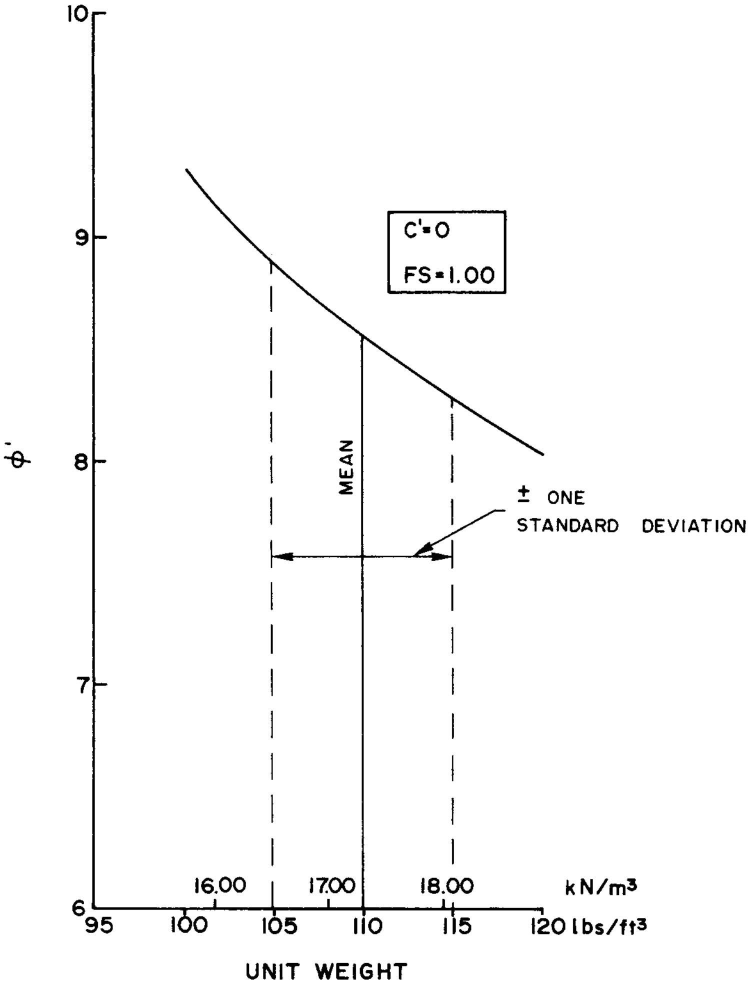


Figure 12. Effect of variation in unit weight on back-calculated value of φ' (After Sauer, 1984).

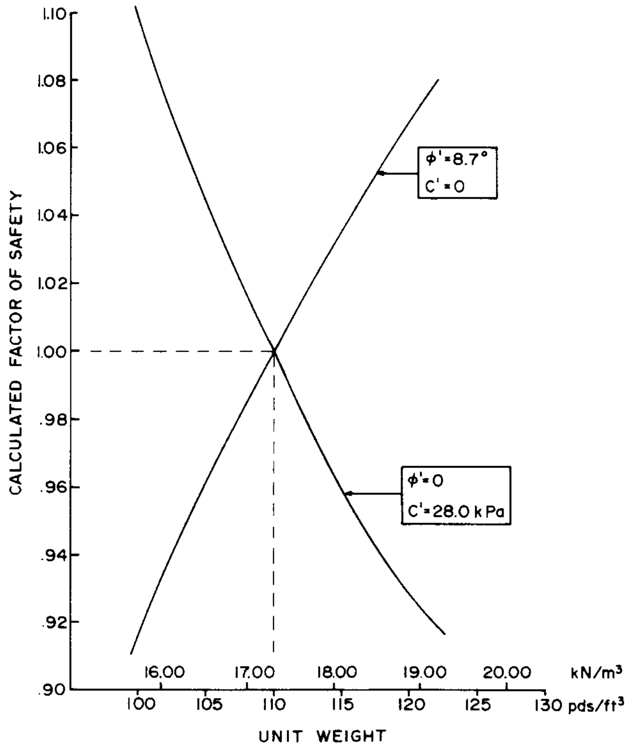


Figure 13. Effect of variations in unit weight on factor of safety for constant values of φ' (c' = 0) and c' (φ' = 0) (After Sauer, 1984)