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A Note on Reinforced Earth Design

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SUMMARY An empirical design of reinforced earth walls based on the working stress state is considered. The coherent gravity method is examined and a modified method is proposed. The governing equations for design resulting from both methods and based on local equilibrium are derived. The results from the application of both methods to reinforced earth wall design are compared. For illustration purposes, results from the application of other semi-analytical design methods based on working stress state are also shown. It is concluded that, using the same factor of safety, the modified coherent gravity method leads to a more economical design than a design based on the original coherent gravity-method.

1 INTRODUCTION

Reinforced earth, a composite material invented by Vidal in 1965, has been widely used for structures such as retaining walls, bridge abutments, railway embankments, landslide repairs, foundation rafts, and other structures. In recent years, reinforced earth design has become a focus of interest for many researchers. Consequently, various analytical, semi-analytical and empirical methods based on the internal stability of the reinforced earth mass have been developed. In general, the design methods fall into two categories:

a) those concerned with the local equilibrium of each strip and the soil in its vicinity, and
b) those concerned with the equilibrium of the failure wedge as a whole.

In the design process, consideration should be given to three possible types of failure:

a) failure due to excessive stress within the reinforcement (i.e., tensile, or break failure);
b) failure due to insufficient friction between the reinforcement and soil (i.e., bond, or pull-out failure);
c) failure due to excessive stress exerted on facing units (i.e., facing failure).

In this paper, attention is focused on an empirical design procedure called the coherent gravity method. In this method, local equilibrium is considered so as to prevent either tensile or bond failures of any individual reinforcement strip.

2 NOTATION

f^* = apparent friction coefficient between the soil and the reinforcement;
f_o^* = (1.2 + log Cu); governing value for the apparent friction coefficient;
Cu = D_{60}/D_{10}; coefficient of uniformity of the soil;
y = thickness of soil above the level under consideration;
ϕ = angle of internal friction of the soil;
x = horizontal distance from the facing elements to the maximum tension line;
H = height of reinforced earth wall (y_{max} = H);
K = coefficient of lateral earth pressure;
K_o = (1-sin ϕ); coefficient of lateral earth-pressure-at-rest;
FS_y = factor of safety against local tensile failure;
K_a = $\tan^2(45^o - \frac{\phi}{2})$; coefficient of active earth pressure;
T_i = tensile force in the reinforcement at level "i";
σ_v = vertical stress within the reinforced earth;
S_v = vertical spacing of the reinforcement;
S_H = horizontal spacing of the reinforcement;
A_i = total cross-section of reinforcement at level "i";
σ_t = allowable tensile stress for the reinforcement;
FS_ϕ = factor of safety against local bond failure;
B_i = total width of reinforcement at level "i";
L_i = total length of the reinforcement strip at level "i";
ψ = angle of friction between soil and reinforcement measured in a direct shear box;
γ = total unit weight of reinforced fill;
L_i' = effective length of the strip.

3 COHERENT GRAVITY METHOD

Simple empirical formulae have been proposed to show the relationship between some of the factors affecting the design of reinforced earth walls (Schlosser, 1978, Schlosser and Segrestin, 1979). The empirical formulae are based on the analysis of field data related to:

a) variations in the apparent friction coefficient between the soil and the reinforcement strip;
b) the shape and location of the maximum tension (or potential failure) line through the reinforcement;
c) the distribution and intensity of the lateral stress within reinforced soil,

as observed in reinforced earth structures under working stress conditions (Schlosser and Elias, 1978; Schlosser, 1978). These empirical formulae have been adopted to form an integral part of the empirical design procedure (McKittrick, 1978) known as the coherent gravity method.

The coherent gravity method is based on the following assumptions:

a) the reinforced earth structures are in a state of safe equilibrium (i.e., working stress conditions where the factor of safety exceeds 1);

b) the maximum tension line separates the structure into two zones. These are the active zone adjacent to the wall face, and the resistant (or passive) zone representing the remaining part of the structure;

c) only the frictional forces between the soil and the reinforcement which develop within the passive zone act to resist the pull-out forces due to the lateral earth pressure;

d) the vertical stress within the reinforced earth mass is described by the Meyerhof distribution (Schlosser, 1978);

$$\sigma_v = \gamma y/(1-K_a y^2/3L_i^2)$$

e) the apparent friction coefficient, the maximum tension line, and the coefficient of lateral earth pressure are given by the following empirical relationships:

$$f^* = \begin{cases} 0.4 & \text{for smooth strips} \\ f_o^*(1-\frac{y}{6})+\frac{y}{6}\tan\phi & \text{for } y\le 6\text{m; ribbed strips} \\ \tan\phi & \text{for } y\ge 6\text{m; ribbed strips} \end{cases} \qquad (1)$$

$$x = \begin{cases} 0.3H & \text{for } y\le 0.5H \\ 0.6(H-y) & \text{for } y\ge 0.5H \end{cases} \qquad (2)$$

$$K = \begin{cases} K_o+(K_a-K_o)\frac{y}{6} & \text{for } y\le 6\text{m} \\ K_a & \text{for } y\ge 6\text{m} \end{cases} \qquad (3)$$

Using the above assumptions, the governing formulae for design can be derived. From local equilibrium, the tensile force in the reinforcement at level "i" is:

$$T_i = K\sigma_v S_v S_H \qquad (4)$$

and the factors of safety against local failure are:

$$FS_y = \frac{A_i\sigma_t}{T_i} \qquad \text{(tensile failure)} \qquad (5)$$

$$FS_\phi = \frac{2B_iL_i'f^*\sigma_v}{T_i} \qquad \text{(bond failure)} \qquad (6)$$

Equations (4), (5), and (6) can be combined with empirical equations (1), (2), and (3) to give the following empressions governing design:

$$FS_y = \begin{cases} \dfrac{A_i\sigma_t(1-K_a\frac{y^2}{3L_i^2})}{\gamma[K_o+(K_a-K_o)\frac{y}{6}]yS_vS_H} & \text{for } 0\le y\le 6\text{m} \\[2ex] \dfrac{A_i\sigma_t(1-K_a\frac{y^2}{3L_i^2})}{\gamma K_a yS_vS_H} & \text{for } y\ge 6\text{m} \end{cases} \qquad (7)$$

for both smooth and ribbed strips,

$$FS_\phi = \begin{cases} \dfrac{0.8B_i(L_i-0.3H)}{[K_o+(K_a-K_o)\frac{y}{6}]S_vSH} & \text{for } 0\le y\le 0.5H,\ 0\le y\le 6\text{m} \\[2ex] \dfrac{0.8B_i(L_i-0.3H)}{K_aS_vS_H} & \text{for } 0\le y\le 0.5H,\ y\ge 6\text{m} \\[2ex] \dfrac{0.8B_i[L_i-0.6(H-y)]}{q[K_o+(K_a-K_o)\frac{y}{6}]S_vS_H} & \text{for } 0.5H\le y\le H,\ 0\le y\le 6\text{m} \\[2ex] \dfrac{0.8B_i[L_i-0.6(H-y)]}{K_aS_vS_H} & \text{for } 0.5H\le y\le H,\ y\ge 6\text{m} \end{cases} \qquad (8)$$

for smooth strips, and

$$FS\phi = \begin{cases} \dfrac{2B_i(L_i-0.3H)[f_o^*(1-\frac{y}{6})+\frac{y}{6}\tan\phi]}{[K_o+(K_a-K_o)\frac{y}{6}]S_vS_H} & \text{for } 0\le y\le 0.5H,\ 0\le y<6\text{m} \\[2ex] \dfrac{2B_i(L_i-0.3H)\tan\phi}{K_aS_vS_H} & \text{for } 0\le y\le 0.5H,\ 0\ge 6\text{m} \\[2ex] \dfrac{2B_i[L_i-0.6(H-y)][f_o^*(1-\frac{y}{6})+\frac{y}{6}\tan\phi]}{[K_o+(K_a-K_o)\frac{y}{6}]S_vS_H} & \text{for } 0.5H\le y\le H,\ 0\le y\le 6\text{m} \\[2ex] \dfrac{2B_i[L_i-0.6(H-y)]\tan\phi}{[K_o+(K_a-K_o)\frac{y}{6}]S_vS_H} & \text{for } 0.5H\le y\le H,\ y\ge 6\text{m} \end{cases} \qquad (9)$$

for ribbed strips.

Using equations (7), (8), (9) and a design factor of safety, the necessary length and cross-section of reinforcement at any level can be calculated.

4 MODIFIED COHERENT GRAVITY METHOD

Empirical relationships (1), (2) and (3) were further studied by Arenicz and Chowdhury (1986) and, as a result, an alternative set of empirical equations were proposed:

$$f^* = \begin{cases} \tan\psi+0.6^y(1.5-\tan\psi) & \text{for smooth strips} \\ \tan\psi+0.6^y(1.7f_o^*-\tan\psi) & \text{for ribbed strips} \end{cases} \qquad (10)$$

$$x = \sqrt{(2.6H)^2-y^2} - 2.3H \qquad (11)$$

$$K = K_a+0.6^y(K_o-K_a) \qquad (12)$$

The above relationships, together with the assumptions listed in the previous section, yield the governing equations for the modified coherent gravity method:

$$FS_y = \frac{A_i\sigma_t(1-K_a\frac{y^2}{3L_i^2})}{\gamma[K_a+0.6^y(K_o-K_a)]yS_vS_H} \qquad (13)$$

$FS\phi$ =

$$\frac{2B_i[L_i-\sqrt{(2.6)H^2-y^2}+2.3H][\tan\psi+0.6^y(1.5-\tan\psi)]}{\gamma[K_a+0.6^y(K_o-K_a)]S_vS_H} \quad (14)$$

for smooth strips, and

FS_ϕ =

$$\frac{2B_i[L_i-\sqrt{(2.6H)^2-y^2}+2.3H][\tan\phi+0.6^y(1.7f_o-\tan\phi)]}{[K_a+0.6^y(K_o-K_a)]S_vS_H} \quad (15)$$

for ribbed strips.

The above formulae are an alternative set of governing equations for the coherent gravity method. The formulae reflect more closely the observed non-linear behaviour of reinforced earth structures.

5 ANALYSIS AND COMPARISON

In order to compare the original and modified coherent gravity methods, the following steps were taken. It was assumed that the reinforcement strips are of uniform lengths but that their total cross-sectional area might vary at every level (i.e., non-uniform reinforcement). This can be practically achieved by varying the number of strips at each level. Later, it was assumed that the reinforcement was uniform (i.e., strips are of the same length and cross-section and the number of strips at each level is constant.)

In each case, local equilibrium calculations were carried out for both methods for various heights and either smooth or ribbed strips. The following set of typical parameters was assumed for calculation purposes:

f_o^* = 2.5 (based on specific requirements for granular fill; Jones, 1985)
H = 5 m; 10 m; 15 m
FS_y = 3
FS_ϕ = 3
σ = 20 KNm^{-3}
σ_t = 20 MNm^{-2}
S_v = 0.5 m
ϕ = 40^o
ψ = 22^o
B_i = 0.08 m (per 1 m run of wall)

For illustration purposes, similar calculations were conducted using two analytical methods based on working stress state conditions, the energy method (Osman et al, 1979) and the elastic analysis method (Banerjee, 1975).

For each design, the total volume (V) of reinforcement was calculated. The results were plotted against the wall height (H) for the four methods. These are shown in Figure 1. The relative saving in volume of reinforcement resulting from the modifications associated with the coherent gravity method and from the use of non-uniform reinforcement was calculated and illustrated in Figures 2 and 3, respectively.

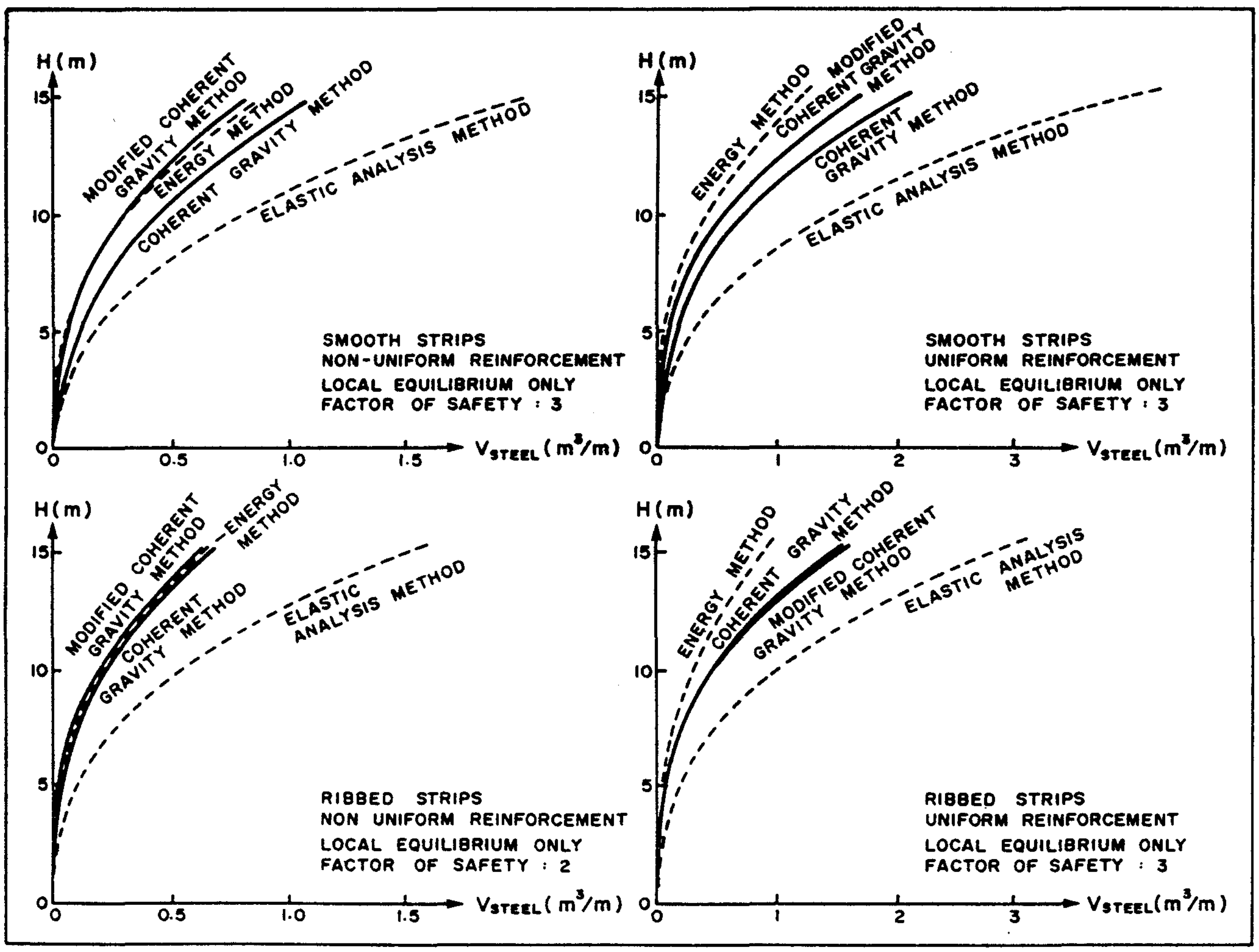

Figure 1 Effect of various design methods on the calculation of reinforcement

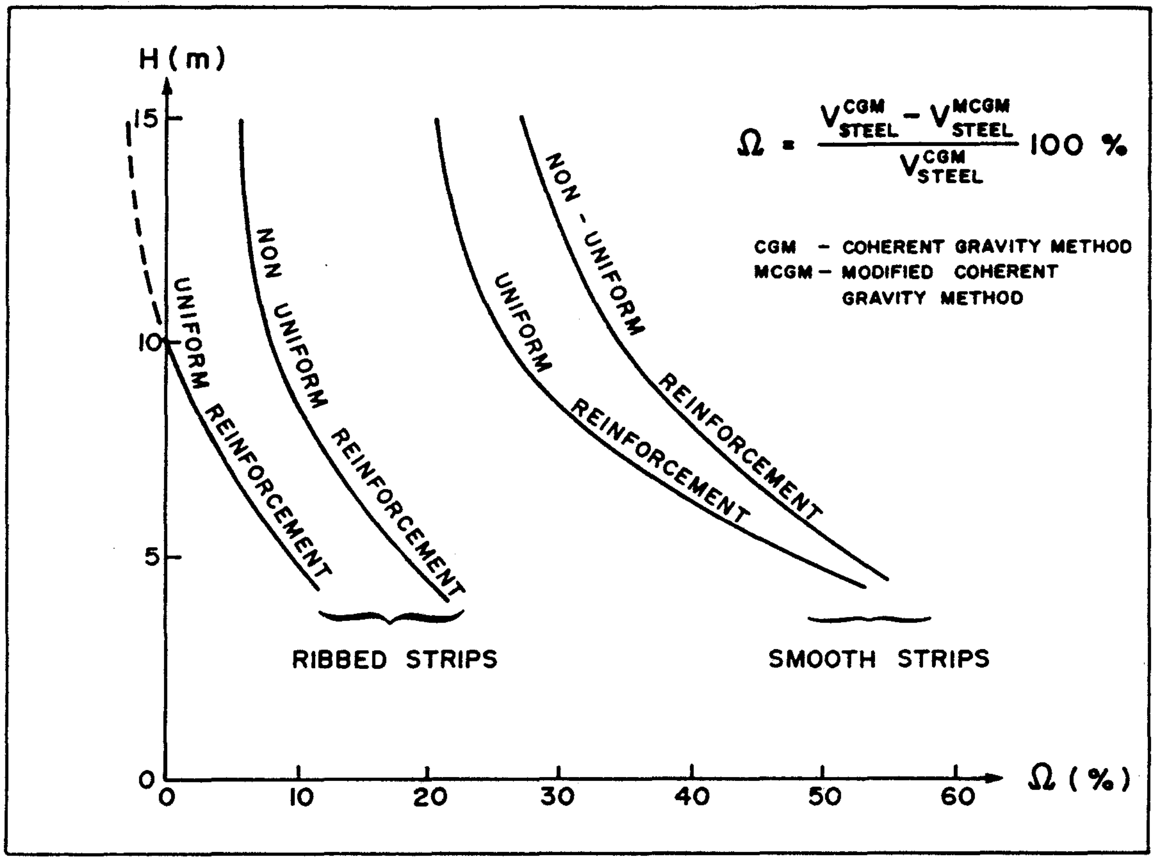

Figure 2 Effect of modifications to the coherent gravity method on the calculation of reinforcement

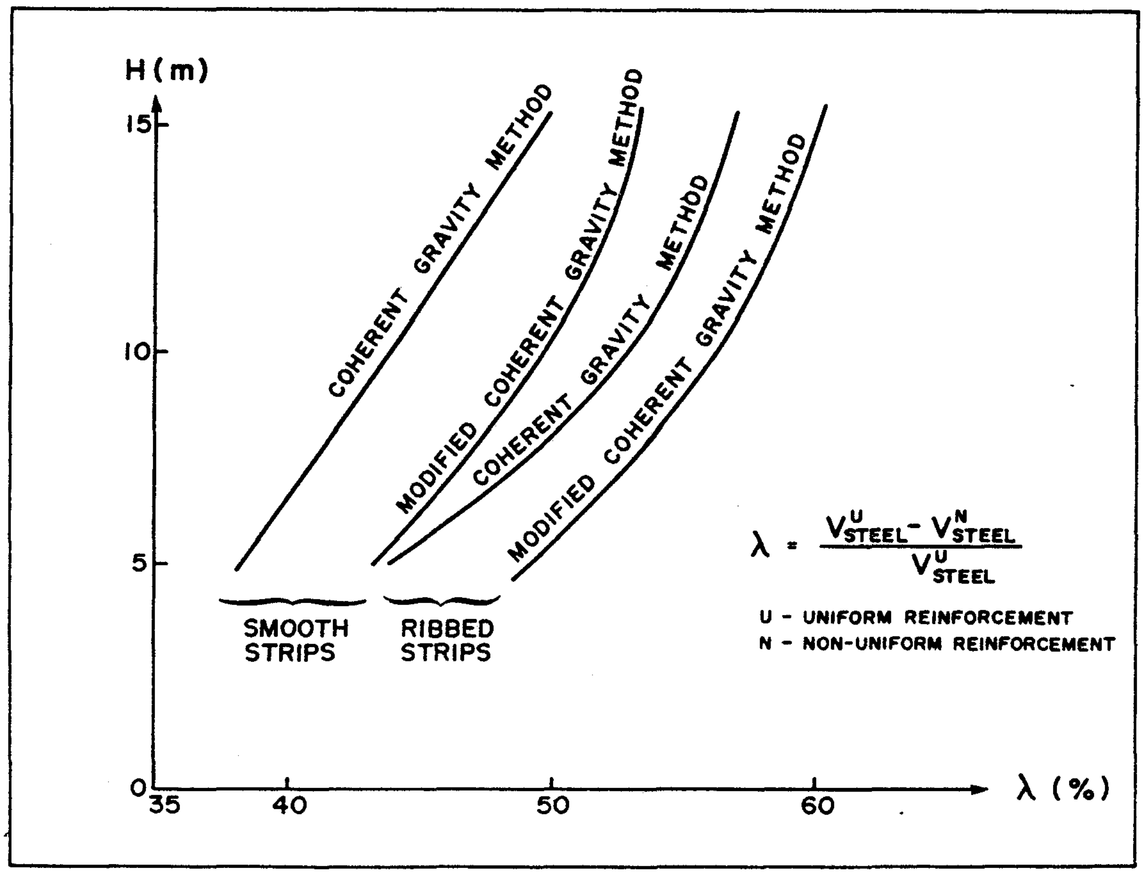

Figure 3 Effect of using non-uniform reinforcement on design economy

6 CONCLUDING REMARKS

The modified coherent gravity hypothesis presented in this paper is based on a set of the empirical relationships formulated by Arenicz and Chowdhury in 1986 which reflect the observed non-linear behaviour of reinforced earth structures. The governing equations for design (Eq's 13, 14, and 15) are in the form of exponential functions with constant parameters. Each of the equations is in a singular form, valid for a significant range of fill depth (y). This is different from governing equations (7), (8), and (9) which have either double or quadruple forms covering up to four regions of y ($0 \leq y \leq 0.5H$; $0.5H \leq y \leq H$; $0 \leq y \leq 6m$; $y \leq 6m$).

The empirical equations of the modified coherent gravity method appear to fit the field data better than those of the original method. The application of the modified method reduces the discrepancy between the local factor of safety within a reinforced earth structure and the factor of safety assumed for purposes of design. This is beneficial from the standpoint of economy in design.

The comparison between the modified and the original coherent gravity methods was shown in Figure 2. It indicates that, for the same factor of safety, a considerable cost saving in reinforcement results from the applicaton of the modified method. Depending upon the wall weight, the required volume of smooth reinforcement is reduced by 20 to 50 % and up to 20% in the case of ribbed reinforcement.

There is close agreement between the design results obtained from the modified coherent gravity method and the energy method, for non-uniform reinforcement. The agreement is also noticeable, although not as close, for uniform reinforcement. This is interesting given that the energy method represents an analytical design attempt entirely different from the modified coherent gravity method.

The results obtained from the application of the elastic analysis method appear to be at odds with those obtained from the other methods. Although all four methods are based on the working stress state, the elastic analysis method consistently leads to volumes of reinforcement at least twice those resulting from the other methods. Given that the coherent gravity method has proven to be a successful and safe method of design, it appears that the elastic analysis method may offer an unnecessary costly conservatism in design. This is particularly true for granular fills.

A considerable saving can result from the design of a non-uniform instead of a uniform reinforcement, with respect to local stability in reinforced earth. Using strips of the same length and cross-section but varying their number at each (or some) level, the required volume of reinforcement can be reduced by 43 - 60% (for $5 \text{ m} \leq H \leq 30 \text{ m}$) as a result of using the modified coherent gravity method. For the other methods considered in the paper, the saving, although smaller, is still substantial: 38 - 57% (coherent gravity method), 45 - 48% (elastic analysis method), and 25 - 30% (energy method).

7 ACKNOWLEDGMENT

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8 REFERENCES

ARENICZ, R.M., CHOWDHURY, R.N., (1986). Empirical formulae in reinforced earth design. Transactions of I.E. Aust (Civil Engineering) - submitted for publication.

BANERJEE, P.K. (1975). Principles of analysis and design of reinforced earth retaining walls. Journal of the Institution of Highway Engineers, Vol. 122 pt 1, pp. 13-18.

JONES, C.J.F.P., (1985). Earth Reinforcement and Soil Structures. London, Butterworths.

MCKITTRICK, D.P. (1978). Reinforced earth: Application of theory and research to practice. Proceedings of the Symposium on Soil Reinforcing and Stabilising Techniques. Sydney, pp. 1-44.

OSMAN, M.A., FINLAY, T.W., SUTHERLAND, H.B. (1979). The internal stability of reinforced earth walls. Proceedings of the International Conference on Soil Reinforcement. Paris, Vol. 1, pp. 107-112.

SCHLOSSER, F. (1978). History, current and future development of reinforced earth. Proceedings of the Symposium on Soil Reinforcing and Stabilising Techniques. Sydney, pp. 5-28.

SCHLOSSER, F., ELIAS, V. (1978). Friction in reinforced earth. Proceedings of the Symposium on Earth Reinforcement, Pittsburgh, pp. 835-763.

SCHLOSSER, F., SEGRESTIN, P. (1979). Local stability analysis method of design of reinforced earth structures. Proceedings of the International Conference on Soil Reinforcement, Paris, Vol. 1, pp. 157-162.