General limit equilibrium method for lateral earth force

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The calculation of the lateral earth force using the limit equilibrium method of slices is an indeterminate problem. An assumption regarding the direction or the magnitude of certain forces, or the position of the line of thrust can be used to render the problem determinate.

A general formulation for the lateral earth force is derived in accordance with the assumptions involved in the general limit equilibrium (GLE) method. An assumption concerning a direction of the interslice forces is utilized to solve the problem of indeterminancy. Horizontal force equilibrium conditions within a sliding mass are used to compute the magnitude of the active and passive forces. The point of application of the lateral earth force is obtained by considering moment equilibrium for each slice.

The coefficient of lateral earth force obtained from the GLE method agrees closely with the results obtained from most other theories. Comparisons are made to the Coulomb theory (i.e., using a planar slip surface) and other theories using a curved or a composite slip surface.

Data are presented for the case of a horizontal cohesionless backfill against a vertical wall. The lateral earth force can be contoured on the grid of centers of rotation. These contours have a bell-shaped characteristic and can be used to locate the critical center of rotation.

The main advantage of this method lies in its capability to analyze arbitrarily stratified soil deposits with complex geometries. Different conditions of pore-water pressure, shear strength, and external loading can be accommodated in the analysis. Factors of safety greater than 1.0 can be applied to the shear strength of the soil for design purposes.

Keywords: lateral earth force, active force, passive force, general limit equilibrium, interslice forces, and coefficient of lateral earth force.

Le calcul des forces de poussée des terres au moyen de la méthode d'équilibre limite par tranche est un problème indéterminé. Une hypothèse concernant la direction ou l'intensité de certaines forces, ou la position de la ligne d'action, peut être faite pour rendre la problème déterminé.

Une formulation générale de la force de poussée des terres est établie en conformité avec les hypothèses incorporées dans la méthode générale d'équilibre limite (GEL). Une hypothèse sur la direction des forces intertranches est utilisée pour résoudre le problème d'interdétermination. Les conditions d'équilibre des forces horizontales dans la masse en glissement sont utilisées pour calculer l'intensité des forces actives et passives. Le point d'application des forces latérales est obtenu en écrivant l'équilibre de moment de chaque tranche.

Le coefficient de poussée des terres obtenu par la méthode GEL est en bon accord avec les résultats obtenus par la plupart des théories. Des comparaisons sont faites avec la théorie de Coulomb (utilisant une surface de rupture plane) et d'autres théories utilisant une surface de rupture courbe ou composée.

Les données sont présentées pour le cas d'un remblai horizontal pulvérulent derrière un mur vertical. La variation de la force de poussée des terres peut être illustrée par lignes de contours sur le maillage des centres de rotation. Ces lignes de contour ont une forme en cloche caractéristique et peuvent être utilisées pour localiser le centre de rotation le plus critique.

L'avantage principal de la méthode réside dans la possibilité d'analyser des dépôts de sols à stratification quelconque et à géométrie complexe. Différentes conditions de pression interstitielle, de résistance au cisaillement et de charge externe peuvent être prises en compte dans l'analyse. Des facteurs de sécurité supérieurs à 1,0 peuvent être appliqués à la résistance au cisaillement du sol pour les fins du projet.

Mots-clés: poussée des terres, poussée, butée, équilibre limite général, forces intertranches, coefficient de poussée.

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Introduction

The lateral earth force theory was first introduced more than 200 years ago by Coulomb in 1776 (Golder 1948). Since then, a number of methods to determine lateral earth forces have been developed. The theories of elasticity and plasticity have also made contributions to the calculation of lateral earth force (Hansen 1953). The soil-structure interaction process has been further understood through experimental and field research (for example, Terzaghi 1934; Tschebotarioff 1979). The classical theories such as the Coulomb and the Rankine theories are limited by the assumptions used in their derivations. Many more rigorous theories are complex in their application.

The limit equilibrium method of slices has been widely used to analyze slope stability problems. The

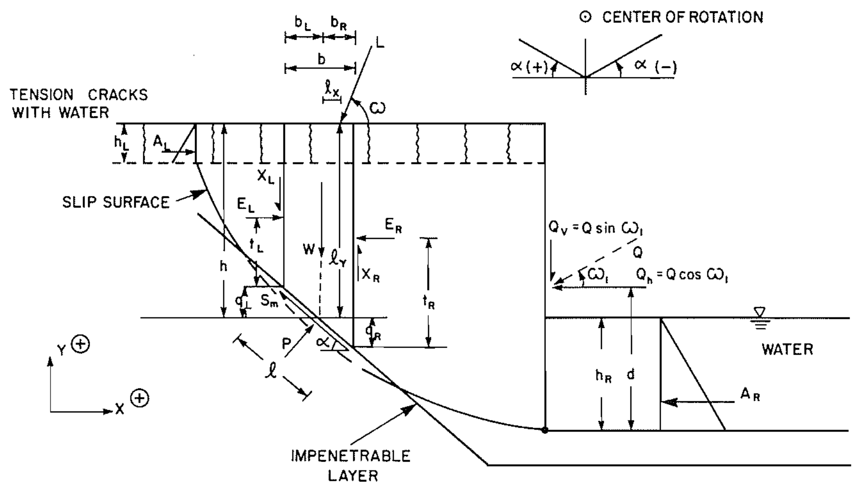

FIG. 1. Forces acting on a slice of a composite slip surface and a vertical wall in the lateral earth force problem.

principles of limit equilibrium used for analyzing slope stability can also be used to analyze lateral earth force problems (Janbu 1957, 1973; Morgenstern and Eisenstein 1970). In general, the lateral earth force problem is indeterminant. Janbu (1957) used an assumption regarding the point at which the interslice forces act (i.e., the line of thrust) to resolve the indeterminancy problem. On the other hand, Shields and Tolunay (1973) utilized an assumption concerning the direction of the interslice forces in computing passive pressure coefficients.

The various methods of limit equilibrium for slope stability can be embraced in a general limit equilibrium method (GLE) of slices as proposed by Fredlund *et al.* (1981). In this paper, the general formulations of the GLE method for lateral earth force computations are presented. An assumption regarding the general direction of the interslice forces is required in the derivation.

A version of the SLOPE-II program at the University of Saskatchewan, Saskatoon (Fredlund 1981), was modified in order to enable lateral earth force calculations in accordance with the GLE method. The results of the calculations presented in this paper were obtained from this version of the SLOPE-II program.

Definition of variables

The GLE method assumes an arbitrary slip surface and the soil mass above the slip surface is divided into vertical slices as shown in Fig. 1. The definition of each variable is given in the List of symbols.

A number of slip surfaces can be considered in either the active or passive case. The slip surface that gives the maximum active force or the minimum passive resistance is considered to be the critical slip surface.

The general formulation derived in this paper is applicable to any shape of slip surface. The use of a curved slip surface affects the critical value of lateral earth force more significantly for the passive case than for the active case (Terzaghi and Peck 1967). This is of particular concern when there is significant wall friction. Many theories use a composite slip surface involving a curved and a straight line portion to analyze the passive case. In this paper, the active and passive forces were computed using a fully circular slip surface.

General limit equilibrium for lateral earth force

The magnitude of the shear force mobilized on the base of a slice is computed according to the Mohr–Coulomb failure criterion:

$$[1] \qquad S_m = (l/F)(c' + (\sigma_n - u)\tan\phi')$$

where c' = effective cohesion intercept, ϕ' = effective angle of internal friction, σ_n = total normal stress (i.e., P/l), u = pore-water pressure, and F = factor of safety (the factor by which the shear strength of the soil must be reduced in order to bring the soil mass into a state of limiting equilibrium along the assumed slip surface).

The factor of safety is assumed to be the same for all slices. In the active and passive cases, the factor of safety is set equal to 1.0 because the shear strength is assumed to be fully mobilized. However, a factor of safety greater than 1.0 could also be used for design purposes. In the active case an upward shear force is mobilized along the slip surface, because of the downward movement of soil mass. The shear force mobilized acts downward in the passive case, because the soil mass is moved upward.

The normal stress distribution along the slip surface and the location of its resultant are unkowns. The lateral earth pressure distribution along the retaining structure and the location of its resultant are also unknowns.

The three static equilibrium equations that can be used are the summation of forces in two directions and the summation of moments about a specified point. These equations of static equilibrium and the Mohr–Coulomb failure criterion are insufficient to render the lateral earth force problem determinate. An assumption regarding the direction or magnitude of some of the forces, or the position of the line of thrust can be used to resolve the indeterminancy problem. The GLE method utilizes an assumption regarding the direction of the interslice force in formulating the method for lateral earth force calculations.

Magnitude of the resultant force

All of the formulations derived in this paper are applicable for slopes facing either to the left or to the right. However, the lateral earth force computations presented herein use a slope that is facing to the right. The lateral earth force is calculated as an external line load applied at the last slice, which is the right-end slice (Fig. 2).

The analysis is commenced by summing forces vertically on each slice to compute the normal force on the base of the slice:

$$[2]\quad P = \left\{ W + (X_L - X_R) - \frac{c'l}{F}\sin\alpha + ul\tan\phi'\sin\alpha/F + [L\sin\omega] + [Q\sin\omega_1] \right\} / m_\alpha$$

where $m_\alpha = \cos\alpha + \sin\alpha\tan\phi'/F$. The terms in the square brackets are only relevant to the slice where the line load, L, or the lateral earth force, Q, acts. For the first iteration when solving for the lateral earth force, the normal force, P, is computed by setting the interslice shear forces, X, to zero and assuming an initial value for the lateral earth force, Q. On subsequent iterations the interslice forces and the lateral earth force are calculated for each iteration. The horizontal equilibrium of forces on each slice is used to compute the interslice normal forces.

$$[3]\quad E_R = E_L + P\sin\alpha - S_m\cos\alpha + kW - [L\cos\omega] - [Q\cos\omega_1] + [A_L] - [A_R]$$

The term $[A]$ is only applied at the extremities of the slip surface if there is an external water force. The interslice shear forces are obtained using an assumption regarding the direction of the interslice force. The interslice force direction is described by an arbitrary interslice force function, $f(x)$ (Morgenstern and Price 1965):

$$[4]\quad X/E = \lambda f(x)$$

where $f(x)$ = a mathematical function that describes the relationship between X and E across the slope and λ = a constant representing the percentage of the function used in the calculation of the lateral earth force.

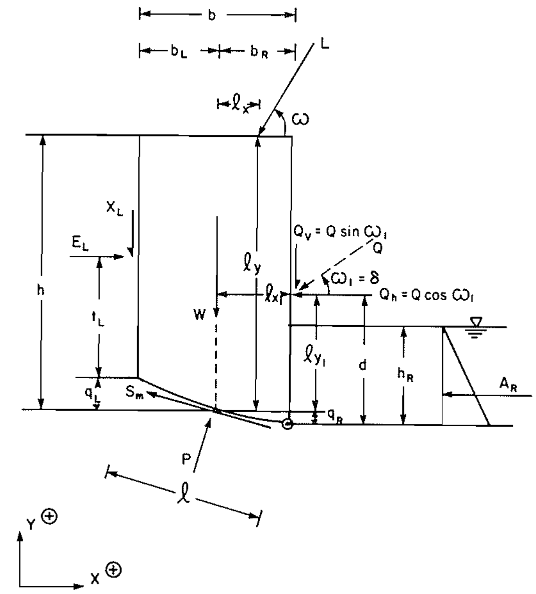

FIG. 2. Forces acting on the last slice where the lateral earth force acts.

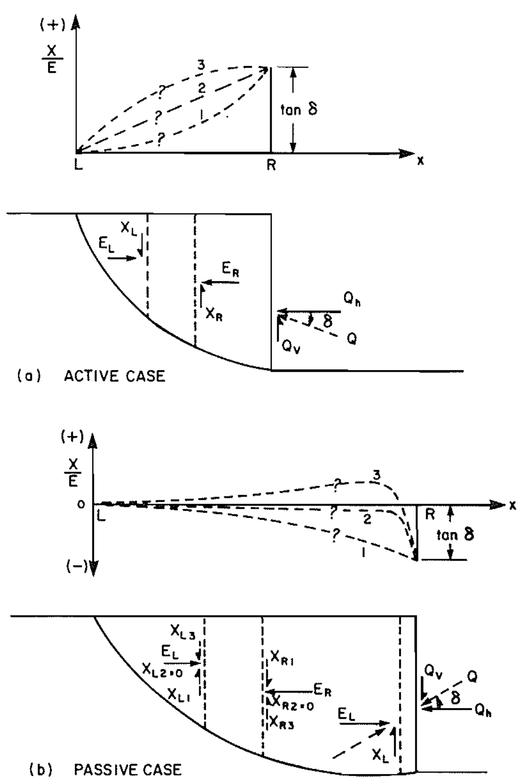

FIG. 3. Possible interslice functions for a horizontal backslope.

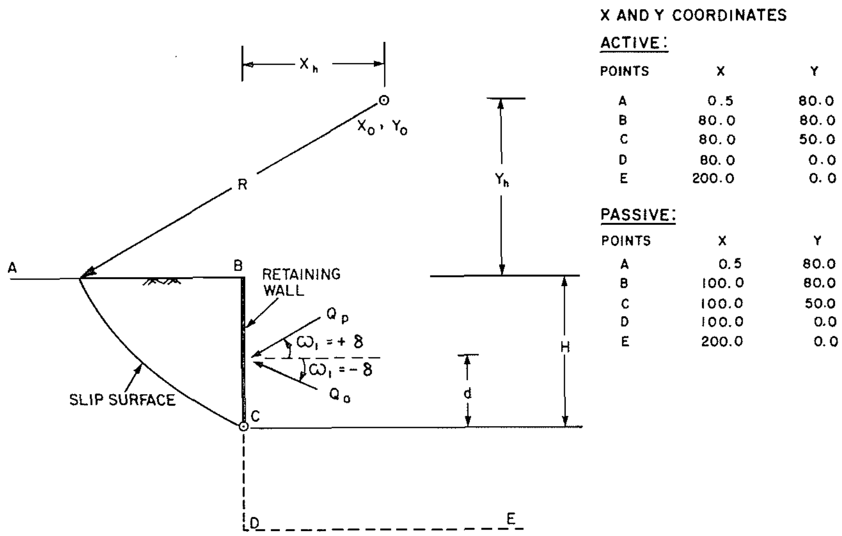

FIG. 4. The geometry used in the calculation of lateral earth force coefficient: $\phi' = 30.0°$, $\delta = 10.0°$. (Q_a = the active lateral earth force; Q_p = the passive lateral earth resistance; (x_0, y_0) = the x and y coordinates of the critical center of rotation of the circular slip surface; R = the radius of the critical center of rotation; X_h, Y_h = the horizontal and vertical distances of the critical center of rotation from the crest of the slope.)

Calculations are commenced at the first left-end slice, where the slip surface intersects the ground surface, and integration proceeds across the slope.

The magnitude of the lateral earth force, Q, can then be calculated from the overall equilibrium of forces in the horizontal direction:

[5] $$Q = \{\Sigma P \sin\alpha - \Sigma S_m \cos\alpha + \Sigma kW + A_L - A_R - \Sigma L \cos\omega\}/\cos\omega_1$$

The horizontal component of the lateral earth force can be expressed as

[6] $$Q_h = Q \cos\omega_1$$

and its vertical component can be written as

[7] $$Q_v = Q \sin\omega_1$$

The inclination angle, ω_1, is dependent on the wall friction angle, δ, and the inclination angle of the wall. When the wall is vertical the inclination angle ω_1 equals δ.

Point of application

The lateral earth force has been assumed as an external force applied at the last or right-end slice. Therefore, its horizontal position (i.e., x coordinate) is implicitly assumed to act at the right edge of the last slice. Hereafter, the point of application refers to the vertical position (i.e., y coordinate) of the lateral earth force.

The points of application of the interslice forces are computed from the summation of moments about the centroid of the base of each slice:

[8] $$t_R = \{E_L(t_L + q_L) - X_L b_L + kWh/2 + E_R q_R - X_R b_R + [A_L h_L/3] - [L \cos\omega l_y] + [L \sin\omega l_x]\}/E_R$$

At the last slice the point of application of the lateral earth force can then be computed:

[9] $$d = \{E_L(t_L + q_L) - X_L b_L + kWh/2 - [A_R h_R/3] - [L \cos\omega l_y] + [L \sin\omega l_x] + Q \sin\omega_1 l_{x1}\}/Q \cos\omega_1 + q_R$$

Interslice force function

The interslice force function is used to render the lateral earth force problem determinate. The interslice force function should be selected such that the boundary conditions and the failure criterion along the slip surface are satisfied. The state of stress within the soil mass should also be kinematically possible (Morgenstern and Price 1965).

Figure 3 shows possible interslice force functions that might be used in the lateral earth force analysis. The resultant of the interslice forces approximates the tangent of the inclination angle, δ, at the boundary between the soil mass and the wall. At the other end where the slip surface cuts the horizontal ground surface, the at-rest stress condition would have approximately zero vertical shear stress. The shear distribution between these two ends can be estimated from theory of elasticity (e.g., finite element method) by considering the boundary conditions and the stress versus strain relationship of the soil (Wilson 1982; Fan 1983).

Study has shown that the interslice force function has a greater effect for the passive case than for the active case (Rahardjo 1982; Rowe 1963). Therefore, the shape of the interslice function is more important in the passive case.

Coefficient of lateral earth force

The coefficient of lateral earth stress, K, is the ratio of horizontal to vertical stress in a soil. A hydrostatic lateral earth stress distribution with K equal to 1.0 results in a magnitude of lateral earth force of $\frac{1}{2}\rho g H^2$ for cohesionless soils, where ρ = total mass density of the soil, g = acceleration due to gravity, and H = height of the retaining structure. In actuality, the magnitude of the lateral earth force can be smaller or greater than $\frac{1}{2}\rho g H^2$ depending upon the failure mode (i.e., active or passive) and the wall friction developed. The magnitude of lateral earth force will be expressed as a ratio of $\frac{1}{2}\rho g H^2$ (Janbu 1957; Shields and Tolunay 1973). This ratio was investigated for cases of a horizontal backslope of cohesionless soil and a vertical wall.

In order to study the effect of wall friction angle, δ, on the coefficient of lateral earth force, some analyses are presented using various combinations of soils and wall friction angles.

Example problems

The active and passive cases for a cohesionless backslope (ϕ' = 30°) against a vertical wall (δ = 10°) are used to demonstrate the computation of the coefficient of the lateral earth force. The soil unit weight, ρg, was taken as 20 kN/m³. The geometries used in the active and passive cases are described in Fig. 4.

For one combination of ϕ' and δ, a number of slip surfaces were investigated. All circular slip surfaces used in the computation were forced to pass through the heel of the wall.

The interslice force function used in the active case was a triangular function (i.e., No. 2 on Fig. 3*a*). This function was selected mainly for its simplicity. However, different functions (Fig. 3*a*) result in little differences in the active coefficients computed.

In the passive case, a zero interslice force function was used which assumes that the wall friction decays rapidly to zero with increasing distance from the wall. This assumption was proposed by Rowe (Lee and Moore 1968) and is considered to give the lowest passive resistance value. Shields and Tolunay (1973) also used this assumption in calculating passive pressure coefficients. It is possible that a more critical passive resistance could be obtained by using the interslice force function No. 3 in Fig. 3*b* (Rahardjo 1982). However, the reasonableness of this function for the passive case still requires further investigation.

Contours of lateral earth force

The lateral earth forces obtained from a number of slip surfaces need to be investigated in order to find the critical slip surface. A grid of centers of rotation with the corresponding lateral earth forces should be plotted as shown in Fig. 5 for the active case and in Fig. 7 for the passive case. Contours of lateral earth force can be plotted on this grid of forces. Figures 6 and 8 present typical contours for the active and passive cases respectively.

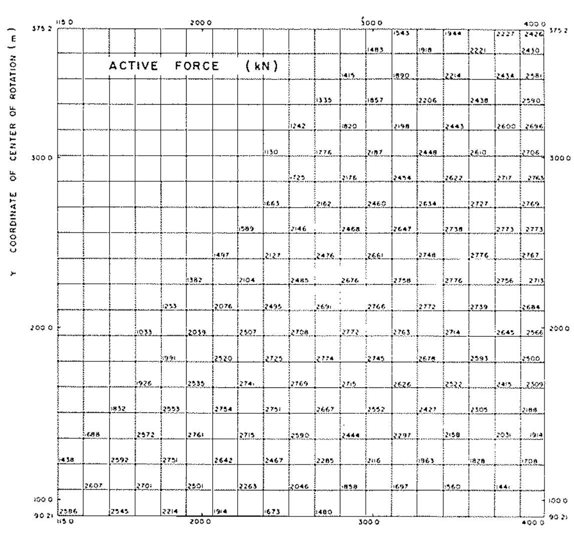

FIG. 5. Grid of centers of rotation for active case: ϕ' = 30.0°, δ = −10.0°.

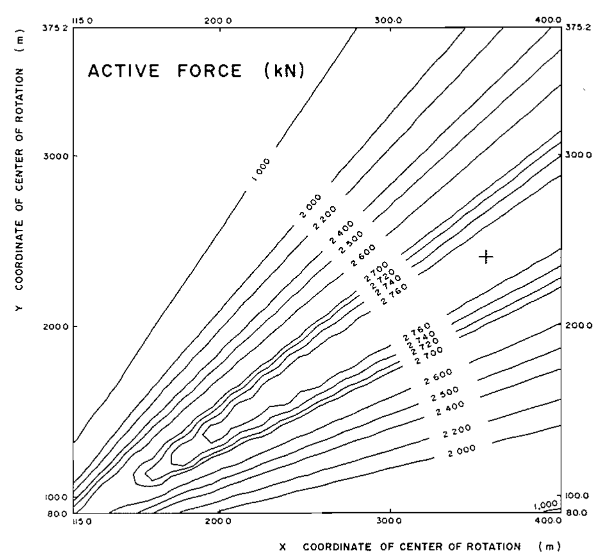

FIG. 6. Contour plot for active case: ϕ' = 30.0°, δ = −10.0°.

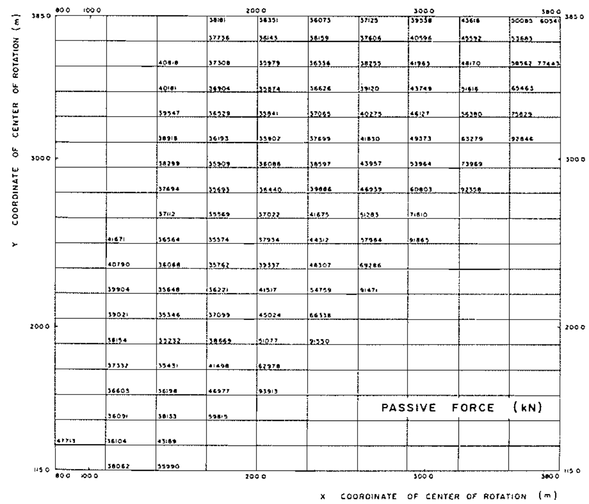

FIG. 7. Grid of centers of rotation for passive case: $\phi' = 30.0°$, $\delta = +10.0°$.

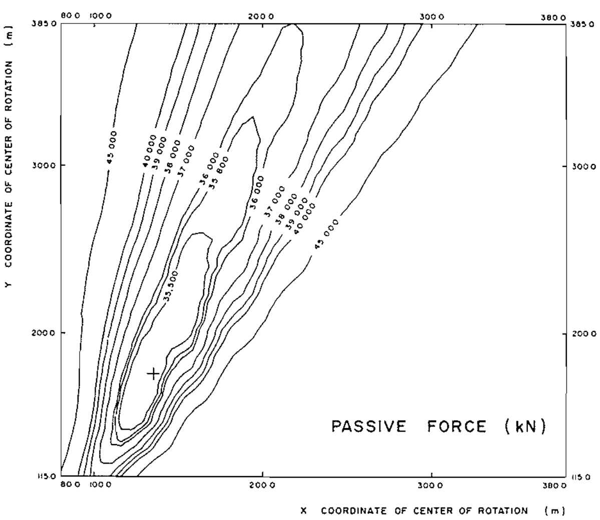

FIG. 8. Contour plot for passive case: $\phi' = 30.0°$, $\delta = +10.0°$.

Contours of both the active and passive forces exhibit a bell shape for the contour lines. The contours are narrow close to the slope and widen at distances away from the slope. The center of rotation, which has the maximum value of active force and is enclosed by the maximum contour line, is considered to be the critical center of rotation (X_0,Y_0). On the other hand, the minimum value of passive resistance surrounded by the minimum contour line is considered to be the critical passive force. These procedures were also performed for other examples with different ϕ' and δ values.

Presentation of analytical data

The maximum active force obtained from the critical

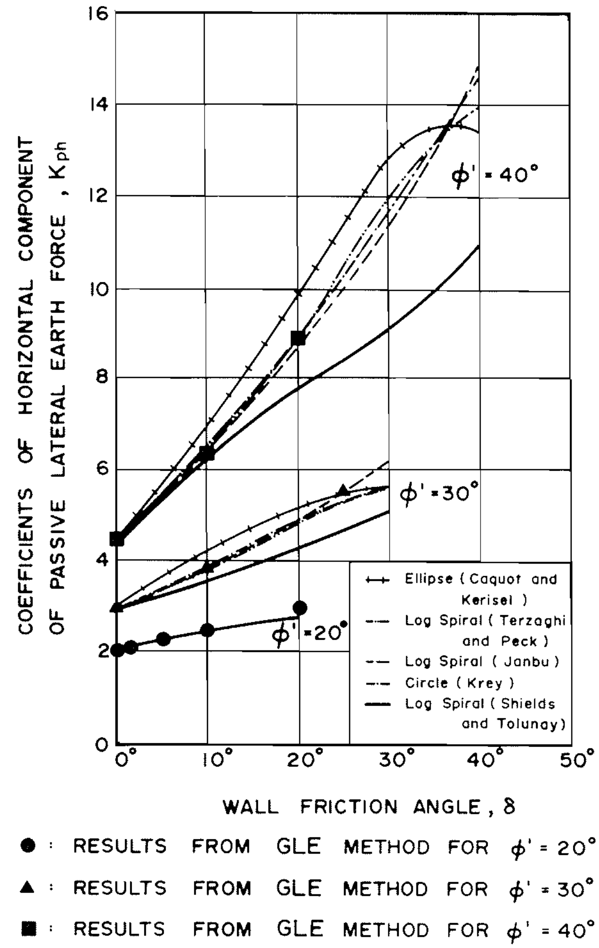

FIG. 9. Comparison of coefficients of horizontal component of passive lateral earth force, K_{ph}, for different types of slip surface (after Shields and Tolunay 1973).

slip surface was used to compute the coefficient of active lateral earth force:

$$[10] \quad K_a = \frac{Q_a}{\frac{1}{2}\rho g H^2}$$

where Q_a = the active lateral earth force and K_a = the coefficient of active lateral earth force. The coefficient of the horizontal component of the active lateral earth force, K_{ah}, can then be expressed as

$$[11] \quad K_{ah} = K_a \cos \delta$$

The coefficient of passive lateral earth force, K_p, can be written in a form similar to that of [10] (i.e., $K_p = Q_p/\frac{1}{2}\rho g H^2$); and the coefficient of the horizontal component of the passive force, K_{ph}, can be written in a form similar to [11] (i.e., $K_{ph} = K_p \cos \delta$).

Comparisons with other methods of analysis

The coefficients of lateral earth force computed from

TABLE 1. Comparisons of coefficients of active lateral earth force, K_a

Angle of internal friction (Degrees)	Wall friction angle (Degrees)	K_a		
		Coulomb (planar slip surface)	Krey (curved slip surface)	GLE (circular slip surface)
20	0	0.49		0.49
	−2	0.48		0.48
	−5	0.47		0.47
	−10	0.45		0.45
	−20	0.43		0.43
30	0	0.33	0.33	0.33
	−10	0.31		0.31
	−20	0.30	0.30	0.30
	−30	0.30	0.31	0.30
40	0	0.22	0.22	0.22
	−10	0.20		0.20
	−20	0.20		0.20
	−40	0.21	0.22	0.21

the GLE method are compared with the coefficients obtained from other theories. Table 1 presents the coefficients of active force, K_a, obtained from the GLE method (i.e., circular slip surface) and the K_a values obtained from the Coulomb theory (i.e., planar slip surface, Jumikis 1962), and from Krey's method (Tschebotarioff 1979) (i.e., curved slip surface). Table 1 shows that the three methods give essentially the same results for different cases of soil and wall friction angles.

Comparisons of the coefficients of the horizontal component of passive force, K_{ph}, are presented in Fig. 9. In these comparisons, a fully circular slip surface was used to compute the K_{ph} values by the GLE method while a composite slip surface was used for theories other than the Rankine and Coulomb methods. The composite slip surface starts with a curved line from the toe of the slope. At some distance from the wall the slip surface becomes a straight line that intersects the soil surface at an angle of $(45° - \phi'/2)$. The curved line portion is a logarithmic spiral (Terzaghi and Peck 1967; Janbu 1957; Shields and Tolunay 1973), an ellipse (Caquot and Kerisel 1948), or a circle (Krey 1936).

Figure 9 shows that the results from the GLE method agree closely with most of the results from other theories. The GLE method, however, gives somewhat different passive pressure results to the Shields and Tolunay method. This difference can be attributed to the different shapes of slip surface used in the GLE and in the Shields and Tolunay methods. In addition, Shields and Tolunay selected a specific slip surface based on the soil and wall friction angles. This produces the critical slip surface for a homogeneous, cohesionless backfill against a vertical wall. However, no solution was proposed for the location of the critical slip surface when a complex geometry, cohesive soil, and/or pore-water pressure are considered.

A convergence problem is encountered when the wall friction angle, δ, approaches the friction angle of the

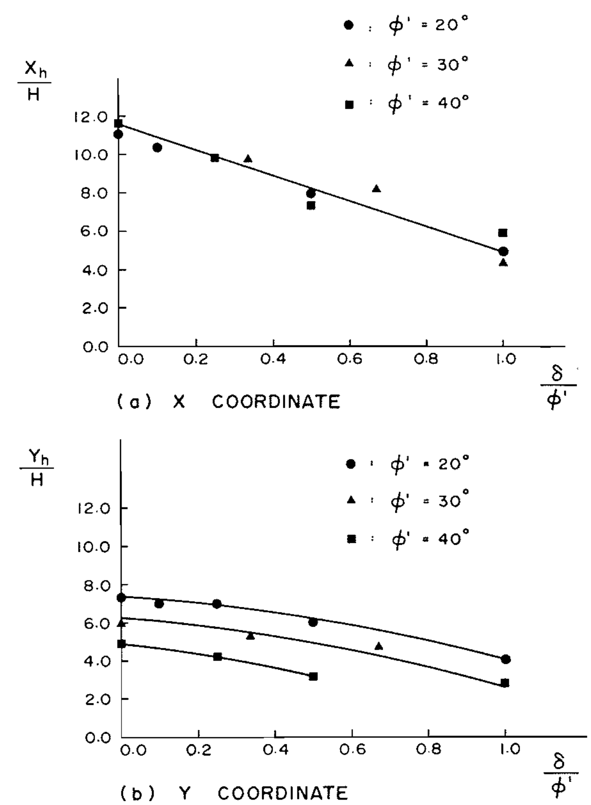

FIG. 10. Location of center of rotation of the critical slip surface for active case.

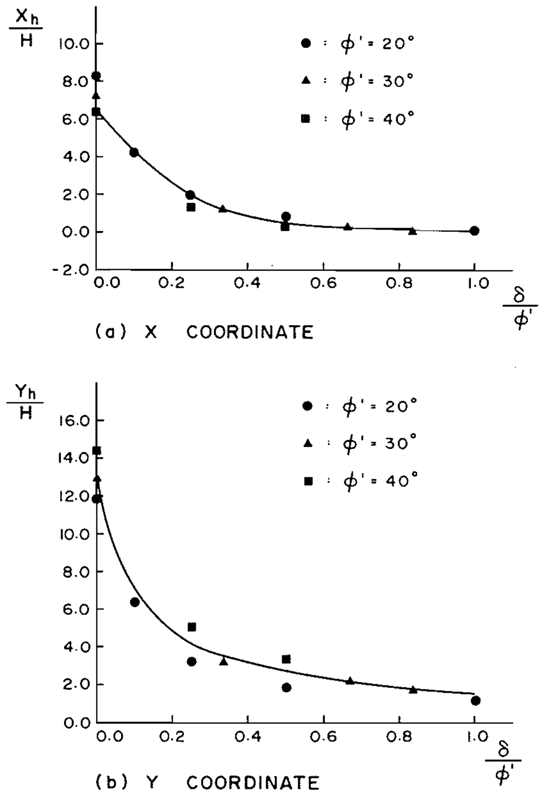

FIG. 11. Location of center of rotation of the critical slip surface for passive case.

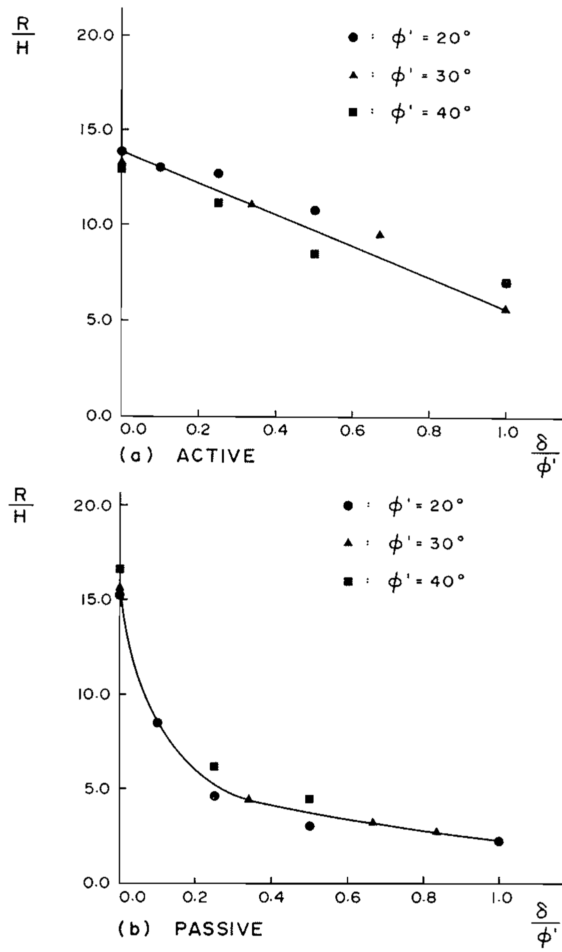

FIG. 12. Radius of the critical slip surface.

soil, ϕ'. In other words the soil friction angle, ϕ', becomes a limiting value for the wall friction angle, δ. This difficulty can be partly attributed to the shape of the slip surface which might not be completely compatible with the zero interslice force function and the high wall friction.

Location of the critical center of rotation

The location of the critical center of rotation is presented in unitless terms, which are X_h/H in the horizontal direction and Y_h/H in the vertical direction. Figure 10 shows the locations of the critical centers of rotation for the active case for different combinations of soil and wall friction angles, δ/ϕ'. This relationship shows that the location of the critical slip surface becomes closer to the slope as the wall friction angle increases. In other words, the wall friction angle results in an increase in the curvature of the critical slip surface.

More pronounced effects of the wall friction angle on the curvature of the slip surface can be observed for the passive case as presented in Fig. 11. This phenomenon can also be explained if the radius of the critical slip surface is plotted against the wall friction angle as shown in Fig. 12. The decrease in the radius is more significant in the passive case (Fig. 12*b*) as compared to the active case (Fig. 12*a*) as the wall friction angle increases. Therefore, disregarding the curvature of slip surface in the passive case produces a more significant error than in the active case.

Location of point of application

The point of application of the lateral earth force for the critical slip surface was computed according to [9]. In both the active and passive cases, the point of application becomes lower when the wall friction angle increases as shown in Fig. 13. The reason can be visualized from [9]. Firstly, the active lateral earth force, Q_a, decreases and passive lateral earth force, Q_p, increases as the wall friction angle, δ, increases. Secondly, the wall friction acts upward in the active case (i.e., ω_l is negative) whereas in the passive case, the wall friction acts downward (i.e., ω_l is positive). Thirdly, the interslice force function also influences the point of application. For example, in the passive case the interslice function No. 1 (Fig. 3*b*) will result in the raising of the point of application as the wall friction

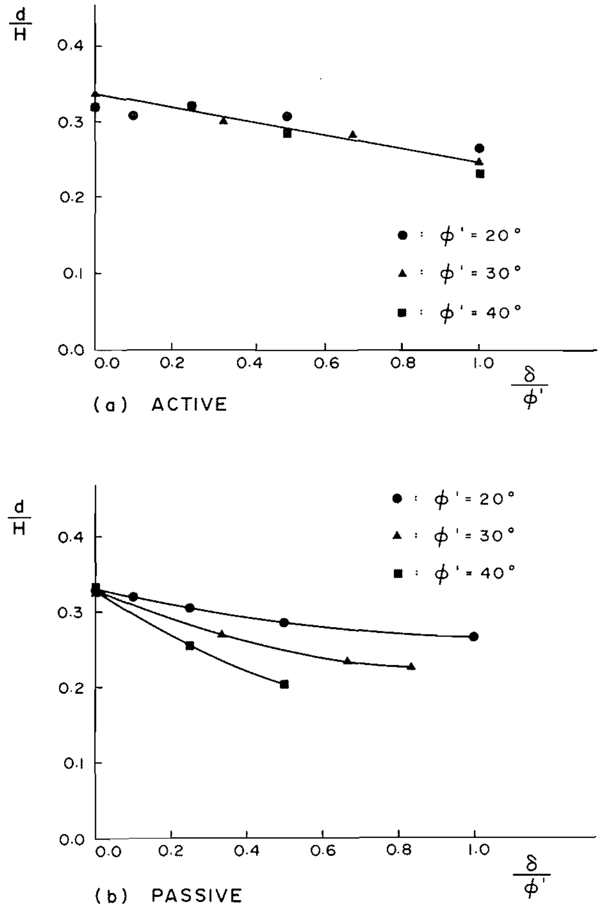

FIG. 13. Point of application of lateral earth force.

angle increases. The lowering of the point of application in the passive case as presented in this paper can be attributed to the use of the zero interslice force function.

Summary

The general formulations for the magnitude and point of application of the lateral earth force are presented. The statics of force and moment equilibrium, together with an assumption regarding the direction of the interslice forces, are used in the formulation. A factor of safety can be applied to the shear strength of the soil.

Contours of the lateral earth force on the grid of centers of rotation exhibit a bell shape and can be used to locate the critical center of rotation. The curvature of the critical slip surface is shown to increase with an increase in wall friction angle. This characteristic agrees with many lateral earth force theories and has a more pronounced effect in the passive case than in the active case.

The fully circular slip surface does not produce a significant difference in the coefficient of active lateral force when compared to the planar slip surface. However, this is not the situation for the passive case. The shape of slip surface, and the interslice force function used in the computation, can alter significantly the coefficient of passive force. The circular slip surface gives results which are in good agreement with most other theories that use a composite slip surface.

The GLE formulation can be applied to any shape of slip surface provided the computer program can accommodate the shape of the slip surface. In addition, complex geometries and different pore-water pressures and loading conditions can also be accommodated using the GLE method. A factor of safety other than unity can also be used for design purposes.

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List of symbols

W the total weight of a slice having width b and height h

P the total normal force on the base of a slice

S_m the shear force mobilized on the base of each slice

E_L, E_R the total horizontal interslice normal forces (the L and R subscripts on the E, X, and A variables denote the left and right sides, respectively)

X_L, X_R the vertical interslice shear forces

A_L, A_R the resultant external water forces

α the angle between the tangent to the centroid of the base of each slice and the horizontal

ω the angle of an externally applied line load from the horizontal. This angle is measured counterclockwise from the positive x axis

b_L, b_R the horizontal distance between the centroid of the base and the left or right edge of the slice, respectively

t_L, t_R the vertical distance between the left or right corners of the base of the slice and the point of application of the interslice forces on the left or right side of the slice, respectively

q_L, q_R the vertical distance between the centroid of the base with the left or right bottom corner of the slice respectively

l the length of the slip surface at the base of each slice

l_x the horizontal distance from the centroid of the base of a slice to the point of application of an external line load

l_y the vertical distance from the centroid of the base of a slice to the point of application of an external line load

Q the active or passive lateral earth force

ω_1 the direction of the lateral earth force which is positive as measured counterclockwise from the positive x axis

Q_h the horizontal component of the lateral earth force (i.e., $Q \cos \omega_1$)

Q_v the vertical component of the lateral earth force (i.e., $Q \sin \omega_1$)

δ the wall friction angle. The wall friction is positive as measured counterclockwise from the normal to the wall

d the vertical distance from the heel of the retaining wall to the point of application of the lateral earth force

h_L, h_R the height of the water which is acting as an external water force on the left or right of the slope, respectively