Some difficulties associated with the limit equilibrium method of slices

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Several commonly encountered problems associated with the limit equilibrium methods of slices are discussed. These problems are primarily related to the assumptions used to render the inherently indeterminate analysis determinate. When these problems occur in the stability computations, unreasonable solutions are often obtained. It appears that problems occur mainly in situations where the assumption to render the analysis determinate seriously departs from realistic soil conditions. These problems should not, in general, discourage the use of the method of slices. Example problems are presented to illustrate these difficulties and suggestions are proposed to resolve these problems.

Keywords: slope stability, limit equilibrium, method of slices, factor of safety, side force function.

Plusieurs problèmes rencontrés couramment dans les méthodes d'équilibre limite par tranches sont discutés. Ces problèmes sont essentiellement reliés aux hypothèses faites pour rendre déterminée une analyse naturellement indéterminée. Lorsque ces problèmes se produisent dans les calculs de stabilité, des solutions non raisonnables sont souvent obtenues. Il semble que les problèmes se produisent surtout dans les situations où l'hypothèse visant à rendre l'analyse déterminée diffère de façon importante de conditions géotechniques réalistes. En général, ces problèmes ne devraient pas empêcher l'usage de la méthode des tranches. Des cas types sont présentés pour illustrer ces difficultés et des suggestions de solutions sont présentées.

Mots-clés: stabilité des pentes, équilibre limite, méthode des tranches, facteur de sécurité, fonction de force intertranche.

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Introduction

Slope stability analyses are most commonly performed using one of several possible limit equilibrium methods of slices. The approach is relatively simple and versatile since these methods can analyze any general shape of slip surface in heterogeneous soil conditions. In all limit equilibrium methods of slices, no consideration is given to the stress versus strain behavior of the soil. Therefore, it is difficult to expect the result calculated by the methods of slices to be an accurate representation of the actual stability conditions. However, documented case histories have led to a relatively high degree of confidence in their use (Sevaldson 1956; Kjaernsli and Simons 1962; Skempton and Hutchinson 1969).

The limit equilibrium method of slices is indeterminate when only the equations of static equilibrium are considered (Morgenstern and Price 1965; Spencer 1967). In order to render the problem determinate, it is necessary to provide additional elements of physics or an assumption in the analytical procedure. The latter approach has been widely adopted in limit equilibrium methods primarily because of its simplicity (Fredlund and Krahn 1977).

Computational difficulties may occasionally be encountered in solving for the factor of safety equations. In this paper, three commonly encountered problems are discussed. The first problem is related to an unreasonably large and/or negative magnitude for the normal force on the base of a slice calculated as a result of the m_α term approaching zero and/or going negative. The second problem is associated with the computation of a negative normal force on the base of a slice if the soil slope is highly cohesive. The third problem deals with convergence difficulties encountered when an unreasonable side force function is assumed. Example problems are presented to demonstrate each situation and suggestions are proposed to resolve these difficulties.

Theoretical aspects

Numerous methods of slices have been proposed. However, for convenience of presentation and comparison, each method will be embraced within a general limit equilibrium formulation called the GLE method (Fredlund *et al.* 1981). Figure 1 presents a composite slip surface (i.e., combined linear and circular segments) together with the forces applied on a typical slice. Definitions of the forces involved are as follows: W = total weight of a slice of width b and height h; P = total normal force acting on the base of a slice, equal to $\sigma_n l$; σ_n is the normal stress on the base of the slice and l is the length of the slip surface at the base of a slice; S_m = mobilized shearing resistance at the base of a slice; E_L, E_R = total interslice normal force on the left and right sides of a slice, respectively; X_L, X_R = interslice shear force on the left and right sides of a slice, respectively; A_L, A_R = resultant external hydrostatic force at the left and right ends of the assumed slip surface, respectively; R = radius or moment arm

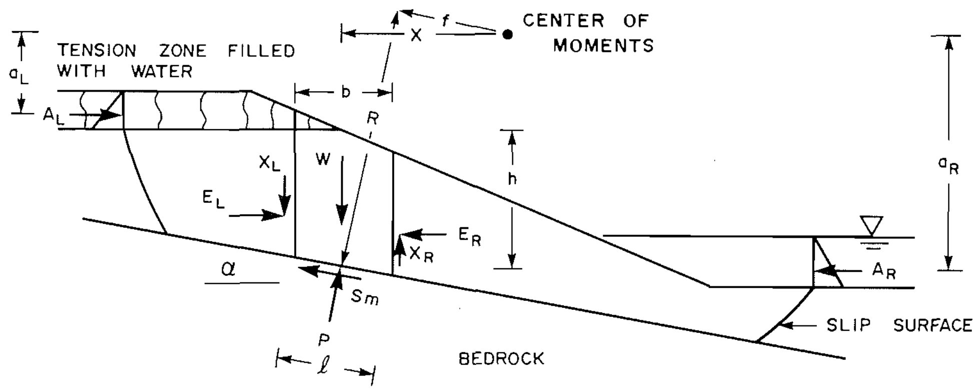

FIG. 1. A composite slip surface with forces applied on a typical slice.

associated with the mobilized shearing resistance, S_m; x = horizontal distance from the centroid of each slice to the centre of moments; f = offset distance from the normal force to the centre of moments; and a_L, a_R = perpendicular distance from the resultant external hydrostatic force to the arbitrary centre of moments.

The mobilized shearing resistance, S_m, is computed from the shear strength criterion of the soil.

$$[1] \quad S_m = \frac{\tau l}{F} = \frac{1}{F}[c'l + (P - ul)\tan\phi']$$

where τ = shear strength, c' = effective cohesion parameter, ϕ' = effective angle of internal friction, u = pore-water pressure, and F = factor of safety.

The normal force, P, acting on the base of the slice can be derived by summing the vertical forces for an individual slice.

$$[2] \quad P = \frac{W - (X_R - X_L) - \dfrac{c'l\sin\alpha}{F} + \dfrac{ul\tan\phi'\sin\alpha}{F}}{m_\alpha}$$

where m_α = a variable whose value depends on the inclination of the base of a slice, α, and $\tan\phi'/F$ as follows:

$$[3] \quad m_\alpha = \cos\alpha + \frac{\sin\alpha\tan\phi'}{F}$$

The factor of safety with respect to moment equilibrium is obtained by the summation of moments about an arbitrary point for all slices within a specified slip surface. The center of rotation for the circular portion of the slip surface is used for the center of moments.

$$[4] \quad F_m = \frac{\Sigma\,[c'lR + (P - ul)R\tan\phi']}{\Sigma\,Wx - \Sigma\,Pf \pm Aa}$$

where F_m = factor of safety with respect to moment equilibrium.

The factor of safety with respect to force equilibrium is derived by summing the forces in the horizontal direction for all slices.

$$[5] \quad F_f = \frac{\Sigma[c'l\cos\alpha + (P - ul)\tan\phi'\cos\alpha]}{\Sigma P\sin\alpha \pm A}$$

where F_f = factor of safety with respect to force equilibrium.

The summation of horizontal forces on each slice can be used to compute the total interslice normal force, E.

$$[6] \quad (E_R - E_L) = [W - (X_R - X_L)]\tan\alpha - \frac{S_m}{\cos\alpha}$$

The interslice shear force, X, can be related to the interslice normal force, E, by a mathematical function (Morgenstern and Price 1965).

$$[7] \quad X = \lambda f(x)E$$

where $f(x)$ = a functional relation which describes the manner in which the magnitude of X/E varies across the slip surface and λ = a scaling constant which represents the percentage of the function, $f(x)$, used for solving the factor of safety equations.

Difficulties associated with m_α values

The normal force at the base of a slice sometimes becomes unreasonable due to the unrealistic values computed for m_α (Whitman and Bailey 1967). The variable, m_α, in [3] is a function of the inclination of the base of a slice, α, and $\tan\phi'/F$, as illustrated in Fig. 2. Computational difficulties occur when m_α becomes small or zero. This situation can occur when α is negative and $\tan\phi'/F$ is large or when α is large and $\tan\phi'/F$ is small. Specifically, the m_α value will become zero when the base inclination of any slice, α, bears the following relationship to the mobilized friction angle, ϕ'_m (Wright 1975):

$$[8] \quad \alpha = \phi'_m - 90°$$

Figure 3 illustrates the relationship between the angle of internal friction, ϕ', and the base angle of the slice, α, as depicted in [8]. When the computed normal force, P, on the slice becomes large, the mobilized shearing

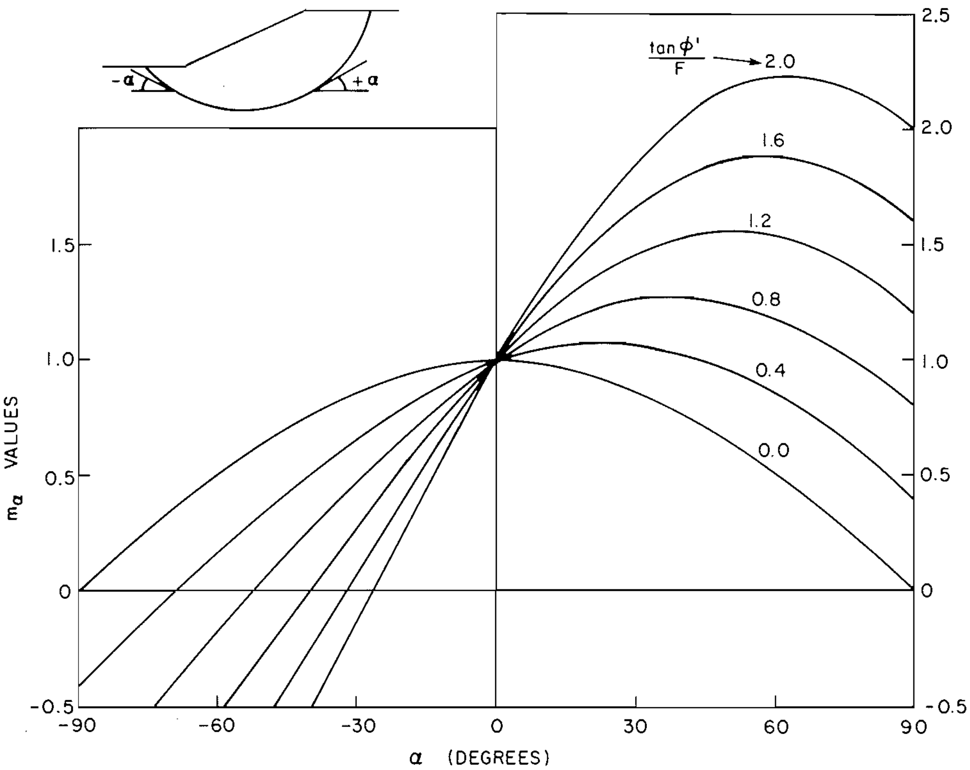

FIG. 2. Values of m_α versus the angle at the base of a slice for various tan ϕ'/F values.

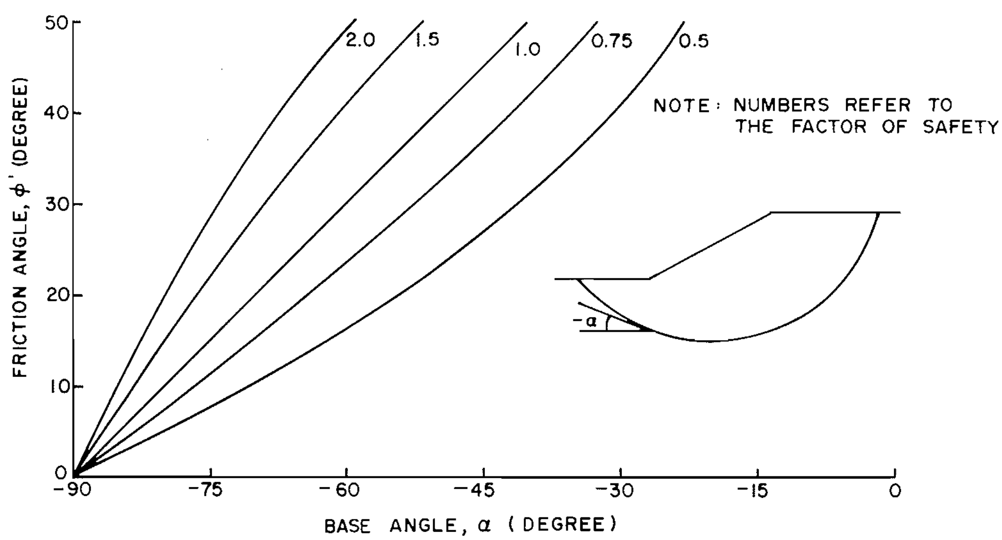

FIG. 3. Relationship between the angle at the base of a slice and soil friction angle that yields m_α equal to zero.

resistance, S_m, also becomes large and exerts a disproportionally large influence on the computation of the factor of safety.

The factor of safety calculation can take on another extreme when m_α is negative. The m_α term can be negative when the base angle of the slice, α, is more negative than the limiting angle computed by [8]. In this case, the computed normal force is negative. Consequently, the computed factor of safety may be underestimated since the total mobilized shearing resistance is reduced. When a particular slice has a small but negative m_α value, its normal force becomes large and

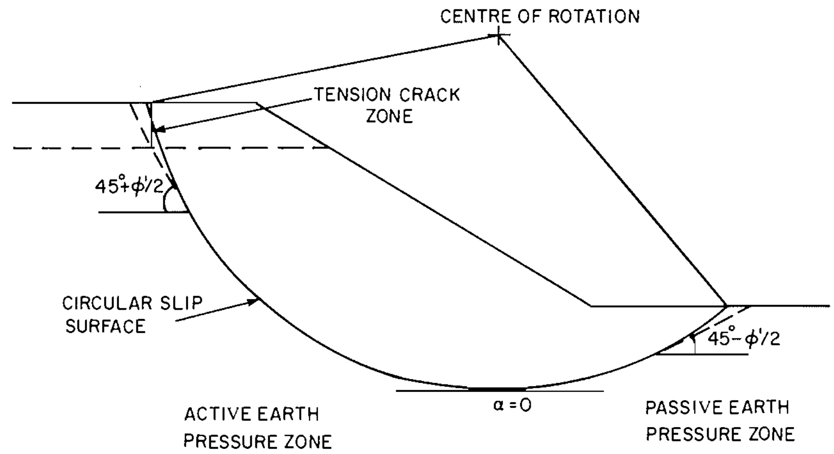

FIG. 4. Soil slope divided into active and passive earth pressure zones.

negative when compared with other slices. The large, negative normal then dominates the stability calculation and the computed factor of safety can go to less than zero, which, of course, is meaningless. Whitman and Bailey (1967) suggested that a low m_α value is unreasonable and that a warning should be printed out by a computer program when its value goes below 0.2.

Suggestions for solving the small m_α *problem*

Problems associated with the magnitude of m_α are mainly the result of an inappropriate assumed shape for the slip surface. The shape and location of the critical slip surface are unknown. Therefore, a slip surface of a specific form (i.e., circular or composite) is assumed in order to locate the critical slip surface. The assumed form can cause conditions that give unreasonable m_α values in the numerical procedure. It is suggested that the classic earth pressure theory can be used in conjunction with the limit equilibrium method to estimate the shape of the rupture surface. In applying the earth pressure theory, the soil slope is divided into two regions, namely, an active earth pressure zone in which the lateral earth pressure decreases due to lateral displacement and a passive earth pressure zone in which the lateral earth pressure increases due to lateral displacement of the soil mass (Fig. 4). In the active zone, the soil mass moves downward which in effect releases the lateral earth pressure, whereas in the passive pressure zone, the soil mass is pushed by the movement of the active soil wedge. The inclination of the slip surface in the passive zone of the sliding mass should be limited to the maximum obliquity for the passive state:

[9] $\alpha = \phi'/2 - 45°$

Likewise, it is suggested that the inclination of the slip surface in the active zone should not exceed the value obtained from the following equation:

[10] $\alpha = \phi'/2 + 45°$

These solutions will resolve the m_α problems. The active zone may also be combined with a vertical tension crack zone to alleviate the m_α problems.

Problem example to demonstrate the m_α *problem*

Example No. 1 (Fig. 5) is presented to illustrate the difficulties associated with small m_α values. Pore-water pressures are calculated using a pore pressure ratio, r_u, equal to 0.4. A composite slip surface (i.e., slip surface #1) is analyzed using the assumption that the interslice forces are horizontal. The analyses were performed using the SLOPE-II computer program (Fredlund 1981). Figure 5 shows the m_α values and the mobilized shearing resistance for all slices. Results indicate that the m_α values become small and negative for slices at the lower end of slip surface #1. The mobilized shearing resistance, S_m, for the corresponding slices becomes large and negative. Their shearing resistance is two to three orders of magnitude larger than those at the central portion of the slide. Consequently, a negative factor of safety is calculated. Slip surface #1 demonstrates the difficulty in attempting to solve for the factor of safety of any arbitrary slip surface. A solution could not be obtained for this slip surface. The assumed slip surface was subsequently changed (i.e., slip surface #2) such that the inclinations of the end slices conform to those calculated by [9] and [10] at their respective locations. The results are also plotted in Fig. 5. The m_α values and the mobilized shearing resistance for slip surface #2 become more reasonable with a computed factor of safety of 0.884.

Difficulties associated with a negative normal force in a cohesive soil slope

When using the limit equilibrium method to analyze a cohesive soil slope, the computed normal force on the base of a slice may become negative. This is especially true for relatively shallow slices in soils where the

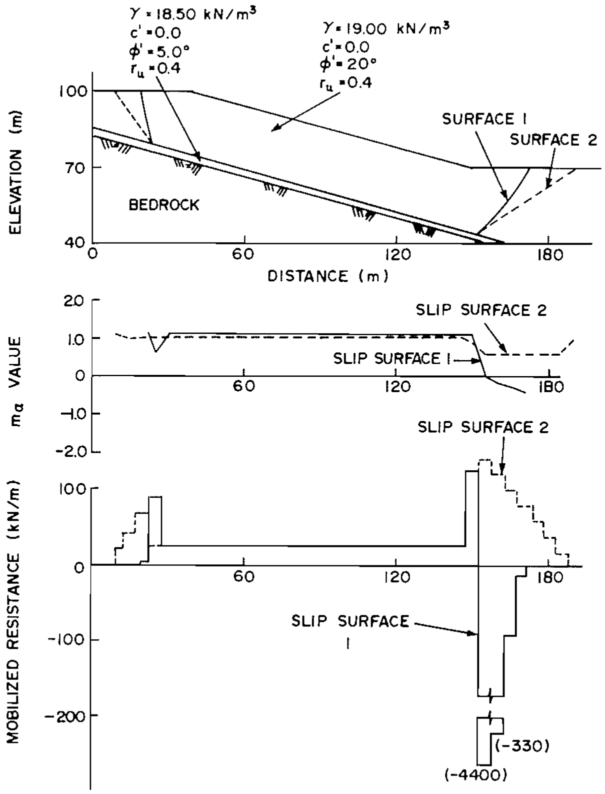

FIG. 5. Problem example No. 1.

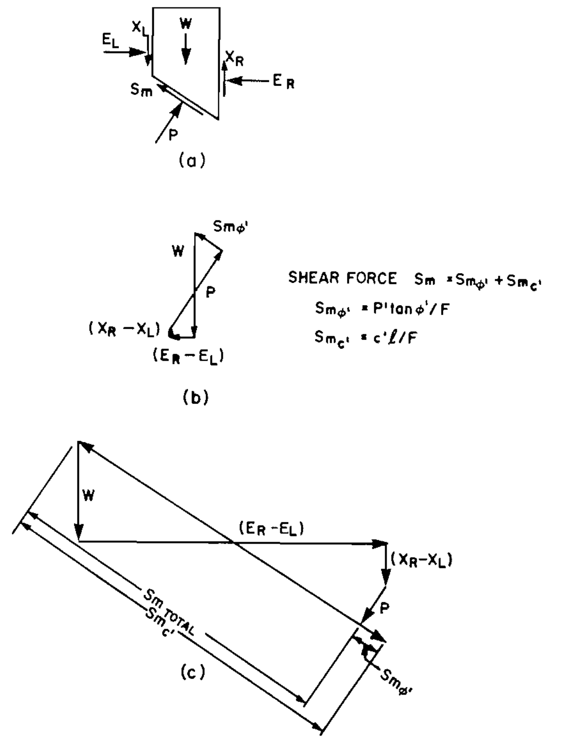

FIG. 6. (*a*) Free body diagram for a typical slice. (*b*) Force polygon for the slice in cohesionless soil. (*c*) Force polygon for the slice in highly cohesive soil.

cohesive component dominates the shear strength of the soil. The presence of a negative normal force denotes the existence of tension within the soil (Spencer 1968). The mechanism and effect of such tensions on the stability analyses have yet to be resolved.

The influence of cohesion on the computation of the normal force can be seen by examining the force equilibrium of a slice. Figure 6*a* shows the free body diagram of a typical slice with its associated forces. Let us first consider a cohesionless soil. In this case, the total and effective normal forces at the base of a slice can be small but positive, and the resultant interslice forces will act opposite to the direction of sliding (Fig. 6*b*). For the same slice in a highly cohesive soil, the mobilized shear resistance due to cohesion is large in comparison with the magnitude of other forces (Fig. 6*c*). In order for this slice to move downward with the rest of the soil mass, the resultant interslice normal and shear forces must act in the down-slope directions. In other words, interslice tension forces are necessary to assist rather than to resist the movement. Consequently, the total and effective normal forces at the base of a slice must become negative in order to maintain force equilibrium. The negative normal force means that the slice is in a state of buoyancy although in reality there will be no tendency for the mass to lift upward. In fact, there is still a positive normal force acting on the base of the slice even though its computed value is negative. This indicates that the use of limit equilibrium principles alone may not be adequate in modelling stability for highly cohesive soil slopes. It is also found that the presence of negative normal forces may often cause instability in the numerical solution for the factor of safety.

Suggestion for solving the problems associated with the negative normal force

Negative normal forces are computed primarily as a result of large mobilized shearing resistance due to cohesion. Spencer (1968, 1973) suggested that a tension crack zone should be located at the top of the cohesive soil slope. The depth of the tension zone, z, was given as equal to the depth of zero active effective stress.

$$[11] \quad z_0 = \frac{2c'}{\gamma F(1 - r_u)}\sqrt{\frac{1 + \sin\phi'_m}{1 - \sin\phi'_m}}$$

where r_u = pore pressure ratio, z_0 = depth to zero active effective stress, γ = total unit weight, F = factor of safety, and ϕ'_m = mobilized angle of internal friction.

Figure 7 illustrates the relationship between the depth of the tension crack zone and cohesion for different angles of internal friction, ϕ', as given in [11]. Values for other parameters are also shown in Fig. 7.

The problem associated with a negative normal force

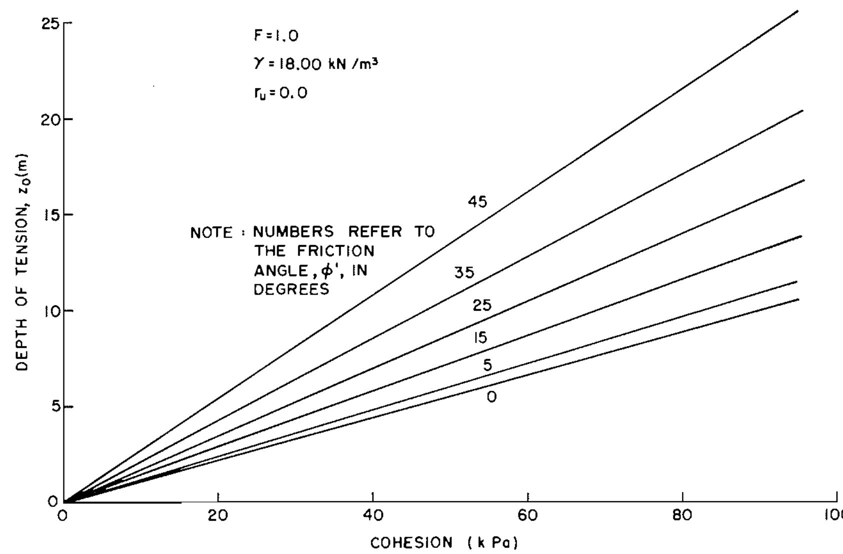

FIG. 7. Depth of tension crack zone versus cohesion for different φ′ angles.

is illustrated by example No. 2 (Fig. 8). In this example, the effect of pore-water pressure is not considered and r_u is equal to zero. Two $c'/\gamma H$ values (i.e., 0 and 0.2) have been used in the analysis of the same slip surface. The normal stresses are calculated satisfying moment equilibrium (i.e., eq. [4]) and assuming that the resultant interslice forces act horizontally. For $c'/\gamma H = 0$, positive normal forces are computed along the entire slip surface, whereas for $c'/\gamma H = 0.2$, the normal forces near the crest and the toe of the slip surface become negative. The existence of negative normal forces in the latter case are clearly the result of having mobilized large shearing resistance. The large resistance is due to cohesion since the weight component, W, is constant and $(X_R - X_L)$ is equal to zero for slices of the same slip surface. To partly overcome this problem, a tension crack zone is placed at the top of the slope. A depth of the tension zone of approximately 5 m is calculated from [11]. The factor of safety was subsequently reduced by 5% from its original value of 3.47.

The use of a tension crack zone eliminates the calculation of negative normals near the crest of the slope. The slip surface is assumed to terminate at the bottom of the tension zone. Thus, it avoids a further subdivision of shallow slices beyond this end of the slip surface where numerical difficulties will likely occur. However, there is a limitation with this method since negative normal forces may be calculated elsewhere along the slip surface below the tension zone (e.g., at the toe of the slip surface in example No. 2). This situation occurs more frequently when the slip surface is relatively shallow and the soil has a high cohesion. For example, it often occurs when analyzing shallow slip surfaces in an unsaturated soil slope where the effect of

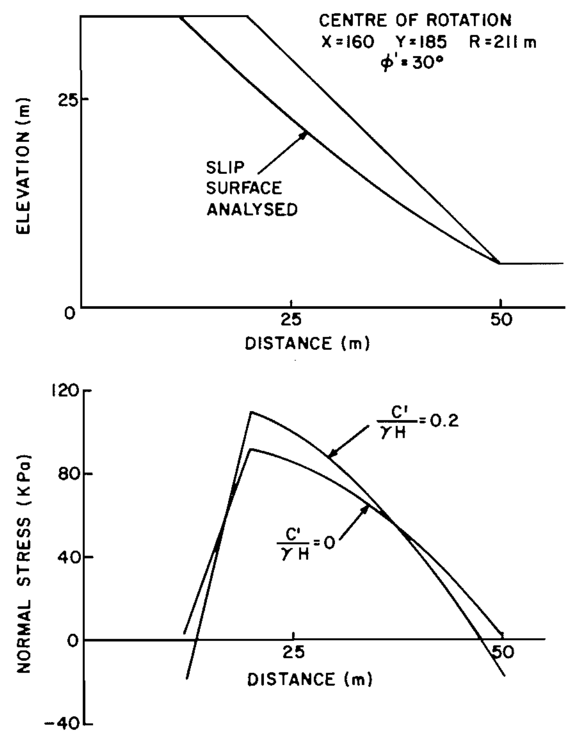

FIG. 8. Problem example No. 2.

matric suction (i.e., $(u_a - u_w)$) is regarded as part of the total cohesion of the soil (Fredlund 1979). The common occurrence of this problem can be illustrated by means of a stress analysis. Figure 9*b* depicts the Mohr circles for soil elements in two slip surfaces at different depths (Fig. 9*a*). For demonstration purposes, the resultant of stress components on the vertical plane is assumed to act

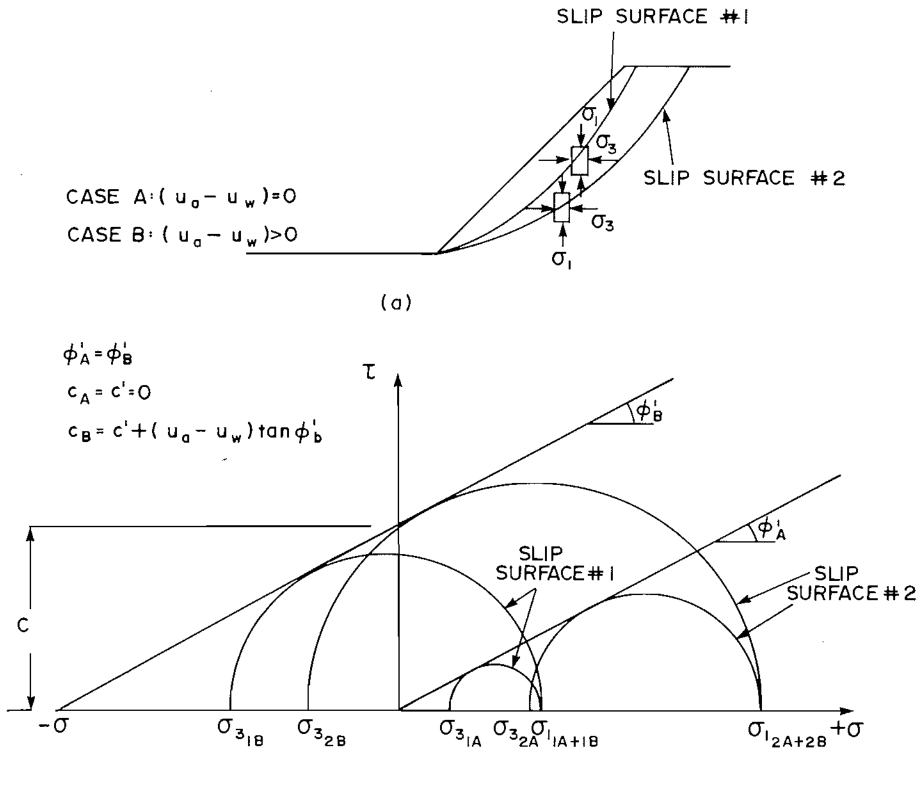

FIG. 9. (*a*) Soil elements for two slip surfaces. (*b*) Mohr circles for elements in soils with and without suction.

horizontally. The horizontal and vertical planes then become the major and minor principal planes. Two cases are investigated, one with and one without soil suction. Figure 9*b* shows that for all cases considered, the normal stress component becomes negative only for the shallow element with a high cohesion component.

The problem of a negative normal force at the base of a specific slice is not as serious as it may first appear. The negative normal is simply a quantity satisfying equilibrium for the specified shear strength parameters. It should be noted that [11] may suggest a greater depth of tension crack than is reasonable to use in an analysis. In reality, the analysis to predict the depth of cracking in a soil mass has not been solved nor verified (Taylor 1948; Pufahl *et al.* 1983).

It might seem reasonable to set the normal force at the base of a slice to zero whenever its magnitude is negative. This approach, however, is not desirable from the theoretical standpoint. From statics, the magnitudes of both $(X_R - X_L)$ and $(E_R - E_L)$ would also need to be adjusted. It follows that the mobilized shearing resistance due to friction is zero and the mobilized shearing resistance due to cohesion must be decreased or increased in order to regain the equilibrium (Fig. 6*c*). This implies that a different factor of safety must be applied to the mobilized shearing resistance for this slice. This would violate the assumption that all slices should have the same factor of safety. It is suggested that the value of the calculated negative normal force should be used in lieu of setting the normal to zero. The calculated factor of safety for the first case would be slightly smaller than for the latter case because the mobilized shearing resistance is reduced.

Numerical difficulties due to the side force assumption

The calculation of interslice forces may sometimes present difficulties in the stability computation. Most limit equilibrium methods of slices make use of an interslice force assumption to render the stability problem determinate (Janbu 1954; Bishop 1955; Morgenstern and Price 1965; Nonveiller 1965; Spencer 1967). Generally, these assumptions are related to the direction or point of application of the interslice forces. Although it is possible to employ a wide range of possible interslice force assumptions, unreasonable solutions or non-converging conditions can result if the assumption is unrealistic. Numerical difficulties may arise and as a consequence the method of analysis may not yield a satisfactory solution.

Equations [4] and [5] are nonlinear since the factor of safety term appears on both sides of the equals sign. In addition, the interslice force equations (i.e., eqs. [6] and [7]) are also nonlinear. All these variables are interrelated. A method of iteration is commonly used to solve for these nonlinear equations. Normally, a converging solution will quickly be obtained within a few iterations. To satisfy the interslice boundary conditions, the computed interslice forces must be such that the summations of forces and moments at both ends of the slip surface are equal to zero or some prescribed values. However, this condition may be difficult to satisfy if an unreasonable interslice force function has been assumed. In this case, a reasonable set of interslice normal and shear forces cannot be obtained to satisfy the boundary requirements for the given interslice force assumption. As a result, numerical instability is introduced and convergence problems are encountered.

Janbu's generalized procedure can be used to demonstrate the convergence problem associated with the use of an unreasonable interslice force assumption (Ching 1981). In this method, an assumption is made regarding the line of thrust which is the locus of the points of application for the resultant interslice forces across the slip surface. The interslice normal forces are calculated from [6] while first assuming that the interslice shear forces are equal to zero. The interslice shear force is then obtained from the summation of moments about the center of the base for an individual slice while using the computed magnitudes of the interslice normal forces and position of the line of thrust.

$$[12] \quad X_R b - (X_R - X_L)b/2 + (E_R - E_L)[t_R - (b/2)\tan\alpha + b\tan\alpha_t] - E_R b\tan\alpha_t = 0$$

where t_R = vertical distance from the base of the slice to the line of thrust on the right side of the slice and α_t = angle between the line of thrust and the horizontal on the right side of the slice.

If the width of the slice, b, is reduced to an infinitesimal value, $(X_R - X_L)b/2$, $(E_R - E_L)(b/2) \times \tan\alpha$ and $(E_R - E_L)b \tan\alpha_t$ become negligible. Equation [12] can be rewritten as:

$$[13] \quad X_R = E_R \tan\alpha_t - \frac{(E_R - E_L)}{b} t_R$$

The interslice normal and shear forces must be computed in an iterative manner since they appear on both sides of [6] and [13]. Equation [13] is similar to the interslice force function [7]. In fact, it can be interpreted as an interslice force function generated from the summation of moments about the center of the base of each slice (Fredlund *et al.* 1981). The second term in [13], which involves the rate of change in the interslice normal forces, is generally small in comparison with the magnitude of the first term and is sometimes omitted. The omission of this term implies that the inclination of the resultant interslice force is approximately equal to the inclination of the line of thrust. In other words, the interslice shear force is a function of the inclination of the line of thrust. If the line of thrust is steep, as in the case of a steeply inclined slip surface, large interslice shear forces are computed. When the slope of the line of thrust exceeds 45°, the computed shear forces will be larger than the normal interslice forces. The computation for the factor of safety may be dominated by these large forces causing convergence difficulties in the numerical scheme.

Suggestions for solving the convergence problems

Numerical problem can be caused by using an unreasonable side force assumption. This is especially true when a given side force assumption is incompatible with a reasonable stress distribution in the soil. Figure 10 demonstrates the approximate shape of a reasonable interslice force ratio distribution. When the ground surface is horizontal, the functional direction should approach zero. The functional direction is anticipated to approach the gradient of the ground surface near the middle of the steepest part of the slope. There should then be a smooth transition between these extremes.

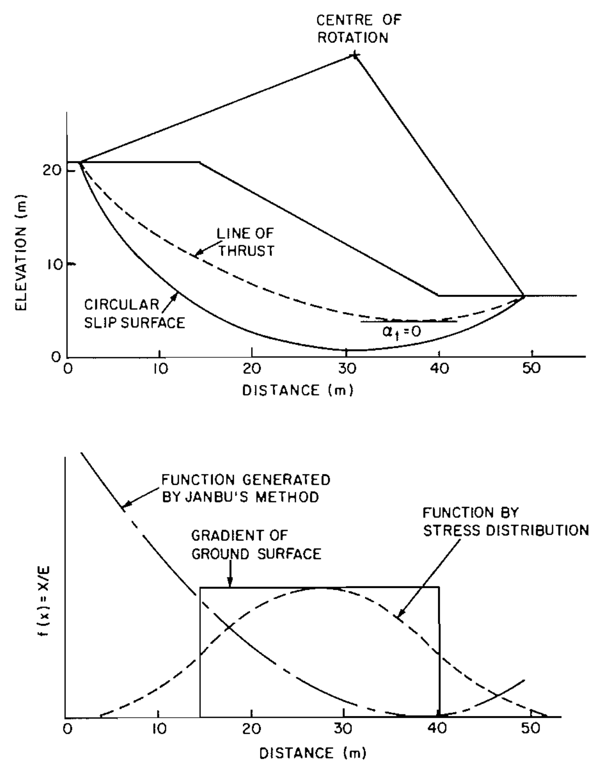

FIG. 10. Interslice force distribution functions.

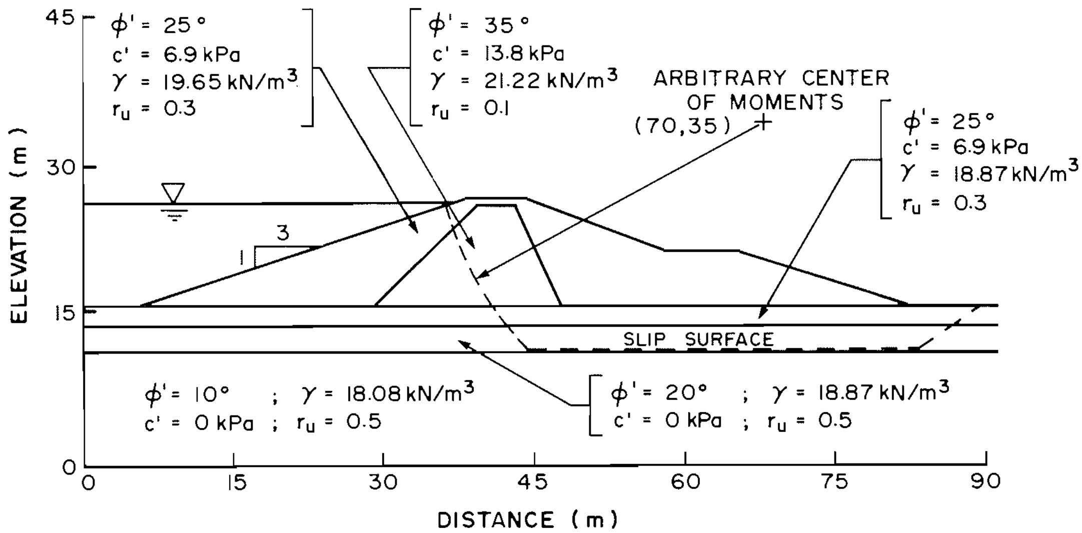

FIG. 11. Problem example No. 3.

This function should, in general, produce results from the moment and force equilibrium equations that satisfy the boundary conditions. Figure 10 also shows the side force ratio distribution that can sometimes be generated using Janbu's generalized procedure. This generated function is almost a reverse of what would appear to be a reasonable function.

In Janbu's generalized method, the convergence problem is primarily due to the assumed line of thrust. This problem may be partially resolved by reducing the inclination of the line of thrust; that is, it can be relocated above the lower third point from the slip surface. It should be noted that this approach may not give the best solution even though convergence is achieved, since the computed interslice forces may be unreasonable. It may also be partly resolved by including a tension crack zone at the crest of the slope where extreme interslice forces are usually calculated. The depth of the tension zone is calculated in accordance with [11].

Example problem to demonstrate the convergence problems

Example No. 3 (Fig. 11) is presented to illustrate the convergence problem associated with Janbu's generalized procedure. For the slip surface shown, large interslice normal and shear forces are computed for those slices along the steeply inclined portion of the slip surface. The interslice forces increase since their increased magnitudes can accumulate as the iterations proceed. The magnitude of $(X_R - X_L)$ and the weight of slices across the prescribed surface are plotted in Fig. 12. The magnitudes of $(X_R - X_L)$ for the first few slices at the upper end of the slip surface are approximately 1 to 2 orders of magnitude larger than their corresponding weights. Such large interslice shear forces are calculated

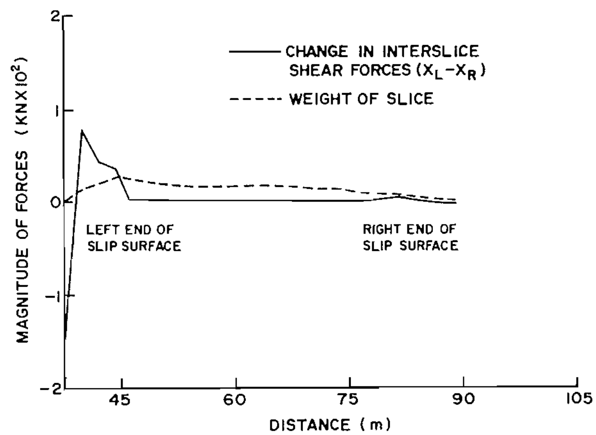

FIG. 12. Magnitudes of slice weight and change in interslice shear forces across the slip surface for example No. 3.

from [13] since the line of thrust is steeply inclined. As a result, the computation is dominated by these exceptionally large forces and the solution diverges.

The convergence problem is not restricted to Janbu's generalized method but is common to any method if an unreasonable side force assumption is used. Example No. 4 (Fig. 13) is used to demonstrate this point. In this case, several different side force assumptions were studied using the GLE method. The first and second side force functions (Fig. 14) are computed using [13] with and without the rate of change of the interslice normal forces (i.e., $(E_R - E_L)t_R/b$), respectively, when calculating the interslice shear forces. These two functions were back-calculated from the interslice shear and normal forces at the last iteration of the calculation although the solution had not converged. As shown in Fig. 14, these two generated functions vary greatly. The

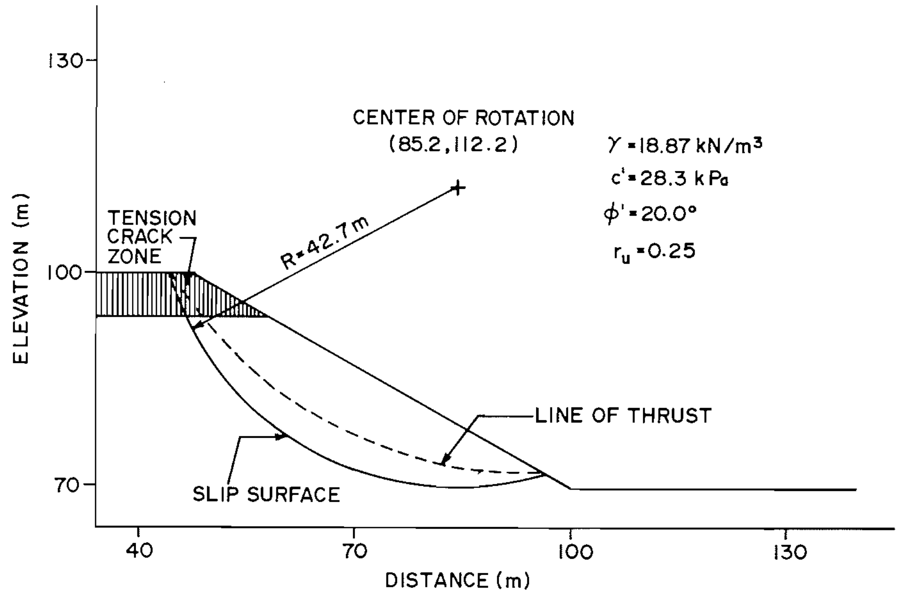

FIG. 13. Problem example No. 4.

TABLE 1. Factors of safety versus values of lambda for four functional variations for example problem No. 4

Value of lambda	Functional variation*							
	$f(x)_1$		$f(x)_2$		$f(x)_3$		$f(x)_4$	
	F_m	F_f	F_m	F_f	F_m	F_f	F_m	F_f
0.0	1.053	0.933	1.053	0.933	1.053	0.933	1.053	0.933
0.2	1.052	0.947	1.049	0.957	1.050	0.955	1.051	0.981
0.4	1.055	N.S.	1.047	0.982	1.049	0.978	1.050	1.029
0.6	N.S.	N.S.	1.045	N.S.	1.049	1.003	1.050	1.080
0.8	N.S.	N.S.	1.067	N.S.	1.050	1.029	1.049	1.135
1.0	N.S.	N.S.	1.044	N.S.	1.052	1.057	1.048	1.193
1.2	N.S.	N.S.	N.S.	N.S.	1.055	1.087	1.048	1.256

*N.S. = no solution because iteration has not converged.

third function was generated by including a tension zone. In this case, a more reasonable function was produced. The fourth function is a half-sine function.

Table 1 summarizes the computations with respect to different lambda values. Lambda represents the percentage of the side force function, $f(x)$, used for solving the factor of safety equations. When the first and second functions are used with large lambda values, significant convergence difficulties are encountered. Convergence difficulty is reduced for calculations using small lambda values since this in effect decreases the magnitude of the interslice shear forces. Solutions are readily obtained within seven to eight iterations when the third and fourth assumptions are used. Factors of safety satisfying force equilibrium are more susceptible to the problem of non-convergence than those for moment equilibrium, as would be expected. The same example problem cannot be solved using Janbu's generalized procedure if the line of thrust is assumed at the lower third position. This problem is overcome, however, by relocating the line of thrust upward. Table 2 gives a summary for the results assuming different positions for the line of thrust. The number of iterations required gives an indication of the ease at which the solution converges. The computed factors of safety generally do not vary significantly for different lines of thrust.

There are other numerical problems associated with the use of an inappropriate side force assumption. For instance, there might not be a solution which simultaneously satisfies both force and moment equilibrium

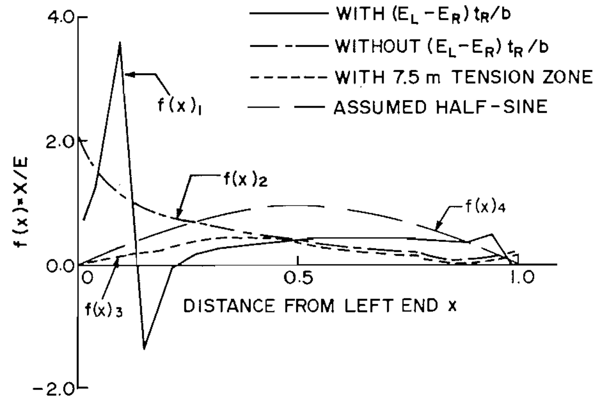

FIG. 14. Generated interslice force functions for example No. 4.

TABLE 2. Factors of safety versus positions for the line of thrust for Janbu's generalized method—example No. 4

Line of thrust*	Factor of safety†	Number of iterations
0.300	N.S.	15
0.333	N.S.	15
0.350	N.S.	15
0.375	N.S.	15
0.400	1.056	9
0.425	1.060	7
0.450	1.059	9
0.475	1.054	6
0.500	1.056	11

*Number indicated the assumed ratio of the vertical distance between the base of the slice and the line of thrust on the right side of the slice to the height of the slice.

†N.S. = no solution because iteration would not converge.

(Soriano 1976). However, more research work is necessary on this problem.

Summary

Several analytical problems associated with the limit equilibrium methods of slices have been discussed. These problems occur primarily in the numerical procedure as a result of the interslice force assumptions and geometric conditions imposed on the stability computations. These conditions have been examined and suggestions have been made to circumvent the problems.

Numerical difficulties may be the result of the normal force calculations when the variable, m_α, approaches zero or becomes negative. Values of m_α may be small and/or negative depending on the angle at the base of a slice and the value of tan ϕ'/F. In these situations, unreasonable normal forces and misleading results may be computed. It is proposed that the inclinations of the slip surface be restricted to values indicated by classical earth pressure theory.

Numerical problems may also originate from the computation of a negative normal force when analyzing highly cohesive soil slopes. Negative normal forces can be due to a large cohesional shearing resistance and the subsequent reversal of interslice normal and shear forces. It is suggested that a tension crack zone be assumed. Any negative normal forces computed beneath the tension crack zone should be used in the computation.

Convergence problems may be encountered in the stability calculation as a result of using an inappropriate interslice force function. The problem is mainly due to the fact that the assumed side force function is not compatible with the stress conditions for the problem. It is suggested that the side force assumption used should be more consistent with the geometry of the slope and the stress distribution within the soil mass.

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