THE PREDICTION OF PORE-PRESSURES FOR SLOPE STABILITY ANALYSES

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by

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PREDICTION OF PORE-WATER PRESSURES FOR SLOPE STABILITY ANALYSES

An independent analysis must be performed in order to obtain the pore-water pressures for a slope stability analysis. These pore-water pressure conditions may results from the effects of construction or they may result from the natural groundwater condition. These two separate conditions may be classified as pore pressure conditions which are a function of total stress, or pore-water conditions which are independent of the applied total stresses and only a function of the natural seepage processes within the soil mass.

1.0 Pore Pressures as a function of Total Stress - End-of-Construction Case

The pore-water pressures in an embankment during and immediately following completion of construction are a function of the applied overburden pressures. The soils are normally placed (i.e., compacted) in an unsaturated state. It is difficult to assess the pore-water pressures for this case.

Procedures that have been used to predict pore-water pressures for the end-of-construction case including the following:

i) the pore pressures coefficient, r_u, procedure

ii) the pore pressure parameter procedure, and

iii) Hilf's procedure

1.1 Pore Pressure Coefficient, r_u

The pore pressure coefficient is, by definition, the ratio of the pore-water pressure to the overburden pressure.

$$r_u = \frac{u_w}{\sum (\gamma_t)_i \, h_i} \qquad [1]$$

where γ_t = total unit weight

h_i = thickness of each layer of overlying soil

The equation is solved for the pore-water pressure.

$$u_w = r_u \sum (\gamma_t)_i \, h_i \qquad [2]$$

There is no theory available to predict the pore pressure coefficient. Rather the value for the pore pressure coefficient is assumed, based on experiments. Design values generally range from 0.3 to 0.45. Experience has shown that problems with instability generally occur when the pore pressure coefficient exceeds approximately 0.35.

Let us suppose that a pore pressure coefficient value of 0.35 is used in design. During construction, piezometers are installed in the embankment to measure the pore pressure. As long as the measured pore pressures are such as to yield an average pore pressure coefficient less than 0.35, no problems with instability would be anticipated. If higher pore pressure coefficients occur, it may be necessary to modify the design or the construction schedule.

1.2 Pore Pressure Parameters

The $\bar{A}$ and B pore pressure parameters can be used to compute design pore water pressures for the end-of-construction case. The A and B pore pressure parameters are determined on the basis of consolidated undrained triaxial tests with pore-water pressure measurements. The actual B pore pressure parameter may range from less than 0.5 up to 1.0. Since its value is also a function of the total stress applied, a relatively high value (i.e., approaching 1) is often used in design. The $\bar{A}$ pore pressure parameter used should be based on the laboratory test results. The theory of elasticity (corresponding to low strains), would indicate an $\bar{A}$ value of approximately 1/3.

The analytical procedure is as follows. The pore-water pressure at any time, u_w, is equal to an initial value, u_{wo}, plus the change in pore-water pressure, Δu_w,

$$u_w = u_{wo} + \Delta u_w \tag{3}$$

The initial pore-water pressure, u_{wo}, is negative. However, often its magnitude is taken as zero. This tends to over-estimate the pore-water pressure.

The change in pore-water pressure can be written:

$$\Delta u_w = B\Delta\sigma_3 + \bar{A}\ (\Delta\sigma_1 - \Delta\sigma_3) \tag{4}$$

where σ_3 = minor total principal stress

σ_1 = major total principal stress

The changes in minor and major principal stresses can be predicted through the use of a finite element stress analysis. The changes in principal stresses are due to 'turning on' the gravity force. This can be done in an incremental manner if the elastic parameters are nonlinear or as one step for constant linear elastic parameters.

If a finite element program is not available it is still possible to estimate the pore-water pressures. Equation [4] can be arranged into a different form. Let us first define A as equal to $\bar{A}/B$. Then substract and add $\Delta\sigma_1$ from equation [4].

$$\Delta u_w = B\,[\Delta\sigma_1 - \Delta\sigma_1 + \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \qquad [5]$$

Rearranging and collecting terms, gives

$$\Delta u_w = B\,[\Delta\sigma_1 - (1 - A)\ (\Delta\sigma_1 - \Delta\sigma_3)] \qquad [6]$$

Divide both sides of the equation by $\Delta\sigma_1$.

$$\frac{\Delta u_w}{\Delta\sigma_1} = B\,[1 - (1 - A)\ (1 - \Delta\sigma_3/\Delta\sigma_1)] \qquad [7]$$

. Let us define $\bar{B}$ as equal to $\Delta u_w/\Delta\sigma_1$. Therefore,

$$\bar{B} = B\,[1 - (1 - A)\ (1 - {}^{\Delta\sigma_3}/\Delta\sigma_1)] \qquad [8]$$

Substituting equation [8] into equation [3], gives

$$u_w = u_{wo} + \bar{B}\ \Delta\sigma_1 \qquad [9]$$

It should be noted that, if u_{wo}, is negligible and $\Delta\sigma_1$ is equal to the overburden pressure, (i.e., $\gamma_t h$), then equation [9] takes on the same form as the pore pressure coefficient equation [1]. However, in this case there is a theoretical basis for estimating a $\bar{B}$ value. Equation [8] can be rearranged and written in terms of the effective coefficient of earth pressure at-rest (i.e., $K = \Delta\sigma_3'/\Delta\sigma_1'$). Equation [8] then takes on the form,

$$\bar{B} = \left[\frac{1 - (1 - A)\ (1 - K)}{1 - B\ (1 - A)\ (1 - K)} \right] B \qquad [10]$$

For a frictional soil on a relatively flat slope, the K coefficient ranges between 0.4 and 0.6. For a frictional soil on a steep slope, the K coefficient ranges between 0.25 and 0.3. An average value of 0.4 is appropriate as a first approximation in equation [10].

1.3 Hilf's Method (or USBR method)

Extensive use has not been made of this method in practice; however, it does appear to best model the actual behavior of compacted soils. The soil is modeled as a laterally confined material, loaded in an undrained manner (Figure 1). This analytical modeling is done through the use of Boyles' and Henry's Laws and the results of conventional one-dimensional oedometer tests.

Boyles' and Henry's Laws are first used to derive a relationship between pore pressure and volume change. This is done by relating the pore-air pressures associated with the initial and final loading conditions.

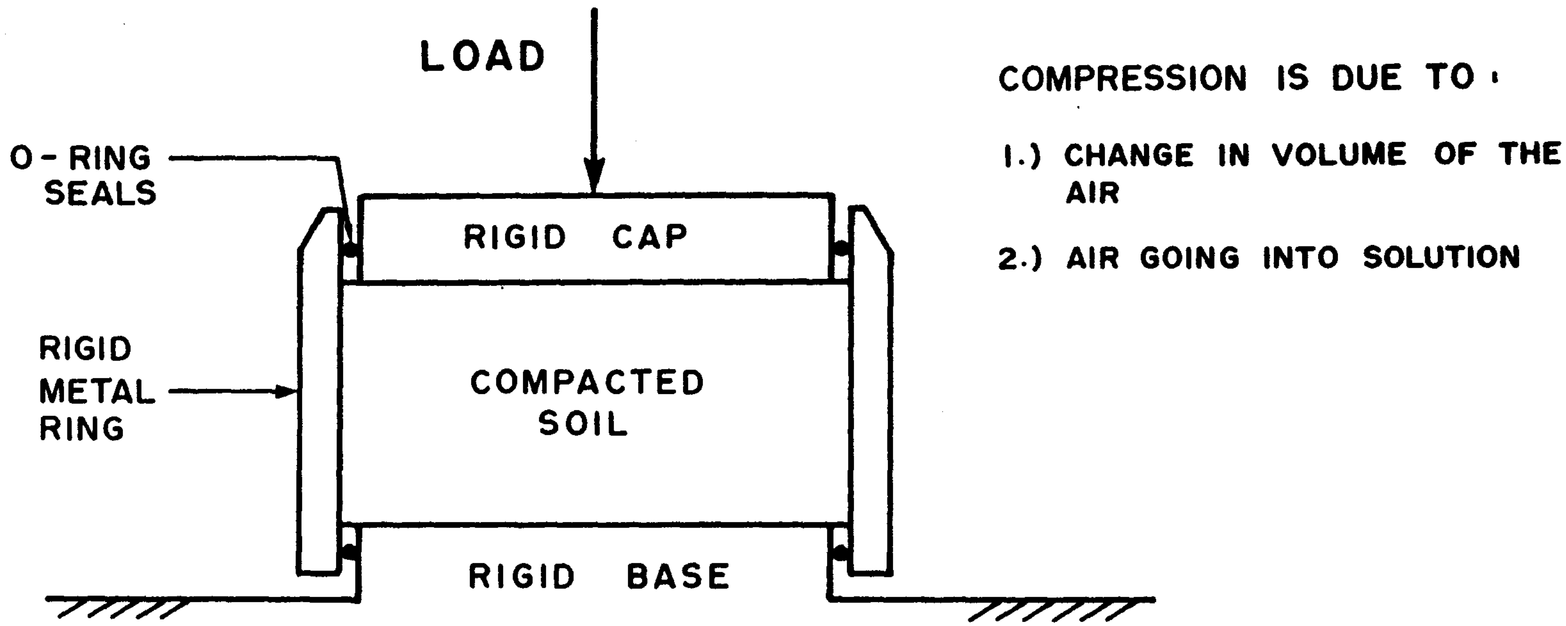


Figure 1 Condition Simulated through Hilf's Analysis

For <u>Initial Conditions,</u> the volume of air, V_{ao} can be written,

$$V_{ao} = (1 - S)\,n + H\,S\,n \qquad [11]$$

where
- S = initial degree of saturation
- n = porosity
- H = Henry's coefficient of solubility by volume. (The value is approximately 0.02).

The $[(1 - S)n]$ term is called the free air volume. The $H\,S\,n$ term is called the dissolved air volume. The initial pore-air pressure, u_{ao}, can be taken as atmospheric conditions (i.e., one atmosphere absolute).

For final conditions, the volume of air, V_f can be written,

$$V_f = (1 - S)n + H\,S\,n - \Delta n \qquad [12]$$

The final pore-air pressure, u_{af}, can be written as the initial pressure plus the change in pore-air pressure (i.e., $u_{ao} + \Delta u_a$).

Boyles' law can be applied to the initial and final conditions of the free and dissolved air.

$$u_{ao}\,V_{ao} = (u_{ao} + \Delta u_a)\,V_f \qquad [13]$$

Substituting the initial and final conditions into equation [13] gives,

$$u_{ao}\,[(1 - S)n + H\,S\,n] = [u_{ao} + \Delta u_a]\,[(1 - S)n + H\,S\,n - \Delta n] \qquad [14]$$

Equation [14] can be solved for the pore-air pressure.

$$\Delta u_a = \frac{u_{ao}\,\Delta n}{(1 - S)n + H\,S\,n - \Delta n} \qquad [15]$$

Equation [15] provides a relationship between the change in pore-air pressure and the change in volume (i.e., Δn). The assumption is now made that a change in pore-air pressure is equal to a change in pore-water pressure (i.e., $\Delta u_w = \Delta u_a$).

The second step is to present the data from a conventional consolidation test (i.e., constant volume oedometer test) as an effective stress versus volume change plot. The data is normally plotted in the form of void ratio versus logarithm of effective stress. Changes in void ratio, e, can be written as a change in porosity, Δn, and plotted as shown in Figure 2.

$$\Delta n = \frac{\Delta e}{1 + e} \qquad [16]$$

Equation [15] and the consolidation test data can now be plotted on a graph of stress versus volume change (Figure 3). The effective stress equation states that the total stress is equal to the effective stress plus the pore-water pressure. Therefore, the effective stress and pore-water pressure can be added together (for various volume changes), to provide a plot of total stress versus volume change. This relationship is also shown on Figure 3. Finally, it is possible

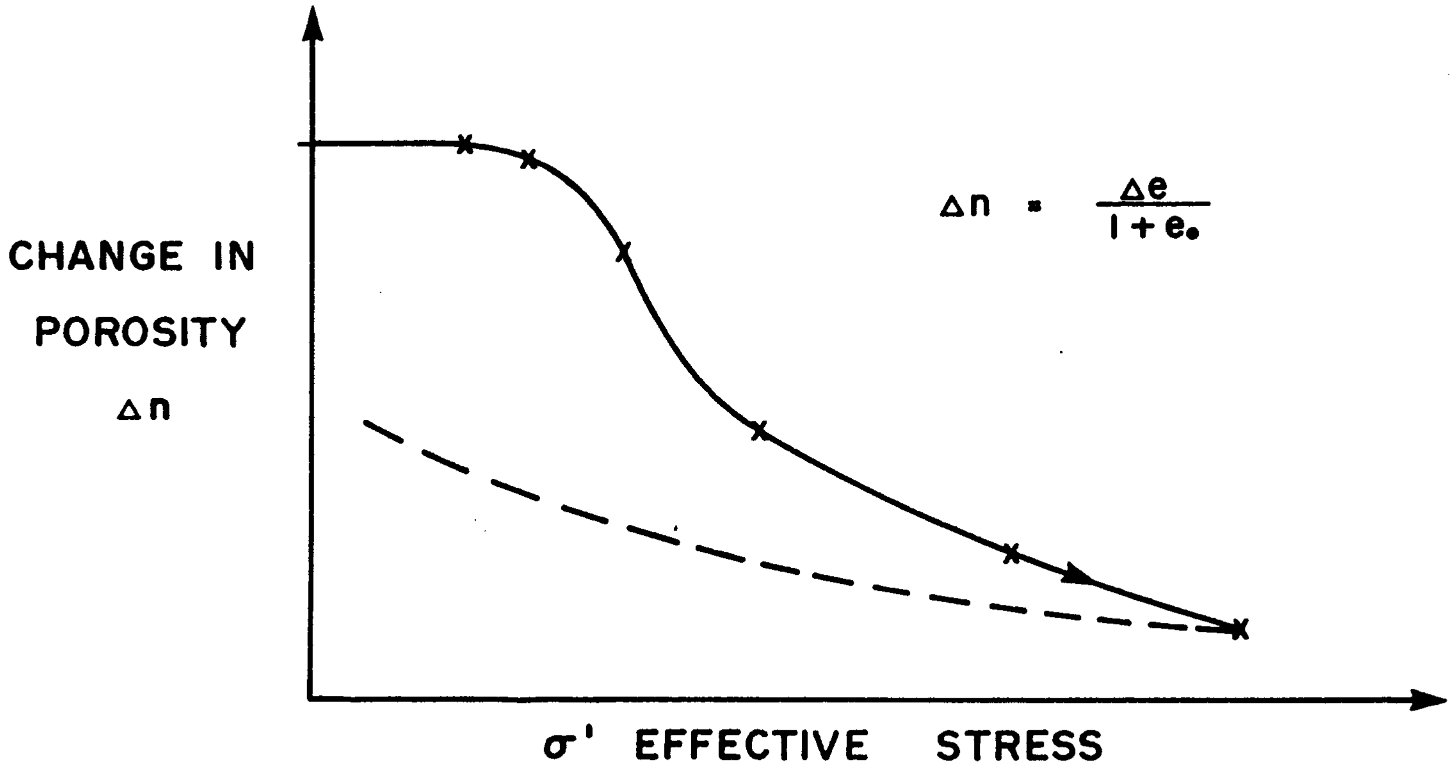


Figure 2 Conventional Consolidation Test Data Presented as a Stress versus Strain Curve

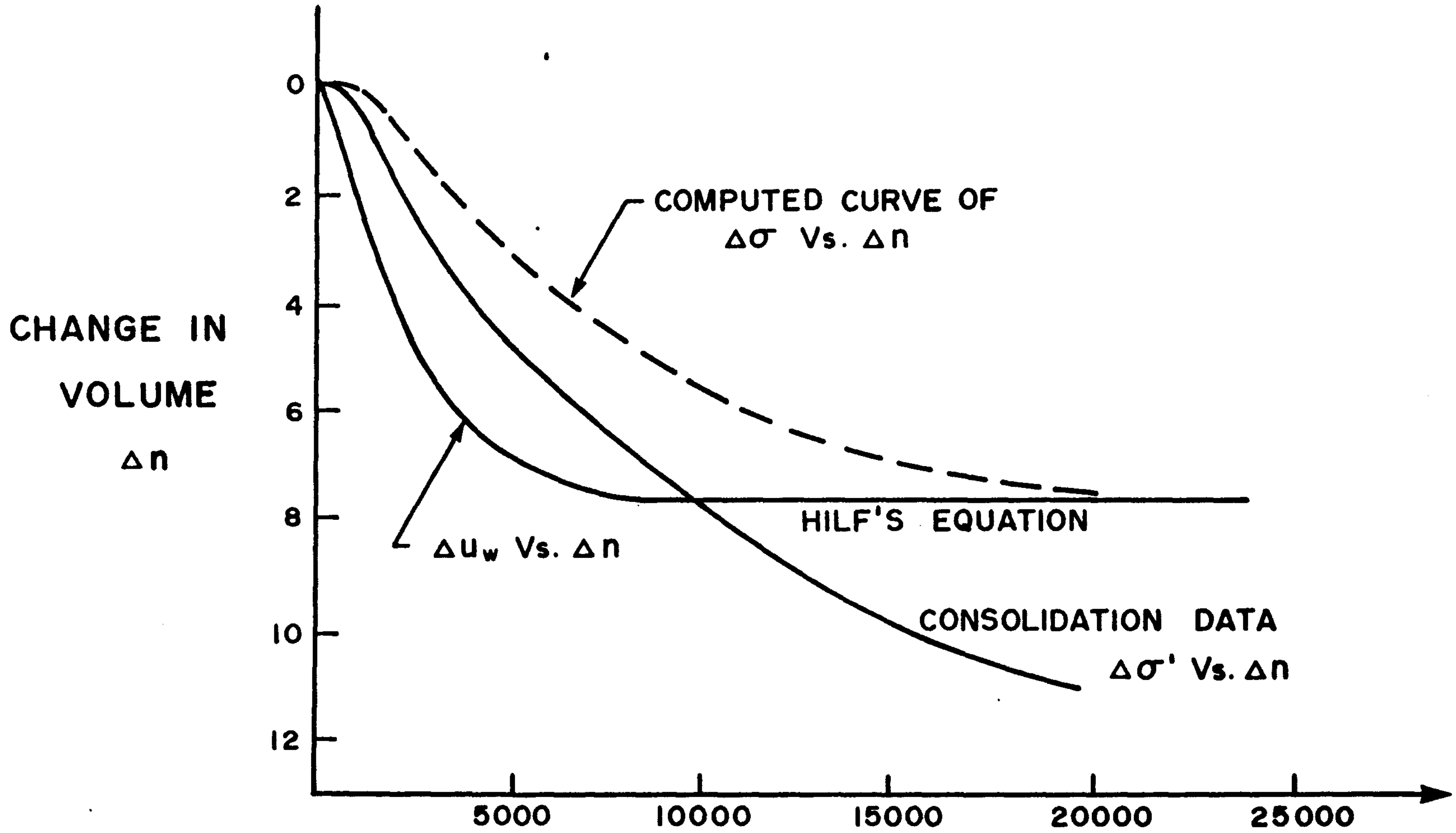


Figure 3 Plots of Stress Components versus Volume Change for Hilf's Analysis

to produce a cross-plot of total stress versus pore-water pressure as shown in Figure 4. The curve is relatively flat for low overburden pressures, increasing to a slope of 45 degrees. At this point the soil is saturated and a change in total stress produces an equal change in pore-water pressure. This curve can be viewed as a nonlinear pore pressure coefficient.

Any one of the above three procedures can be used to predict the pore-water pressures at the end-of-construction. However, Hilf's analysis appears to best duplicate the actual field experience.

2.0 Pore Pressures as a function of Seepage Processes

In order to assess the long term stability of compacted earth slopes or natural slopes the distribution of pore-water pressures throughout the slope must be known. This pressure distribution will result from the groundwater flow or seepage that occurs through the slope.

Groundwater flow results from variations of hydraulic head and is governed by Darcy's law which is stated as:

$$q = ki = k\, dh/dx \qquad [17]$$

where q is the Darcy flux ($m^3/s/m^2$), k is permeability (m/s), and h is the hydraulic head (m). The hydraulic head is written as:

$$h = \frac{u_w}{\gamma w} + z \qquad [18]$$

where u_w is the pore water pressure, γw is the unit weight of water and z is the elevation above some datum.

The pore-water pressures must be determined from the seepage analysis for use in the slope stability analysis. The seepage

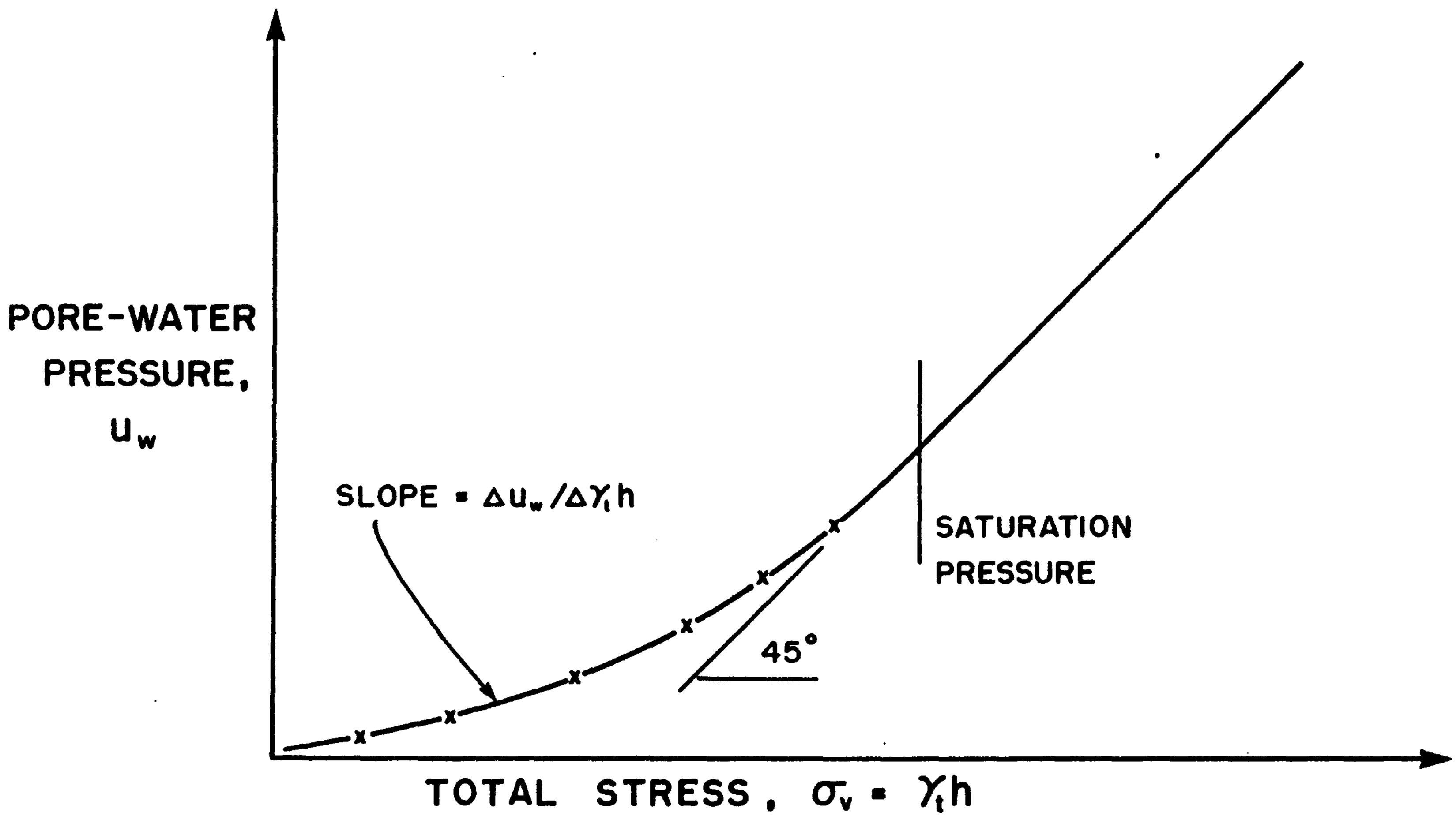


Figure 4 Nonlinear Pore-Water Pressure versus Total Stress derived using Hilf's Analysis

analysis results in a knowledge of the equipotential heads at all points. The equipotential lines designate total head which is the sum of the elevation head and the pressure head. Only the pressure head is required for the slopes stability analysis and it must be converted to a pore-water pressure as follows;

$$u_w = (h - z)\,\gamma w \qquad [19]$$

where γw is the unit weight of water.

Two cases of seepage that are of interest in stability analysis is steady state seepage as it relates to the long term stability of a slope, and transient seepage conditions, such as rapid drawdown, which may cause critical pore pressures to occur during a time when the pore pressures are changing very rapidly within the slope.

2.1 Steady State Seepage

The equation governing steady state seepage through a slope in two dimensions is written as follows:

$$kx\partial^2h/\partial x^2 + ky\partial^2h/\partial y^2 = 0 \qquad [20]$$

Where kx, and ky are the permeability in the x and y directions and h is the hydraulic head. Solution of this equation for a particular problem can be obtained using graphical, analogue or numerical techniques. Most commonly used are graphical techniques by flow net construction and numerical models using finite element or finite difference techniques.

Flow Nets

Pore pressures have been classicially assessed through the drawing of flownets. For example, to assess the long-term stability of the downstream slope of a dam, the steady state pore-water pressures must be known. Pore-water pressure distributions determined by flownets have proved to be quite reliable for slope stability analysis. Figure 5 shows the flownets for several homogeneous type dam cross-sections with isotropic soils. The line of seepage (i.e., uppermost flowline) is first determined in accordance with some empirical rules. The flownet is then constructed ensuring that flowlines and equipotential lines cross at 90 degrees and form 'squares'.

Flownets can also be drawn for more complex situations as the one shown in Figure 6. Here, each soil is isotropic but there are two different types of soil, the downstream soil having a permeability of 5 times the upstream soil. Anisotropy can also be taken into account, however, the drawing of flownets becomes complex. When complex cross-sections are encountered, it is often possible to consider only the portion of the dam through which most of the water will be forced to flow. The flownet is then completed for this material alone, as shown in Figure 7. This procedure is often adequate for assessing quantities of seepage and for predicting pore-water pressures for a slope stability analysis.

The manner in which the pressure head can be determined from the flownet is shown on Figure 8. Consider a point at the base of a slice for a particular slip surface as shown. Follow an equipotential line through this point to the line of seepage. The vertical distance between the base of the slice and the line of seepage gives the pressure head. The pressure head is then multiplied by the density of water and gravity acceleration to give a pore-water pressure. If this procedure is followed for several points along the slip surface, it is

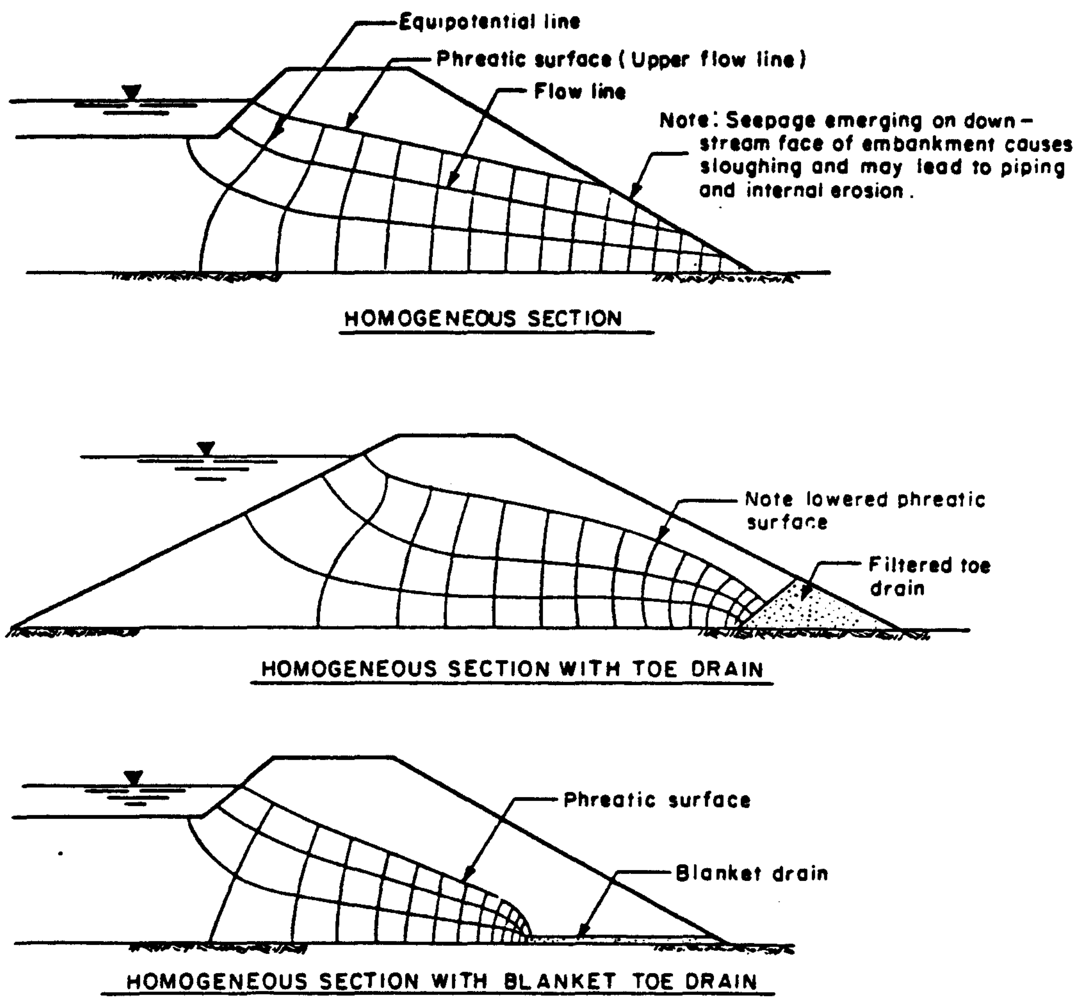


Figure 5 Typical Flownets for Homogeneous Embankments

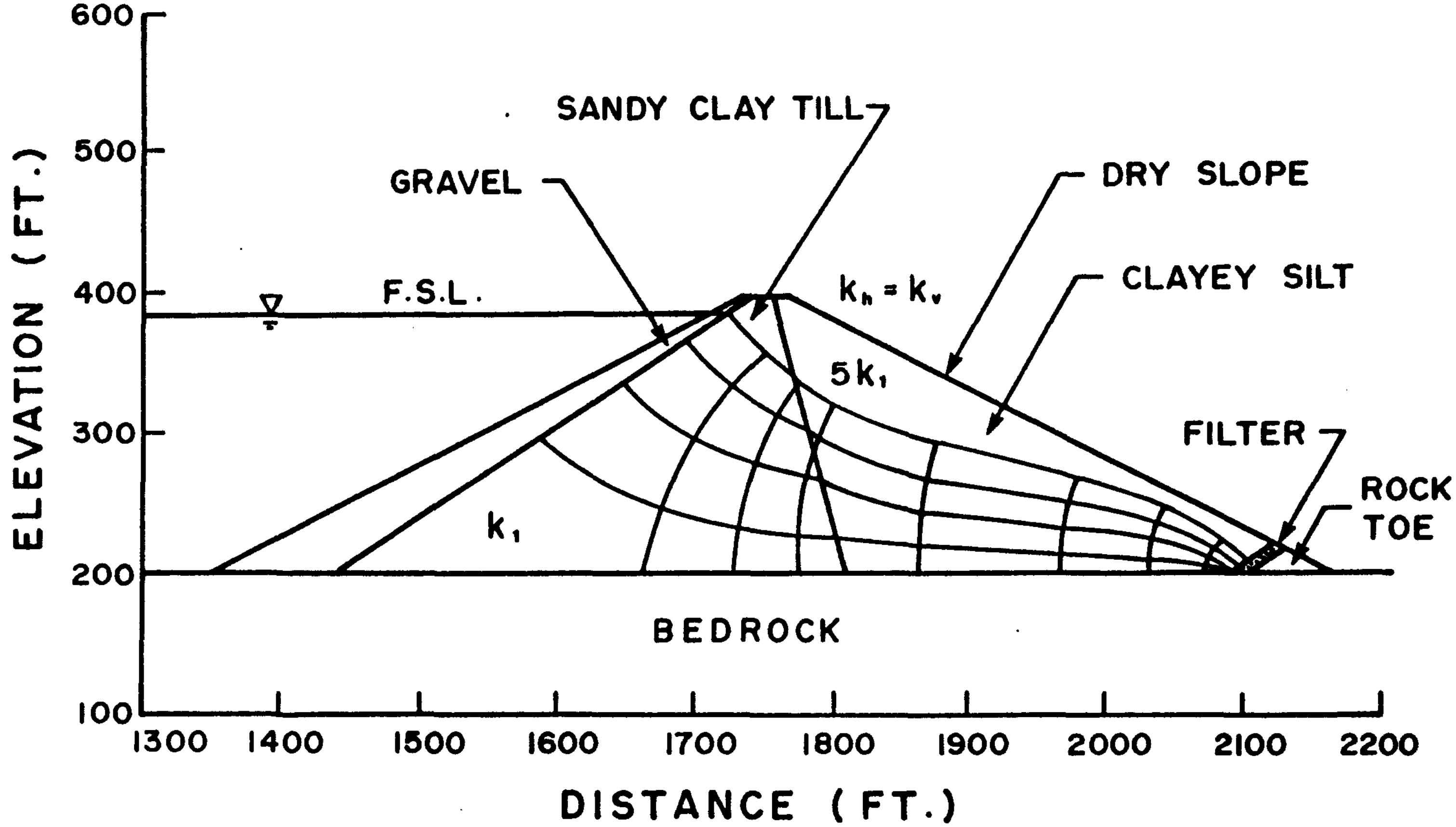
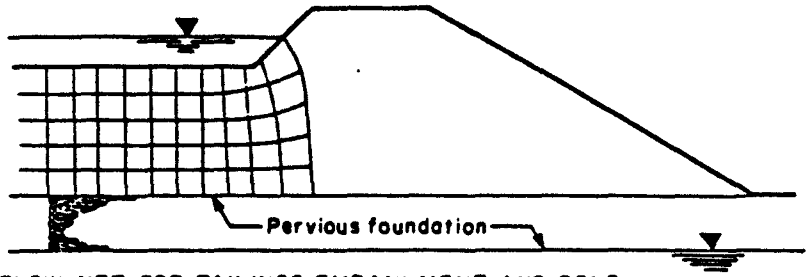
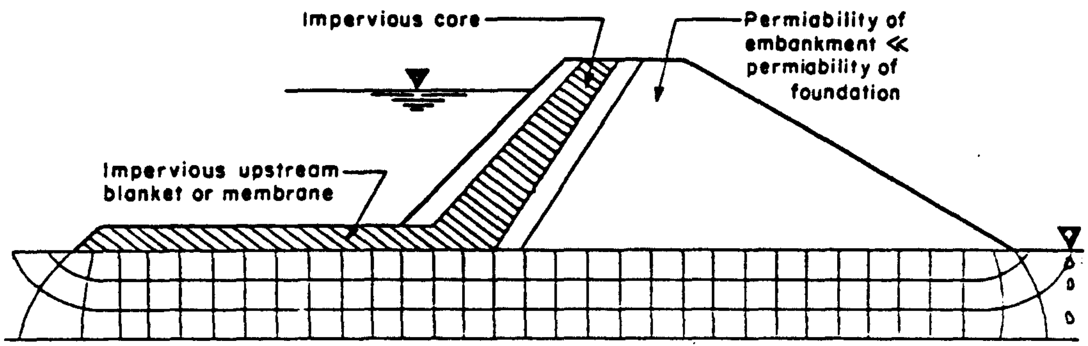


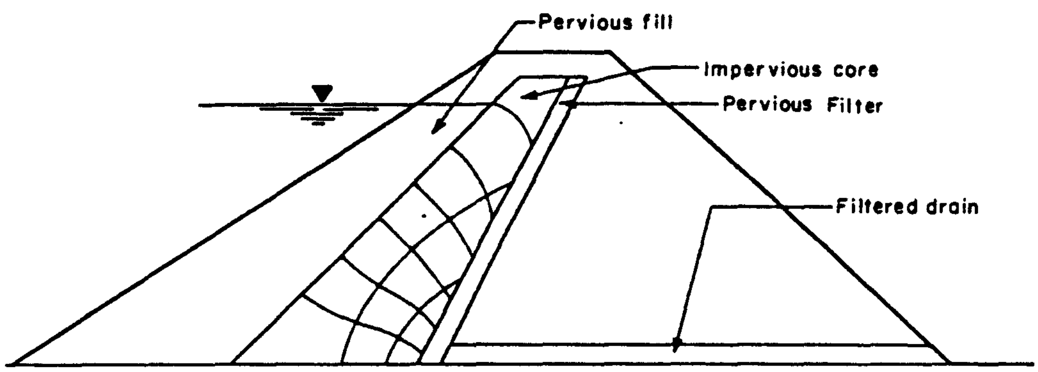
Figure 6 Flownet for a Case of Two Soils



FLOW NET FOR TAILINGS EMBANKMENT AND POND
UNDERLAIN BY PERVIOUS FOUNDATION



FLOW NET FOR SEEPAGE THROUGH PERVIOUS FOUNDATION
IMPERVIOUS-UPSTREAM BLANKET OR MEMBRANE USED
TO INCREASE LENGTH OF SEEPAGE PATH



APPROXIMATE FLOW NET FOR
SEEPAGE THROUGH IMPERVIOUS CORE

Figure 7 Examples of Flownets for Tailings Embankments with Complex Cross-sections

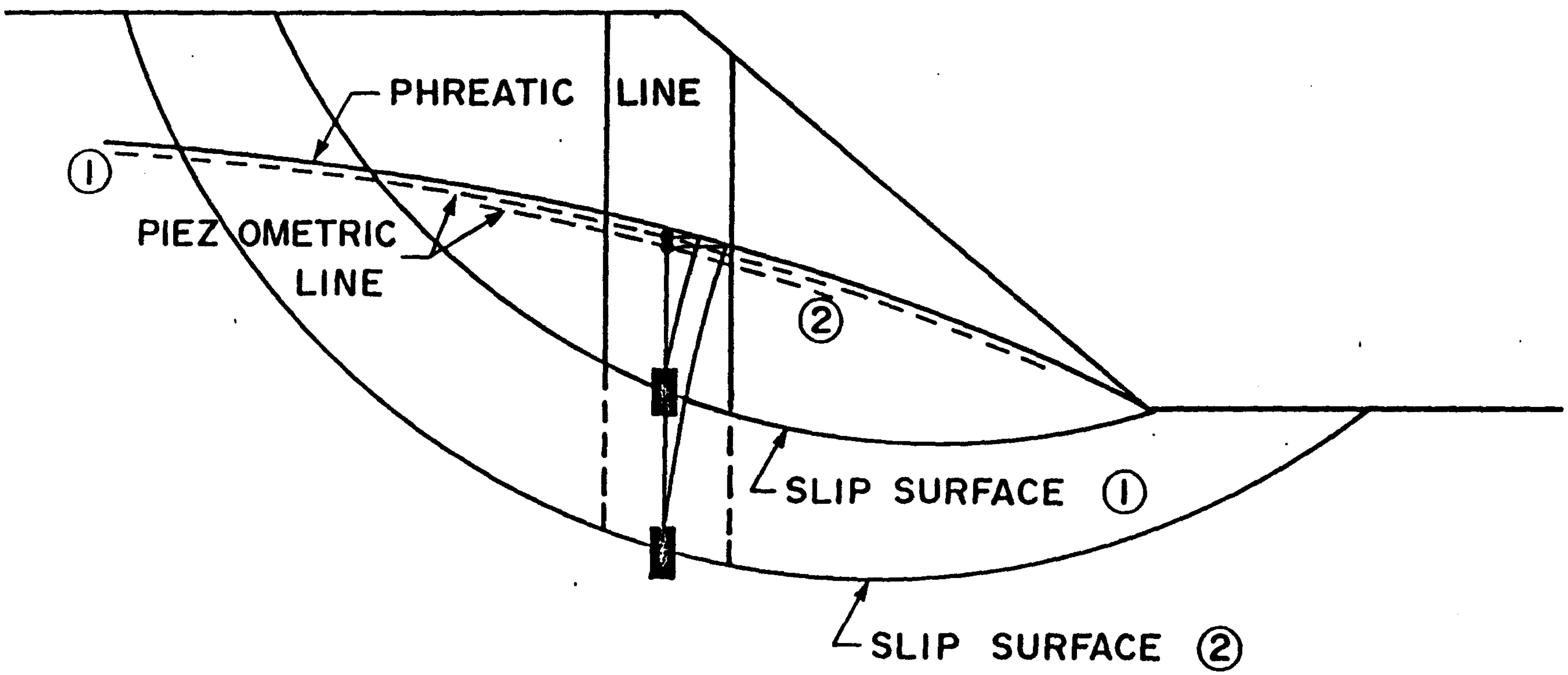


Figure 8 The Relationship between a Phreatic Line and Piezometric Line

possible to define a 'piezometric line' for the slip surface under consideration. This is the line that should be used to compute pore-water pressures during a slope stability analysis. It differs slightly from the phreatic line or the line of seepage. In other words, the piezometric line is a function of the slip surface being considered.

Numerical Models

In recent years there have been substantial advances made on the use of numerical models to predict the seepage conditions within slopes. These procedures are not restricted by conditions of nonhomogenieity and anisotropy. These procedures have also been extended to include the unsaturated zone and consequently do not require any predefinition of the phreatic surface through the slope as flow net construction requires. Darcy's law for saturated/unsaturated flow is of the same form as equation [17], however the permeability is now a function of the negative pressure head, or matrix suction, as shown in Figure 9.

Numerical modelling of saturated/unsaturated seepage allows the soil to be analyzed as a continuum, encompassing both the saturated and unsaturated zones, and also allows the effects of surface fluxes, such as infiltration, to be incorporated into the analysis.

An example of these kind of numerical models is TRASEE (Lam 1983, 1984). TRASEE is a two-dimensional transient finite element seepage model developed by the Department of Civil Engineering, University of Saskatchewan. The output from TRASEE can be manipulated and presented in graphical forms. Seepage fluxes, hydraulic heads or pressure heads can all be obtained automatically from the computed

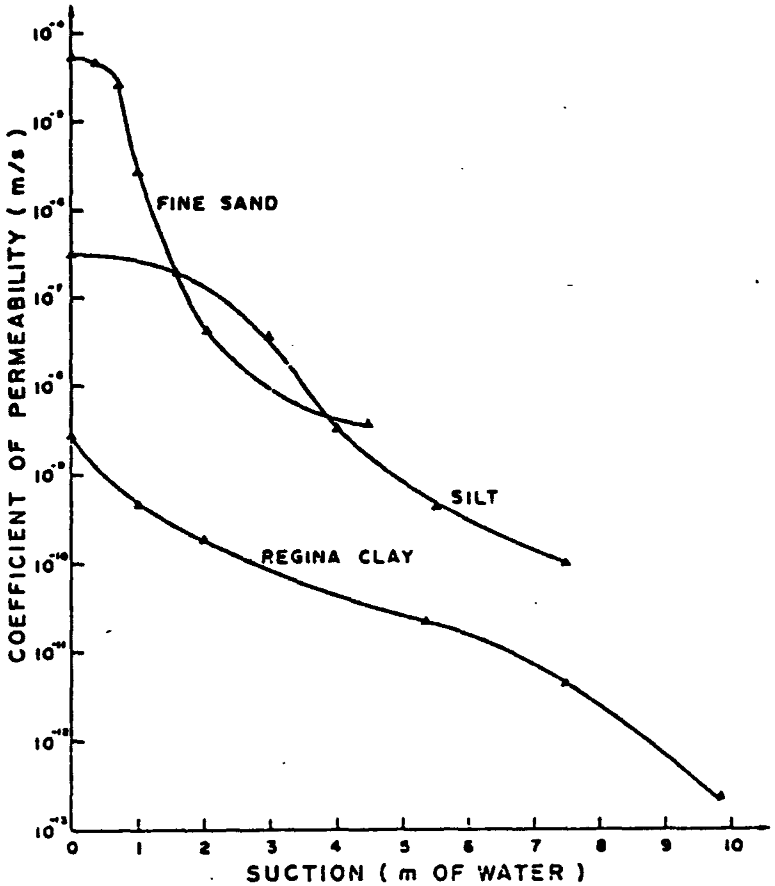


Figure 9 A Typical Unsaturated Permeability Function for Fine Sand, Silt and Clay

results. Examples of these types of graphical output are illustrated in Figures 10 through 13. In each example two plots are presented; one shows a plot of vectors depicting the direction and magnitude of the seepage velocities throughout the flow system superimposed on a plot of the phreatic (solid line) and equipotential lines (dashed lines), the second plot is a set of equal pressure contours for the flow system. The value of the intersection of the pressure contours with a trial slip circle would define the piezometric line for a stability analysis.

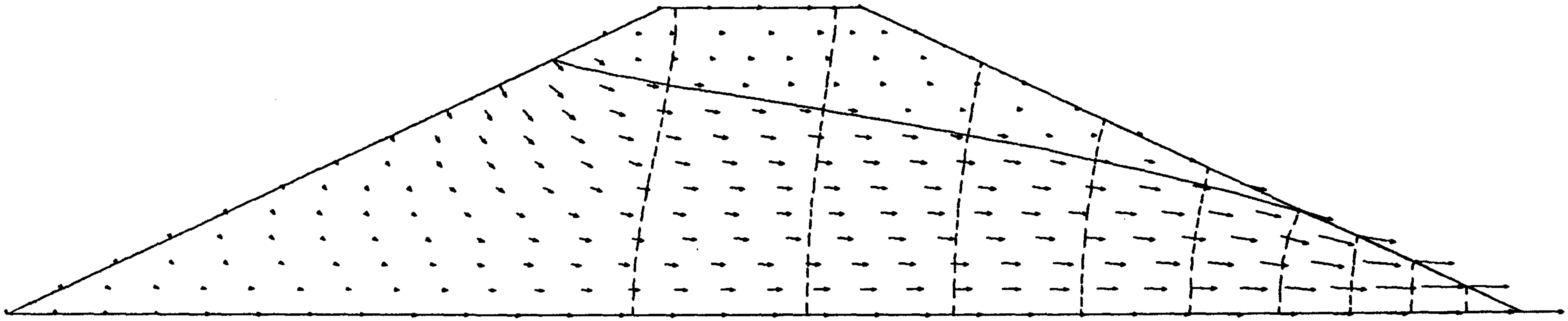
Figure 10 through 12 depict the seepage conditions through several homogeneous type dam cross sections with isotropic soils ($K = 1.0 \times 10^{-7}$ m/s), similar to those depicted in Figure 5 for flow net construction. Figure 13 illustrates the effect of anisotropy.

A flow system similar to the two soil systems depicted in Figure 6 is shown in Figure 14. In addition, the effect of surface infiltration on the flow system within the embankment is also presented in Figure 15.

7.2.2 Transient Seepage Conditions

In some cases the critical condition of stability for a slope occurs during a period in which the pore-pressures through the slope are changing quite dynamically due to a change in head or seepage flux along the surface. Examples of these types of situations include rapid drawdown of the reservoir behind a dam, or surface infiltration into a slope due to severe rainfall or runoff conditions.

In these cases the hydraulic head or flux along the boundary of the flow system is changing. This results in a variation in pore



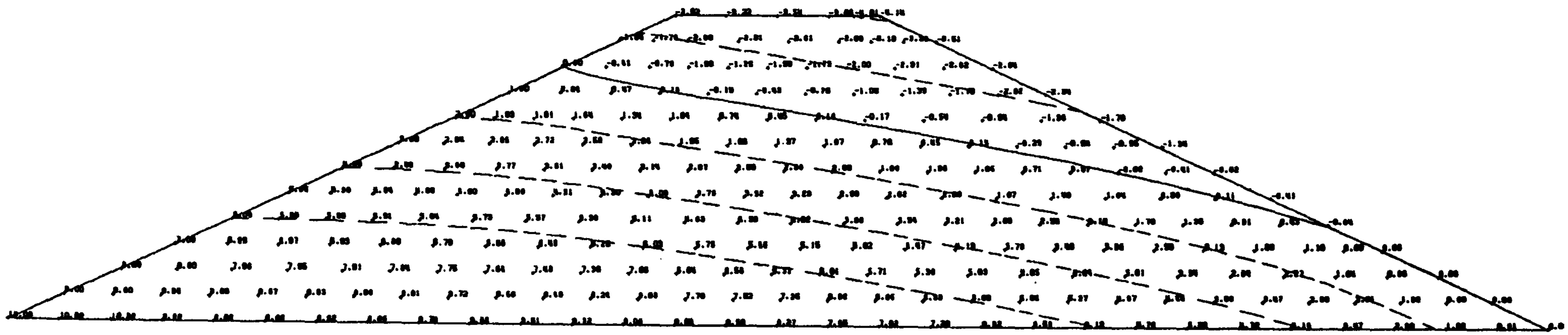
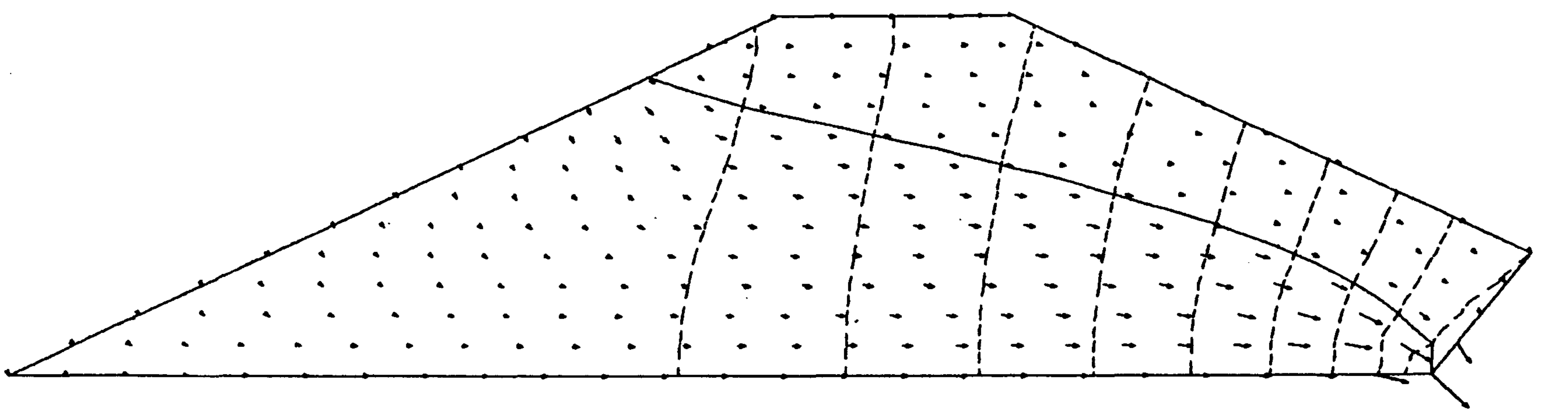


Figure 10 Numerical Solution for Homogeneous Dam Cross-section

a) Velocity Vectors and Equipotential Contours

b) Piezometric Contours



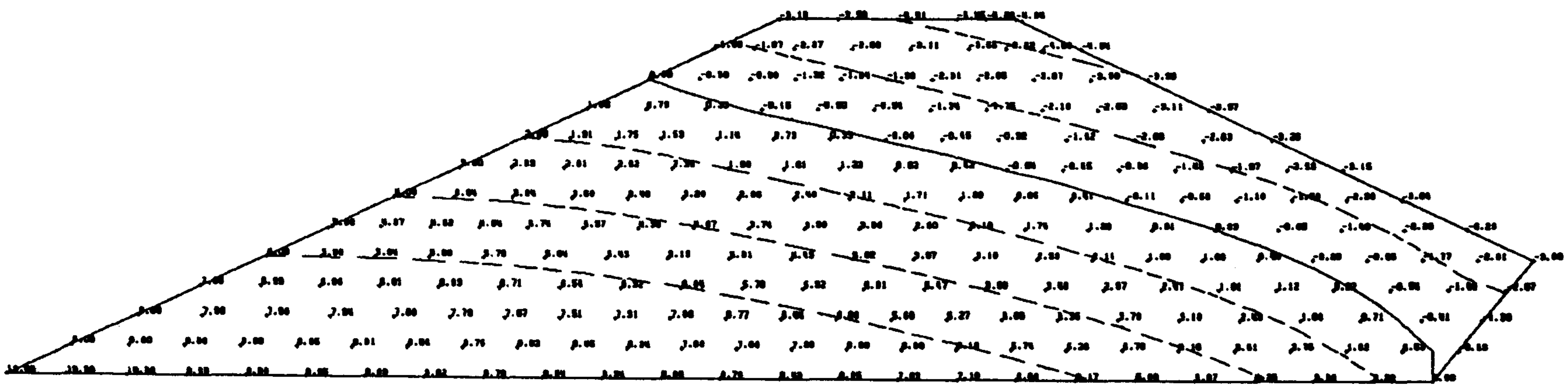


Figure 11 Numerical Solution for Homogeneous Dam Cross-section with Toe Drain

a) Velocity Vectors and Equipotential Contours

b) Piezometric Contours

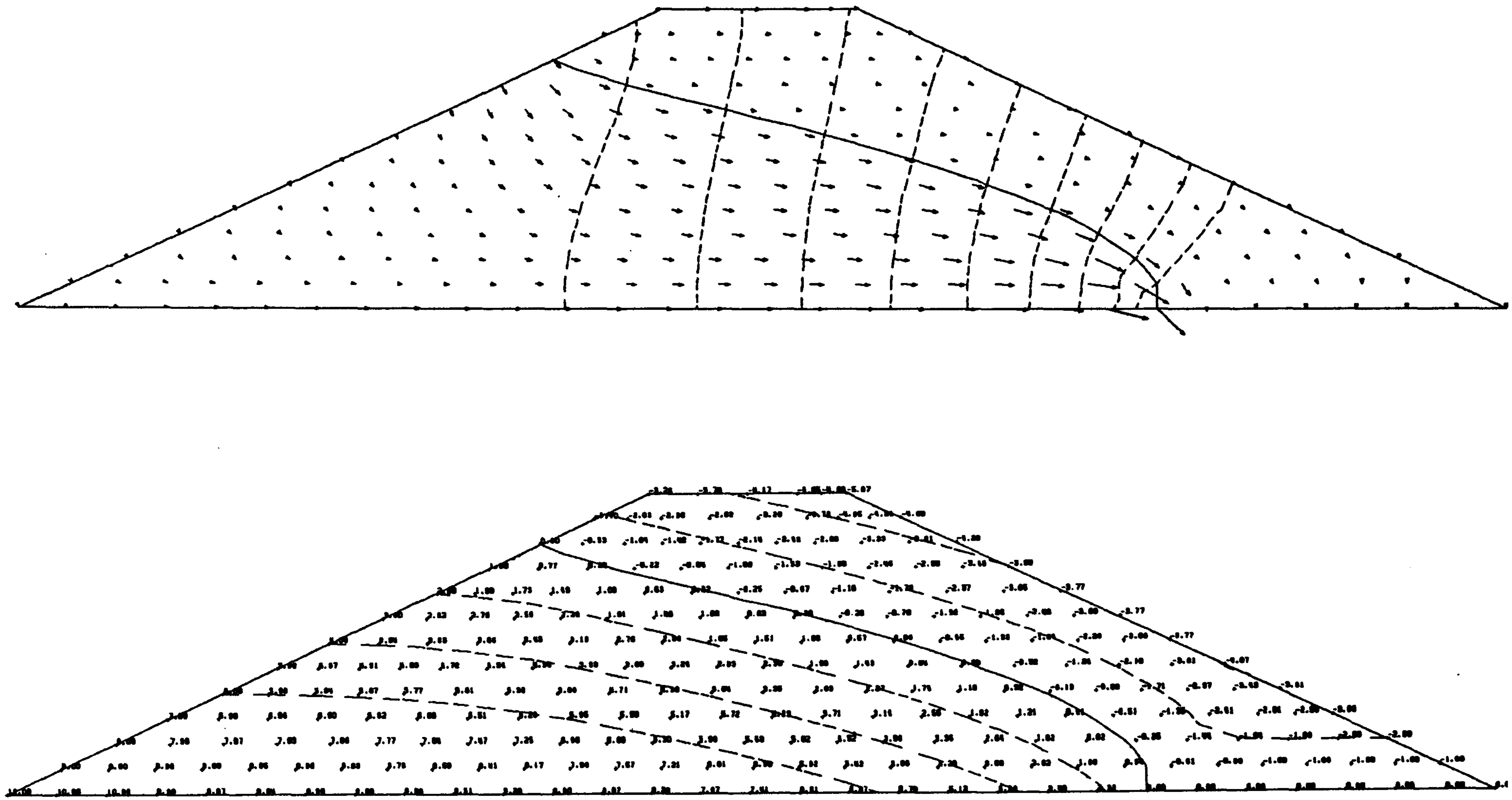


Figure 12 Numerical Solution for Homogeneous Dam Cross-section with Blanket Drain

a) Velocity Vectors and Equipotential Contours

b) Piezometric Co... s

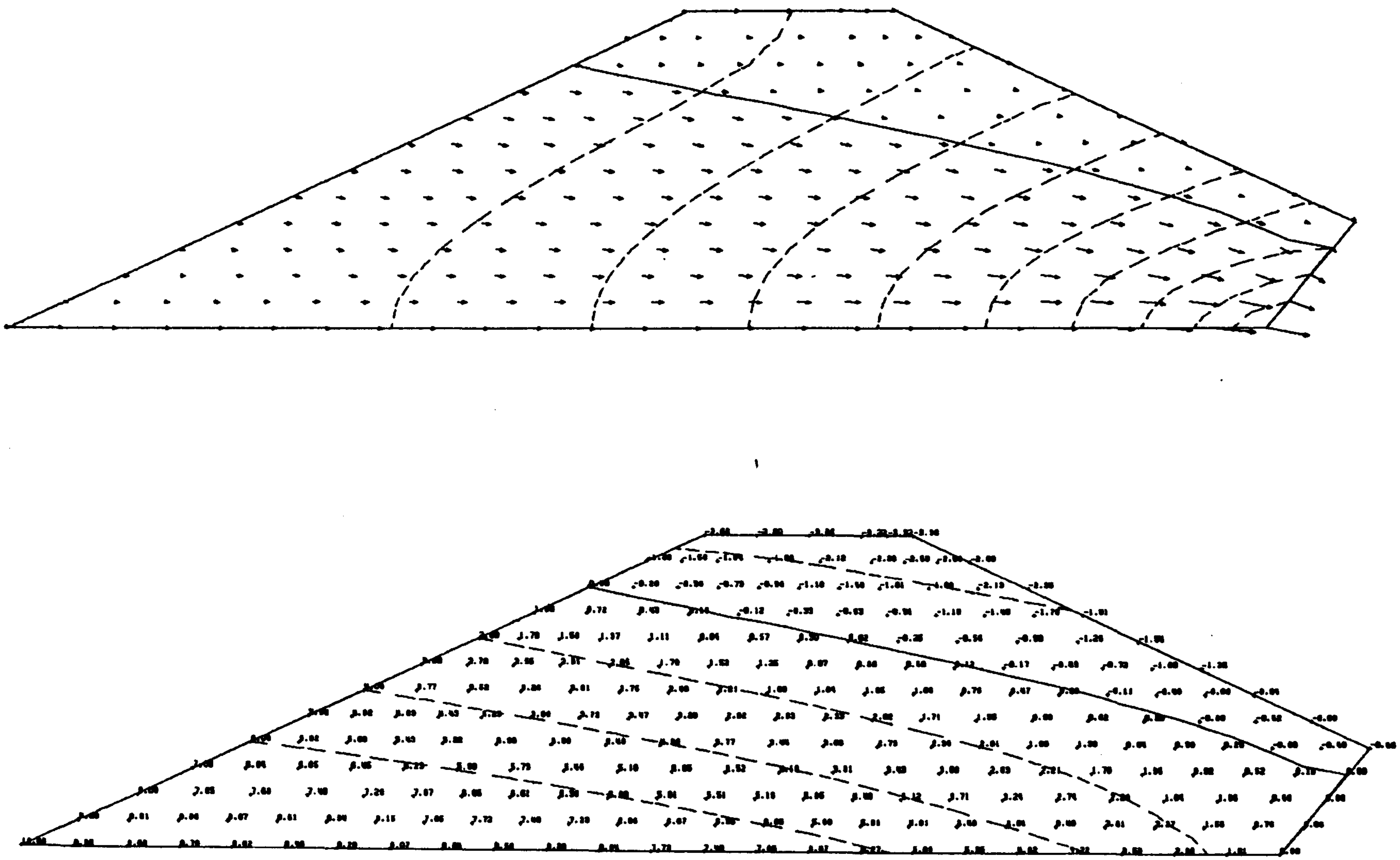
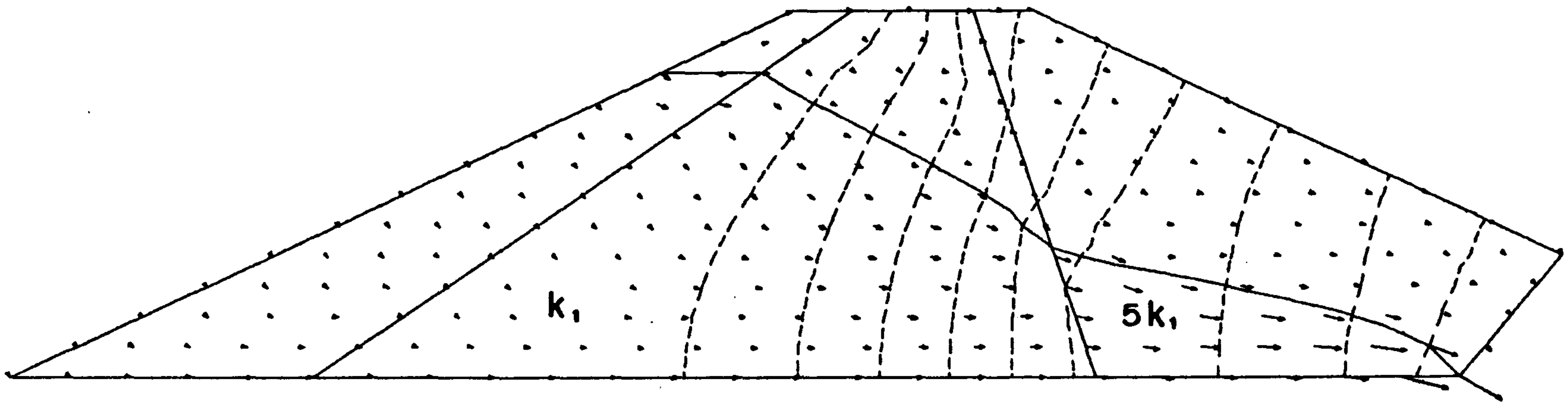


Figure 13 Numerical Solution for Homogeneous, Anisotropic Dam Cross-section with Toe Drain, Anisotropy Kh = 10 Kv

a) Velocity Vectors and Equipotential Contours

b) Piezometric Contours



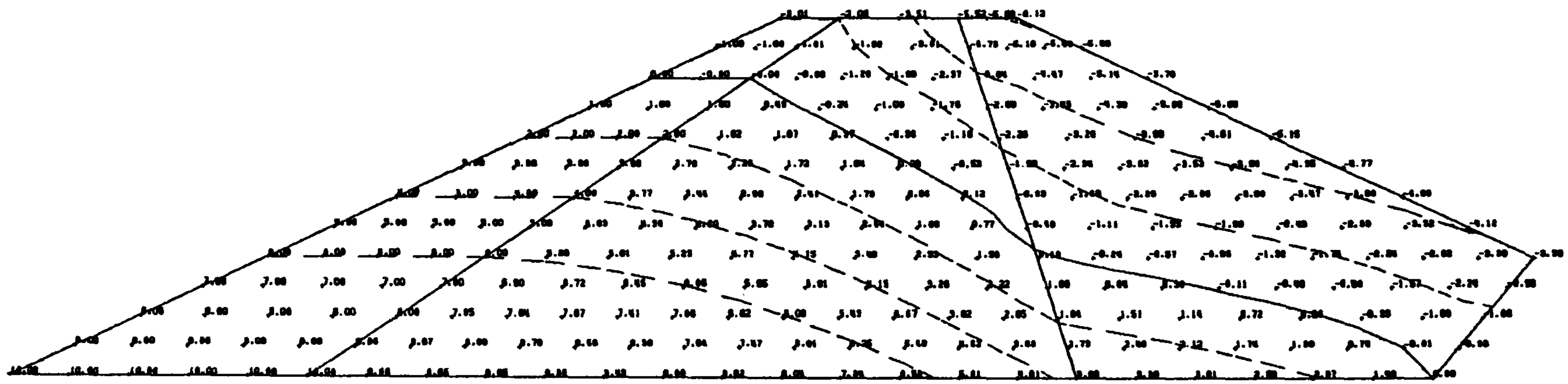


Figure 14   Numerical Solution for Two Material Dam Cross-section

a) Velocity Vectors and Equipotential Contours

b) Piezometric Cont

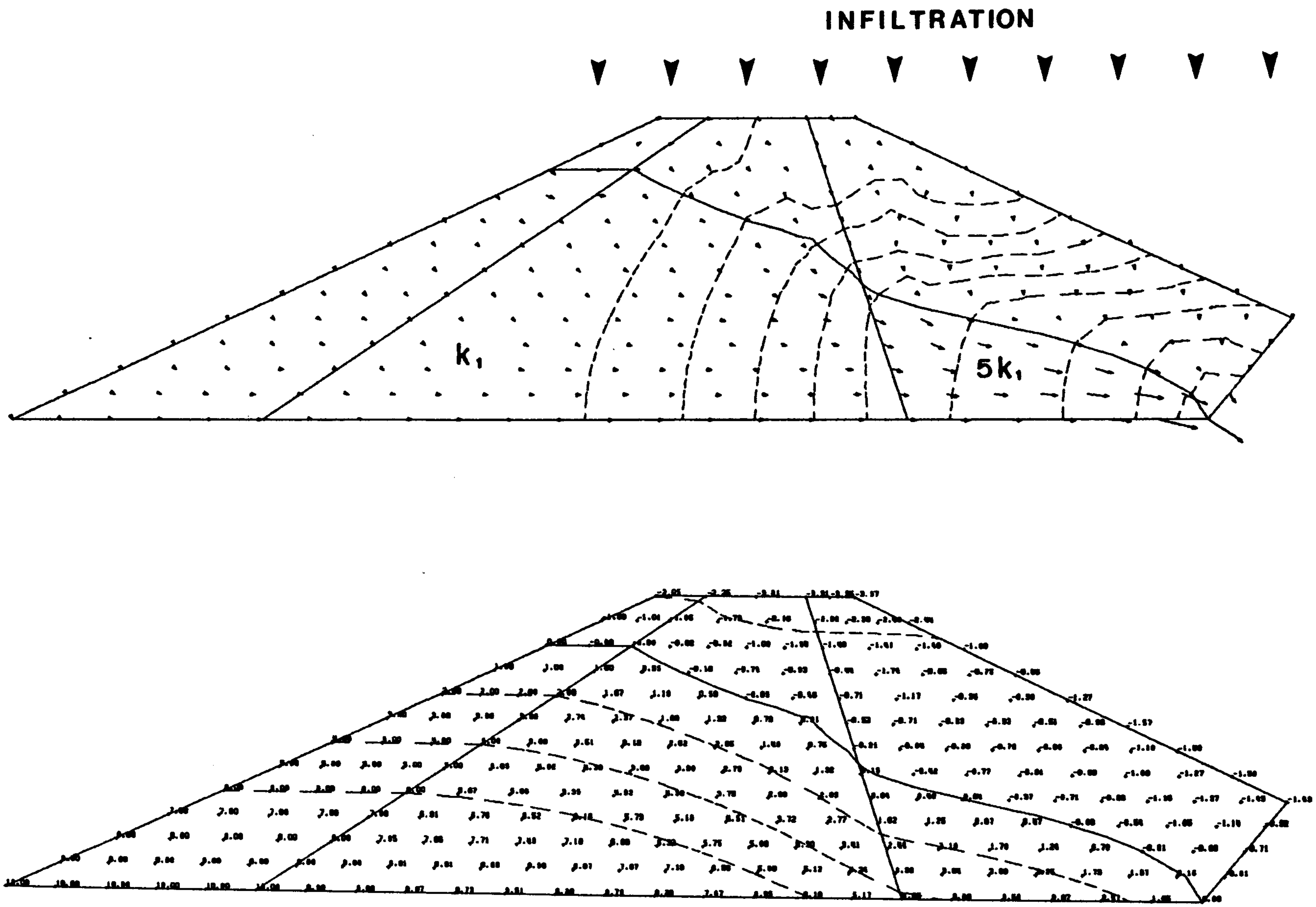


Figure 15 Numerical Solution for Two Material Dam Cross-section with Infiltration along the Surface

a) Velocity Vectors and Equipotential Contours

b) Piezometric Contours

pressures within the flow system as pore water is released or taken into storage in response to this new boundary condition. A particular example of transient flow is saturated/unsaturated flow of water in an unconfined flow system.

The governing differential equation for flow in this case can be written as follows:

$$kx \frac{\partial^2 h}{\partial x^2} + Ky \frac{\partial^2 h}{\partial y^2} = m_2^w \rho_w g \frac{\partial h}{\partial t} \qquad [21]$$

where m_2^w is a storage coefficient which relates the release or uptake of water by the flow system due to a change in pore pressure.

Flow net construction can be used to evaluate this condition for particular cases. An example of the flownet corresponding to 50% drawdown of the reservoir is shown in Figure 16. This situation often produces instability because the pore-water pressures have not had time to dissipate but the lateral support of the reservoir has been removed. A piezometric line for stability analysis follows the upstream face (i.e., of the impervious zone), joining the previous line of seepage. This line is often used to assess the pore-water pressures for the rapid drawdown case. Actual pore-water pressures at the base of the slice in a slope stability analysis, can also be determined through use of the equipotential lines.

Numerical modelling can be particularly powerful in the analyses of the transient seepage case. Figures 17 and 18 illustrate the output from TRASEE for a rapid drawdown case similar to that depicted in Figure 16, but for two different elapsed times.

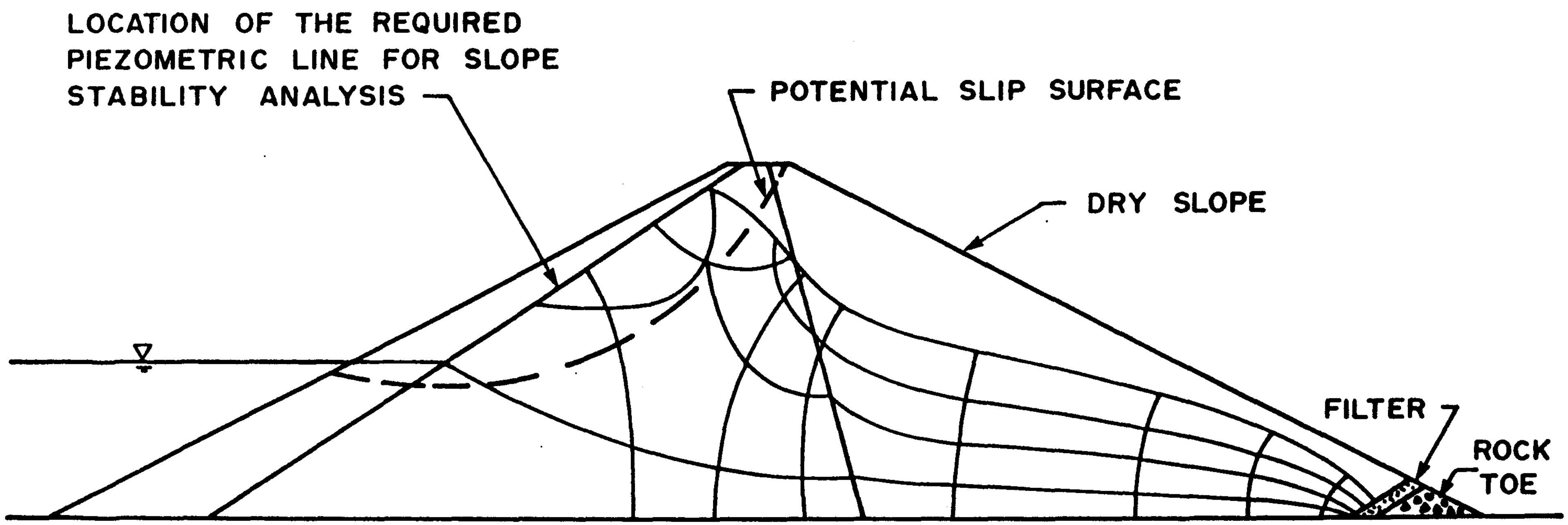


Figure 16 Flownet Corresponding to 50% Drawdown of the Reservoir

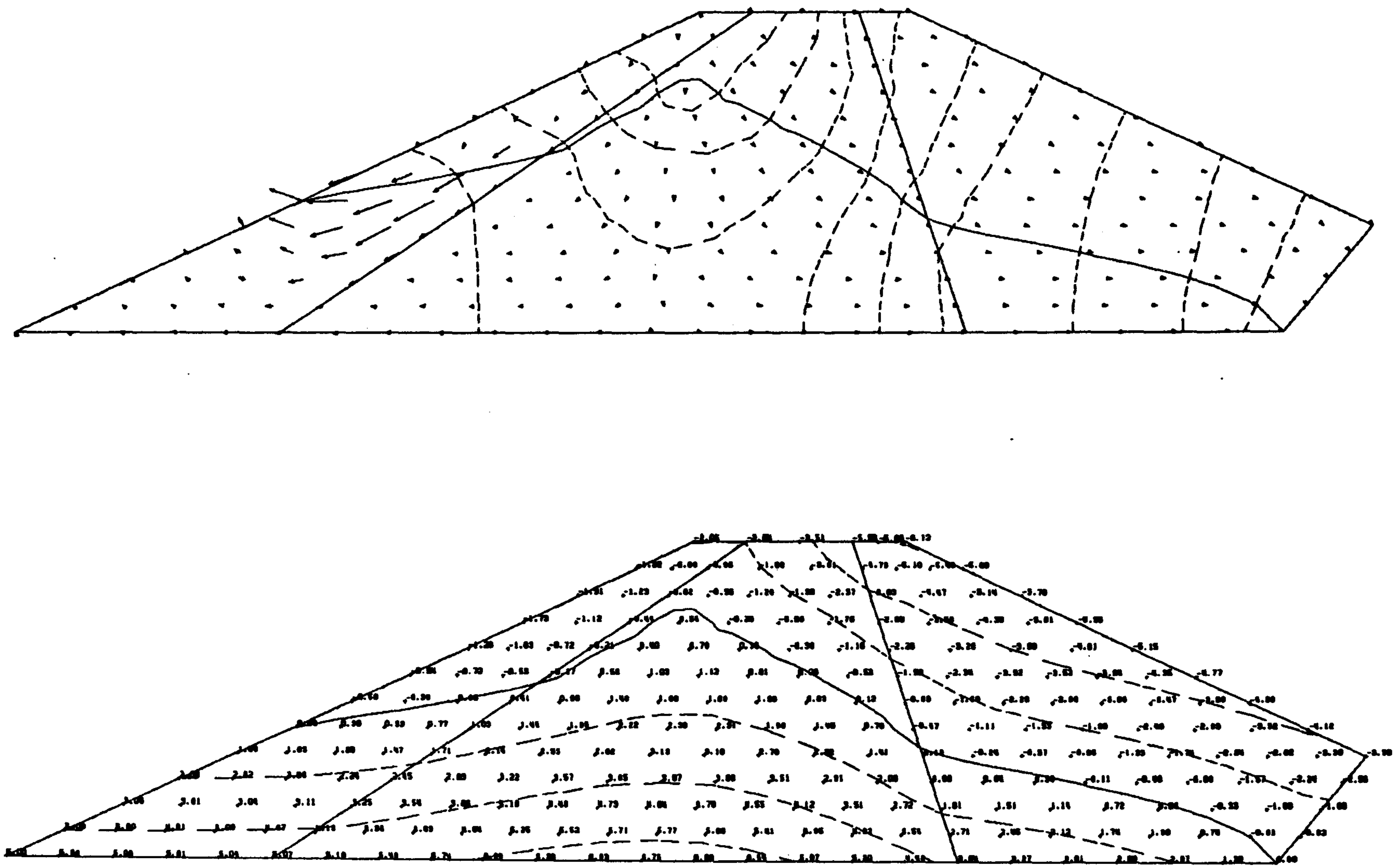


Figure 17 Numerical Solution for an Elapsed Time of 11 Minutes from a 50% Drawdown of the Reservoir

a) Velocity Vectors and Equipotential Contours

b) Piezometric Contours

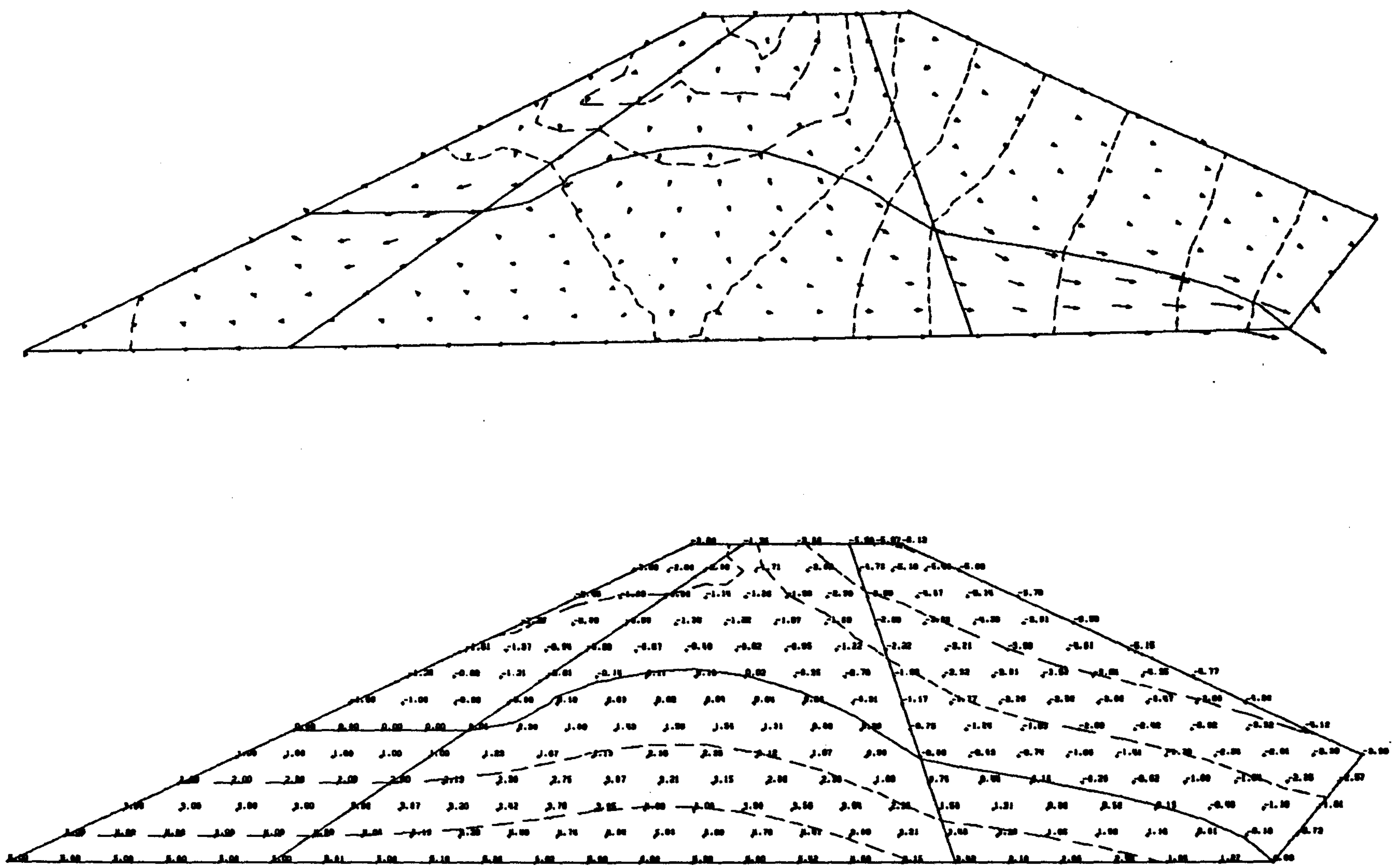


Figure 18 Numerical Solution for an Elapsed Time of 61 Minutes from a 50% Drawdown of the Reservoir

a) Velocity Vectors and Equipotential Contours

b) Piezometric Contours

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