PROCEEDINGS OF THE ELEVENTH INTERNATIONAL CONFERENCE ON SOIL MECHANICS AND FOUNDATION ENGINEERING

SAN FRANCISCO / 12-16 AUGUST 1985

EDITOR: PUBLICATIONS COMMITTEE OF XI ICSMFE

VOLUME 2
TECHNICAL PAPERS
EXPOSES TECHNIQUES

COMPTES RENDUS DU ONZIEME CONGRES INTERNATIONAL DE MECANIQUE DES SOLS ET DES TRAVAUX DE FONDATIONS

SAN FRANCISCO / 12-16 AOUT 1985

EDITEUR: COMITE DES PUBLICATIONS DU XI CIMSTF

A.A.BALKEMA / ROTTERDAM / BOSTON / 1985

Soil mechanics principles that embrace unsaturated soils

Les principes de la mécanique du sol qui comprennent les sols non-saturés

D. G. FREDLUND, Professor of Civil Engineering, Department of Civil Engineering,
University of Saskatchewan, Saskatoon, Saskatchewan, Canada

SYNOPSIS The classic saturated soil mechanics principles and equations have been extended to embrace the behavior of unsaturated soils. Equations are presented for fluid flow, shear strength and volume change. These equations are applied to practical problems such as steady state seepage, the consolidation process and slope stability analysis. In each case the accepted equations for saturated soils become a special case of the more general unsaturated soil mechanics equations.

INTRODUCTION

An understanding of the behavior of unsaturated soils is important in dealing with numerous problems encountered in geotechnical engineering. For example, the heave associated with unsaturated, expansive soils sustains an enormous economic drain on society (Krohn and Slosson, 1980). For another example, the shear strength of unsaturated, residual soils is important in understanding slope stability problems related to changes in soil suction. There are numerous other applications related to compacted soils in highways, airport runways, railroads and other earth structures.

In retrospect, the first ISSMFE conference in 1936 provided a forum for the establishment of principles and equations relevant to saturated soil mechanics. These principles and equations have remained pivotal throughout subsequent decades of research. This same conference was also a forum for numerous research papers on unsaturated soil behavior. Unfortunately a parallel set of principles and equations did not immediately imerge for unsaturated soils. As a result, a science and technology for unsaturated soils has been slow to develop (Fredlund, 1979). Not until the research at Imperial College in the late 1950's did a science for unsaturated soils begin to appear (Bishop, 1959).

Saturated soil mechanics theory and technology have developed around a few classic equations. These can be summarized as: i) the flow equation, ii) the shear strength equation, and iii) the volume change constitutive equation. The shear strength and volume change equations are written in terms of a stress variable controlling the behavior of the soil structure. This stress variable, $(\sigma - u_w)$, is referred to as the effective stress of the soil where σ = total stress and u_w = pore-water pressure. Uniqueness has been demonstrated for both the shear strength and volume change equations relative to specific stress paths. The flow equation was written in terms of the total head in the water phase.

This paper summarizes a set of classic equations for unsaturated soils which are a logical extension of those accepted for saturated soils. The research confirming each of the equations has been presented elsewhere and this paper synthesizes the results in terms of a limited number of equations. The objective of this paper is to provide a consistent set of classic equations for unsaturated soil behavior which can be widely accepted.

RELATING SOIL BEHAVIOR TO STRESS VARIABLES

The shear strength and volume change behavior of saturated soils has been related to the $(\sigma - u_w)$ stress variable. The complete description of the stress state takes the form of a matrix.

$$\begin{bmatrix} (\sigma_x - u_w) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & (\sigma_y - u_w) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & (\sigma_z - u_w) \end{bmatrix} \quad [1]$$

$\sigma_x, \sigma_y, \sigma_z$, are the total stress in the x, y and z - directions, respectively and $\tau_{xy}, \tau_{yx}, \tau_{xz}, \tau_{zx}, \tau_{zy}, \tau_{yz}$ are shear stresses. In practice, it is not necessary to relate mechanical behavior of the saturated soil to all components of the stress state when specific stress paths are selected. Theoretically, the first, second and third stress invariants form a more fundamental basis for describing mechanical behavior.

The shear strength and volume change behavior of an unsaturated soil can best be described in terms of two independent stress variables; namely, $(\sigma - u_a)$ and $(u_a - u_w)$. The term $(u_a - u_w)$ is referred to as matric suction where u_a = pore-air pressure (Fredlund and Morgenstern, 1977). The complete description of the stress state takes the form of two matrices.

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & (\sigma_y - u_a) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & (\sigma_z - u_a) \end{bmatrix} \quad [2]$$

and

$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix} \quad [3]$$

The first, second and third stress invariants can be written for each matrix. However, in practice it is not necessary to relate mechanical behavior to all the stress variables provided a selected stress path is used for both the analysis and for measuring the soil parameters.

The above stress variables for an unsaturated soil also yield a smooth transition to the saturated case. As the degree of saturation approaches 100 percent, the pore-air pressure approaches the pore-water pressure. Therefore, the matric suction term goes to zero and the pore-air pressure term in the first stress matrix becomes the pore-water pressure.

FLOW LAWS AND STEADY STATE SEEPAGE

The driving potential for flow of the water phase of a saturated soil is the hydraulic head (i.e., elevation head plus pressure head). This is also true for an unsaturated soil.

Flow of water in a saturated soil is commonly described using Darcy's law.

$$v_w = - k_w \frac{\partial h_w}{\partial y} \quad [4]$$

where v_w = velocity; k_w = coefficient of permeability for water, and y = coordinate direction.

Darcy's law can also be used for unsaturated soils. However, the coefficient of permeability varies significantly with negative pore-water pressure head and must be assumed to be a variable in formulations. The most common form for the variation of permeability with negative pressure head is that proposed by Gardner (1958) (Figure 1):

$$k_w = \frac{k_s}{1 + a \left(\frac{u_a - u_w}{\rho_w g}\right)^n} \quad [5]$$

where k_s = saturated coefficient of permeability; ρ_w = density of water; g = gravity acceleration, and a and n = material parameters. Equation [5] can be indirectly computed from the results of a suction versus water content test and a saturated permeability test. The form of the equation is particularly useful for sands and silts. More research is required on clayey soils which tend to crack and form a secondary structure as suction increases.

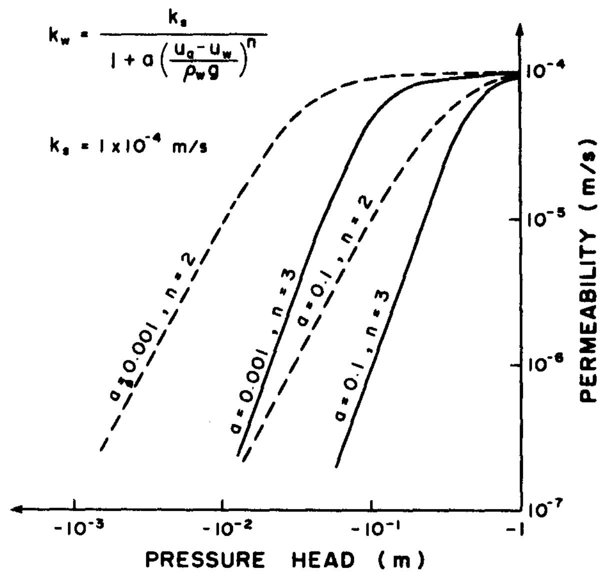


FIG. 1 GARDNER'S EQUATION RELATING PERMEABILITY TO NEGATIVE PORE-WATER HEAD

For some problems it may be necessary to describe the flow of air through an unsaturated soil. This requires an additional flow law; namely, Fick's law.

$$v_a = -D^* \frac{\partial u_a}{\partial y} \quad [6]$$

where v_a = mass rate of air flow and, D^* = constant of proportionality.

The classic, two-dimensional, steady state seepage equation for an isotropic saturated soil is called the La Placian equation.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad [7]$$

For an unsaturated soil, the variation in permeability with negative head results in an expansion of the above equation.

$$k_w \frac{\partial^2 h}{\partial x^2} + \frac{\partial k_w}{\partial x}\frac{\partial h}{\partial x} + k_w \frac{\partial^2 h}{\partial y^2} + \frac{\partial k_w}{\partial y}\frac{\partial h}{\partial y} + \frac{\partial k_w}{\partial y} = 0 \quad [8]$$

The second and fourth terms in equation [8] account for the spatial variation in permeability while the fifth term is referred to as a gravity term. Equation [8] is a nonlinear partial differential equation which must also satisfy the permeability equation (e.g., equation [5]).

SHEAR STRENGTH AND SLOPE STABILITY ANALYSIS

The classic shear strength equation for a saturated soil is written in terms of the effective normal stress and takes the form of a Mohr-Coulomb failure criteria.

$$\tau = c' + (\sigma_n - u_w) \tan \phi' \quad [9]$$

where τ = shear strength; c' = effective cohesion intercept; σ_n = total normal stress, and ϕ' = effective angle of internal friction.

For an unsaturated soil, the shear strength equation takes the form of a three-dimensional extension of the Mohr-Coulomb failure criteria (Figure 2).

$$\tau = c' + (u_a - u_w)\tan \phi^b + (\sigma_n - u_a) \tan \phi' \quad [10]$$

where ϕ^b = angle of shear strength increase with an increase in $(u_a - u_w)$.

The angle, ϕ^b, has been measured on numerous soils from various countries of the world. Table I summarizes a few of the experimental results.

The value for ϕ^b is consistently less than ϕ' and appears to commonly be in the range of 15 degrees. Various laboratory testing procedures have been used but further research is still required.

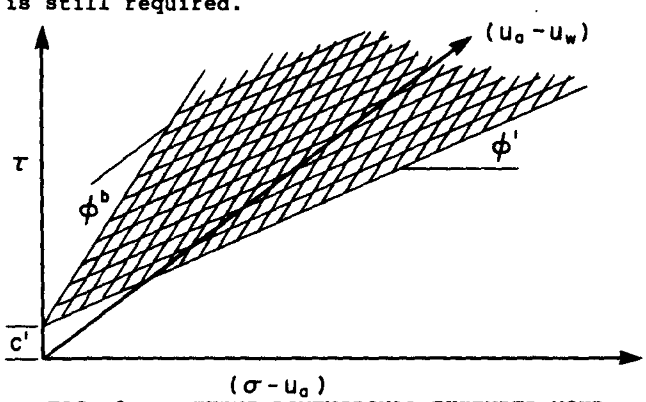


FIG. 2a THREE-DIMENSIONAL EXTENDED MOHR COULOMB FAILURE SURFACE

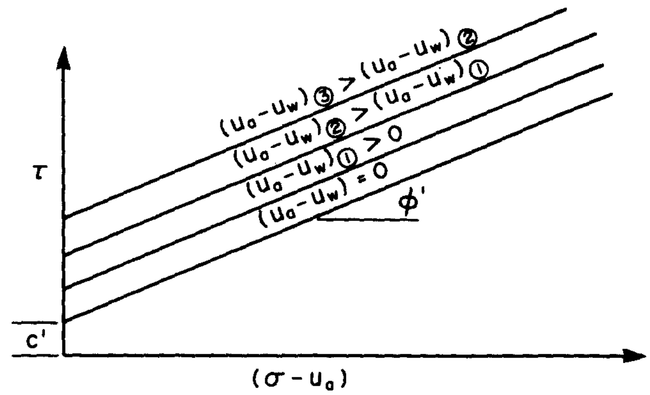


FIG. 2b EFFECT OF MATRIC SUCTION CONTOURED ON FAILURE ENVELOPE

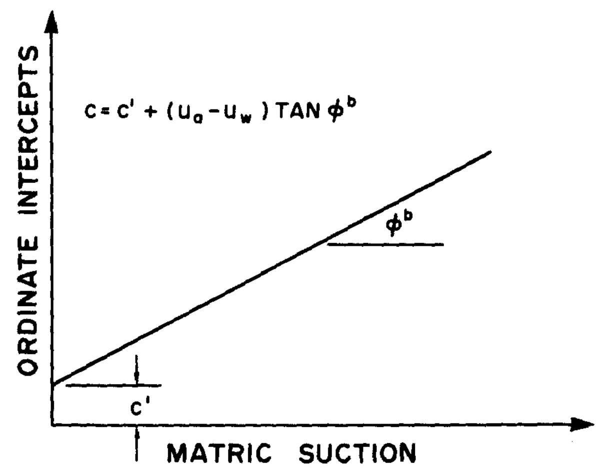


FIG. 2c INCREASE IN SHEAR STRENGTH DUE TO SUCTION

FIG. 2 EXTENDED MOHR COULOMB FAILURE CRITERIA FOR AN UNSATURATED SOIL

The unsaturated soil can be visualized as having two components of cohesion

$$c = c' + (u_a - u_w) \tan \phi^b \quad [11]$$

When the matric suction goes to zero, equation [10] reverts to equation [9] for a saturated soil.

Not only is there a smooth transition between the saturated and unsaturated soil cases; the shear strength equation takes on the same form in both cases. This means that the same factor of safety equations used for a saturated soil can also be used for an unsaturated soil. It is merely necessary to make the cohesion of the soil a function of the negative pore-water pressure. Such analysis have already been applied to unsaturated soil slopes in Hong Kong (Ching, Sweeney and Fredlund, 1984) and other regions (Fontoura, DeCompos and Costa Filho, 1984).

TABLE I

Experimental Values for ϕ^b

Soil Type	ϕ^b [Degrees]	Test Procedure	Reference
Compacted Shale; w = 18.6%	18.1	Constant Water Content Triaxial	Bishop, Alpan, Donald and Blight, 1960
Boulder Clay; w = 11.6%	21.7	Constant Water Content Triaxial	Bishop, Alpan, Donald and Blight, 1960
Dhanauri Clay; w = 22.2%, γ_d = 15.5 kN/m^3	16.2	Consolidated Drained Triaxial	Satija, 1978
Dhanauri Clay; w = 22.2%, γ_d = 14.5 kN/m^3	12.6	Consolidated Drained Triaxial	Satija, 1978
Dhanauri Clay; w = 22.2%, γ_d = 15.5 kN/m^3	22.6	Constant Water Content Triaxial	Satija, 1978
Dhanauri Clay; w = 22.2%, γ_d = 14.5 kN/m^3	16.5	Constant Water Content Triaxial	Satija, 1978
Madrid Gray Clay; w = 29%, γ_d = 13.1 kN/m^3	16.1	Consolidated Drained, Direct Shear	Escario, 1980
Undisturbed Decomposed Granite; Hong Kong	15.3	Consolidated Drained, Multi-Stage Triaxial	Ho and Fredlund, 1982
Undisturbed Decomposed Rhyolite; Hong Kong	13.8	Consolidated Drained, Multi-Stage Triaxial	Ho and Fredlund, 1982
Cranbrook Silt	16.5	Consolidated Drained, Multi-Stage Triaxial	Fredlund (unpublished)

The results have shown that relatively small magnitudes of $(u_a - u_w)$ can produce substantial increases in the computed factor of safety.

CONSTITUTIVE RELATIONS

The classic volume change constitutive relations for saturated soils can be presented in one of several forms; namely the elasticity form, the compressibility form and the soil mechanics terminology forms. The elasticity form for saturated soils defines the linear strain of the soil structure in three orthogonal directions for a linear, elastic material.

$$\varepsilon_x = \frac{(\sigma_x - u_w)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_w)$$

$$\varepsilon_y = \frac{(\sigma_y - u_w)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_w) \qquad [12]$$

$$\varepsilon_z = \frac{(\sigma_z - u_w)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_w)$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ = strain in the x, y and z - directions; E = Young's modulus, and μ = Poisson's ratio.

The compressibility form for a saturated soil is as follows:

$$\varepsilon = m_v\, d(\sigma - u_w) \qquad [13]$$

where m_v = coefficient of volume compressibility and ε = volumetric strain; equal to $\varepsilon_x + \varepsilon_y + \varepsilon_z$. The compressibility modulus, m_v, can be written in terms of the elastic parameters, E and μ, for K_o and other loading conditions.

Using soil mechanics terminology, the volume change for a saturated soil is written as:

$$de = a_v\, d(\sigma_y - u_w) \qquad [14]$$

where e = void ratio; a_v = coefficient of compressibility.

Constitutive relations for unsaturated soils have been written which are extensions of those presented above. It is also necessary to use more than one constitutive relation for an unsaturated soil because of the volumetric continuity requirement for a referential type element.

$$\varepsilon = \theta_w + \theta_a \qquad [15]$$

where θ_w = net inflow or outflow of water for an element; θ_a = change in volume of air due to flow or compression.

The elasticity form for linear strain in the soil structure of an unsaturated soil can be written as a linear combination of the effects of each stress variable.

$$\varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H}$$

$$\varepsilon_y = \frac{(\sigma_y - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H} \quad [16]$$

$$\varepsilon_z = \frac{(\sigma_z - u_a)}{E} - \frac{\mu}{E}(\sigma_x + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H}$$

where H = elastic modulus with respect to $(u_a - u_w)$. The E modulus must now be restricted to a change in the $(\sigma - u_a)$ stress variable.

A second independent, elasticity form of constitutive equation can be written for the water phase.

$$\theta_w = \frac{(\sigma_x + \sigma_y + \sigma_z - 3u_a)}{3H'} + \frac{(u_a - u_w)}{R} \quad [17]$$

where H' = water phase (elasticity type) parameter with respect to $(\sigma_y - u_a)$; R = water phase (elasticity type) parameter with respect to $(u_a - u_w)$. The form of the above equations is the same as that first presented by Biot (1941) and Coleman (1962) and later by Fredlund and Morgenstern (1976).

The compressibility form of the constitutive equation for an unsaturated soil can be written as follows.

$$\varepsilon = m_1^s\, d(\sigma - u_a) + m_2^s\, d(u_a - u_w) \quad [18]$$

where m_1^s = compressibility of the soil structure with respect to a change in $(\sigma - u_a)$; m_2^s = compressibility of the soil structure with respect to a change in $(u_a - u_w)$. A second compressibility form constitutive equation can be written for the water phase.

$$\theta_w = m_1^w\, d(\sigma_y - u_a) + m_2^w\, d(u_a - u_w) \quad [19]$$

where m_1^w = slope of the $(\sigma_y - u_a)$ versus θ_w plot; m_2^w = slope of the $(u_a - u_w)$ versus θ_w plot.

A compressibility form of constitutive equation for the air phase can be written as the difference between the soil structure and water phase constitutive equations.

Using soil mechanics terminology, the change in void ratio, de, of an unsaturated soil can be written:

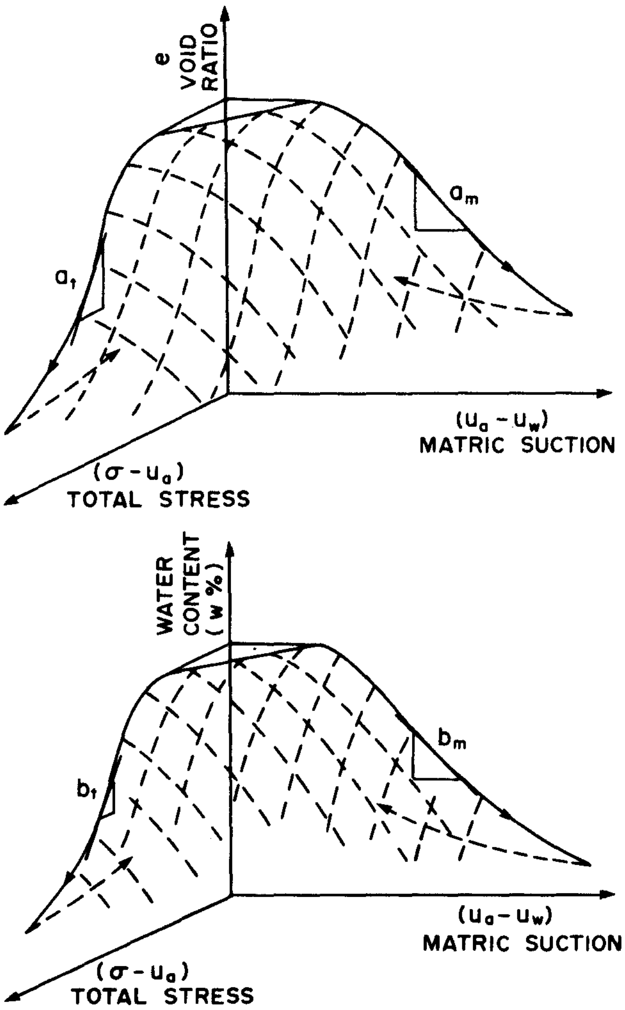
$$de = a_t\, d(\sigma_y - u_a) + a_m\, d(u_a - u_w) \quad [20]$$

where a_t = coefficient of compressibility with respect to a change in $(\sigma_y - u_a)$; a_m = coefficient of compressibility with respect to a change in $(u_a - u_w)$. A second equation describing the change in water content, dw, of the soil can be written as:

$$dw = b_t\, d(\sigma_y - u_a) + b_m\, d(u_a - u_w) \quad [21]$$

where b_t = coefficient of water content change with respect to $(\sigma_y - u_a)$; b_m = coefficient of water content change with respect to $(u_a - u_w)$. The above constitutive equations can be visualized as three-dimensional plots with each abscissa representing a stress variable and the ordinate representing the soil property (Figure 3). These can also be reduced to two-dimensional plots which readily show the relationship between the various moduli (Figure 3). It is also possible to plot both two-dimensional plots on one plot by using the variables Se and wG_s as ordinate variables where S = degree of saturation; w = water content and G_s = specific gravity. The void ratio and water content constitutive relations can be largely linearized by plotting the logarithm of the stress variables.

The loading curve for $(\sigma_y - u_a)$ versus void ratio can readily be measured using conventional soil testing equipment (i.e., a_t or m_1^s). Equipment commonly used in soil science can be used to measure the relationship between water content and $(u_a - u_w)$ (i.e., b_m or m_2^w). Some research has been done on the remaining two relationships (i.e., (a_m or m_2^s) and (b_t or m_1^w) (Fredlund, 1979). Sufficient research has already been done to verify the uniqueness of the proposed forms for the constitutive equations (Matyas and Radhakrishna, 1968; Barden, Madedor and Sides, 1969; Fredlund and Morgenstern, 1976).



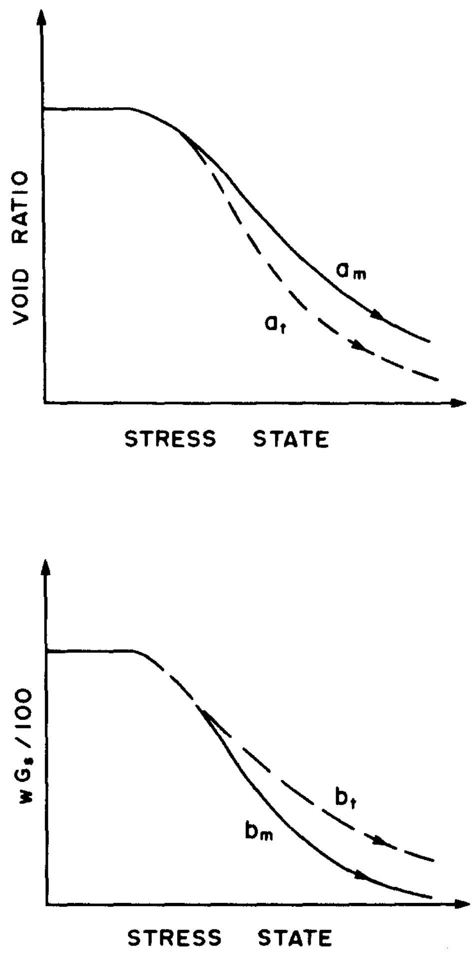


FIG. 3a THREE-DIMENSIONAL VOID RATIO AND WATER CONTENT CONSTITUTIVE SURFACES

FIG. 3b TWO-DIMENSIONAL COMPARISON OF COMPRESSIBILITY MODULI

FIG. 3 CONSTITUTIVE SURFACES FOR AN UNSATURATED SOIL

CONSOLIDATION THEORY

The flow laws and constitutive equations provide the necessary physical relations for the formulation of transient flow problems. The one-dimensional formulation is presented to demonstrate the extension of the saturated soil case to that for an unsaturated soil. The Terzaghi equation for a saturated soil is derived by equating the time differential of the constitutive equation to the divergence of velocity.

$$\frac{\partial u_w}{\partial t} = c_v \frac{\partial^2 u_w}{\partial y^2} \qquad [22]$$

where t = time; c_v = coefficient of consolidation. For an unsaturated soil there is the possibility of flow in both the water and air phases. Two partial differential equations can be derived considering the continuity of the water and air phases respectively (Fredlund and Hasan, 1979; Fredlund, 1982). The water phase partial differential equation is as follows:

$$\frac{\partial u_w}{\partial t} = c_v^w \frac{\partial^2 u_w}{\partial y^2} - C_w \frac{\partial u_a}{\partial t} + \frac{c_v^w}{k_w} \frac{\partial k_w}{\partial y} \frac{\partial u_w}{\partial y} + c_g \frac{\partial k_w}{\partial y} \qquad [23]$$

where c_v^w = coefficient of consolidation with respect to the water phase; C_w = interactive constant associated with the water phase equation; c_g = gravity term constant. The second term to the right of the equal sign arises because of an interaction between simultaneous air and water flow from the soil. The third term to the right of the equal sign must be retained since the permeability can vary significantly with space during the consolidation process.

A smooth, mathematical transition to the saturated case occurs when the coefficient of permeability becomes a constant and the gradient in the air phase disappears.

The air phase partial differential equation is as follows:

$$\frac{\partial u_a}{\partial t} = c_v^a \frac{\partial^2 u_a}{\partial y^2} - C_a \frac{\partial u_w}{\partial t} + \frac{c_v^a}{D^*} \frac{\partial D^*}{\partial y} \frac{\partial u_a}{\partial y} \qquad [24]$$

where c_v^a = coefficient of consolidation with respect to the air phase; C_a = interactive constant associated with the air phase equation. The pore-water and pore-air pressures can be solved simultaneously for all depths and elapsed times. In many practical cases, the gradient in the air phase can be assumed to be negligent in which case only the water phase partial differential equation needs to be solved.

PORE PRESSURE PARAMETERS

The A and B pore pressure parameters are used to predict excess pore pressures induced during the undrained loading of a soil. For an unsaturated soil, pore pressure parameters are required for both the water and air phases (Hasan and Fredlund, 1980). The general form of these equations is the same as for a saturated soil (Skempton, 1954; Bishop and Henkel, 1962).

$$\Delta u_w = B_w \{\Delta\sigma_3 + A_w(\Delta\sigma_1 - \Delta\sigma_3)\} \qquad [25]$$

$$\Delta u_a = B_a \{\Delta\sigma_3 + A_a(\Delta\sigma_1 - \Delta\sigma_3)\} \qquad [26]$$

The B pore pressure parameter can be written in terms of the constitutive equations and the density and compressibility equations for air-water mixtures (Fredlund, 1976). These, in turn, make use of Boyles' and Henry's gas laws. The effect of Henry's law may or may not be included depending on whether the undrained loading is short or long term.

The B_w and B_a parameters can be solved for simultaneously when equations [25] and [26] are written in terms of the compressibility of the soil and air-water mixtures. Earlier equations published by Hilf (1949) and Bishop (1957) are simplifications of the more rigorous formulation by Hasan and Fredlund (1980). Once again the more rigorous equations show a smooth transition to the case of a saturated soil.

MEASUREMENT OF PORE-WATER PRESSURE

A technology cannot survive and grow simply on sound, theoretical equations. It must also be possible to measure the necessary stress variables and soil properties in an economically viable manner. This has been possible in the case of saturated soils.

For unsaturated soils, the pore-water pressure is negative and its measurement has remained the primary stumbling block in advancing a suitable technology. However, in recent years significant advances have been made in the measurement of negative pore-water pressures. A detailed review of the research in this area is outside the scope of this paper. However, the author would encourage that review and research be conducted on this subject.

Recent research on the thermal conductivity type gauges has shown promise for geotechnical engineering (Picornell, Lytton and Steinberg, 1983; Lee and Fredlund, 1984). However, further studies are required prior to its endorsement.

The axis-translation technique (Hilf, 1949) has proven to be satisfactory for numerous laboratory studies. However, the technique cannot be used for insitu measurements. The measurement of the (corrected) swelling pressure of soils has also been used as an indirect measurement of matric suction (Fredlund, Hasan and Filson, 1980).

CONCLUSION

A suitable science for unsaturated soils is presently available. It can be visualized as a logical extension of classic saturated soil mechanics principles and equations. The greatest need in the ensuing decade is to improve our ability to measure negative pore-water pressure. Also requiring further studies are the procedures for the measurement of relevant soil parameters. These stress and soil parameter measurements, combined with the summarized equations can form the basis for much needed case histories concerning all aspects of unsaturated soil behavior.

REFERENCES

Barden, L., Madedor, A.O., and Sides, G.R. (1969) "Volume Change Characteristics of Unsaturated Clay," ASCE Journal of the Soil Mechanics and Foundation Division, 95 (SM1), pp. 33-52.

Biot, M.A. (1941) "General Theory of Three-Dimensional Consolidation," Journal of Applied Physics, 12(2), p. 155-164.

Bishop, A.W., (1959) "The Principle of Effective Stress," Technisk Ukeflad, No. 39.

Bishop, A.W., Alpan, I., Blight, G.E., and Donald, I.B. (1960) "Factors Contolling the Strength of Partly Saturated Cohesive Soils," Proceedings of the ASCE Research Conference on Shear Strength of Cohesive Soils, Boulder, Colorado, U.S.A., pp. 503-532.

Bishop, A.W. and Henkel, D.J. (1962) "The Measurement of Soil Properties in the Triaxial Test," Edward Arnold Ltd., London, England, 2nd Edition.

Ching, R.K.H., Sweeney, D.J. and Fredlund D.G. (1984) "Increase in Factor of Safety Due to Soil Suction for Two Hong Kong Slopes," Proceedings of the IV Int. Symposium on Landslides, Toronto, Canada.

Coleman, J.D. (1962) "Stress/Strain Relations for Partly Saturated Soils," Correspondence in Geotechnique, 12(4), pp. 348-350.

Escario, V., (1980) "Suction Controlled Penetration and Shear Tests," Proceedings of the Fourth International Expansive Soil Conference, Denver, Colorado, U.S.A.

Fontoura, A.B., De Compos, L.E., and Costa Filho, L.M. (1984) "A Reanalysis of some Slides in Gneissic Residual Soils", Proceedings of the IV Int. Symposium on Landslides, Toronto, Canada.

Fredlund, D.G. (1976) "Density and Compressibility Characteristics of Air-Water Mixtures," Canadian Geotechnical Journal, 13, pp. 386-396.

Fredlund, D.G. (1979) "Appropriate Concepts and Technology for Unsaturated Soils," Second Canadian Geotechnical Colloquium, Canadian Geotechnical Journal, Vol. 16, No. 1, pp. 121-139.

Fredlund, D.G. and Morgenstern, N.R. (1976) "Constitutive Relations for Volume Change in Unsaturated Soils," Canadian Geotechnical Journal, 13, pp. 261-276.

Fredlund, D.G. and Morgenstern, N.R. (1977) "Stress State Variables for Unsaturated Soils," ASCE Journal of the Geotechnical Engineering Division, 103 (GT5), pp. 447-466.

Fredlund, D.G., Morgenstern, N.R. and Widger, R.A. (1978) "Shear Strength of Unsaturated Soils," Canadian Geotechnical Journal, 15, pp.313-321.

Fredlund, D.G. and Hasan, J.U. (1979) "One-Dimensional Consolidation Theory: Unsaturated Soils," Canadian Geotechnical Journal, Vol. 1, No. 3, pp. 521-531.

Fredlund, D.G., Hasan, J.U. and Filson, H.L. (1980) "The Prediction of Total Heave," Fourth International Conference on Expansive Soils, Denver, Colorado, U.S.A., pp. 1-17.

Fredlund, D.G. (1982) "Consolidation of Unsaturated Porous Media," Proceedings of the NATO Advanced Study Institute, Mechanics of Fluids in Porous Media, University of Delaware, U.S.A., July.

Gardner, W.R. (1958) "Laboratory Studies of Evaporation from Soil Columns in the Presence of Water Table," Soil Science of America, vol. 85, p. 244.

Hasan, J.U. and Fredlund, D.G. (1980) "Pore Pressure Parameters for Unsaturated Soils," Canadian Geotechnical Journal, Vol. 17, No. 3, pp. 395-404.

Hilf, J.W. (1949) "Estimating Construction Pore Pressures in Rolled Earth Dams," Proceedings, 2nd International Conference on Soil Mechanics and Foundation Engineering, Rotterdam, Vol. III, pp. 234-240.

Ho, D.Y.F. and Fredlund, D.G. (1982) "Increase in Strength Due to Suction for Two Hong Kong Soils," Proceedings of the Conference on Engineering and Construction in Tropical and Residual Soils, ASCE, Honolulu, Hawaii, January 11-15.

Krohn, J.P. and Slosson, J.E., (1980) "Assessment of Expansive Soils in the United States," Fourth International Conference on Expansive Soils, Denver, Colorado, pp. 596-608.

Lee, R.K.C. and Fredlund, D.G. (1984) "Measurement of Soil Suction Using the MCS 6000 Sensor," Fifth International Conference on Expansive Soils, Adelaide, South Australia, May 21-23, pp. 50-54.

Matyas, E.L., and Radhakrishna, H.S. (1968) "Volume Change Characteristics of Partially Saturated Soils," Geotechnique, 18(4), pp. 432-448.

Picornell, M., Lytton, R.L., and Steinberg, M., (1983) "Matrix Suction Instrumentation of a Vertical Moisture Barrier," Journal of Transportation, ASCE.

Satija, D.J. (1978) "Shear Behavior of Partially Saturated Soils," Unpublished Ph.D. thesis, Indian Institute of Technology, Delhi, India.

Skempton, A.W. (1954) "The Pore Pressure Coefficients, A and B," Geotechnique, 4(4), pp. 143-147.