Theoretical context for understanding unsaturated residual soil behavior

D.G. Fredlund and H. Rahardjo

Abstract: The basic concepts for understanding the behavior of unsaturated residual soils are outlined. The available theory for fluid flow, shear strength and volume change of unsaturated soils are described. The application of the theory and technology in resolving shear strength problems of unsaturated residual soils is presented using case histories. The results of *in situ* measurements, laboratory tests and theoretical analyses have proven satisfactory in analysing problems involving unsaturated residual soils.

Key words: residual soils, unsaturated soil behavior, soil suction, shear strength.

Introduction

A large portion of the earth's surface is covered with residual soils which can be either saturated or unsaturated. The theory and technology for the saturated soils have been well developed and applied in engineering practice for over 5 decades. However, weathering processes have left over eighty percent of earth surface desiccated. As a result, cracks start to develop, and the soil becomes unsaturated. Many geotechnical problems have been encountered involving unsaturated soils, but our understanding of the behavior of these soils is far behind our knowledge of saturated soils. Most of the engineering problems involving heave, consolidation, collapse and dramatic changes in shear strength are directly related to the behavior of unsaturated soils. Cyclic variations in environmental conditions increase the complexity of the problems.

The objective of this paper is to summarize the theoretically based principles, and mathematical equations necessary for establishing a practical science and technology for unsaturated soils. The theory is consistent with multiphase continuum mechanics and has been directed towards resolving observable phenomena in unsaturated soils. Equations are presented for fluid flow, shear strength and volume change behavior of unsaturated soils. The equations for unsaturated soil behavior are extensions of the equations commonly used for saturated soils. The unsaturated soil theory is also applied to the slope stability problem in Hong Kong residual soils. The application involves *in situ* measurements of soil suction, laboratory tests for shear strength and theoretical slope stability analyses.

Stress state variables

An unsaturated soil can be considered as a four phase system (Fredlund and Morgenstern 1977). The solid phases (i.e., soil particles and contractile skin) will reach an equilibrium under applied stress gradients, whereas the fluid phases (i.e., water and air) flow under applied stress gradients. The theoretical analysis of this four phase system using the multiphase continuum mechanics yields several combinations of independent stress state variables. However, the combination of $(\sigma - u_a)$ and $(u_a - u_w)$ as stress state variables are superior for practical applications because the effects of changes in total stress and pore-water pressure can be separated. The term $(u_a - u_w)$ is referred to as matric suction where u_a is the pore-air pressure; u_w is the pore-water pressure; and σ is the total stress. The stress state variables can be extended to a matrix form when multi-directional analyses are being attempted.

As a soil approaches saturation, the pore-water pressure approaches the pore-air pressure. Therefore, the matric suction term goes to zero, and there is a smooth transition to the saturated soil stress state variable, $(\sigma - u_w)$.

Flow laws and seepage

Darcy's Law can be used to describe the flow of water through an unsaturated soil (Childs and Collis-George 1950; Freeze and Cherry 1979),

$$[1] \quad v_w = -k_w \frac{\partial h_w}{\partial y}$$

where:

v_w = the velocity of water,

k_w = the coefficient of permeability with respect to the water phase,

h_w = the total head in the water phase, and

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y = the coordinate direction.

The coefficient of permeability, k_w, is highly variable and can be written as a function of negative pore-water pressure head (Gardner 1958).

$$[2] \quad k_w = \frac{k_s}{1 + a\left(\frac{u_a - u_w}{\rho_w g}\right)^n}$$

where:

k_s = the saturated coefficient of permeability,
ρ_w = density of water,
g = the gravitational acceleration, and
a, n = material properties.

The flow of air through an unsaturated soil may be required in the analysis of certain problems. In this case the flow of air can be described using Fick's Law (Blight 1971),

$$[3] \quad v_a = -D^* \frac{\partial u_a}{\partial y}$$

where:

v_a = the mass rate of air flow, and
D^* = the transmission constant of proportionality for the air phase.

The steady seepage equation for the water phase in an unsaturated soil becomes an expansion of the common La Placian equation,

$$[4] \quad k_w \frac{\partial^2 h}{\partial x^2} + \frac{\partial k_w}{\partial x}\frac{\partial h}{\partial x} + k_w \frac{\partial^2 h}{\partial y^2} + \frac{\partial k_w}{\partial y}\frac{\partial h}{\partial y} + \frac{\partial k_w}{\partial y} = 0$$

The second and fourth terms in eq. [4] account for the variation of the coefficient of permeability of water with respect to space. The fifth term is called the gravity term. As a soil approaches saturation, the spatial variations in permeability go to zero, resulting in a smooth transition to the saturated soil case.

Shear strength

The shear strength equation for an unsaturated soil can be written in terms of the $(\sigma - u_a)$ and $(u_a - u_w)$ stress state variables (Fredlund et al. 1978).

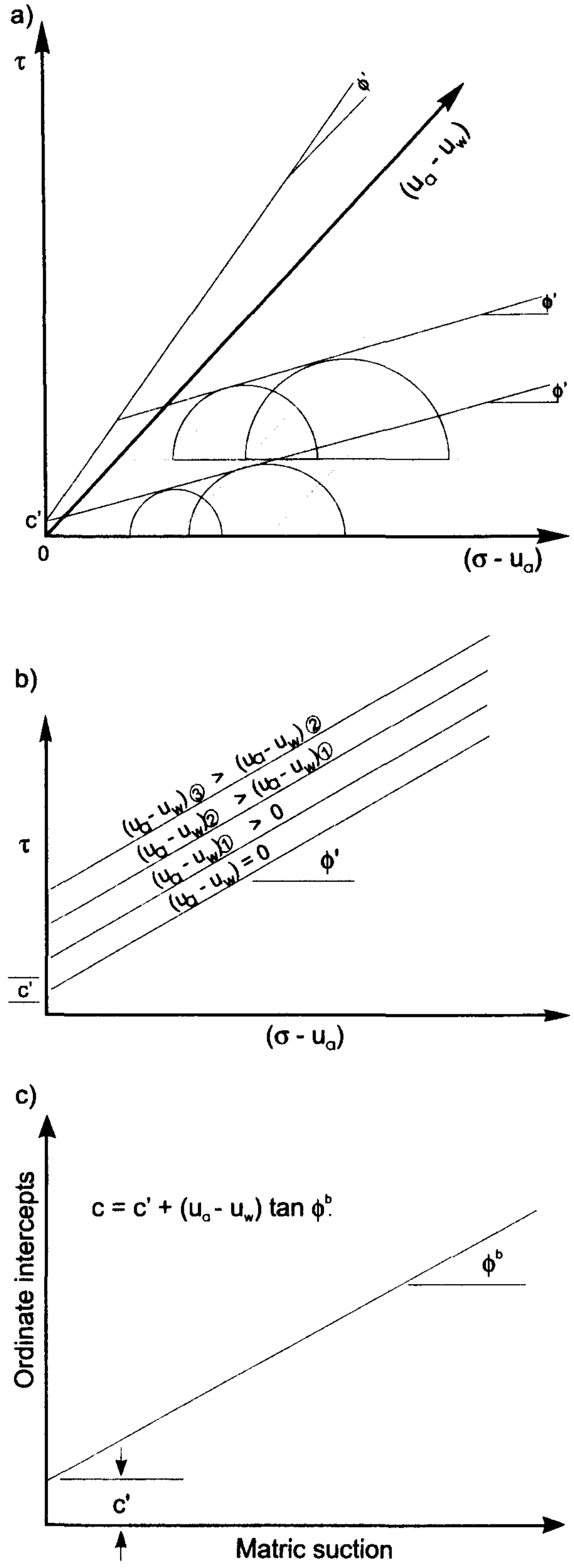
$$[5] \quad \tau = c' + (u_a - u_w)\tan\phi^b + (\sigma_n - u_a)\tan\phi$$

where:

τ = the shear strength,
c' = the effective cohesion intercept,
ϕ^b = the angle of shear strength increase with an increase in $(u_a - u_w)$,
σ_n = the total normal stress, and
ϕ' = the effective angle of internal friction.

Equation [5] is an extension of the conventional Mohr-Coulomb failure criteria, and can be plotted in a three-dimensional form using $(\sigma - u_a)$ and $(u_a - u_w)$ as abscissas (Fig. 1*a*). The equation can also be visualized as a two-

Fig. 1. *a*) Three-dimensional failure surface for unsaturated soils; *b*) Projection of failure surface on the τ and $(\sigma - u_a)$ plane by looking parallel to the $(u_a - u_w)$ axis; *c*) The ordinate intercepts on the τ and $(u_a - u_w)$ plane for various matric suction contours.



dimensional graph with matric suction contoured as the third variable (Fig. 1*b*). A plot of the ordinate intercepts of the various matric suction contours on a τ versus $(u_a - u_w)$ graph gives the friction angle, ϕ^b (Fig. 1c).

The unsaturated soil is therefore visualized as having two components of cohesion (Figs. 1*b* and 1c).

$$[6] \quad c = c' + (u_a - u_w) \tan \phi^b$$

The second term in eq. [6] accounts for the increase in shear strength due to suction (Fredlund 1979). Once again, as a soil approaches saturation, the second component goes to zero and the shear strength equation takes the form commonly used for saturated soils.

Volume change relations

The volume change constitutive relations for an unsaturated soil can be represented using the $(\sigma - u_a)$ and $(u_a - u_w)$ stress state variables and the volume - mass soil properties. Volumetric continuity requires that two independent constitutive relations be written. One constitutive equation describes the deformation of the soil structure in terms of the change in void ratio, d*e*.

$$[7] \quad de = a_t \, d(\sigma - u_a) + a_m \, d(u_a - u_w)$$

where:

e = the void ratio,
a_t = the coefficient of compressibility with respect to a change in $(\sigma - u_a)$, and
a_m = the coefficient of compressibility with respect to a change in $(u_a - u_w)$.

A second constitutive relation describes the change in water content, d*w*,

$$[8] \quad dw = b_t \, d(\sigma - u_a) + b_m \, d(u_a - u_w)$$

where:

w = the water content,
b_t = the coefficient of water content change with respect to a change in $(\sigma - u_a)$, and
b_m = the coefficient of water content change with respect to a change in $(u_a - u_w)$.

Changes in the volume of air in the soil can be obtained by computing the difference between the change in void ratio and the change in water content.

The uniqueness of the proposed constitutive relations has been verified by several researchers (Matyas and Radhakrishna 1968; Barden et al. 1969; Fredlund and Morgenstern 1976). The volume change constitutive relations are particularly useful for heave predictions in swelling soils (Fredlund et al. 1980; Yoshida *et al.* 1983). Transient flow problems such as consolidation can also be formulated utilizing the volume change equations together with the flow laws (Fredlund and Hasan 1979; Fredlund 1982; Dakshanamurthy et al. 1984).

Heave or swelling is a problem encountered with some residual soils. However, equally as common is the seldom addressed problem of soil collapse in residual soils. Although little research has been conducted in this area, it is the authors' opinion that the soil collapse phenomenon can best be understood in terms of changes in the stress state variables.

Application of unsaturated soil theory to slope stability analysis

The following portion of this paper demonstrates the application of the unsaturated soil theory to the analysis of slope stability problems in residual soils in Hong Kong.

Hong Kong case histories

The surficial soils in Hong Kong are unsaturated, residual soils mainly consisting of decomposed granite and decomposed volcanic rock (rhyolite). The residual soils extend to a considerable depth due to active weathering processes. The water table is located far below ground surface (e.g., 20 metres).

The stability of numerous steep slopes in residual soils has been a major problem in Hong Kong. Stability analyses performed on steep slopes, assuming saturated conditions, often resulted in a factor of safety less than 1. However, the slopes may not show signs of distress. The soil suction throughout a considerable depth of the soil profile has been shown to be the significant factor in maintaining the stability of these slopes (Sweeney and Robertson 1979; Ho and Fredlund 1982; Ching et al. 1984). Nevertheless, slope failures have frequently occurred during or after periods of heavy and prolonged rainfall. This can be attributed to the reduction in soil strength as a result of a loss of suction due to infiltration (Lumb 1975).

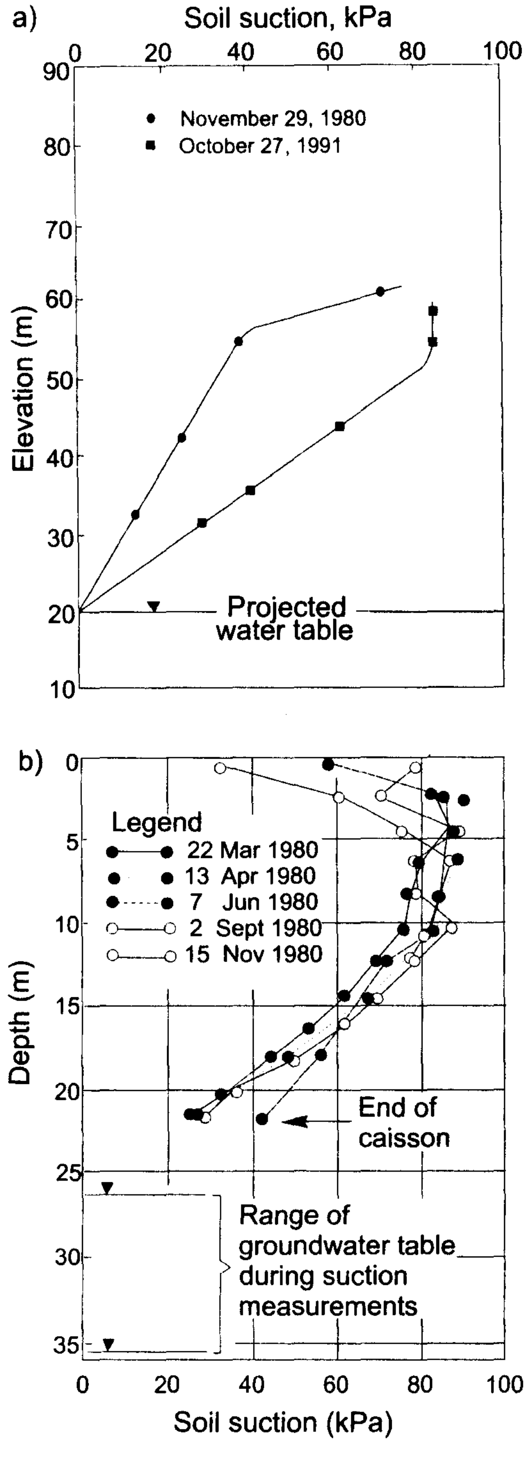
In situ measurements

The *in situ* measurement of soil suction was performed at two sites in Hong Kong in order to better understand the range in the magnitude of the soil suction and its variation with respect to time and environment. Sweeney (1982) reported the measurement of *in situ* soil suctions using tensiometers at three cut slopes in Hong Kong.

Exploratory concrete-lined shafts were made in order to measure suction at different depths at the Thorpe Manor site. Suction measurements were made in small holes perpendicular to the slope at the Fung Fai Terrace site. The matric suction measurement using a tensiometer is obtained from equilibrium across a high air entry porous medium. Two types of tensiometers (i.e., "Quick Draw" Soilmoisture Probe and Soilmoisture Probe (No. 2100) with a flexible tube) were used in the measurement of soil suction. Figures 2*a* and 2*b* show the matric suction profiles obtained from Fung Fai Terrace and Thorpe Manor sites, respectively. The suction values reached magnitudes of approximately 80 to 100 kPa which is the limit of the tensiometer. In general, the position of the water table and rainfall variations determined the characteristic of the measured suction profiles.

The measured suctions at the Thorpe Manor site were relatively constant throughout the seasons, with variations

Fig. 2. *a*) Suction measurements on Fung Fai Terrace; *b*) Suction measurements on Thorpe Manor.



occurring primarily in the extreme upper portion of the profile and at the water table. This observation may not be applicable to all cases.

Laboratory tests

Laboratory tests were performed on undisturbed samples to measure the effect of suction on shear strength. In other words, the shear strength parameter, ϕ^b, as shown in Fig. 1c was quantified for the two Hong Kong residual soils.

Multi-stage triaxial tests were conducted on ten decomposed granites, and seven decomposed volcanics by Ho and Fredlund (1982). The tests were performed by controlling the pore-air and pore-water pressures so as to maintain a constant suction in the sample during each stage of the test. The axis-translation technique was used in order to apply suctions higher than one atmosphere.

The results showed that the increase in shear strength due to an increase in soil suction was in accordance with eq. [6]. The average angle of ϕ^b was found to be 15.3° for the decomposed granite, and 13.8° for the decomposed volcanic rock.

Slope stability analysis

The effect of soil suction on the computed factor of safety for the two slopes in Hong Kong, (i.e., Fung Fai Terrace and Thorpe Manor), was demonstrated by Ching et al. (1984). There is no need to reformulate the factor of safety equations when dealing with unsaturated soils, since the matric suction term can be considered as part of the cohesion of the soil.

Values of c' and ϕ' for the Hong Kong residual soils have been reported by Lumb (1962, 1965), and ϕ^b values measured by Ho and Fredlund (1982) were used in the analysis.

A cross section of the Fung Fai Terrace is shown in Fig. 3*a*, and the strength properties of each soil stratum are listed in Table 1. Several slope stability analyses were performed for various percentages of the negative hydrostatic profile. The analyses (Fig. 4) show an increase in the factor of safety as the matric suction value increases. For the saturated case the factor of safety computed was 0.86, which indicated instability of the slope. The slope, however, had remained stable, likely due to matric suction.

Several slope stability analyses were also performed based on the *in situ* suction measurements (Fig. 2*a*). The factor of safety was 1.07 based on the suction measurements on November 29, 1980, and 0.98 based on the suction measurements on October 27, 1981.

Figure 3*b* shows a cross-section of Thorpe Manor, and Table 2 gives the strength properties of each soil stratum. The results of the analyses are presented in Fig. 4, and indicate the increase in factor of safety with respect to an increase in matric suction. The analysis, using a saturated soil parameters resulted in a factor of safety of 1.05. When the *in situ* matric suction values are used (Fig. 2*b*), the factor of safety increased to 1.25.

Conclusions

The theory of unsaturated soil behavior provides a general conceptual framework for analyzing practical problems involving residual soils. It takes the form of an extension of saturated soil concepts with a smooth transition between the two cases.

Table 1. Strength properties of soils at Fung Fai Terrace.

Soil type	Unit weight (kN/m^3)	c' (kPa)	ϕ' (degrees)	ϕ^b (degrees)
Colluvium	19.6	10.0	35.0	15.0
Completely weathered granite	19.6	15.1	35.2	15
Completely to highly weathered granite	19.6	23.5	41.5	15.0

Table 2. Strength properties of soils at Thorpe Manor.

Soil type	Unit weight (kN/m^3)	c' (kPa)	ϕ' (degrees)	ϕ^b (degrees)
Completely weathered rhyolite	18.4	10.1	42.6	12.0
Completely to highly weathered rhyolite	21.4	12.0	43.9	12.0

Fig. 3. *a*) Fung Fai Terrace cross section; *b*) Thorpe Manor cross section.

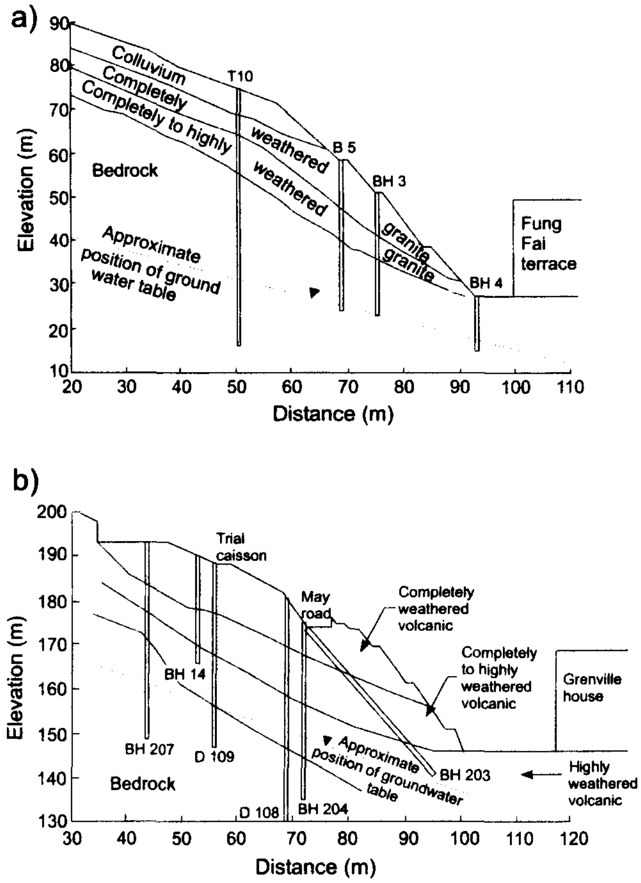
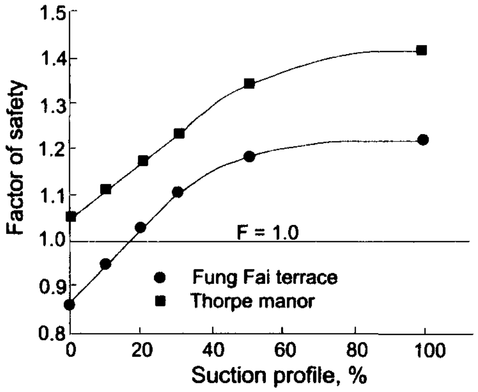


Fig. 4. Factor of safety versus suction profile for two Hong Kong slopes.



The application of the proposed theory in solving geotechnical problems in unsaturated, residual soils has been limited but has proven to be extremely useful. The influence of soil suction on shear strength can be quantified and applied to the slope stability problem. More case histories are required to further verify the proposed theory in practice. In addition, there is a great need to improve our ability to measure negative pore-water pressures.

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References

Barden, L., Madedor, A.O., and Sides, G.R. 1969. Volume change characteristics of unsaturated clay. ASCE, Journal of Soil Mechanics and Foundation Division, **95**(SM1): 33–52.

Blight, G.E. 1971. Flow of air through soils. ASCE, Journal of Soil Mechanics and Foundation Division, **97**: 607–624.

Childs, E.C., and Collis-George, N. 1950. The permeability of porous materials. Proceedings of the Royal Society of London, Series A, **201**: 392–405.

Ching, R.K.H., Sweeney, D.J., and Fredlund, D.G. 1984. Increase in factor of safety due to soil suction for two Hong Kong slopes. Proceedings, Fourth International Symposium on Landslides, Toronto, Canada, Vol. 1, pp. 617–623.

Dakshanamurthy, V., Fredlund, D.G., and Rahardjo, H. 1984. Coupled three-dimensional consolidation theory of unsaturated porous media. Proceedings, Fifth International Conference on Expansive Soils, Adelaide, South Australia, Institute of Engineers, Australia, pp. 99–103.

Fredlund, D.G. 1979. Second Canadian Geotechnical Colloquium: Appropriate concepts and technology for unsaturated soils. Canadian Geotechnical Journal, **16**(1): 121–139.

Fredlund, D.G. 1982. Consolidation of unsaturated porous media. Proceedings, Symposium on the Mechanics of Fluid in Porous Media-New Approaches in Research, NATO Advance Study Institute, University of Delaware, Newark, DL.

Fredlund, D.G., and Hasan, J.U. 1979. One-dimensional consolidation theory: Unsaturated soils. Canadian Geotechnical Journal, **16**(3): 521–531.

Fredlund, D.G., and Morgenstern, N.R. 1976. Constitutive relations for volume change in unsaturated soils. Canadian Geotechnical Journal, **13**(3): 261–276.

Fredlund, D.G., and Morgenstern, N.R. 1977. Stress state variables for unsaturated soils. ASCE, Journal of Geotechnical Engineering Division, **103**(GT5): 447–466.

Fredlund, D.G., Hasan, J.U., and Filson, H. 1980. The prediction of total heave. Proceedings, 4th International Conference on Expansive Soils, Denver, CO, Vol. 1, pp. 1–17.

Fredlund, D.G., Morgenstern, N.R., and Widger, R.A. 1978. The shear strength of unsaturated soils. Canadian Geotechnical Journal, **15**(3): 313–321.

Freeze, R.A., and Cherry, J.A. 1979. Groundwater. Prentice-Hall Inc., Englewood Cliffs, NJ.

Gardner, W.R. 1958. Some steady state solutions of the unsaturated moisture flow equations with application to evaporation from a water-table. Soil Science, **85**(4):228–232.

Ho, D.Y.F., and Fredlund, D.G. 1982. A multi-stage triaxial test for unsaturated soils. ASTM Geotechnical Testing Journal, **5**(1/2): 18–25.

Lumb, P. 1962. The properties of decomposed granite. Géotechnique, **12**: 226–243.

Lumb, P. 1965. The residual soils of Hong Kong. Géotechnique, **15**: 180–194.

Lumb, P. 1975. Slope failures in Hong Kong. Quarterly Journal of Engineering Geology, **8**: 31–65.

Matyas, E.L., and Radhakrishna, H.S. 1968. Volume change characteristics of partially saturated soils. Géotechnique, **18**(4): 432–448.

Sweeney, D.J. 1982. Some *in situ* soil suction measurements in Hong Kong's residual soil slopes. Proceedings, 7th Southeast Asia Geotechnical Conference, Hong Kong, Vol. 1, pp. 91–106.

Sweeney, D.J., and Robertson, P. 1979. A fundamental approach to slope stability in Hong Kong. Hong Kong Engineering, pp. 35–44.

Yoshida, R., Fredlund, D.G., and Hamilton, J.J. 1983. The prediction of total heave of a slap-on-ground floor on Reginal clay. Canadian Geotechnical Journal, **20**(1): 69–81.