An interslice force function for limit equilibrium slope stability analysis

K. FAN
Chaminade University of Honolulu, Honolulu, HI 96701, U.S.A.
D. G. FREDLUND
Department of Civil Engineering, University of Saskatchewan, Saskatoon, Sask., Canada S7N 0W0
AND
G. W. WILSON
Clifton Associates Ltd., Saskatoon, Sask., Canada S7L 6A8

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The equations of statical equilibrium and the failure criterion for the soil are insufficient to render a slope stability analysis determinate. Generally, the problem is resolved by making assumptions regarding the interslice forces. Calculated safety factors generally do not vary widely as a result of different assumptions. However, difficulties are sometimes associated with the convergence of the nonlinear solution when assumptions deviate excessively from realistic *in situ* stress conditions.

This paper presents the results of an elastic analysis for the computation of interslice normal and shear forces acting within a soil mass. The directions of the resultant interslice forces were computed using a two-dimensional, finite element analysis with constant strain elements. Simpson's method of integration was used to develop a function, $f(x)$, describing the directions of the resultant interslice forces. The function was found to be bell-shaped with the peak occurring approximately at mid-slope (i.e., halfway between the crest and the toe). A generalized empirical function has been proposed. The equation takes the form of an extended error function in which the constants c and n are functions of the slope inclination, and the K-constant is a function of the slope inclination and the depth of the slip surface. Slope analyses using the proposed function show a line of thrust near the one-third point. No problems have been observed with convergence of the solution.

Key words: slope stability, safety factor, method of slices, interslice forces, finite elements, interslice force functions, clay, slopes.

Les équations d'équilibre statique et le critère de rupture du sol sont insuffisants pour rendre l'analyse de stabilité des pentes déterminée. En général, le problème est résolu en faisant des hypothèses quant aux forces inter-tranches. Les coefficients de sécurité calculés ne varient pas beaucoup en fonction des différentes hypothèses. Cependant, des difficultés de convergence des solutions non linéaires sont parfois rencontrées lorsque les hypothèses varient de façon excessive par rapport aux conditions réalistes de contraintes en place.

Cet article présente les résultats d'une analyse élastique pour le calcul des forces normales et de cisaillement entre les tranches agissant à l'intérieur d'un massif de sol. Les directions des forces inter-tranches ont été calculées au moyen d'une analyse bi-dimensionnelle en éléments finis avec éléments à déformation constante. La méthode d'intégration de Simpson a été utilisée pour développer une fonction, $f(x)$, décrivant la direction des forces résultantes inter-tranches. Il a été trouvé que le fonction était en forme de cloche avec un pic localisé approximativement à mi-pente. Une fonction généralisée empirique est proposée. L'équation prend la forme d'une fonction d'erreur dans laquelle les constantes c et n sont fonctions de l'inclinaison de la pente, et la constante K est fonction de l'inclinaison de la pente et de la profondeur de la surface de rupture. Les analyses de stabilité des pentes utilisant la fonction proposée indiquent une ligne de forces près du tiers. Aucun problème de convergence de la solution n'a été observé.

Mots clés : stabilité des pentes, coefficient de sécurité, méthode des tranches, forces inter-tranches, éléments finis, fonction de forces inter-tranches, argile, pentes.

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Introduction

Limit equilibrium methods of slices have been widely used in the analysis of the stability of slopes and embankments. These methods provide an effective means for quantitative analysis; however, all methods must in some way deal with the problem of statical indeterminacy. The solution for the factor of safety requires at least one assumption regarding the interslice forces. Most commonly, an assumption is made regarding the direction, magnitude, or point of application of the interslice forces.

Numerous methods of slices have been proposed based on differing assumptions regarding the interslice forces (Fredlund and Krahn 1977). These assumptions have generally been made on the basis of intuition and experience. Several examples are as follows: Fellenius (1936) set the magnitude of the interslice forces on each slice as zero. Bishop (1955) set the magnitude of the net interslice shear force as zero, thereby making the resultant interslice force horizontal. Janbu (1954), in his simplified method, also set the net interslice shear forces on each slice as zero. Spencer (1967) set the interslice resultant forces to a constant angle and independently solved the moment and force equilibrium factor of safety equations. By selecting different angles for the resultant interslice forces, it was possible to find one angle that yielded the same moment and force factors of safety. The U.S. Army Corps of Engineers assumed that the direction of the interslice force was either (i) parallel to the ground surface or (ii) equal to the average slope from the beginning to the end of the slip surface. Morgenstern and Price (1965) allowed any arbitrarily defined function to be used to describe the direction of the resultant interslice forces.

Morgenstern and Price (1965) suggested that the interslice forces should be related to the shear and normal stresses on vertical slices through a sliding mass. In 1979 Maksimovic used the finite element method and a nonlinear characterization of the soil to compute stresses in a soil mass. These stresses were subsequently used in a limit equilibrium analysis. The effort to evaluate these stresses is substantial and it could be questioned

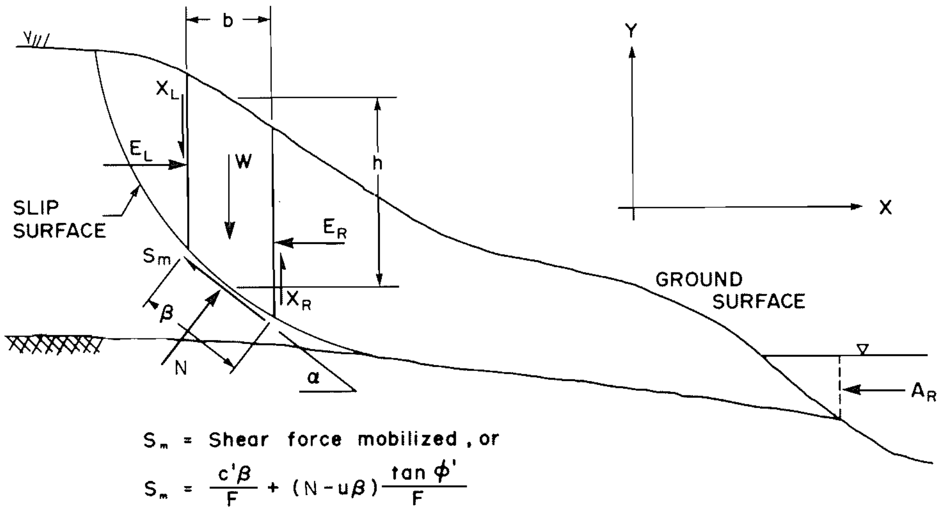


FIG. 1. The forces acting on each slice of a slip surface.

whether the effort is justifiable on a routine basis. On the other hand, it would appear that there is need for an understanding of the general shape of the interslice force function.

This paper uses an elastic theory approach (i.e., the finite element method) to compute the normal and shear stresses along vertical planes through a sliding mass. The stresses are then integrated vertically and the ratio of the shear to normal interslice forces is computed. Subsequently, this ratio is used to propose a generalized function for the direction of the resultant interslice forces, $f(x)$. The objective of the research was to propose an interslice force function that would provide a realistic approximation of the state of stress in the soil mass and thereby minimize convergence problems when computing the factor of safety.

Justification for a generalized interslice force function

Limit equilibrium analyses would be unnecessary if it were possible to perform a rigorous stress analysis on a slope, using truly representative stress–strain relations. Our inability to perform such an analysis, even with the advances in the finite element technique, continues to justify the application of limit equilibrium methods.

The statical indeterminacy associated with a limit equilibrium slope stability analysis arises from a lack of knowledge of the stresses within the soil mass. This lack has resulted in three classes of assumptions being made to render the problem statically determinate. The first class assumes a distribution of the normal stress along the slip surface. Studies have shown that it is superior to place the burden of indeterminacy on the internal forces in the soil mass (Morgenstern and Price 1965). The second class assumes a position for the "line of thrust." Studies (Ching and Fredlund 1983) have illustrated the numerical difficulties that often arise when using this approach. The third class assumes a relation between the interslice shear and normal forces. This approach has proven to be the most successful, primarily owing to the observation that "the factor of safety is relatively insensitive to the distribution of the internal forces" (Bishop 1955). Morgenstern and Price (1965) also demonstrated this insensitivity through use of example problems. Numerical instability problems are less common when this third class of assumptions is used.

The third class of assumptions (i.e., use of an interslice force function) was advocated by Morgenstern and Price (1958). It was suggested that integration of the interslice shear and normal forces acting along vertical planes through the soil mass could provide the forces necessary for an appropriate interslice force function. The function could take any form but it should be controlled by reasonableness based on theory and field observation. They state, "Estimates of the function can be obtained from elastic theory," and also, "more reliable field observations of the internal stresses in dams will also be useful in estimating the distribution of internal forces." In general, reasonableness of the function has largely controlled its choice in practice. The authors also outlined conditions for physical acceptability such as a check on the location of the computed "line of thrust."

Few theoretical stress analyses have been subsequently performed (Maksimovic 1979). One reason is the extensive time and computing efforts required to obtain an "approximation" of a function that, in turn, does not produce much change in the computed factor of safety. Another reason is related to the difficulty in approximating the stress–strain relations for the soil mass, particularly as failure conditions are approached. It has never been shown whether this latter reason is real or perceived.

The following assumptions are made in this study to produce a general, approximate, interslice force function.

(i) Slight variations in the interslice force function have an insignificant affect on the computed factor of safety.

(ii) The local factor of safety along vertical planes within a sliding mass is higher than the factor of safety along the base of the sliding mass.

(iii) The "turning on" of gravity will reasonably reproduce the *in situ* stress state. This means that complex stress histories are not being modelled. Complex geometries involving numerous soils with complex stress–strain relations could be modelled on an individual basis. However, the purpose of this study is to provide the slope stability analyst with a reasonable approximation for an interslice force function, based on a limited number of assumptions.

(iv) A linear stress–strain relation will produce a reasonable approximation of the internal stress state within the sliding mass. This assumes that nonlinear stress–strain relations and nonhomogeneity are of secondary importance. These conditions can be modelled using the finite element technique; however, the function being investigated in this study is based on the assumption that the stress–strain relation is linear and the soil is homogeneous.

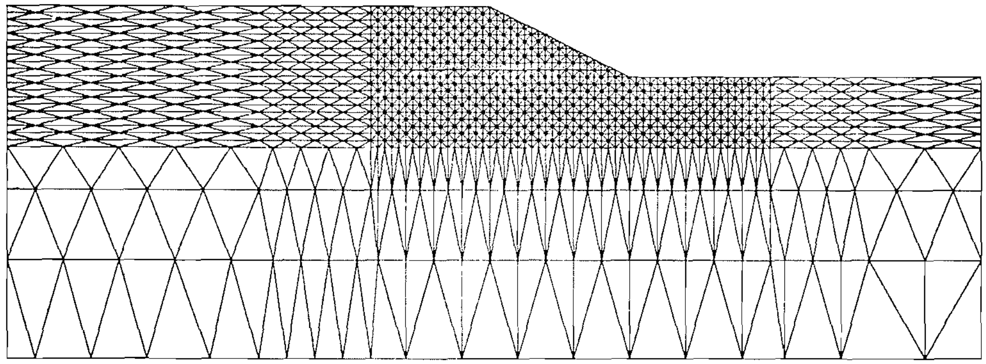


FIG. 2. The finite element mesh used for the 2:1 slope.

(v) Using a total stress finite element analysis will produce reasonable stresses to approximate an interslice force function in terms of total stresses.

Slope stability theory

The slope stability theory is presented within the context of the general limit equilibrium theory, GLE (Fredlund *et al.* 1981). Figure 1 shows the forces on a slice within the sliding mass. The interslice normal forces E_L and E_R act on the left and right sides of the slice, respectively. Correspondingly, the interslice shear forces are X_L and X_R. These forces cancel in any summation equation for the entire sliding mass (e.g., $\Sigma F_H = 0$ or $\Sigma M = 0$).

The normal force N on the base of each slice is obtained by summing forces in the y-direction:

$$[1] \quad N = \frac{W - (X_R - X_L) - \dfrac{c'\beta \sin\alpha}{F} + \dfrac{u\beta \tan\phi' \sin\alpha}{F}}{\cos\alpha + \dfrac{\sin\alpha \tan\phi'}{F}}$$

where W = total weight of a slice of width b and height h, c' = effective cohesion, ϕ' = effective angle of internal friction, u = pore-water pressure at the base of a slice, F = factor of safety, β = length of the slip surface at the base of a slice, and α = angle between the slip surface and the horizontal.

In order to compute the normal force at the base of each slice N, it is necessary to know the interslice shear forces X. The shear forces are assumed to be a function of the interslice normals E in the Morgenstern–Price and GLE methods. The interslice normals are obtained by integrating from one end of the sliding mass to the other. The summation of forces in a horizontal direction for each slice can be written as follows:

$$[2] \quad E_R = E_L + N \sin\alpha + S_m \cos\alpha$$

where S_m = shear force mobilized.

Since the interslice shear forces also appear in [2] (i.e., in the normal force N), it is necessary to first set them to zero and then have them lag in magnitude by one iteration in the solution for the interslice normals. The interslice shear forces are then related to the interslice normal forces by an empirical equation (Morgenstern and Price 1965)

$$[3] \quad X_R = E_R \lambda f(x)$$

where $f(x)$ = the interslice force function and λ = a scaling factor representing the percentage of the function used when solving for the factor of safety.

The interslice force function is a mathematical relationship that gives the relative direction for the resultant interslice forces. The actual direction of the resultant interslice force is obtained by multiplying the function by the scaling factor λ. The interslice force functions have commonly been assumed to take any one of a variety of forms. Some examples are (i) a constant, (ii) a half sine, (iii) a clipped sine, and (iv) a trapezoid. In reality, the behavior of the soil mass imposes certain restrictions on the shape of the function (Ching and Fredlund 1983). The purpose of this study is to provide more definitive guidance regarding the shape of the interslice force function.

Finite element stress analysis

The finite element method is used to compute an approximate directional function for the resultant interslice forces. In this way, the interslice shear forces computed in the limit equilibrium analysis should also be reasonable. The attempt to formulate a single generalized interslice force function is based on the assumption that the primary factors involved are the geometry of the slope and the force due to gravity. It was assumed that differing moduli of elasticity and Poisson's ratio are of secondary relevance and, therefore, do not significantly affect the shape of the function $f(x)$. It was also assumed that it was not necessary to subdivide the interslice normal force into effective and pore-water components.

A two-dimensional, finite element computer program that used constant strain, triangular elements was used to compute the internal stresses in a soil mass (Fredlund 1978). The elements were arranged in a symmetrical manner, which would provide ease in integrating along vertical surfaces. The finite element analysis was performed with the gravity force applied, and the stresses on all elements were computed. Various slip surfaces were then selected and the interslice force function was computed for each slip surface.

The study consisted primarily of linear stress analyses, with a few nonlinear stress analyses. For the linear analysis, a modulus of elasticity of 15 000 kPa was used. A unit weight of 18 kN/m^3 was used and Poisson's ratio values ranged from 0.35 to 0.45. A two-constant, hyperbolic curve (Kondner 1963) was used to represent the nonlinear stress–strain behavior of the soil. This was modelled in the finite element analysis by assigning a unique modulus to each element in accordance with the element stress level (Duncan and Chang 1970). Several iterations were performed to ensure that the soil system was conforming to the hyperbolic constitutive relationship.

The slopes were assumed to be homogeneous, deforming under the influence of gravity. Slope inclinations investigated ranged from gentle (i.e., 3 horizontal:1 vertical) to steep (i.e., 1 horizontal:2 vertical). In addition, vertical slopes were examined. Figure 2 shows the finite element mesh for a 2 horizontal:1

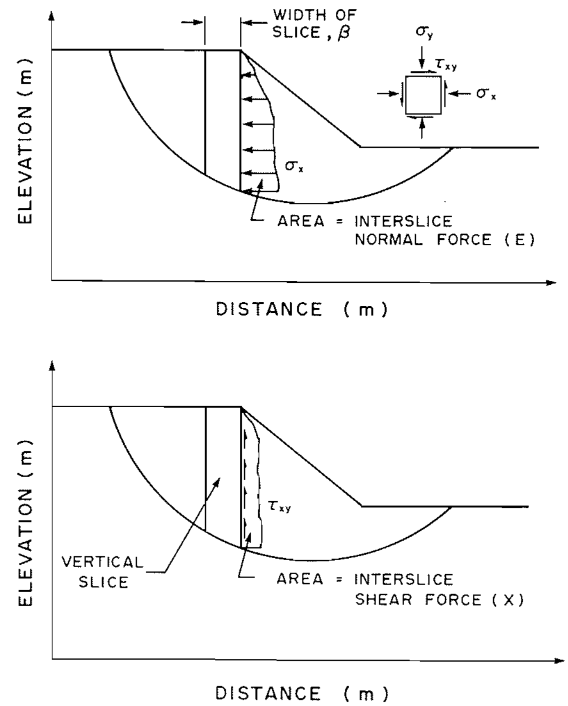


FIG. 3. The normal and shear forces acting on the right side of a slice.

vertical slope. The slope was divided into vertical slices, which were subsequently divided into symmetrical triangular elements. The mesh of elements was set up in this manner in order to provide ease in integrating the stresses at the finite element nodes along vertical planes. Potential sliding masses were defined mainly by circular slip surfaces; however, some composite or noncircular slip surfaces were also examined.

The following boundary conditions were applied. First, the lower left corner was fixed at zero displacement in the x- and y-directions. Second, the vertical side boundaries were assumed to undergo zero displacement in the x-direction and have zero force in the y-direction. Third, the horizontal base boundary was assumed to have zero force in the x-direction and zero displacement in the y-direction,

The forces of interest are the interslice normal and shear forces (i.e., E and X). The interslice forces were computed using Simpson's method of integration for the horizontal and shear stresses over the vertical sides of each slice (Fig. 3).

Presentation of results

The interslice force functions $f(x)$ were computed for four slope inclinations as well as for a vertical slope (Wilson 1982; Fan 1983). A wide range of circular slip surfaces were investigated for each slope geometry by varying the center of rotation and the radius. In total, several hundred slip surfaces were investigated. All of the slip surfaces analyzed showed a similar functional relationship between the interslice normal and interslice shear forces. Figures 4–7 illustrate typical interslice force functions for one slip surface from each of the four slope geometries. The side force function for a slip surface passing through the toe of a vertical slope is shown in Fig. 8. All the above analyses were performed assuming a linear elastic behavior for the soil.

Nonlinear finite element analyses were performed for two slope inclinations (i.e., 2:1 slope and 1:2 slope). The functional relationship between the interslice shear and interslice normal forces is similar to those obtained when using a single modulus value.

The side force function for a typical composite-shaped slip surface plotted in Fig. 9. In this case the slope was analyzed as a homogeneous mass but a lower limit of integration equal to the base of the composite slip surface was imposed.

Discussion and analysis

The interslice force functions for the four slope geometries investigated are bell-shaped. The bells are flattest for the gentle slopes (i.e., 3:1 slopes) and sharpest for the steep slopes (i.e., 1:2 slopes). The magnitude of the interslice force function is relatively constant between the crest and toe for both the 3:1 and 2:1 slopes. Sharper peaks occur at mid-slope for the 1:1 and 1:2 slopes. These peaks are sharpest when the slip surface is shallow-seated.

The interslice force functions show points of inflection located slightly beyond the crest and the toe of the slope. The maximum values for the side force ratio are observed to occur at mid-slope, halfway between the crest and the toe. Furthermore, the magnitude of the peak side force ratio approaches the tangent of the slope inclination for shallow slip surfaces.

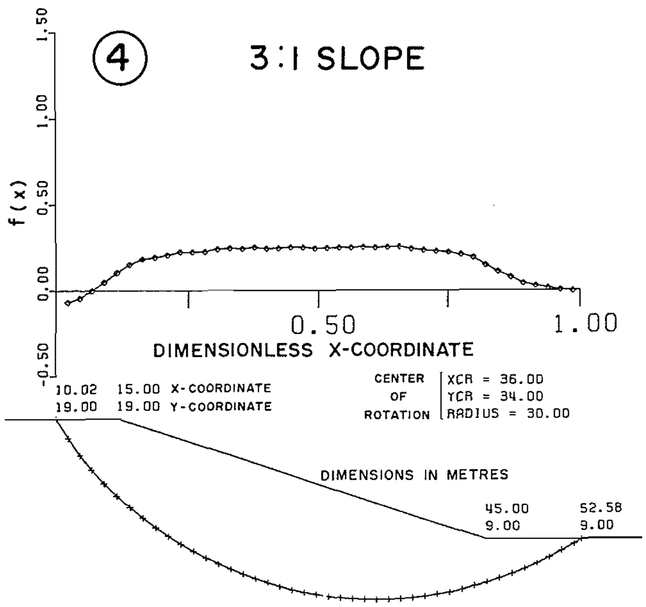
The interslice force function for a deep-seated slip surface in a vertical slope is also bell-shaped with a pronounced peak at the edge of the slope. For slip surfaces passing through the toe, the failure mode resembles that of an active earth pressure analysis. As shown in Fig. 8, the interslice force function increases steadily, reaching a peak just back from the edge of the slope and then dropping rapidly to zero. Numerical instability in the finite element method and the related numerical integration for the interslice forces did not allow the computed function to be zero at the face of the slope.

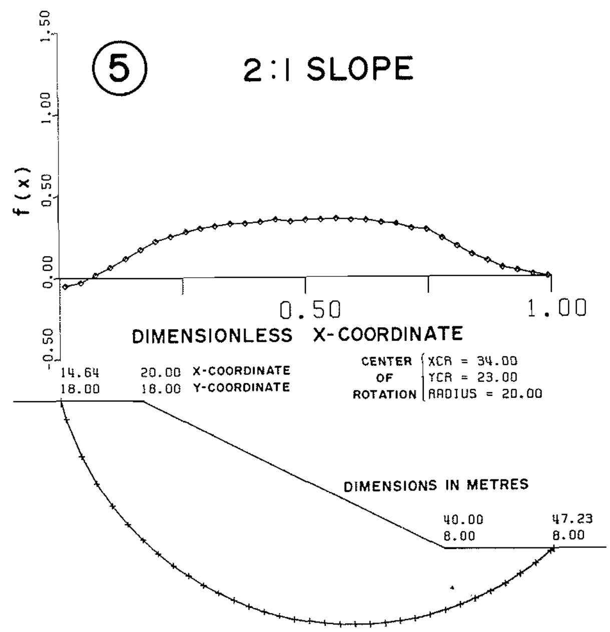
The differences between the interslice force functions produced by a linear and a nonlinear stress analysis were minor. Varying the modulus values according to stress level has little effect on the interslice force function provided the stress levels do not exceed the strength of the soil on the vertical interfaces. In general, the use of a nonlinear constitutive relation leads to a slight lowering and lateral spreading of the interslice force function. It should be noted that the factor of safety used in some examples was quite high (i.e., greater than 3). The use of lower factors of safety slightly influences the magnitude of the function but its characteristic shape is retained. Additional studies could be performed to assess the influence of highly nonlinear stress–strain relations.

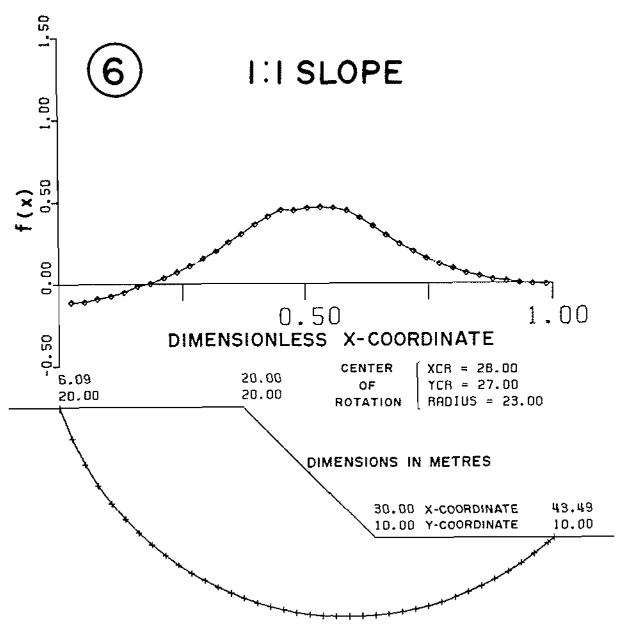
In the case of a composite or noncircular slip surface (Fig. 9), the depth of integration is reduced for those slices intercepting the impenetrable layer. This results in a larger reduction in the interslice normal forces than in the interslice shear forces for slices in this region. Consequently, the peaks of the interslice force function are more pronounced and higher in magnitude for composite slip surfaces. However, the general shape of the interslice force function is retained. It is difficult to do a detailed, general study on composite slip surfaces owing to the large number of shapes that could be assumed.

Formulation of the generalized function

Based on the examination of several hundred interslice force functions obtained from the linear stress analysis, a generalized function is proposed. The function is empirical and takes the







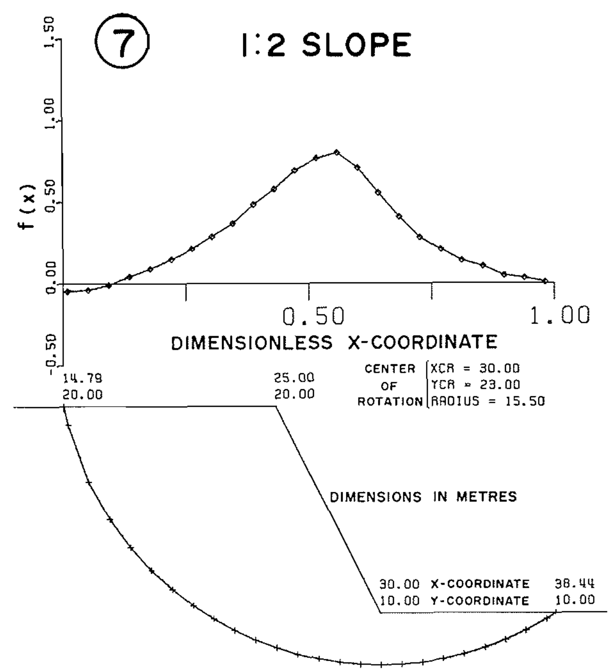


FIG. 4. The interslice force function for a deep-seated slip surface through a 3:1 slope (linear elastic).
FIG. 5. The interslice force function for a deep-seated slip surface through a 2:1 slope (linear elastic).
FIG. 6. The interslice force function for a deep-seated slip surface through a 1:1 slope (linear elastic).
FIG. 7. The interslice force function for a deep-seated slip surface through a 1:2 slope (linear elastic).

form of an extended error function:

[4] $f(x) = Ke^{(-c^n\omega^n)/2}$

where K = the magnitude of the interslice force function at mid-slope (i.e., maximum value), c = a variable defining the inflection points near the crest and toe of a simple slope, n = a variable specifying the flatness or sharpness of curvature of the function, and ω = the dimensionless position relative to the midpoint of each slope.

The dimensionless position ω is measured from the midpoint of the slope in either direction (Fig. 10). The points of inflection of the function are slightly outside the scarp and toe of the slope.

The effect of varying n and c is shown in Figs. 11 and 12, respectively. The K-variable is set at 0.5 for both plots. In Fig. 11, the inflection points are fixed while the shape in the proximity of the midpoint is varied by changing the n-variable. In Fig. 12, the n-variable is set at 5 and the variation in c shows how the function can be extended in a horizontal direction.

The maximum value of the interslice force function K varies with the slope inclination and the depth factor D_f for the slip

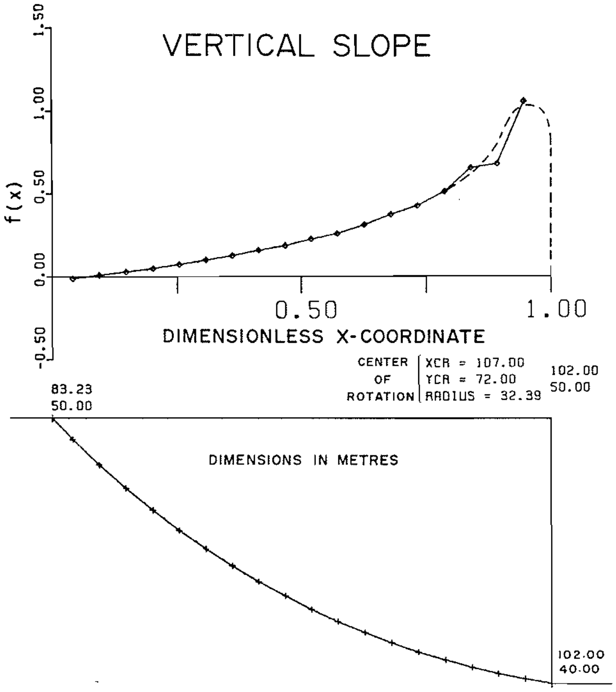


FIG. 8. The interslice force function for a slip surface passing through the toe of a vertical slope (linear elastic).

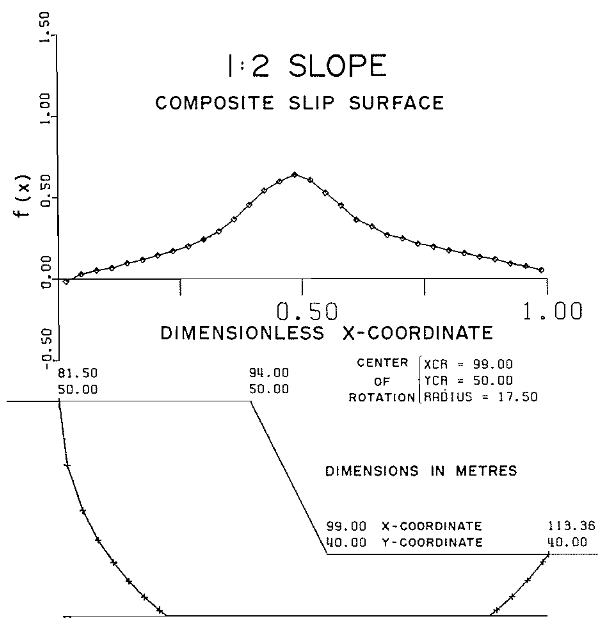


FIG. 9. The interslice force function for a deep-seated, composite slip surface through a 1:2 slope.

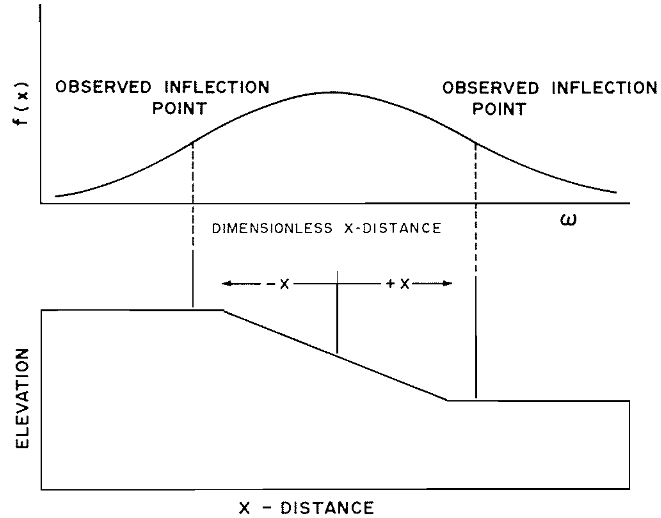


FIG. 10. The definition of the dimensionless distance ω for a slope.

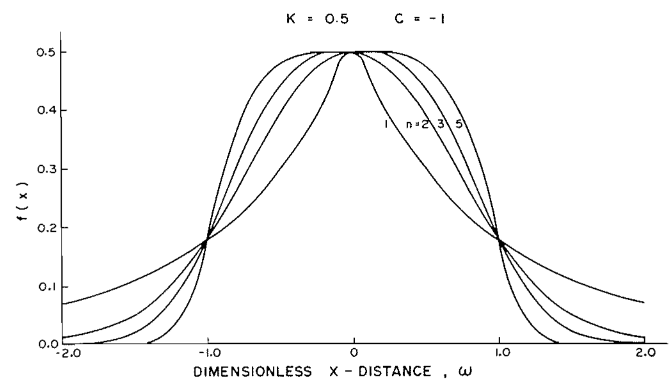


FIG. 11. Changes in the shape of the proposed interslice force function when n is varied.

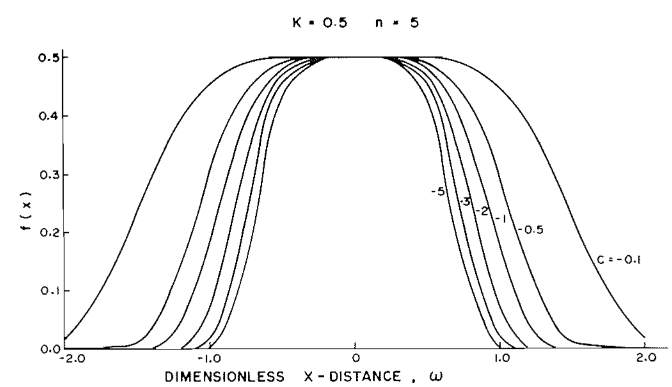


FIG. 12. Changes in the shape of the proposed interslice force function when c is varied.

surface (Fig. 13). The logarithm of K plots as a linear function of the depth factor and can be defined by the following equation:

$$[5] \quad K = e^{(D_i + D_s(D_f - 1.0))}$$

where D_f = depth factor, D_i = the natural logarithm of the y-intercept for $D_f = 1.0$, and D_s = the slope of the K versus depth factor relationship.

The values of D_i and D_s for various slope inclinations are shown in Figs. 14 and 15, respectively. The values of K for the case of a vertical slope are shown in Fig. 16.

Figure 17 shows that K varies with Poisson's ratio. However, the variation is not serious since the K-variable will be adjusted by the λ-factor during a slope stability analysis.

The magnitudes of c and n are functions of the slope inclination and are shown in Fig. 18. The variable n was obtained through a trial-and-error process. The inflection points, variable c, were obtained by setting the second derivative of [4] to zero. The results listed in Table 1.

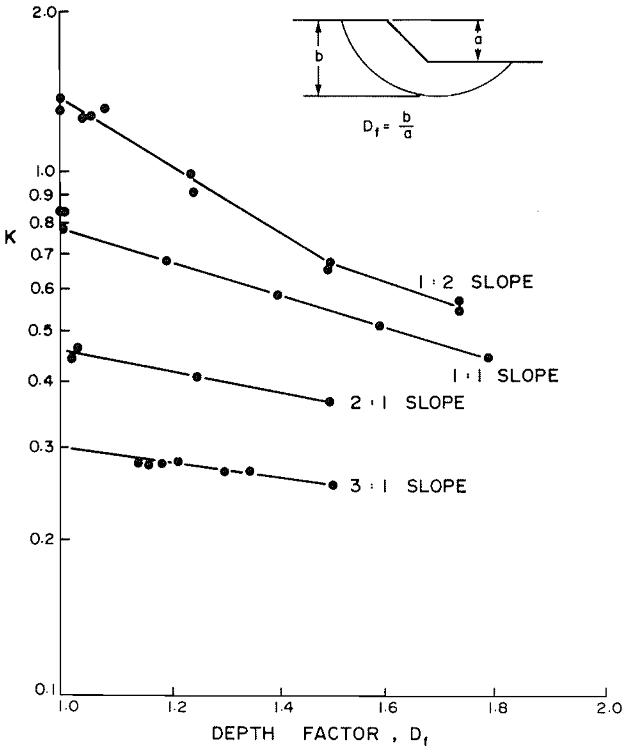


FIG. 13. The value of K for various slope inclinations and depth factors.

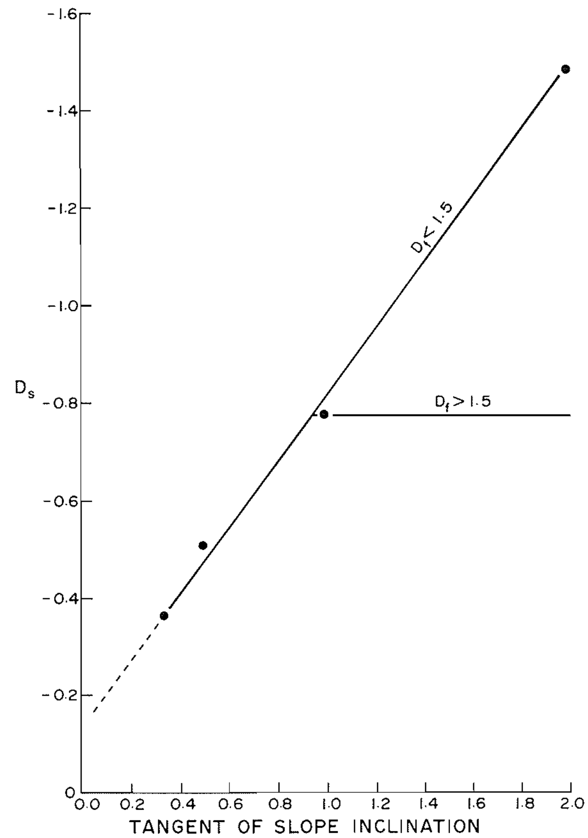


FIG. 15. The value of D_s for various depth factors and slope inclinations.

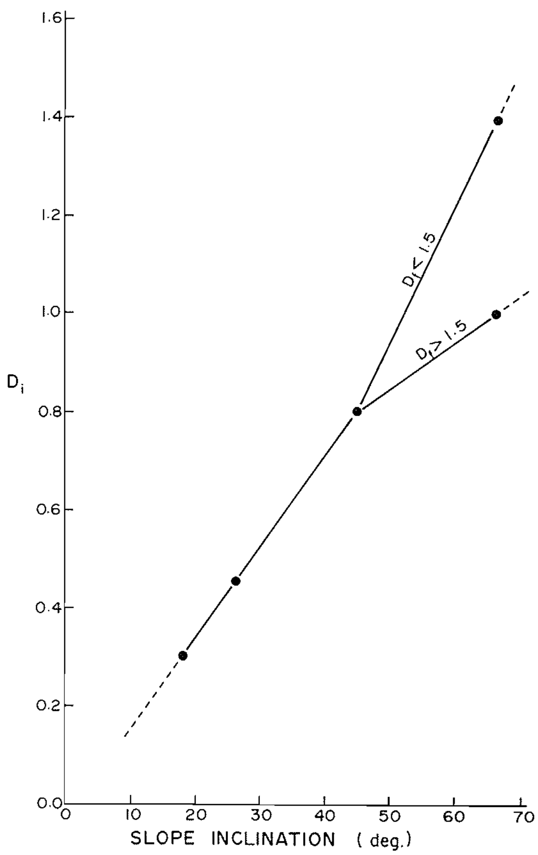


FIG. 14. The value of D_i for various depth factors and slope inclinations.

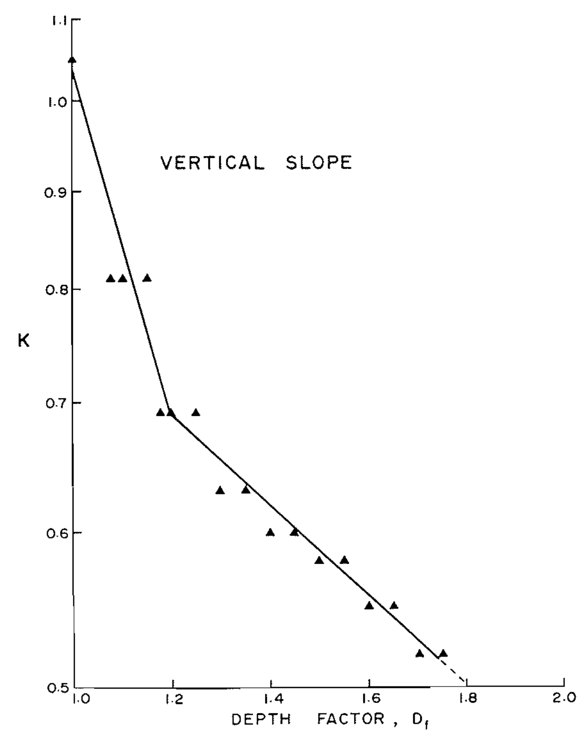


FIG. 16. The value of K for various depth factors for a vertical slope.

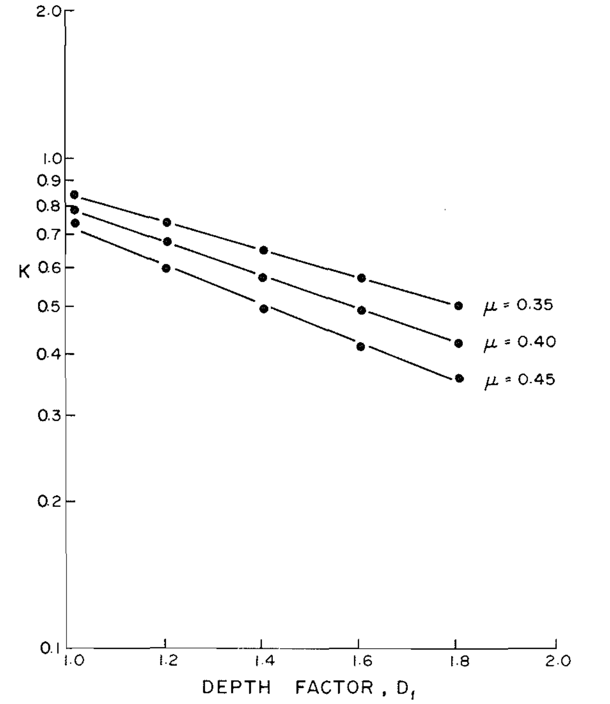


FIG. 17. The variation in K-value for changing Poisson's ratio.

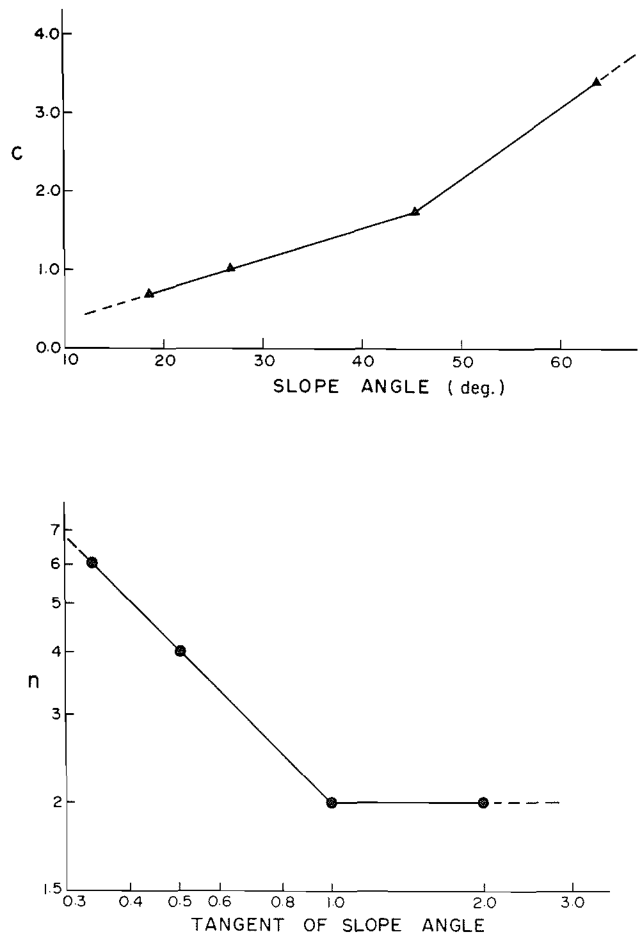


FIG. 18. The magnitude of c and n for various slope inclinations.

TABLE 1. Values of D_i, D_s, c, and n for various slope inclinations

Slope inclination	D_i	D_s	$-c^n/2$	n
3:1	−1.204	−0.365	−0.052	6
2:1	−0.755	−0.512	−0.583	4
1:1	−0.223	−0.776	−1.509	2
1:2 ($D_f < 1.5$)	0.336	−1.484	−5.664	2
1:2 ($D_f > 1.5$)	0.00	−0.776	−2.000	2
Vertical (deep seated)	*	*	−2.000	1.3
Vertical (toe circle)	*	*	−1.650	1.65

*Not applicable.

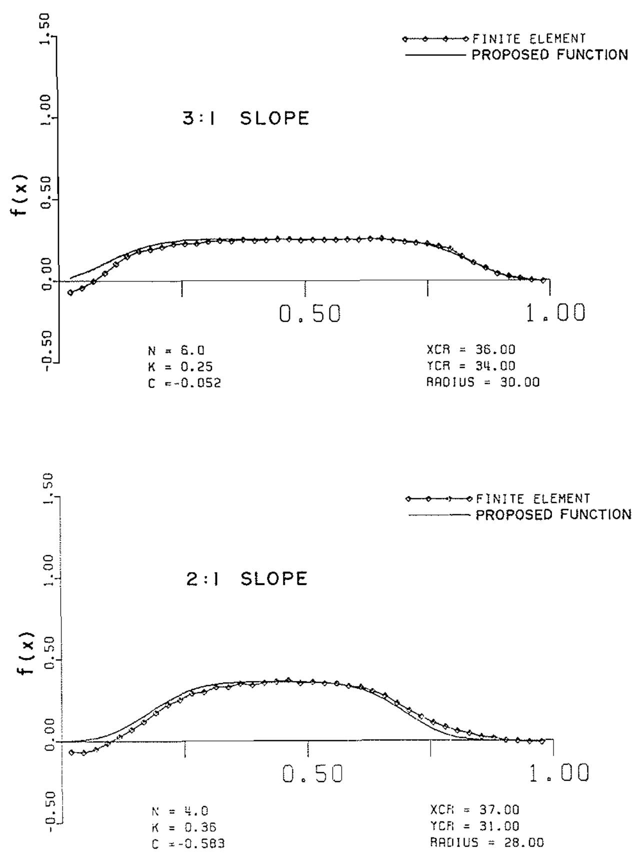


FIG. 19. Comparison of the proposed function to the finite element results (3:1 slope and a 2:1 slope).

Evaluation of the proposed function

The proposed function was evaluated in two ways. First, [4] was programmed on the computer to allow a check against the finite element results. This allowed a visual comparison of several hundred finite element analyses. Second, [4] was added to the computer code for SLOPE-II (Fredlund 1984). In this way, the proposed function could be evaluated with respect to its use in solving for the factor of safety.

Typical results from the first evaluation are shown in Figs. 19 and 20. The proposed function agrees closely with the results obtained from the finite element analysis. A slight discrepancy is observed near the crest of the slope where the interslice force function computed by the finite element analysis becomes

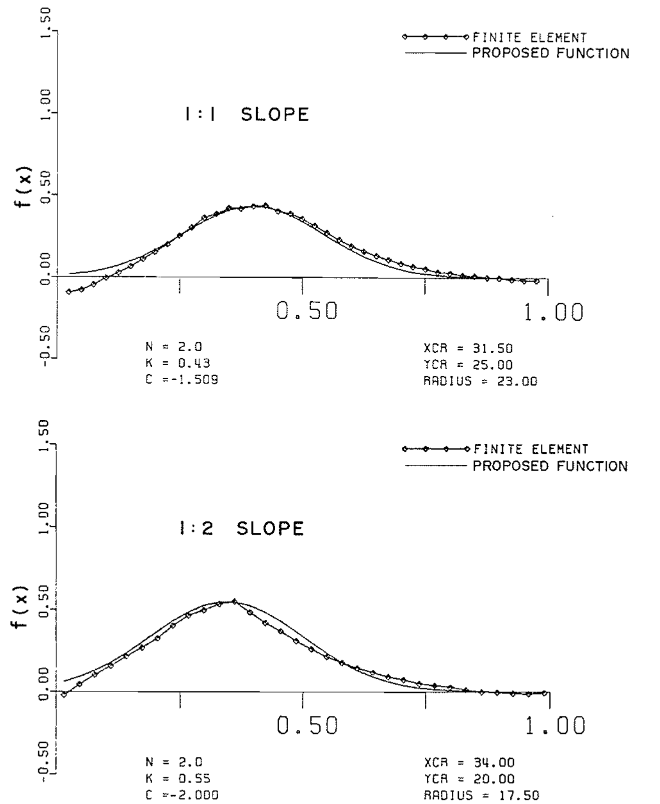


FIG. 20. Comparison of the proposed function to the finite element results (1:1 slope and a 1:2 slope).

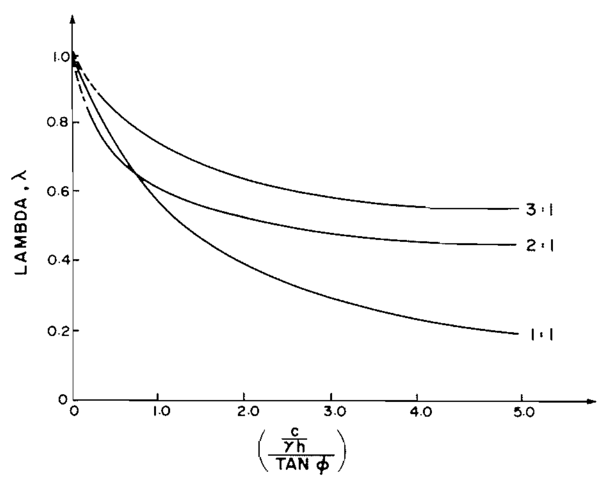


FIG. 21. The λ-value required to satisfy moment and force equilibrium using the proposed function in a slope stability analysis.

slightly negative. This is attributed to tensile forces originating from the imposed lateral boundary conditions. The proposed function values are considered to be more correct in this region.

When the proposed function is used in conjunction with the GLE method in SLOPE-II, λ-values of approximately unity are obtained for cohesionless soils (Fig. 21). As cohesion increases, the scaling factor λ decreases. In other words, the percentage of the proposed interslice force function required to bring the moment and force factors of safety to a single value decreases. The most significant drop in λ occurs with increasing cohesion for steep slopes. The proposed interslice force function yields reasonable values even when λ drops below unity.

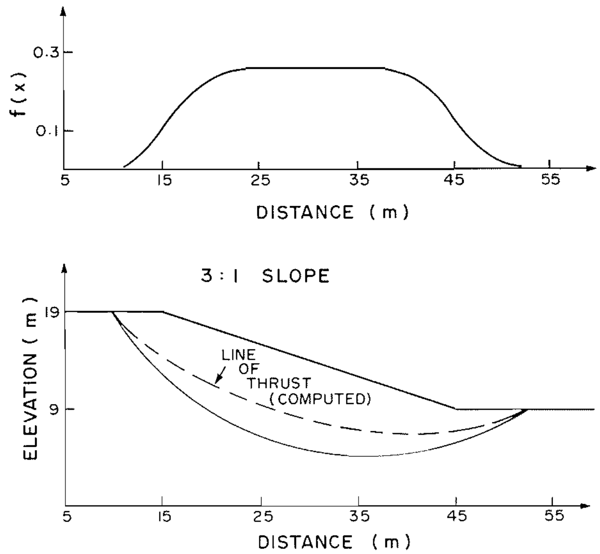


FIG. 22. The computed "line of thrust" for a 3:1 slope using the proposed interslice force function.

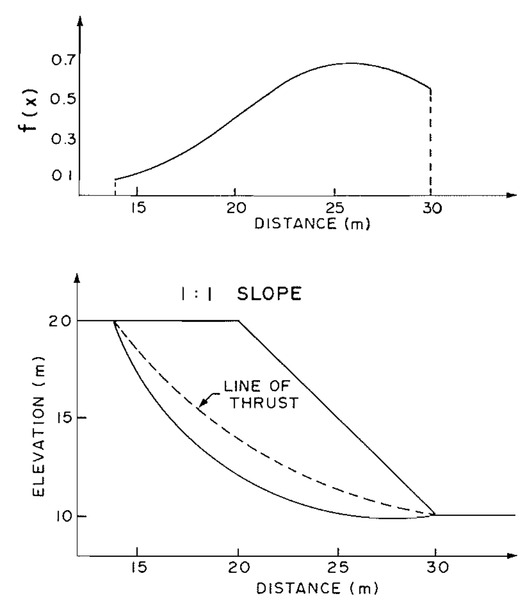


FIG. 23. The computed "line of thrust" for a toe circle through a 1:1 slope using the proposed interslice force function.

The computed "lines of thrust" for two slip surfaces are plotted in Figs. 22 and 23. The "lines of thrust" fall near the lower one-third point on the sides of the slices, as would be expected from earth pressure theory. On the basis of the examples checked, it would appear that one of the requirements for a "physically acceptable" solution outlined by Morgenstern and Price (1965) has been met (i.e., the line of thrust must fall within the potential sliding mass). In addition, no difficulties have been observed in obtaining convergence of either the moment or force equilibrium equations.

Summary

The interslice force function for simple homogeneous slopes with circular slip surfaces are bell-shaped. The functions are flattest for the 3:1 slope and sharpest for the 1:2 slopes. Inflection points are slightly beyond the crest and the toe of the slope. The maximum interslice force function occurs at mid-slope, halfway between the crest and the toe. For a vertical slope, the interslice force function retains the characteristic bell shape with the peak near the edge of the slope.

The differences between the interslice force functions for a linear and a nonlinear stress analysis appear to be insignificant provided the stresses do not produce a local failure condition on the interfaces between vertical slices.

A composite slip surface results in a more pronounced peak in the interslice force function. However, the general shape of the interslice force function is retained.

The developed computer software could be used to compute the interslice force function for a slope having several soil types and any shape of slip surface. The proposed approximate function should generally be adequate for slope stability analyses.

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