Proceedings of the First Canadian Symposium on Micro-computer Applications to Geotechnique, Regina, Saskatchewan, Canada. pp. 173-181. October 22-23. 1987

The PC-SLOPE Family of Software for Slope Stability Analysis

D.G. Fredlund
University of Saskatchewan, Saskatoon

and

L.D. Fredlund
Alberta Energy Board, Calgary

SYNOPSIS

PC-SLOPE is a family of computer programs designed to assist the geotechnical engineer in assessing the stability of man-made and natural slopes. This paper describes the PC-SLOPE family of programs which can be categorized as preprocessor, processor, and post-processor programs. The theory and main algorithm for computing the factor of safety are described.

INTRODUCTION

There is probably no analysis conducted by geotechnical engineers which has received more programming attention than the limit equilibrium methods of slices used to compute a factor of safety (Fredlund, 1980). There may be several reasons for this, but there appears to be two main reasons. First, the limit equilibrium method has proven to be a useful and reliable tool in assessing the stability of slopes. Its "track-record" is impressive for most cases where the shear strength properties of the soil and the pore-water pressure conditions have been assessed by a knowledgeable engineer (Chowdhury, 1980). Second, the limit equilibrium methods of slices require a limited amount of input information but can quickly perform extensive trial and error searches for the critical slip surface.

There is danger in the indiscriminate usage of a "black-box" computer program by an inexperienced engineer. This will always be a problem and certain steps should be adhered to in performing a slope stability study. The following steps are suggested with respect to the use of slope stability software: 1) Obtain an approximate solution for the problem either from stability charts or from hand calculations on a simplified form of the problem. 2) Perform the computer analysis of the actual problem. 3) Compare the 'approximate' and 'computer' results and resolve anomolies. 4) Modify appropriate variables (i.e., geometry, soil properties, or pore-water conditions) and check the sensitivity of the factor of safety to these variables. 5) Make the necessary engineering decisions and plot the significant results.

The computer must be regarded as only one of the tools required in analyzing slopes, keeping in mind, however, that it is an extremely important tool.

HISTORY OF PC-SLOPE

A study was undertaken by GEO-SLOPE Programming Limited in 1982 to assess the feasibility of 'paring-down' the SLOPE-II computer program for use on a microcomputer (Fredlund and Rahardjo, 1983). The study revealed that although this was feasible, the program was inefficient in terms of execution time. As a result of this study, GEO-SLOPE decided to undertake the development of a new slope stability computer program, one specifically designed for microcomputers. The objective was to have as efficient a program as possible while retaining most of the features of SLOPE-II. To optimize the computing speed, the factor of safety equations were rearranged and a heirarchy was established on the order in which variables were calculated. Each variable was computed one time, if possible. Only the nonlinear portions of the factor of safety equations were computed during each iteration as the solution tended towards convergence. The end result was a powerful, efficient, single algorithm which allows the simulation of most of the commonly used methods of slices.

The design of the overall slope stability computer package involves three programs. A preprocessor called PC-PROMSL which allows the use of a specially designed free-format system for the creation, modification, and verification of a datafile. The factor of safety computation processor is called PC-SLOPE. A post-processor called PC-DOT is used to provide graphic displays of pertinent output from the slope stability analysis.

LAYOUT AND FEATURES OF THE PC-SLOPE FAMILY OF SOFTWARE

The relationship between the members of the PC-SLOPE family of software are shown in

Fig. 1. (i.e., PC-PROMSL, PC-SLOPE, PC-DOT1/2/3/A).

PC-SLOPE Family of Software

PREPROCESSOR	PROCESSOR	POST-PROCESSOR
PC-PROMSL Data Input, Editing, and Verification	PC-SLOPE Computation of Factor of Safety	PC-DOT1 Plot of the geometry, including pore-pressure conditions. The contoured factors of safety and critical slip surface can also be plotted
		PC-DOTA Plot of the geometry along with the point numbers
		PC-DOT2 Plot of the grid of factors of safety along with contours
		PC-DOT3 Plots of various dimensions and forces associated with the critical slip surface

Fig. 1 Layout of the PC-SLOPE Software System

Figure 2 shows the layout of a typical data file consisting of several keywords, followed by pertinent data. A large part of the datafile is made up of the x and y-coordinates of points followed by a series of points which must be joined to form lines. PC-DOT1/2/3/A assist the user in handling the computer output in a graphical form. Other programs involving techniques such as the finite element method can be used in an independent manner, to assist in preparing pore-pressure data for PC-SLOPE.

PC-SLOPE utilizes the limit equilibrium method of slices theory to solve for the factor of safety of an earth or rock slope. The main algorithm simultaneously computes the force and moment equilibrium factor of safety equations for various interslice force conditions. The nonlinear, force and moment equilibrium equations are solved simultaneously, resulting in an efficient and general algorithm for computing factors of safety.
Some of the main features of the PC-SLOPE program are as follows:

i) The commonly used methods of slices are available in the PC-SLOPE program. These are:
- the Fellenius or Ordinary method,
- the Bishop Simplified method (Bishop, 1955)
- the Janbu's Simplified method with no f_0 factor applied (Janbu, 1954),
- the Spencer method (Spencer, 1967),
- the General Limit Equilibrium, GLE, method, (Fredlund, Krahn and Pufahl, 1981)

```
QUESTIONS
     BEN.SET           = DATA FILE NAME
     PC-SLOPE BENCHMARK (BEN)
     NO. 1             = TRIAL NUMBER
     MAR. 6,1985       = DATE
     EXCAVATION IN SILTY CLAY (CIRCULAR SLIP SURFACES)
          20,          = # OF SLICES / SLIP SURFACE
       .01000,         = TOLERANCE
       .00000,         = SEISMIC COEFFICIENT
     62.40000,         = UNIT WEIGHT OF WATER
POINT     ,   22
    1,    10.000,   160.000
    2,   160.000,   160.000
    3,   174.000,   153.000
    4,   190.000,   145.000
    5,   240.000,   120.000
    6,   400.000,   120.000
    7,    10.000,   153.000
    8,    10.000,   145.000
    9,    10.000,   113.000
   10,   400.000,   113.000
   11,    10.000,   100.000
   12,   400.000,   100.000
   13,    10.000,   143.000
   14,   160.000,   141.000
   15,   200.000,   134.000
   17,   170.000,   170.000
   18,   210.000,   170.000
   19,   192.500,   260.000
   20,   100.000,   130.000
   21,   300.000,   130.000
   22,   100.000,   102.000
   23,   300.000,   102.000
LINE      ,   5
  1,  1,  2,  3,  4,  5,  6,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  2,  7,  3,  4,  5,  6,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  3,  8,  4,  5,  6,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  4,  9, 10,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  5, 11, 12,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
SOIL      ,   5
    1,  115.0000,  700.0000,   22.0000,     .0000,     .0000,     .0000
     UPPER PORTION OF SILTY CLAY
    2,  112.0000,  600.0000,   21.0000,     .0000,     .0000,     .0000
     LOWER PORTION OF SILTY CLAY
    3,  110.0000,  300.0000,   20.0000,     .0000,     .0000,     .0000
     SOFT SILTY CLAY
    4,  123.0000,  600.0000,   27.0000,     .0000,     .0000,     .0000
     SANDY CLAY TILL
    5,   -1.0000,     .0000,     .0000,     .0000,     .0000,     .0000
     BASE OF PROBLEM
TENSION   ,   0
GRID
   17,   18,   19
    2,    3
RADIUS
   20,   21,
   22,   23,
    2,
SIDE
    1,                 1-CONSTANT FUNCTION
LAMBDA    ,   1
     .0000,    .0000     .0000     .0000     .0000     .0000
LOAD      ,   0
PIEZ      ,   5
  1,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  2,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  3, 13, 14, 15,  5,  6,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  4, 13, 14, 15,  5,  6,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
  5,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,  0,
END
```

Fig. 2 Typical input data file created by PC-PROMSL

- the Morgenstern-Price method, (Morgenstern and Price, 1965),
- the Lowe-Karafiath method,
- the Corps of Engineers method,
- Other methods such as the Nonveiller (Nonveiller, 1965) and the Sarma (Sarma, 1973) method can also be simulated using the General Limit Equilibrium method solver.

All of the above methods require the calculation of similar basic forces. For example, the cohesive force, the frictional resistance force, the pore-water force and numerous other forces are the same for all methods. However, the manner in which the various forces are combined varies from one method to another. It is reasonable, therefore, to incorporate several methods of analysis in one computer program. This procedure also allows for an accurate comparison of the different methods since the basic forces are computed in a similar manner.

ii) Three failure modes (i.e., circular, composite, and point-specified slip surfaces) can be examined using any of the above methods of slices. Grid search capabilities are available for both circular and composite slip surfaces. Figure 3 shows a composite slip surface with the forces and moment arms defined. The slip surface generally deviates from a circular shape as a result of a 'hard' or competent soil layer at some depth. Therefore, it is still reasonable to use a grid of centers to investigate various slip surfaces in an attempt to find the most critical slip surface. The slip surface is assumed to be circular through the upper soil strata and then assumed to follow the top of the 'hard' layer in the central portion of the slip surface. Figure 4 shows the forces and moment arms associated with a point-specified slip surface passed through a cross-section. Each specified slip surface consists of the coordinates of a series of points which start and end above the geometry. In other words, the slip surface becomes a series of straight line segments.

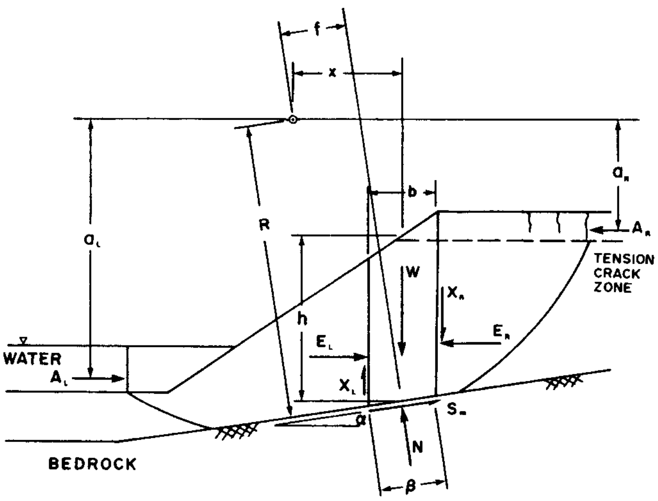


Fig. 3 Forces acting on a slice through a sliding mass defined by a composite slip surface

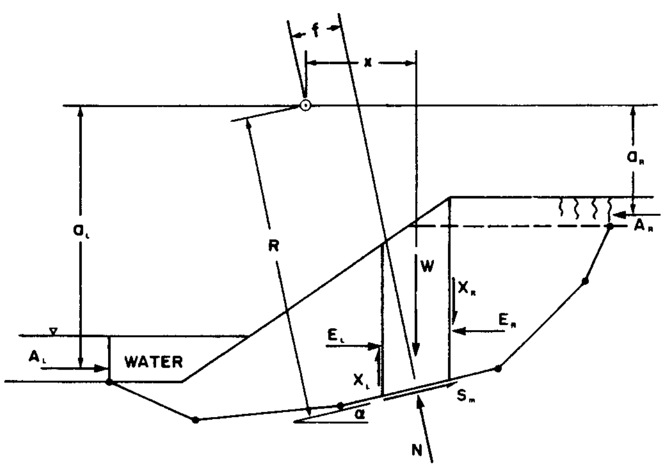


Fig. 4 Forces acting on a slice within a sliding mass defined by a point-specified slip surface

iii) Complicated geometric surfaces and stratigraphic conditions can be simulated. One of the most difficult geometric conditions to represent is that of a discontinuous soil strata. This conditon often occurs in a man-made structure such as a dam but can also occur in natural soil strata. The discontinous strata can be modelled as two overlapping lines extended to the extremity of the geometric section. The geometry or cross-section can be input in any one of the four quadrants.

iv) Externally applied line loads, berms and surcharges can be taken into account in all methods of slices. The line loads may be due to a machine such as a dragline. In this case, an equivalent load per unit width of slope must be calculated and applied to the slope. Berms and surcharges can be modelled as an independent soil strata.

v) Bilinear shear strength envelopes can be used in addition to the conventional Mohr-Coulomb type of shear strength failure envelopes. As well, the extended Mohr-Coulomb failure envelope accounting for negative pore-water pressures in unsaturated soils can be used.

vi) Partial or complete submergence of a slope or tension cracks in the soil can be accommodated. Both of these cases are modelled in a similar manner.

vii) Several methods are available for handling the pore pressure data. These are:
- a pore pressure coefficient, r_u
- a series of piezometric lines,
- a series of constant pressure contours
- a grid of pore-water pressures
- a grid of pore-water heads
- a grid of pore pressure coefficients.

Each of these methods are shown in Fig. 5. When using a grid of pore-pressure heads, pressures or pore-pressure coefficients, the pore-water pressure at the base of a slice is obtained using a 4-Way interpolation technique as illustrated in Fig. 6. Values at a maximum of 800 points can be specified. More recently, a Kriging technique has also been added as an interpolation technique to compute the pore-water pressures.

viii) Seismic or earthquake loadings can be taken into account in all methods.

ix) Any consistent set of units can be used to input the data.

x) PC-SLOPE can be executed either in an interactive or batch mode. In the batch mode, up to five problems can be computed consecutively (Fig. 7). The

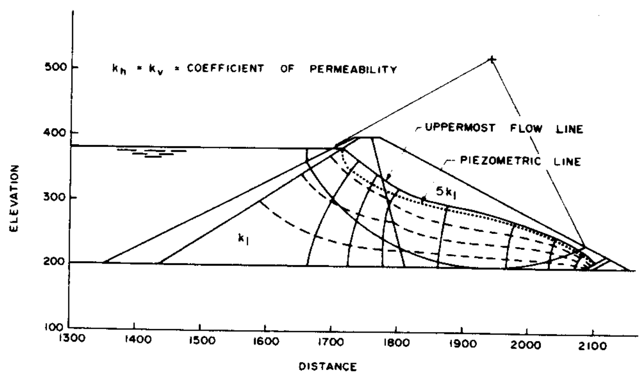


Fig. 5.1 Use of piezometric lines

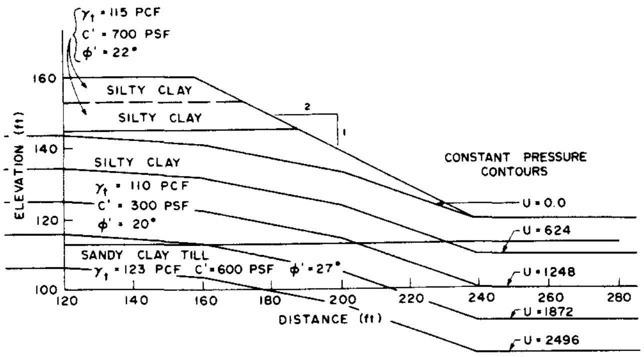


Fig. 5.2 Contours of pore-water pressure

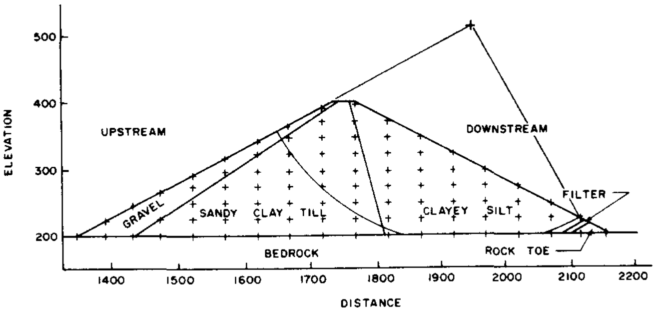


Fig. 5.3 Grid of pore-water heads

Fig. 5 Several procedures for computing pore-water pressures

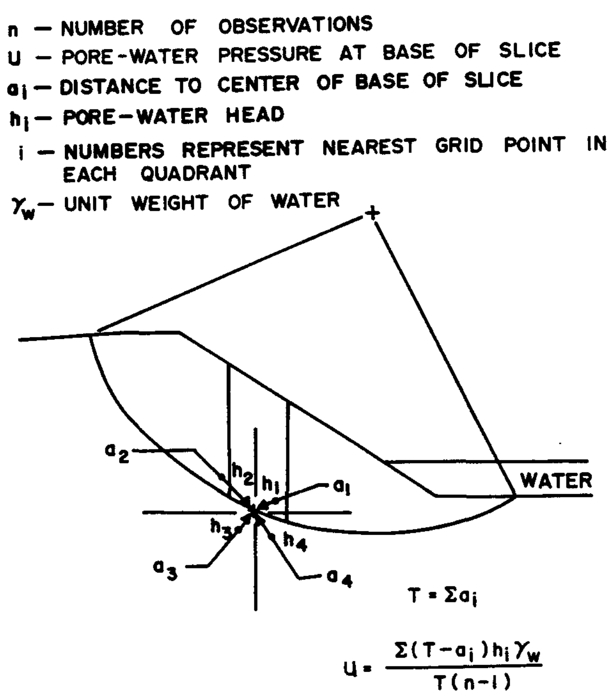


Fig. 6 The four-way interpolation for the pore-water pressure at the base of a slice

```
PPPPPPP  CCCCCCC         SSSSSSS  LL       0000000  PPPPPPP  EEEEEEE
PPPPPPP  CCCCCCC         SSSSSSS  LL       0000000  PPPPPPP  EEEEEEE
PP   PP  CC              SS       LL       OO   OO  PP   PP  EE
PP   PP  CC         ===  SSSSSSS  LL       OO   OO  PP   PP  EEEEE
PPPPPPP  CC         ===  SSSSSSS  LL       OO   OO  PPPPPPP  EEEEE
PPPPPPP  CC         ===       SS  LL       OO   OO  PPPPPPP  EE
PP       CCCCCCC         SSSSSSS  LLLLLLL  0000000  PP       EEEEEEE
PP       CCCCCCC         SSSSSSS  LLLLLLL  0000000  PP       EEEEEEE

PC-SLOPE is a general limit equilibrium, method of slices
    computer program to calculate the factor of safety.
(C) COPYRIGHT 1985,1986,1987 by GEO-SLOPE Programming Ltd.
                    Version 3.0,  1987

SERIAL NO. 86042 is licensed to:


------- CONTROL MENU -------
1 - INTERACTIVE MODE
2 - BATCH MODE
3 - EXIT

OPTION ? 2

Job No. 1
---------------------------------------------------------------------
INPUT DATAFILE NAME ( -------.SET)................: A:BEN
Drive for FACTOR OF SAFETY File (e.g., C) ........: C
Create DETAILED FORCES File (Y/N)?................: N

Job No. 2  Press RETURN to end Batch
---------------------------------------------------------------------
INPUT DATAFILE NAME ( -------.SET)................: A:KRIG1
Drive for FACTOR OF SAFETY File (e.g., C) ........: C
Create DETAILED FORCES File (Y/N)?................: Y
Drive for DETAILED CALCULATIONS File (e.g., C) ...: A

Job No. 3  Press RETURN to end Batch
---------------------------------------------------------------------
INPUT DATAFILE NAME ( -------.SET)................: A:BEN10
Drive for FACTOR OF SAFETY File (e.g., C) ........: C
Create DETAILED FORCES File (Y/N)?................: N

Job No. 4  Press RETURN to end Batch
---------------------------------------------------------------------
INPUT DATAFILE NAME ( -------.SET)................:

List INPUT & OUTPUT Data (Y/N)....................: N

JOB NO.  1
==========================================

FOLLOWING FILES ARE OPEN
------------------------------------------
INPUT DATA FILE..............:A:BEN.SET
FACTOR OF SAFETY FILE........:C:BEN.FAC
DETAILED FORCES FILE.........:

INPUT DATAFILE BEING READ AND COMPUTATIONS PERFORMED
---------------------------------------------------------------------

JOB NO.  2
==========================================

FOLLOWING FILES ARE OPEN
------------------------------------------
INPUT DATA FILE..............:A:KRIG1.SET
FACTOR OF SAFETY FILE........:C:KRIG1.FAC
DETAILED FORCES FILE.........:A:KRIG1.DET

INPUT DATAFILE BEING READ AND COMPUTATIONS PERFORMED
---------------------------------------------------------------------

JOB NO.  3
==========================================

FOLLOWING FILES ARE OPEN
------------------------------------------
INPUT DATA FILE..............:A:BEN10.SET
FACTOR OF SAFETY FILE........:C:BEN10.FAC
DETAILED FORCES FILE.........:

INPUT DATAFILE BEING READ AND COMPUTATIONS PERFORMED
---------------------------------------------------------------------
```

Fig. 7 Example illustrating interactive and batching options in PC-SLOPE

input and output can be taken from and directed to any of the drives on the computer. The input data and computer results can either be printed to the screen or suppressed.

xi) High resolution Dot Matrix plotting of pertinent data is available using the PC-DOT programs. Figure 8 shows a typical dot matrix plots of the geometry and the critical slip surface while Fig. 9 shows the factors of safety contoured on a grid of centers.

The PC-DOT3 program allows for the plotting of all forces and dimensions associated with the critical slip surface. Possible plots are as follows:

i) x and y-coordinates of the top and bottom of each slice,
ii) the width of each slice and the base length versus the x-coordinate,
iii) angle of internal friction versus

x-coordinate,

iv) interslice force function and lambda times the function versus x-coordinate,

v) radius, weight moment arm, base normal force moment arm, and seismic force moment arm versus x-coordinate,

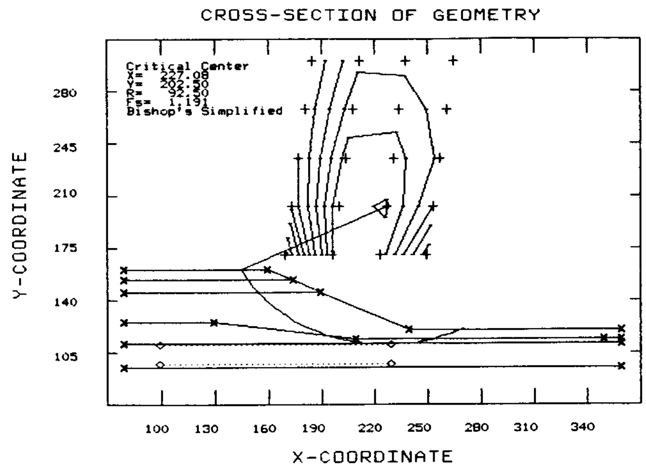
vi) unit cohesive force and stress versus x-coordinate,

vii) unit pore-water force and pressure versus x-coordinate,

viii) weight versus x-coordinate,

ix) normal force and stress at the base of a slice for the first and last iterations for both moment and force equilibrium versus x-coordinate,

x) interslice normal and shear forces versus x-coordinate.



UNIT WEIGHT	COHESION	PHI	DESCRIPTION
115.0000	700.0000	22.0000	UPPER PORTION OF SILTY CLAY
112.0000	600.0000	21.0000	LOWER PORTION OF SILTY CLAY
110.0000	300.0000	20.0000	SILTY CLAY
109.0000	100.0000	18.0000	SOFT CLAY
-1.0000	.0000	.0000	SANDY CLAY TILL
-1.0000	.0000	.0000	BASE OF PROBLEM

Fig. 8 Dot matrix plot of the geometry and factor of safety grid using PC-DOT1

LAYOUT OF A SLOPE STABILITY PROBLEM

The GEOMETRY of the problem (i.e., the ground surface and stratigraphy) can be laid out in any one of four quadrants and may face either towards the right or left. Figure 10 shows the general slope stability problem layout. The geometry is defined by means of the POINT and LINE keywords. The keyword GRID refers to a specified, regular grid of points that can be used to describe circular and composite slips surfaces. The keyword RADIUS refers to the specified zone where the circular slip surfaces will be tangent to a series of lines. The GRID and RADIUS keywords are not used when the location of the slip surface is totally specified by a series of straight lines (i.e., keyword SLIP).

When a slip surface (i.e., circular, composite, or of point-specified shape) is passed through a soil mass, the geometry is divided into a series of SECTIONS. Figure 11 shows the definition of the SECTIONS for a composite slips surface.

The right and left intercepts between the slip surface and the ground surface are first computed for the definition of SECTIONS.

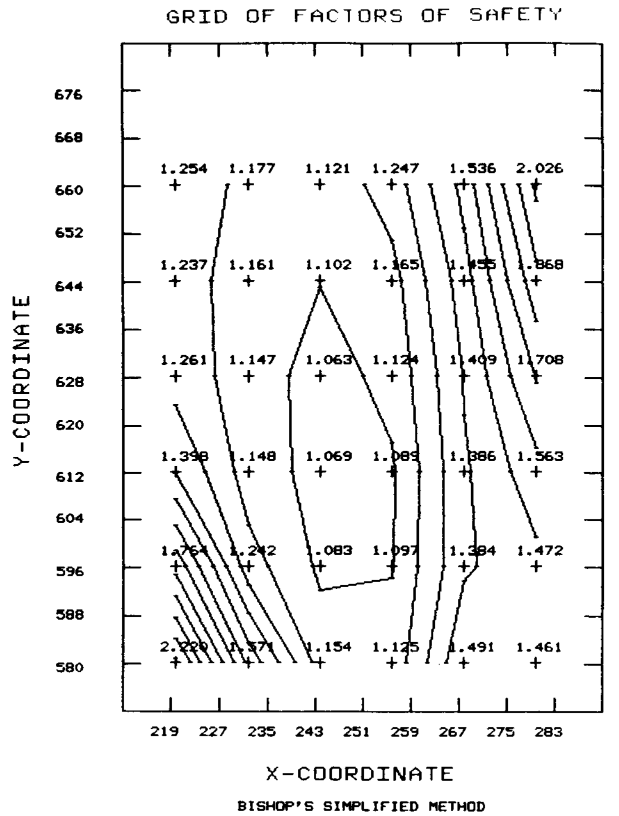


Fig. 9 Dot matrix plot of factors of safety using PC-DOT2

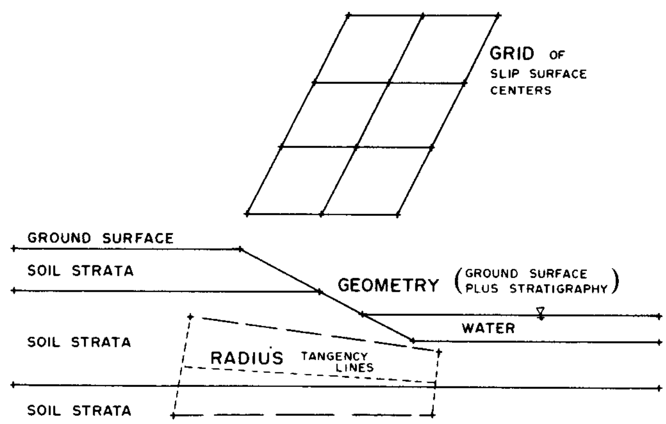


Fig. 10 The main aspects of a slope stability problem

Secondly, all vertices or points along the top line, within the range of the intercepts, are identified. Thirdly, the intersection points between a soil strata and the slip surface are computed. Fourthly, if the slip surface is composite or specified, additional points along the linear portions are identified. All the above points are arranged according to their ascending x-coordinate magnitude. These vertices define the SECTIONS into which the sliding mass is divided. The SECTIONS are subsequently subdivided into smaller vertical slices.

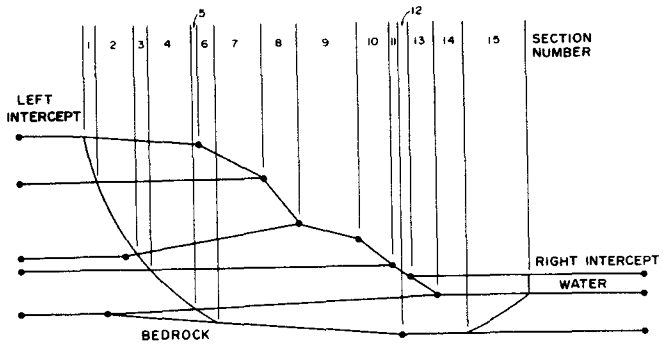


Fig. 11 Definition of SECTIONS for a composite slip surface

An 'average width of slice' is first computed by dividing the x-coordinate distance between the left and right intercepts into the requested number of slices. This 'average width of slice' is then used to compute the number of slices within each section. As a result of using the above technique, the computed factors of safety do not vary significantly with the number of slices used in the analysis.

LIMIT EQUILIBRIUM THEORY

The PC-SLOPE computer program utilizes the theory of limit equilibrium of forces and moments to compute the factor of safety against movement. The General Limit Equilibrium theory (i.e., GLE) is used as the context for relating the factors of safety for all commonly used methods of slices (Fredlund and Krahn, 1977, Fredlund, Krahn and Pufahl, 1981).

The stability analysis involves passing a slip surface through the earth mass and dividing the inscribed portion into vertical slices. The center for moment equilibrium is immaterial when both moment and force equilibrium are satisfied. However, when only moment equilibrium is satisfied, it is important to select a reasonable center for moment equilibrium. Figure 12 shows the procedure used in PC-SLOPE to compute the center for moment equilibrium. First, the length of the 'chord' across the slip surface is computed (i.e., a line joining the starting and ending points of the slip surface with the ground surface). Second, the perpendicular bisector of the 'chord' is defined. Third, the perpendicular bisector is drawn for the first and last line segments defining the slip surface. These will generally intersect the 'chord bisector' at two different points. The lower (i.e., in terms of y-coordinate) of these two intersection points is used as the center for moment equilibrium.

The elements of statics that can be used to derive the factor of safety are the summations of forces in two directions and the summation of moments. These, along with the failure criteria, are insufficient to make the problem determinate. More information must be known to render the problem determinate. All methods solved using PC-SLOPE make an assumption regarding the interslice forces in order to solve for the factor of safety.

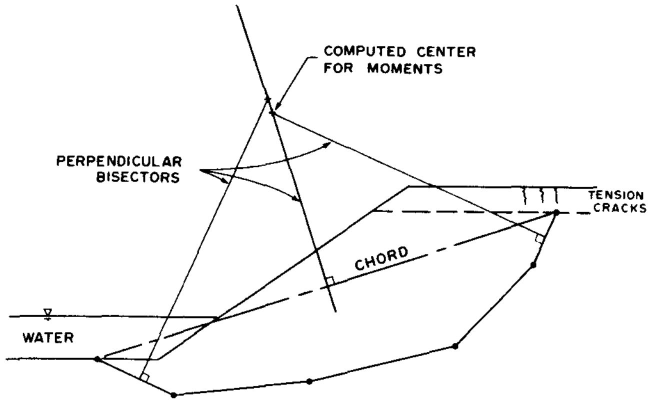


Fig. 12 Procedure for locating the center of moments for a point-specified slip surface

The General Limit Equilibrium method, GLE, utilizes the following equations of statics in solving for the factor of safety:

i) The summation of moments about a common point for all slices. The equation can be rearranged and solved for the moment equilibrium factor of safety, F_m.
ii) The summation of forces in a horizontal direction for all slices which gives rise to a force equilibrium factor of safety, F_f.
iii) The summation of forces in a vertical direction for each slice. The equation is solved for the normal force at the base of the slice, N.
iv) The summation of forces in a horizontal direction for each slice is used to compute the interslice normal force, E. This equation is applied in an integration manner across the sliding mass.

The analysis is still indeterminate and a further assumption is made regarding the direction (or magnitude) of the resultant interslice forces. The factor of safety satisfying both moment and force equilibrium can then be computed. It is also possible to satisfy only the moment or force equilibrium conditions.

FACTOR OF SAFETY WITH RESPECT TO MOMENT EQUILIBRIUM, F_m

Reference can be made to Figs. 3 or 4 for deriving the moment equilibrium factor of safety equation. In each case, moments can be summed about a common point for all slices. Substituting the shear strength equation into the moment equilibrium equation and solving for the factor of safety gives,

$$F_m = \frac{\sum\{c'\beta R + (N - u\beta) R \tan \phi'\}}{\sum Wx - \sum Nf \pm Aa} \qquad [1]$$

Equation [1] is nonlinear since the normal force, N, is also a function of the factor of safety.

FACTOR OF SAFETY WITH RESPECT TO FORCE EQUILIBRIUM, F_f

The forces in a horizontal direction can be summed for all slices. Substituting the shear strength equation into the horizontal force equilibrium equation and solving for the factor of safety gives,

$$F_f = \frac{\sum(c'\beta \cos\alpha + (N - u\beta)\tan\phi' \cos\alpha)}{\sum N \sin\alpha \pm A} \qquad [2]$$

Equation [2] is also nonlinear since the normal force, N, includes the factor of safety.

NORMAL FORCE AT THE BASE OF A SLICE, N

The normal force at the base of a slice is derived from the summation of forces in a vertical direction on each slice. Substituting the shear strength equation into the vertical force equilibrium equation and solving for the normal force, N, gives,

$$N = \frac{W + (X_R - X_L) - c'\beta \sin\alpha/F + u\beta \sin\alpha \tan\phi'}{\cos\alpha + \sin\alpha \tan\phi'/F} \qquad [3]$$

The denominator in Eq. [3] is commonly given the variable name, m . The factor of safety, F, in Eq. [3] is equal to the moment equilibrium factor of safety, F_m, when solving moment equilibrium, and equal to the force factor of safety, F_f, when solving force equilibrium.

Equation [3] cannot be solved directly since the interslice shear forces, (i.e., X_L and X_R) are unknown. These forces can be related to the interslice normal forces.

In order to commence the solution for the factor of safety, it is possible to neglect the interslice shear and normal forces on each slice. When forces are summed in a direction perpendicular to the base of each slice, the following equation is obtained for the normal force.

$$N = W \cos\alpha \qquad [4]$$

Using equation [4] in solving Eqs. [1] and [2] provides a starting value for the factor of safety computations. The factor of safety computed using Eq. [1] corresponds to the Fellenius or Ordinary method.

Next, it is possible to assume that only the interslice shear forces are equal to zero in equation [3]. Applying the computed normal forces at the base of each slice to Eq. [1] gives the factor of safety for Bishop's Simplified method. The formulation used in PC-SLOPE has also been extended to noncircular slip surfaces.

Equation [3] can also be used in solving the force equilibrium factor of safety equation. The solution is Janbu's Simplified method without the empirical correction factor, f_o, applied.

INTERSLICE NORMAL AND SHEAR FORCES

The interslice shear forces are required in order to calculate the normal force at the base of each slice. The interslice shear force is computed as a percentage of the interslice normal force in accordance with an empirical equation (Morgenstern and Price, 1965)

$$X = E \; \lambda f(x) \qquad [5]$$

where:

λ = the percentage (in decimal form) of the function utilized, and

$f(x)$ = any functional distribution for the relative direction of the resultant interslice force.

The type of function used in calculating the factor of safety is the perogative of the program user. Figure 13 shows a reasonable interslice force function based on a finite element stress analysis (Fan, Fredlund and Wilson, 1986). Before the interslice shear force is computed, it is necessary to calculate the interslice normal force.

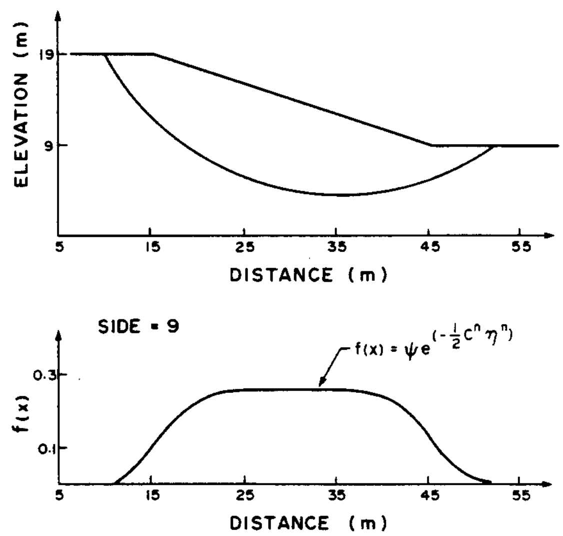


Fig. 13 Example of the element-based interslice function

The summation of forces in a horizontal direction can be written for each slice. Substituting the equation for the shear strength of the soil into the horizontal force equilibrium equation and solving for the interslice normal force on the right side of each slice gives,

$$E_R = E_L + (c'\beta - u\beta\tan\phi')\cos\alpha/F + N(\tan\phi' \cos\alpha - \sin\alpha)/F = 0 \qquad [6]$$

The interslice normal forces are solved using an intergration procedure commencing at the left end of each slip surface.

SOLVING FOR THE FACTORS OF SAFETY

Several factors of safety can be computed for each slip surface. Figure 14 shows a flow diagram where the various factors of safety are placed in 'Stages'. It is possible to solve for only the factors of safety associated with Stages 1 and 2. If further calculations are desired, a choice must be made between using Stage 3 or 4.

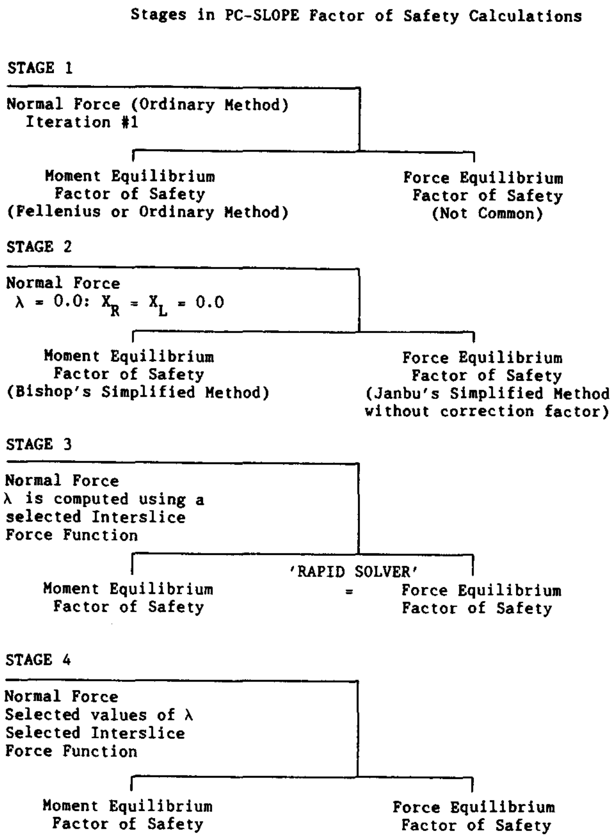


Fig. 14 Outline for the stages involved in solving for the factors of safety

Stage 1 Solution

The first iteration of the factor of safety equations is solved using a linear equation for the normal force at the base of a slice (i.e., Fellenius' assumption). Both the moment and force equilibrium factors of safety are solved to provide approximate factors of safety for further computations.

Both factors of safety should be considered as approximate. The force equilibrium factor of safety has received little mention in the literature and is of little relevance. Both the interslice normal and shear forces are set to zero in these solutions.

Stage 2 Solution

Stage 2 starts the solution of the nonlinear factor of safety equations. Lambda, λ , is set to zero and therefore, the interslice shear forces are set to zero. Usually four to six iterations are required to ensure convergence of the moment and force equilibrium factor of safety equations. The answer from the moment equation corresponds to Bishop's Simplified method. The answer from the force equilibrium equation corresponds to Janbu's Simplified method without the application of the empirical correction factor, f_o. A simple back-substitution technique is used to solve both nonlinear factor of safety equations.

Stage 3 Solution

Stages 3 and 4 compute the moment and force equilibrium factors of safety for any general side force function conditions. In Stage 3, a lambda value, λ , is computed which provides an equal value for the force and moment equilibrium factors of safety (i.e., $F_m = F_f$). The procedure used is referred to as the 'Rapid Solver' and is similar in concept to a Newton-Raphson procedure. The 'Rapid Solver' procedure can be described as follows: The initial value for lambda, λ, is computed internally to PC-SLOPE as being equal to 2/3 of the chord slope. The moment and force equilibrium factors of safety are computed using this estimate of lambda. These factors of safety along with the factors of safety corresponding to a lambda equal to 0.0 are used to predict a lambda value where the force and moment factors of safety will be equal. The above procedure for computing new lambda values is repeated until the force and moment equilibrium factors of safety are within the selected tolerance. Any one of the interslice force functions, f(x), can be used when solving for the factor of safety.

Stage 4 Solution

If stage 4 is used, a series of lambda values are selected and the moment and force equilibrium factors are solved. These factors of safety can be plotted versus lambda, and the factor of safety satisfying both moment and force equilibrium can be selected from the plot.

Stage 4 provides the complete relationship between the moment and force equilibrium factors of safety. It also allows the simulation of slope stability methods which specify the interslice force function and satisfy only one of the overall equilibrium conditions (e.g., Corps of Engineers method).

SIMULATION OF VARIOUS METHODS OF SLICES

The General Limit Equilibrium, GLE, formulation and solution can be used to simulate most of the commonly used methods of slices (Fredlund, Krahn and Pufahl, 1981). From a theoretical standpoint, the various methods of slices can be categorized in terms of the conditions of statical equilibrium satisfied and the assumption regarding the interslice forces. Table 1 shows the procedure for simulating various methods of slices using PC-SLOPE.

ONGOING MAINTENANCE OF PC-SLOPE

The PC-SLOPE software package brings together almost two decades of experience with slope stability analysis. Concerns pertaining to slope stability analysis, expressed to GEO-SLOPE over the years, have been given consideration in the development of PC-SLOPE. The desire behind the development of PC-SLOPE has been to produce a high quality, efficient, and comprehensive software program for slope stability analysis. New releases of PC-SLOPE have been issued on an annual basis. Each release has provided a number new features in PC-SLOPE.

TABLE I

Simulation of Commonly Used Methods of Slices

Method of Slices	Stage	Interslice Force Function	Lambda
Fellenius or Ordinary	1	N.A.*	N.A. (Set to 0.0)
Bishop's Simplified	2	Any f(x)	0.0
Janbu's Simplified**	2	Any f(x)	0.0
Janbu's Generalized	Cannot be Directly Simulated		
Spencer	3 or 4	f(x) = 1.0	Computed
Morgenstern-Price	3	Any f(x)	Computed
GLE	3 or 4	Any f(x)	Computed
Corps of Engineers	4	f(x) = Corps assumtions	λ = 1.0
Love-Karafiath	4	f(x) = Love-Karafiath Assumption	λ = 1.0

* N.A. means Not Applicable
** Must be multiplied by empirical correction factor, f_o

SELECTED REFERENCES

Bishop, A.W. (1955). "The Use of the Slip Circle in the Stability Analysis of Slopes", Geotechnique, Vol. 5, pp. 7-17.

Chowdhury, R.N. (1980). "A Reassessment of Limit Equilibrium Concepts in Geotechnique", Proceedings of the Symposium on Limit Equilibrium, Plasticity and Generalized Stress Strain Applications in Geotechnical Engineering, ASCE, 1980 Annual Conventional, Hollywood, Florida.

Fan, K., Fredlund, D.G., and Wilson, G.W. (1986). "An Interslice Force Function for Limit Equilibrium Slope Stability Analysis", Can. Geot. Jour., Vol. 23, No. 3, pp. 287-296.

Fredlund, D.G. (1980). "Use of Computers for Slope Stability Analysis", State-of-the-Art, Int. Symposium on Landslides (ISL 1980), April, New Delhi, India, pp. 129-138.

Fredlund, D.G. (1984). "Analytical Methods for Slope Stability Analysis", State-of-the-Art Paper, Proc. IVth Int. Symposium on Landslides, Toronto, Vol. 1, pp. 229-250.

Fredlund, D.G. and Krahn, J. (1977). "Comparison of Slope Stability Methods of Analysis", Can. Geot. Jour., Vol. 14, No. 3, pp. 429-439.

Fredlund, D.G., Krahn, J. and Pufahl, D.E. (1981). "The Relationship Between Limit Equilibrium Slope Stability Methods", Int. Conf. on Soil Mech. and Found. Eng. June, Stockholm, Sweden, Vol. 3, pp. 409-416.

Fredlund, D.G. and Rahardjo, H. (1983). "A Comprehensive Limit Equilibrium Software Package for Geotechnical Engineering Presented to the Canadian Society for Civil Engineering, Annual Conference, Computer Systems in Civil Engineering, Ottawa, Canada, June, pp. 39-57.

Janbu, N. (1954). "Application of Composite Slip Surfaces for Stability Analysis", Proc. European Conf. on Stability of Slopes, Stockholm, Discussion, Vol. 3, pp. 43-49.

Morgenstern, N.R. and Price, V.E. (1965). "The Analysis of the Stability of General Slip Surfaces", Geotechnique, Vol. 14, pp. 79-93.

Nonveiller, E. (1965). "The Stability Analysis of Slopes with a Slip Surface of General Shape", Sixth Int. Conf. on Soil Mech. and Found. Eng., Toronto, Vol. 2, pp. 522-525.

Sarma, S.K. (1973). "Stability Analysis of Embankments and Slopes", Geotechnique, Vol. 23, pp. 423-433.

Spencer, E. (1967). "A Method of Analysis of the Stability of Embankments Assuming Parallel Interslice Forces", Geotechnique, Vol. 17, pp. 11-26.