LATERAL EARTH FORCE THEORY

USING LIMIT EQUILIBRIUM

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Presented at the
Ninth Canadian Congress of Applied Mechanics
(CANCAM 83)
College of Engineering
University of Saskatchewan
Saskatoon, Canada

May 30 - June 3, 1983

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ABSTRACT

This paper presents a general limiting equilibrium formulation (i.e., GLE formulation) for the lateral earth force analysis. Indeterminancy in the lateral earth force analysis occurs when the slip surface is curved. An assumption regarding the direction of the interslice forces is made to resolve the indeterminancy.

The results of the GLE formulation agree closely with most of the results obtained from other theories. The assumed distribution for the direction of the interslice forces has a greater influence in the results for the passive case than for the active case.

The main advantage of the GLE method lies in its capability to analyze complex geometric conditions. Different soil strata, pore-water pressures and external loading conditions can also be accommodated. Factors of safety greater than unity can also be applied to the shear strength of the soil for design purposes.

INTRODUCTION

The lateral earth force theory allows the computation of the magnitude of the lateral earth force exerted against an earth-retaining structure. The theory has been developed for over 200 years and is considered as the earliest theory developed in soil mechanics.

The magnitude of lateral earth force is related to the mode of shear failure that occurs within the soil mass. A soil mass can fail downward in the active sense due to its own gravity (W) which acts as the driving force. The failure takes place along a slip surface where the shear strength mobilized, S_m, along this plane reaches its limiting value. Although the shear strength mobilized is acting upward as a resisting force, there is still an external force required to bring the soil mass into a state of limiting equilibrium along the slip surface (Figure 1). This external force is provided by the earth-retaining structure and its maximum value is called the active lateral earth force, Q_a.

A soil mass can also be failed in the passive sense due to the application of an external force which is much greater than the active force. In the passive case the shear strength, S_m, is mobilized downward along the slip surface because the soil mass is moved upward (Figure 1). The minimum value of the external force required to develop a state of limiting equilibrium in the passive case is called the passive earth force, Q_p.

The development of the lateral earth force theory was originated by Coulomb (1776). Since then, a number of plasticity theories (Rankine 1857 and Kötter 1903) and elasticity theories (Boussinesq

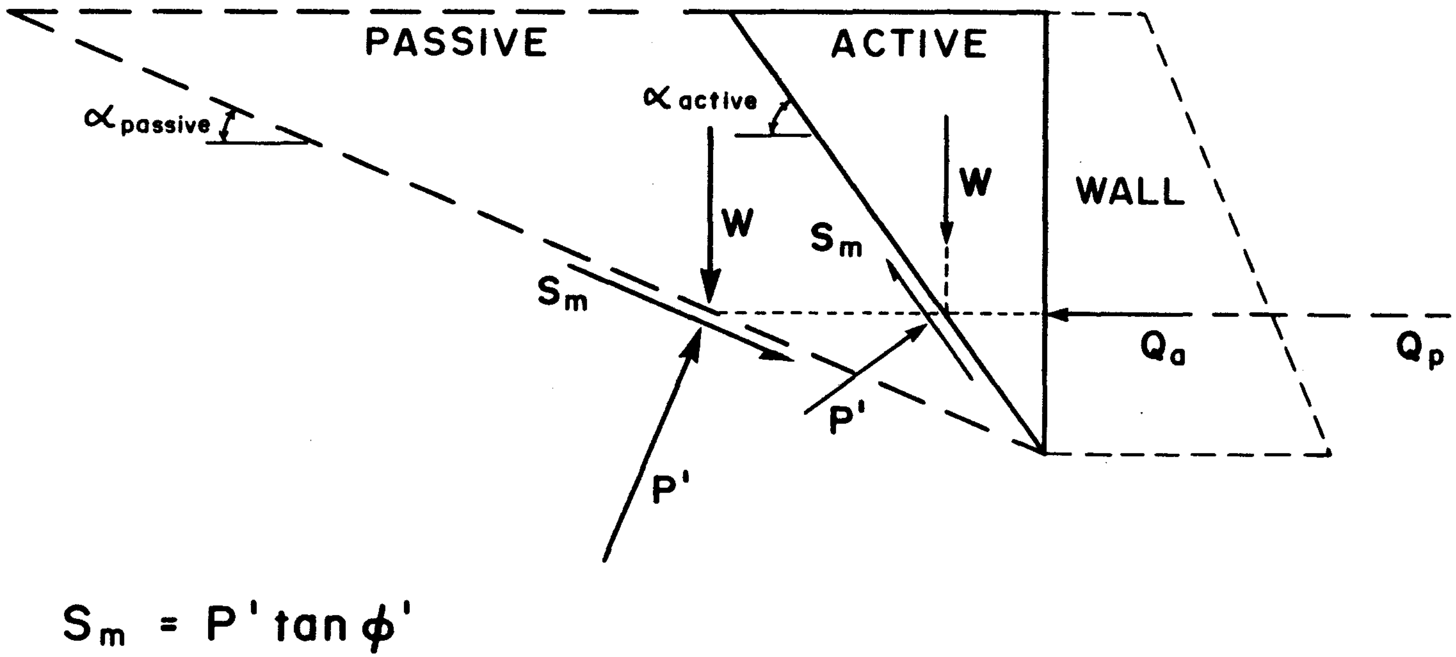


Figure 1 Forces Acting on a Cohesionless Soil Wedge in a Dry Condition for Active and Passive Cases

1885) have contributed to the understanding of lateral earth force calculations (Hansen 1953). The experimental and field works performed by Terzaghi (1934) and Tschebotarioff (1949) have further provided the theory with a better knowledge of soil-structure interaction processes. Many classic theories are limited by the assumptions used in the derivations, and other rigorous theories are complex in their applications.

The limit equilibrium method of slices is a numerical method commonly used in the slope stability analysis. The method is based upon the principles of kinematics only (i.e., static equilibrium of forces and/or moments) without giving any consideration to the displacement of the soil mass. These principles can also be used for analyzing lateral earth force problems. In general, the lateral earth force problem is indeterminant. Janbu (1957) utilized an assumption regarding the point at which the interslice forces act (i.e., the line of thrust) to resolve the indeterminancy problem. Shields and Tolunay (1973) computed passive pressure coefficients by assuming the direction of the interslice forces.

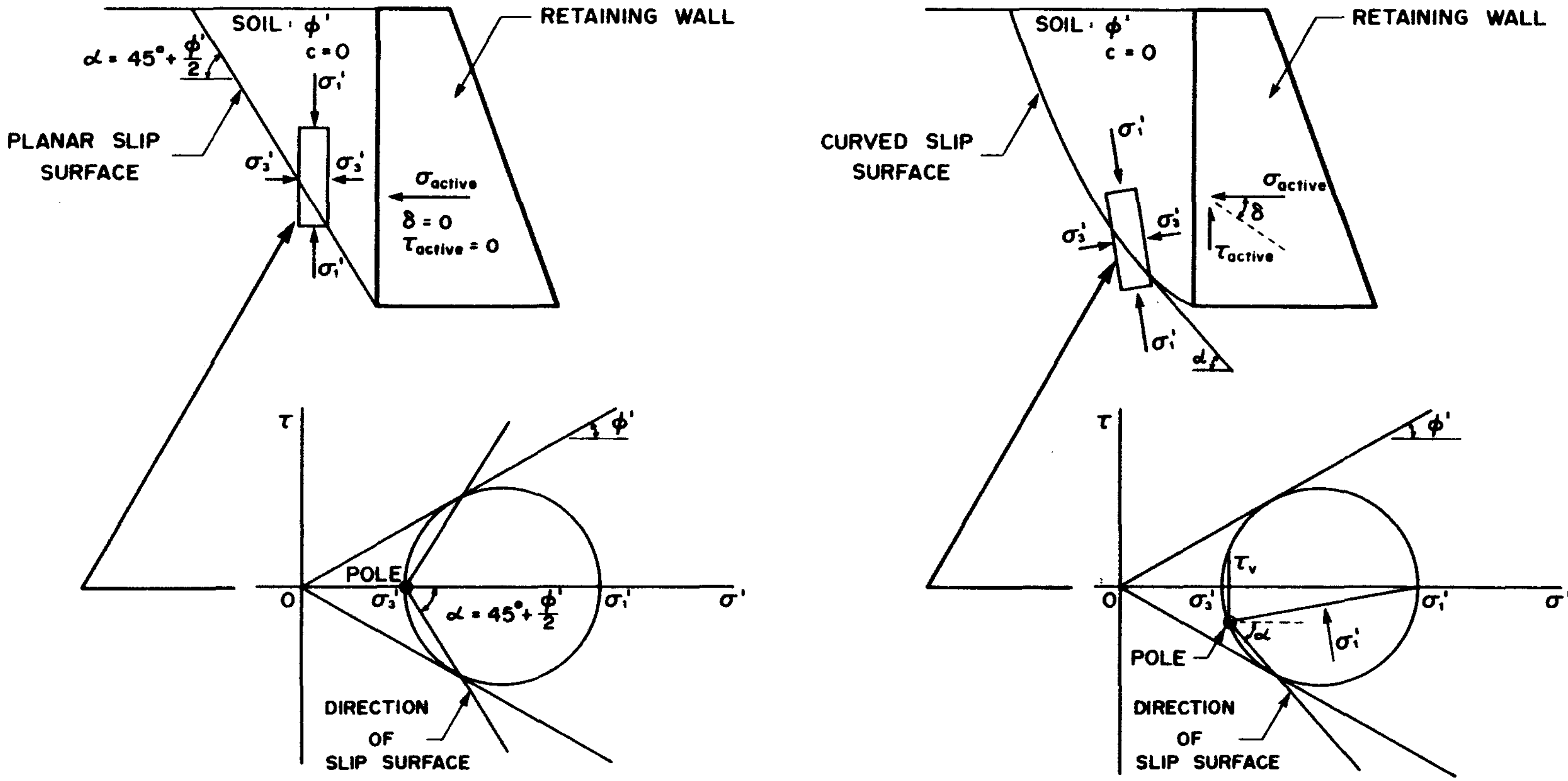
In 1981, Fredlund, Krahn and Pufahl proposed a general formulation of the limit equilibrium method which has been shown to be a common formulation for various limit equilibrium methods of slope stability. This general limit equilirium method will hereafter be referred to as the GLE method. The GLE method satisfies force and moment equilibriums and makes use of an assumption regarding the direction of the interslice forces. This paper attempts to formulate the GLE method for lateral earth force problems. A version of the SLOPE-II program at the University of Saskatchewan, Saskatoon, has been modified in order to accommodate the lateral earth force computation in accordance with the GLE formulation.

PRINCIPLES OF LIMIT EQUILIBRIUM ANALYSIS

Limit equilibrium method is a numerical analysis which only uses static equilibrium of the soil mass in its computations. Several basic assumptions and principles are used in formulating the limit equilibrium analysis. Assumptions are made regarding the shape and location of the potential slip surface. The soil mass is assumed to fail along the slip surface according to the Mohr-Coulomb failure criteria. Analysis is performed by investigating the force and/or moment equilibrium of the soil mass above the slip surface.

The shape of the slip surface was assumed to be planar (Figure 1) in the Coulomb theory (1776) for the active or passive case. The assumption of a planar slip surface is rigorously correct only when there is no shear force developed within the soil mass (Figure 2.a). In other words, the vertical and horizontal planes are the principal planes. For the active case of a horizontal slope and a vertical wall, the critical slip surface (i.e., gives the maximum active force) has an inclination angle, α, of $(45^{o} + \phi'/2)$ from the horizontal.

In order to reach the active or passive case, deformation must occur within the soil mass to fully mobilize the shear strength along the slip surface (Figure 3). Considerably greater deformation is required to mobilize the passive case than to mobilize the active case. This deformation produces shear forces between the wall and soil mass, and the shear force is induced into the soil mass. In this case, the horizontal and vertical planes are not the principal planes.



(a) Active Case With No Wall Friction

(b) Active Case With Wall Friction

Figure 2 Mohr-Coulomb Diagram for Cases With and Without Wall Friction

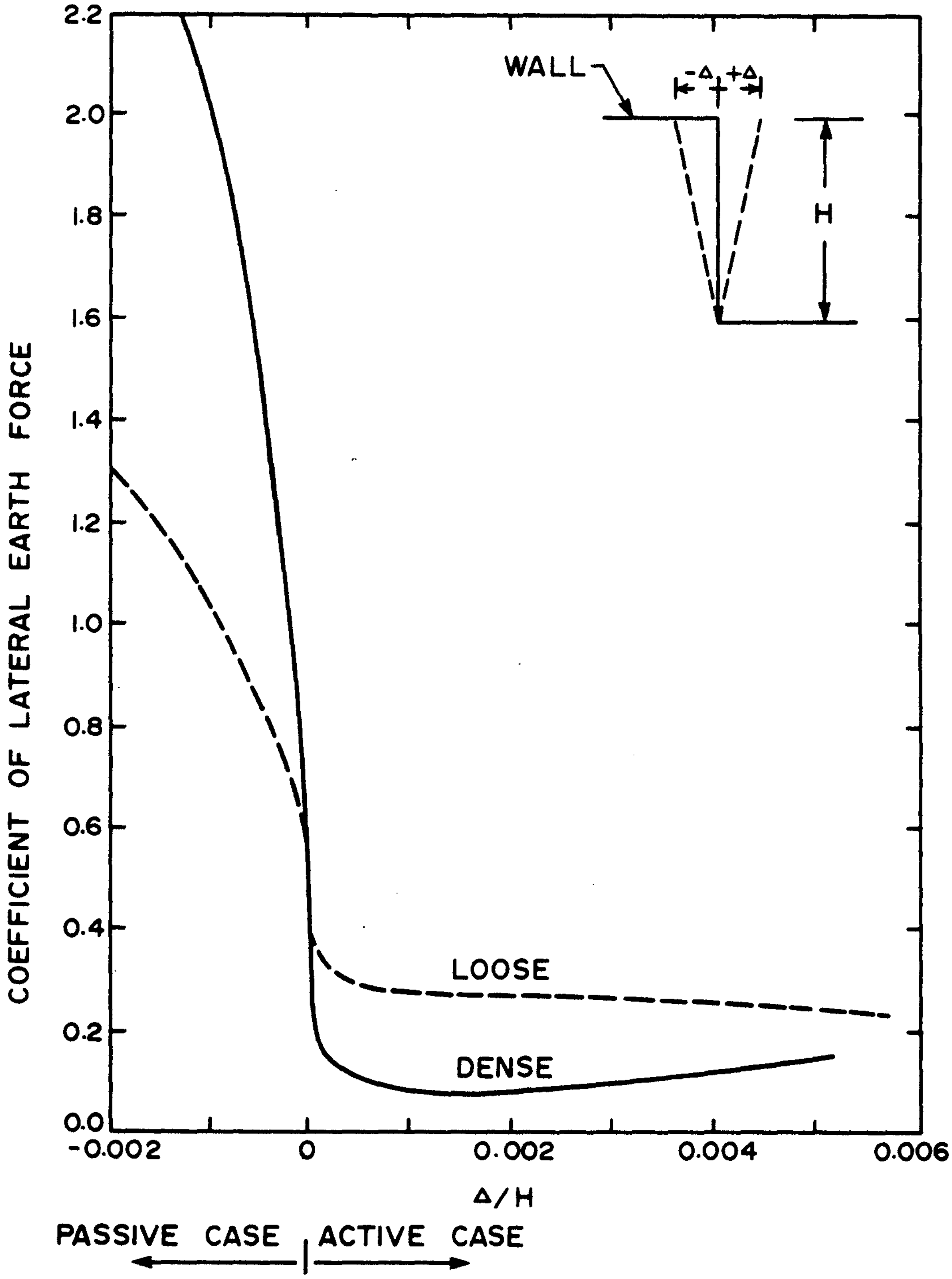
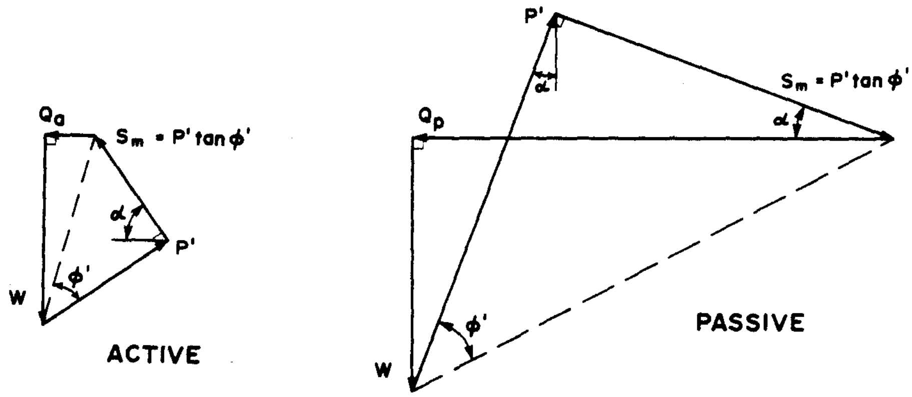


Figure 3 Deformation Required to Develop Active and Passive Cases (After Terzaghi 1954)

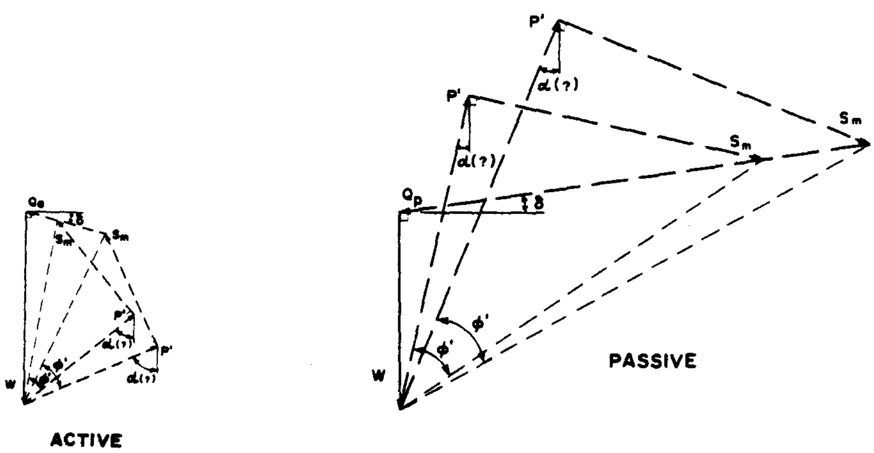
The inclination angle of the slip surface, at a certain distance from the wall, rotates corresponding to the amount of shear force developed at that location (Figure 2.b). As a result, a curved slip surface is developed instead of a planar slip surface. Therefore, the analysis based on the assumption of a planar slip surface (Coulomb 1776 and Rankine 1886) often results in over-estimating the active force and in under-estimating the passive resistance.

The curved slip surface has been used in many lateral earth force theories. This curved slip surface can be a logarithmic spiral (Terzaghi and Peck 1943, Janbu 1957, Shields and Tolunay 1973), or an ellipse (Caquot and Kerisel 1948) or a circle (Krey 1936). The shear force developed in the passive case is greater than in the active case; therefore, the slip surface is more curved in the passive case. This means that the use of a planar slip surface in the analysis is more erroneous for the passive case than for the active case (Terzaghi and Peck 1967).

The lateral earth force analysis using a planar slip surface is a determinant problem. On the other hand, the use of a curved slip surface causes the lateral earth force analysis to become indeterminant (Rahardjo 1982). The indeterminancy problem in the analysis using a curved slip surface is due to the inclination angle, α, which is not constant along the slip suface. On the planar slip surface, the α angle is constant (Figure 1); therefore, the magnitude of the lateral earth force, Q, can be obtained directly from a force polygon for the whole wedge of the soil mass above the slip surface (Figure 4.a). However, this procedure cannot be applied to the analysis using a curved slip surface. The changing angle, α , along



(a) Planar Slip Surface



(b) Curved Slip Surface

Figure 4 Force Polygons for a Cohesionless Soil in a Dry Condition Using Planar and Curved Slip Surfaces the curved slip surface makes it impossible to establish the force polygon for the whole free body of the soil mass above the slip surface (Figure 4.b).

The analysis using a curved slip surface can be performed by dividing the soil mass into vertical slices. The analysis will use the alpha angle at the centroid of the base of each slice (Figure 5). The problem, however, is still indeterminant because the interslice forces (i.e. X and E), which are unknowns, will appear in the analysis of each slice. Either additional elements of physics or an assumption regarding the direction or magnitude of some of the forces is required to render the problem determinate.

The GLE method for slope stability analysis utilizes an assumption regarding the direction of the interslice forces (Fredlund, Krahn and Pufahl 1981). This assumption has been found to be satisfactory in resolving many slope stability problems and is commonly used in practice (Bishop 1955, Morgenstern and Price 1965, and Spencer 1967). The direction of the interslice forces can be described by an arbitrary mathematical function provided that the computed forces are physically admissible (Morgenstern and Price 1965).

Several attempts have been made to estimate the direction of the interslice force using theory of elasticity (e.g., finite element method) by considering the boundary conditions and the stress-strain relationship of the soil (Wilson 1982, Fan 1983). Therefore, the GLE method for lateral earth force analysis was formulated utilizing an assumption regarding the direction of the interslice forces (Rahardjo 1982).

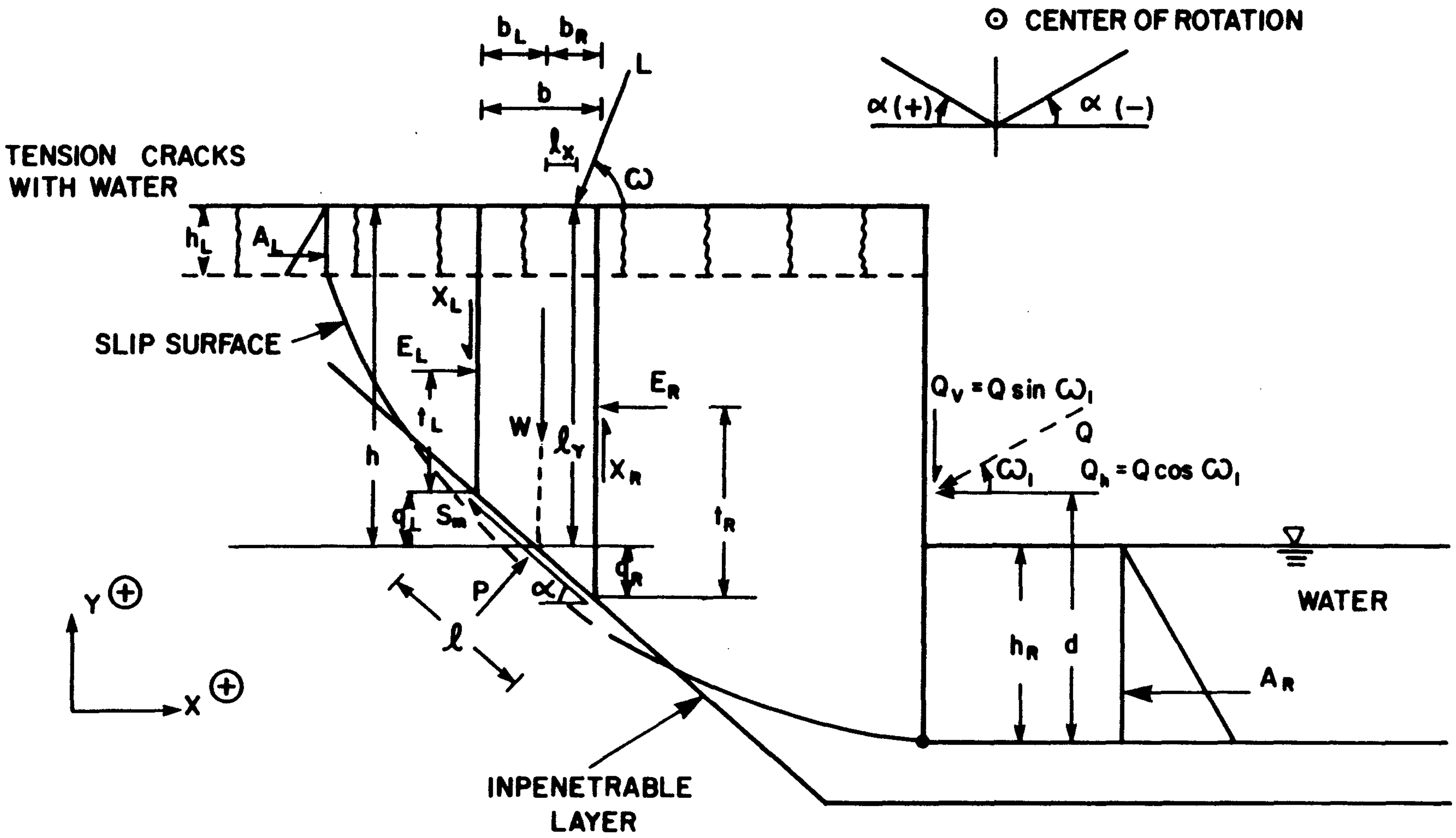


Figure 5 Forces Acting on the Overall Slope and on Each Slice of a Composite Slip Surface

LATERAL EARTH FORCE FORMULATIONS

The assumed potential slip surface with vertical slices through the soil mass is shown in Figure 5. The definition of each variable used is as follows:

W = the total weight of a slice having width 'b' and height 'h'

P = the total normal force on the base of a slice

S_m = the shear force mobilized on the base of each slice

E_L, E_R = the total horizontal interslice normal forces (the 'L' and 'R' subscripts on the 'E', 'X' and 'A' variables denote the left and right sides, respectively)

X_L, X_R = the vertical interslice shear forces

A_L, A_R = the resultant external water forces

α = the angle between the tangent to the centroid of the base of each slice and the horizontal

ω = the angle of an externally applied line load from the horizontal. This angle is measured counter-clockwise from the positive x-axis.

b_L, b_R = the horizontal distance between the centroid of the base and the left or right edge of the slice, respectively.

t_L, t_R = the vertical distance between the left and right corners of the base of the slice and the point of application of the interslice forces on the left or right side of the slice, respectively.

q_L, q_R = the vertical distance between the centroid of the base with the left or right bottom corner of the slice, respectively.

ℓ = the length of the slip surface at the base of each slice.

ℓ_x = the horizontal distance from the centroid of the base of a slice to the point of application of an external line load.

ℓ_y = the vertical distance from the centroid of the base of a slice to the point of application of an external line load.

Q = the active or passive lateral earth force.

ω_1 = the direction of the lateral earth force which is positive as measured counter-clockwise from the positive x-axis.

Q_h = the horizontal component of the lateral earth force (i.e., $Q \cos \omega_1$)

Q_v = the vertical component of the lateral earth force (i.e., $Q \sin \omega_1$)

δ = the wall friction angle. The wall friction is positive as measured counter-clockwise from the normal to the wall.

d = the vertical distance from the heel of the retaining wall to the point of application of the lateral earth force.

h_L, h_R = the height of the water which is acting as an external water force on the left or right of the slope, respectively.

The general limit equilibrium method, GLE, derived in this paper is applicable to any shape of slip surface: circular, planar or a combination of both circular and planar slip surfaces (i.e., composite).

The soil is assumed to fail according to the Mohr-Coulomb failure criteria. The shear strength of the soil for the effective stress analysis (Terzaghi 1936) can be expressed as:

[1] $$s = c' + (\sigma_n - u) \tan \phi'$$

where:

s = shear strength

c' = effective cohesion

ϕ' = efective angle of internal friction

σ_n = total normal stress

u = pore-water pressure

The magnitude of the shear force mobilized on the base of a slice can be computed according to equation [2].

$$[2] \qquad S_m = \frac{\ell}{F} \{c' + (\sigma_n - u) \tan \phi'\}$$

where:

F = factor of safety; the factor by which the shear strength of the soil must be reduced in order to bring the soil mass into a state of limiting equilibrium along an assumed slip surface.

The factor of safety is assumed to be the same for all slices. In the active and passive cases, the factor of safety is set equal to 1.0 because the shear strength is assumed to be fully mobilized. However, a factor of safety greater than 1.0 could also be used for design purposes.

The normal stress distribution along the slip surface and the location of its resultant are unknowns. The lateral earth pressure distribution along the retaining structure and the location of its resultant are also unknowns.

The three static equilibrium equations that can be used are the summation of forces in two directions and the summation of moments. These equations of static equilibrium and the Mohr-Coulomb failure criteria are insufficient to render the lateral earth force problem determinate.

All of the formulations derived in this paper are applicable for slopes facing either to the left or to the right. However, the lateral earth force computations presented herein use a slope that is facing to the right. The lateral earth force, Q, is considered as an external load required to bring the soil mass into a state of limiting equilibrium along an assumed slip surface. This lateral earth force is assumed to act at the last slice, which is the right end slice (Figure 6).

The summation of forces in the vertical direction for each slice is used to compute the normal force on the base of the slice.

[3] $$P = \frac{W + (X_L - X_R) - \frac{c'\ell}{F} \sin\alpha + \frac{u\ell \tan\phi' \sin\alpha}{F}}{m_\alpha}$$

$$+ \frac{[L \sin\omega] + [Q \sin\omega_1]}{m_\alpha}$$

where: $m_\alpha = \cos\alpha + \frac{\sin\alpha \tan\phi'}{F}$

The terms in the square brackets, [ ], are only relevant to the slice where the line load, L, or the lateral earth force, Q, acts.

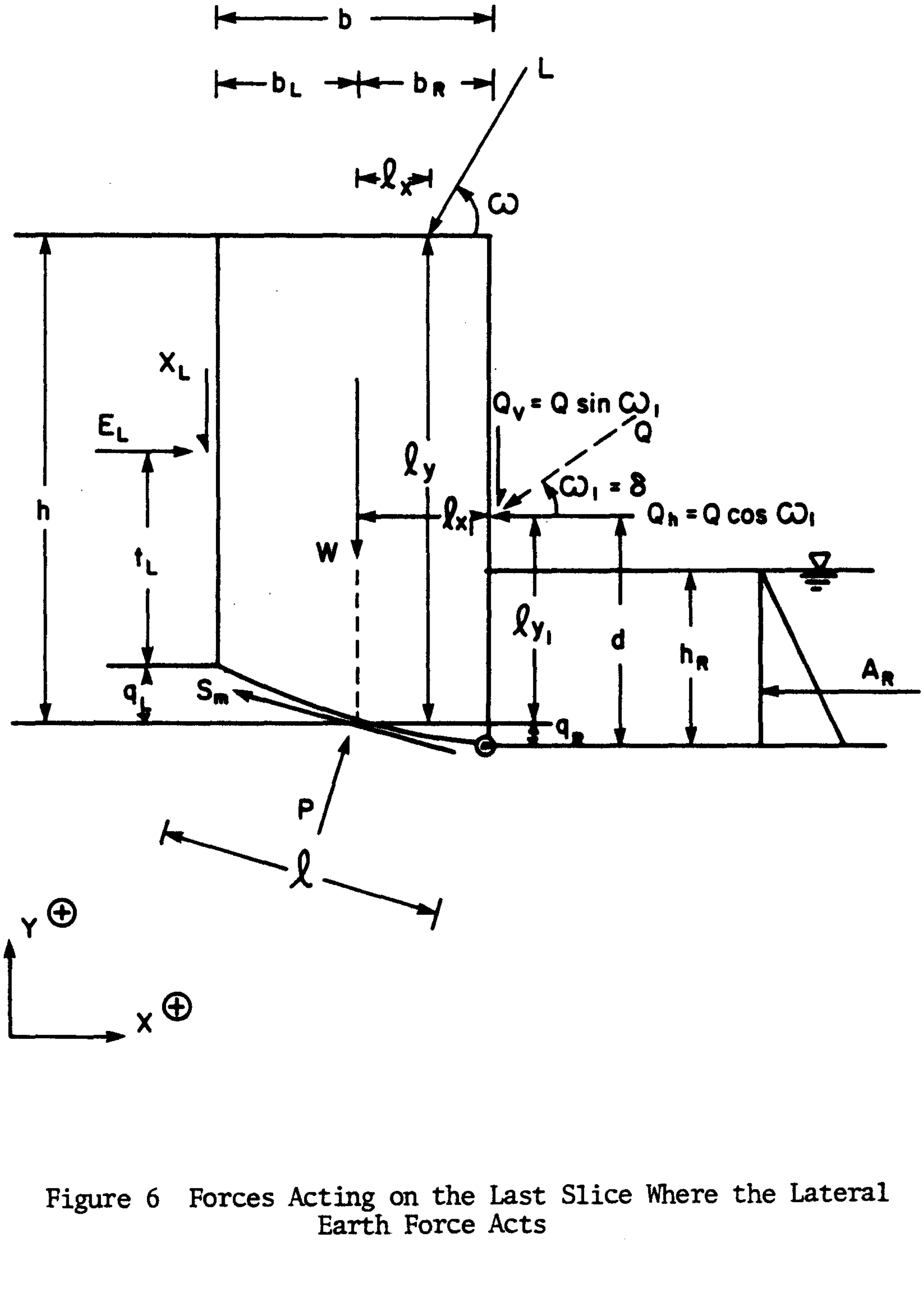


Figure 6 Forces Acting on the Last Slice Where the Lateral Earth Force Acts

The normal force is computed for the first iteration by setting the interslice shear forces, X, to zero and assuming an initial value for the lateral earth force, Q. On subsequent iterations the interslice forces and the lateral earth force are calculated for each iteration.

The interslice nomal forces, E, are obtained by analyzing the horizontal equilibrium of forces on each slice.

[4] $$E_R = E_L + P \sin \alpha - S_m \cos \alpha + kW - [L \cos \omega] - [Q \cos \omega_1] + [A_L] - [A_R]$$

The term [A] is only applied at the extremities of the slip surface if there is an external water force. The interslice shear forces, X, are computed by utilizing an arbitrary mathematical function to describe the direction of the interslice force (Morgenstern and Price 1965).

[5] $$\frac{X}{E} = \lambda f(x)$$

where:

f(x) = a mathematical function that describes the relationship between X and E across the slope, and

λ = a constant representing the percentage of the function used in the calculation of the lateral earth force.

The right interslice shear forces, X_R, can be computed using equation [6].

[6] $$X_R = E_R \cdot \lambda f(x)$$

The interslice normal forces for each slice, E, are first computed from equation [4]. The interslice shear forces, X, are then computed using equation [6]. Calculations are commenced at the first or left-end slice, where the slip surface intersects the ground surface and integration proceeds across the slope.

The magnitude of the lateral earth force, Q, is calculated from the overall equilibrium of forces in the horizontal direction.

$$Q = \frac{\Sigma P \sin \alpha - \Sigma S_m \cos \alpha + \Sigma kW + A_L - A_R - \Sigma L \cos \omega}{\cos \omega_1} \quad [7]$$

Equations [8] and [9] can be used to compute the horizontal and vertical components of the lateral earth force, respectively.

$$Q_h = Q \cos \omega_1 \quad [8]$$

$$Q_v = Q \sin \omega_1 \quad [9]$$

The inclination angle, ω_1, is dependent on the wall friction angle, δ, and the inclination angle of the wall. When the wall is vertical the inclination angle, ω_1, equals δ.

The points of application of the interslice forces (i.e., its vertical distance from the base of the slice), t, are calculated from the summation of moments about the centroid of the base of each slice.

$$t_R = \frac{E_L (t_L + q_L) - X_L b_L + kW h/2 + E_R q_R - X_R b_R}{E_R} \quad [10]$$

$$\frac{+ [A_L h_L/3] - [L \cos \omega \ell_y] + [L \sin \omega \ell_x]}{E_R}$$

The computation is started at the left-end slice and proceeds across the slope. At the last slice the vertical distance of the lateral earth force from the base of the slice, d, is obtained using equation [11].

[11] $$d = \frac{E_L\ (t_L + q_L) - X_L\ b_L + kW\ h/2 - [A_R\ h_R/3]}{Q \cos \omega_1}$$

$$- \frac{[L \cos \omega\ \ell\ _y] + [L \sin \omega\ \ell\ _x] + Q \sin \omega_1\ \ell x_1}{Q \cos \omega_1} + q_R$$

The lateral earth force is assumed to act at the right edge of the last slice.

The force equilibrium in the horizontal and vertical directions together with the assumption regarding the direction of the interslice force have been used in order to compute the magnitude of the lateral earth force. In case of a planar slip surface, the magnitude of the lateral earth force can be computed from the force equilibrium in two directions only. The assumption regarding the interslice force is not needed in the analysis of a planar slip surface because it is a determinate problem. Therefore, a closed form equation for the lateral earth force of a planar slip surface is possible as shown in equation [12] (Rahardjo 1982).

[12] $$Q = \frac{\Sigma\ [kW \sin \alpha + A_L \sin \alpha - A_R \sin \alpha - W \cos \alpha - L \sin (\alpha + \omega)] \left(\sin \alpha - \frac{\tan \phi'}{F} \cos \alpha\right) + \Sigma\left(\frac{c'\ell}{F} \cos \alpha - \frac{u\ell}{F} \tan \phi' \cos \alpha - kW - A_L + L \cos \omega\right)}{\sin (\alpha + \omega_1)\ (\sin \alpha - \tan \phi' \cos \alpha) - \cos \omega_1}$$

RESULTS OF ANALYSES

Some analyses of active and passive cases were performed for comparing the results obtained from the GLE method with the results from other theories. A horizontal slope against a vertical wall was analyzed using various combinations of cohesionless soils with friction angle, ϕ', and wall friction angle, δ . For one combination of ϕ' and δ , a number of circular slip surfaces were investigated. The slip surface which gives the maximum active force, Q_a, or the minimum passive resistance, Q_p, is considered as the critical slip surface. The interslice force function used in the active case was a triangular function (i.e., No. 2 on Figure 8.a). A zero interslice force function was used in the passive case as proposed by Rowe (Lee and Moore 1968) and is considered to give the minimum passive resistance.

The results are presented as the coefficient of lateral earth force which is defined in equation [13] for the active case.

[13] $$K_a = \frac{Q_a}{1/2\ \rho g\ H^2}$$

where:

Q_a = the maximum active lateral earth force

K_a = the coefficient of active lateral earth force

The coefficient of the horizontal component of the active lateral earth force can then be expressed as:

[14] $$K_{ah} = K_a \cos \delta$$

The coefficient of passive lateral earth force can be written in a form similar to that of equation [13] (i.e., $K_p = Q_p / 1/2 \rho g H^2$); and the coefficient of the horizontal component of the passive force can be written in a form similar to equation [14] (i.e., $K_{ph} = K_p \cos\delta$). where:

Q_p = the minimum passive lateral earth resistance

Table 1 presents the coefficients of active force, K_a, obtained from the GLE method (i.e., circular slip surface) and the K_a values obtained from the Coulomb theory (i.e., planar slip surface), and from Krey's method (Tschebotarioff 1979) (i.e., curved slip surface). Table 1 shows that the three methods give essentially the same results for different cases of soil and wall friction angles.

Table 1 Comparisons of Coefficients of Active Lateral Earth Force, Ka

ϕ' [Degrees]	δ [Degrees]	K_a		
		Coulomb (planar slip surface)	Krey (curved slip surface)	GLE (circular slip surface)
20	0	0.49		0.49
	- 2	0.48		0.48
	- 5	0.47		0.47
	-10	0.45		0.45
	-20	0.43		0.43
30	0	0.33	0.33	0.33
	-10	0.31		0.31
	-20	0.30	0.30	0.30
	-30	0.30	0.31	0.30
40	0	0.22	0.22	0.22
	-10	0.20		0.20
	-20	0.20		0.20
	-40	0.21	0.22	0.21

These findings show that the use of a planar instead of a curved slip surface does not give any difference in results for the active case.

In the passive case, comparisons are made for the coefficient of the horizontal component of passive force, K_{ph}, as presented in Figure 7. The results from the GLE method are in a good agreement with most of the results from other theories which used a curved slip surface. However, if the GLE method (i.e., circular slip surface) is compared with the Coulomb theory (i.e., planar slip surface) significant differences in K_p values are found (Table 2). This difference increases as the wall friction angle, δ, increases. In other words, the curved slip surface gives more critical values of passive resistance than the planar slip surface especially when the shear force developed is large. Therefore, the use of a curved slip surface is necessary in the passive case.

Table 2 Comparisons of Coefficients of Passive Lateral Earth Force, K_p

ϕ' Degrees	δ Degrees	K_p Coulomb (planar slip surface)	K_p GLE (circular slip surface)
20	0	2.04	2.04
	+ 2	2.15	2.13
	+ 5	2.31	2.23
	+10	2.64	2.35
	+20	3.53	2.64
30	0	3.00	3.00
	+10	4.14	3.65
	+20	6.11	4.17
40	0	4.60	4.62
	+10	6.95	5.97
	+20	11.77	7.31

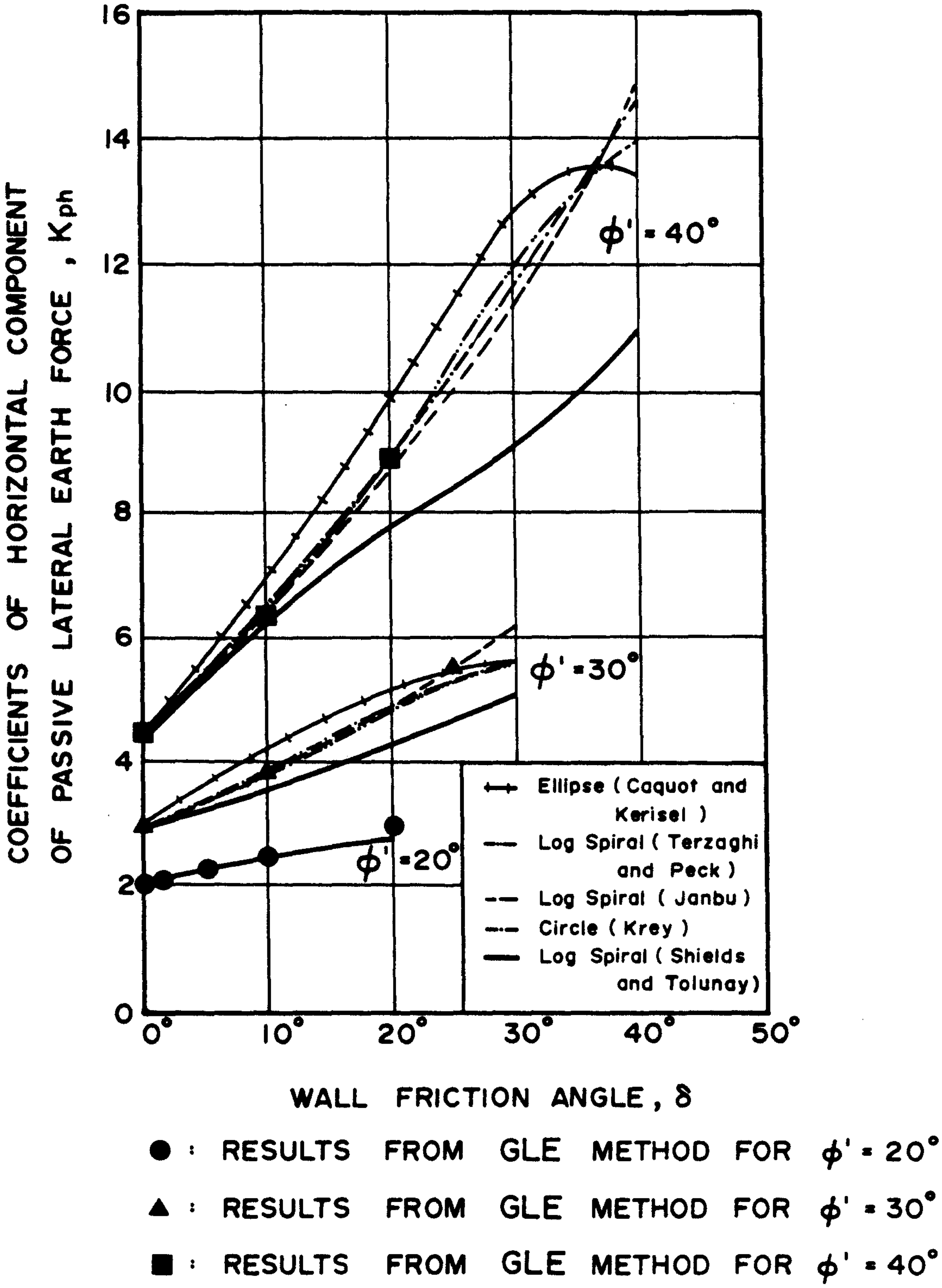


Figure 7 Comparisons of K_{ph} Values for Different Methods of Computations (After Shields and Tolunay 1973)

EFFECT OF INTERSLICE FORCE FUNCTION

The direction of the interslice force has been assumed to render the lateral earth force problem determinate. This assumption, however, should be selected such that the boundary conditions and the failure criterion along the slip surface are satisfied. Figure 8 shows some interslice force functions that might be used in the active and passive cases. The direction of the resultant of interslice forces approaches the wall friction angle, δ , at the boundary between the soil mass and the wall. At the other end where the slip surface cuts the horizontal ground surface, the at-rest stress condition which has approximately zero vertical shear stress is assumed. The distribution of shear between these two ends can be estimated from theory of elasticity (e.g., finite element method) by considering the boundary conditions and the stress-strain relationship of the soil (Wilson 1982, Fan 1983).

Some study of the effect of interslice functions on the lateral earth force computations have been performed by Rahardjo (1982). Table 3 presents the different interslice force functions used in the active case. The difference in K_a values obtained from different functions is small. However, this is not true for the passive case. Table 4 shows that different interslice force functions used in the passive case produce significant differences in K_p values. Therefore, the shape of the interslice force function has a greater effect for the passive case than for the active case.

In general, the magnitude of the lateral earth force decreases as the interslice force function used increases towards a positive value (i.e., the left shear interslice force has a downward direction). For

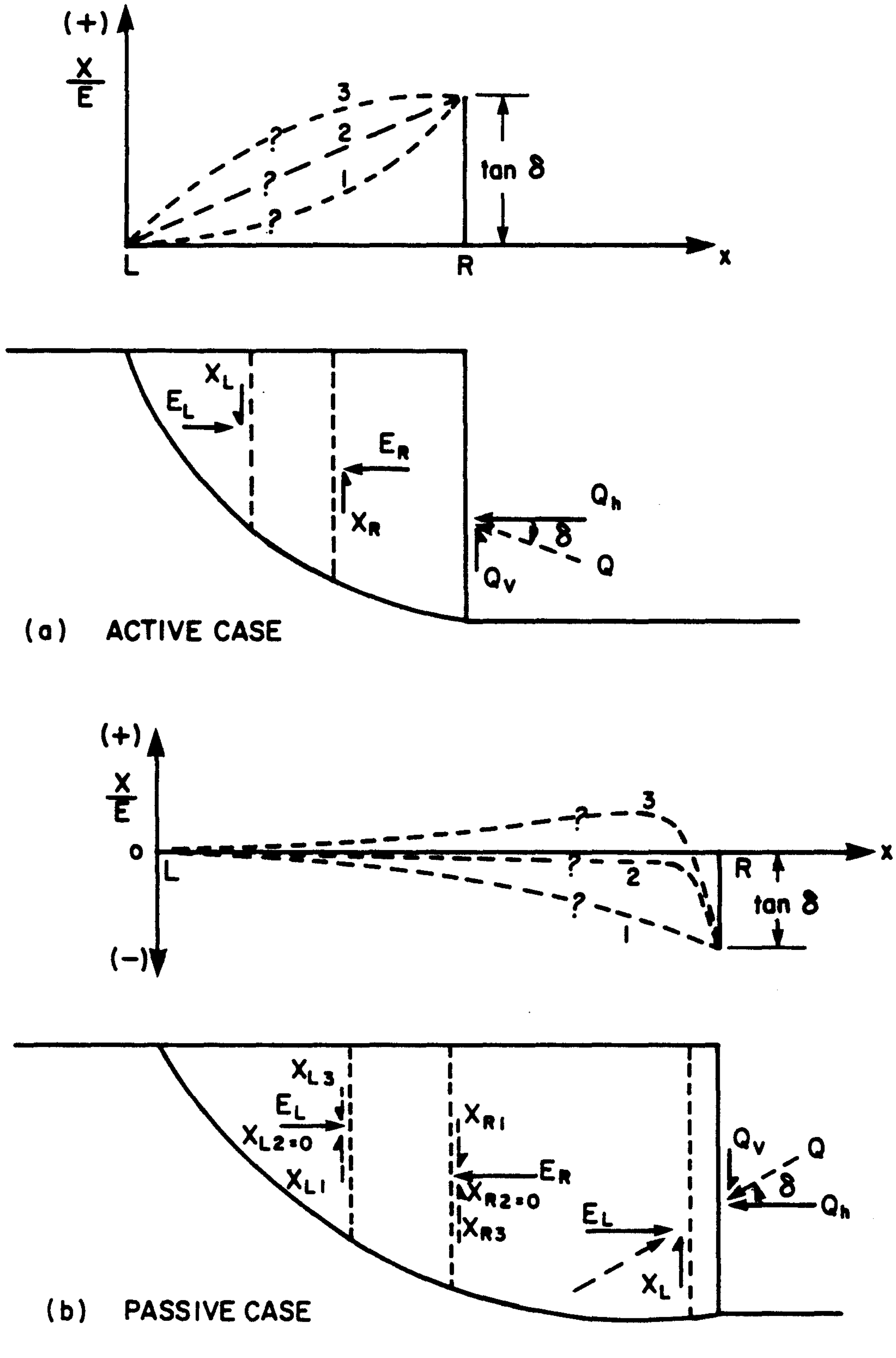
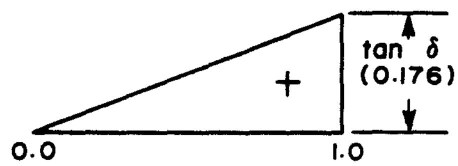


Figure 8 Assumed Interslice Force Functions for Active and Passive Cases

Table 3 Trial Interslice Force Functions for Active Case
(ρg = 20 kN/m^3, ϕ' = 30.0°, δ = - 10.0°
height of wall = 30.0 m)

<table>
<tr><th>No</th><th>INTERSLICE FUNCTION, $\frac{X}{E}$</th><th>Q_{active} (kN)</th><th>Ka</th></tr>
<tr><td>1</td><td>0</td><td>2790</td><td>0.310</td></tr>
<tr><td>2</td><td>
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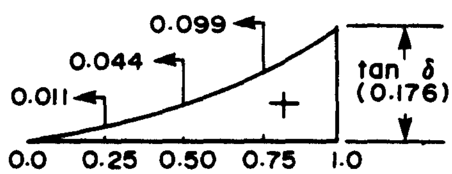
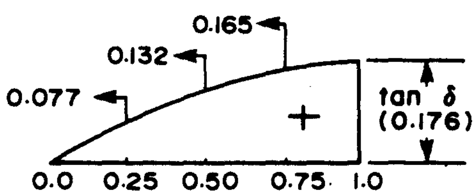
<tr><td>3</td><td>
</td><td>2779</td><td>0.309</td></tr>
<tr><td>4</td><td>
</td><td>2773</td><td>0.308</td></tr>
</table>

Table 4 Trial Interslice Force Functions for Passive Case
(ρg = 20 kN/m^3, ϕ' = 30.0°, δ = + 10.0°,
height of wall = 30.0 m)

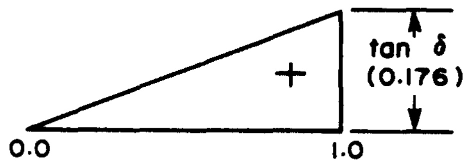
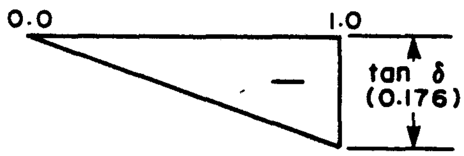
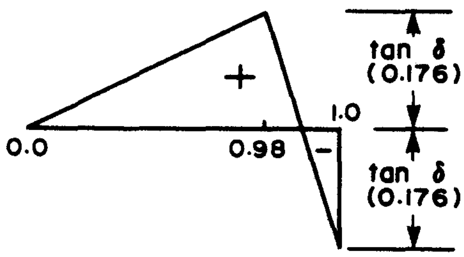
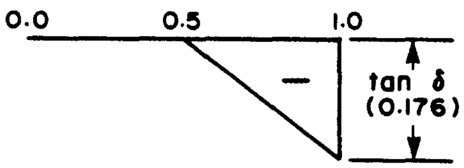
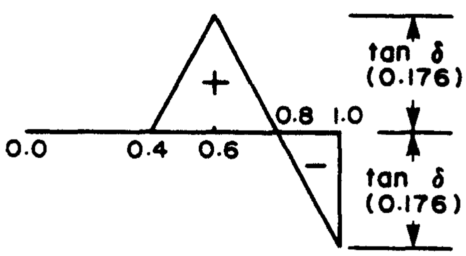
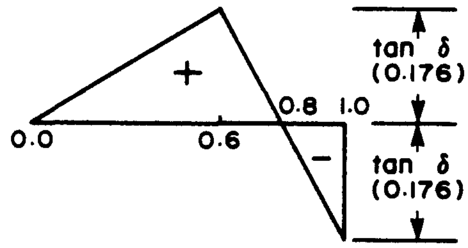
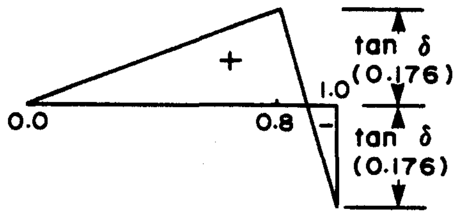
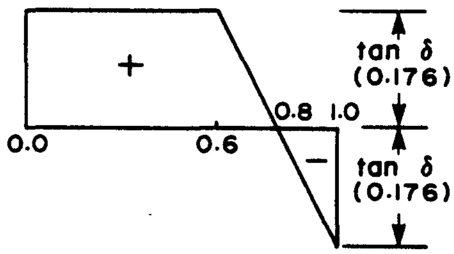
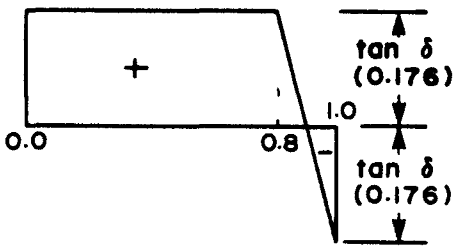
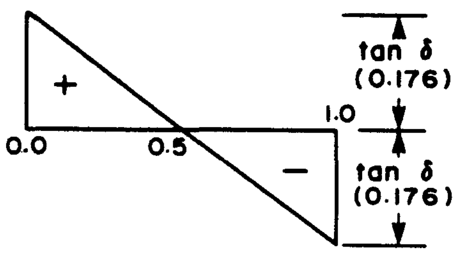
<table>
<tr><th>No</th><th>INTERSLICE FUNCTION, $\frac{X}{E}$</th><th>$Q_{passive}$ (kN)</th><th>K_p</th></tr>
<tr><td>1</td><td>0</td><td>35557</td><td>3.951</td></tr>
<tr><td>2</td><td>
</td><td>32924</td><td>3.658</td></tr>
<tr><td>3</td><td>
</td><td>38912</td><td>4.324</td></tr>
<tr><td>4</td><td>
</td><td>32885</td><td>3.654</td></tr>
<tr><td>5</td><td>
</td><td>37226</td><td>4.141</td></tr>
<tr><td>6</td><td>
</td><td>35125</td><td>3.913</td></tr>
</table>

Table 4 (continued)

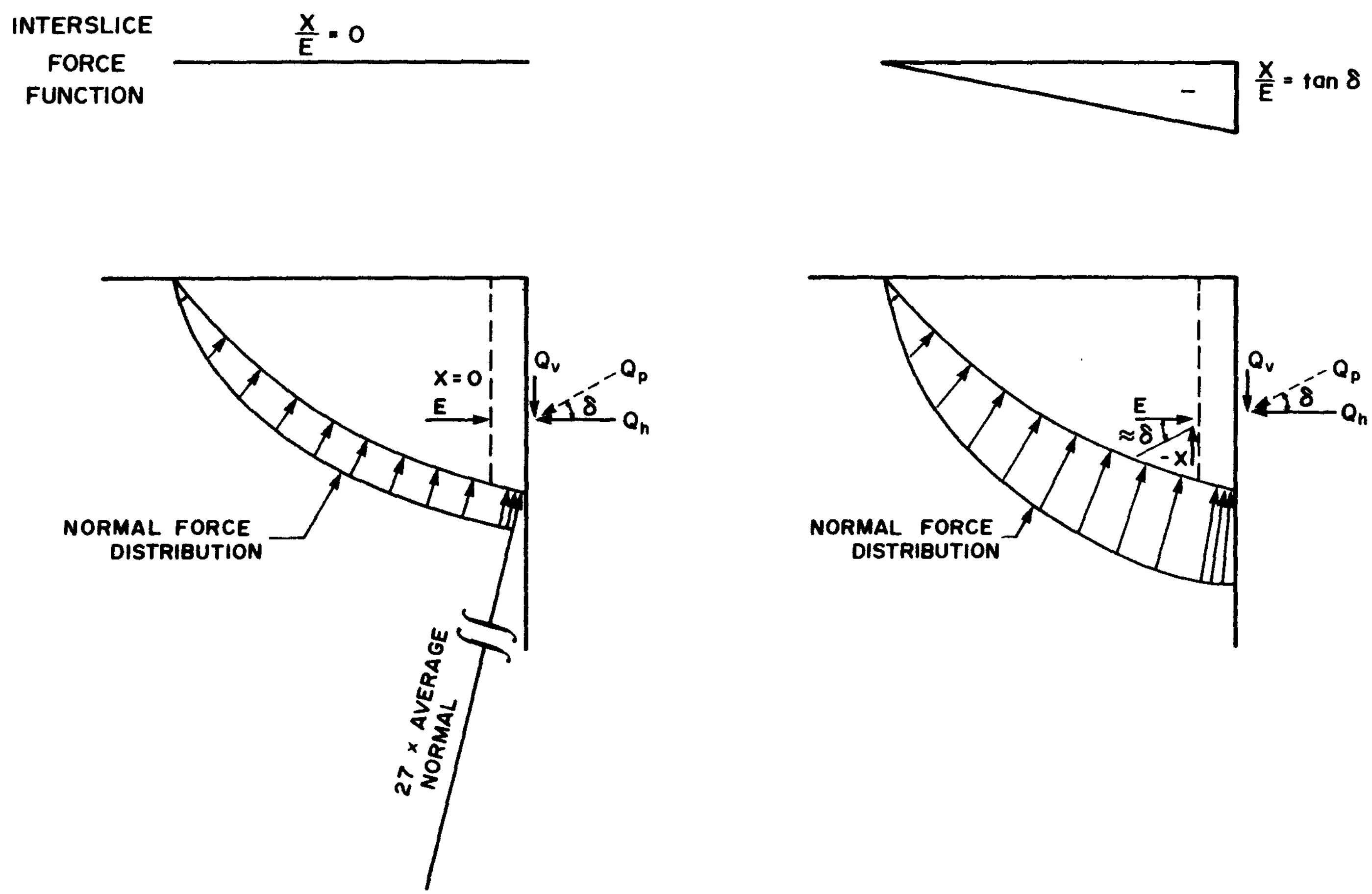
No	INTERSLICE FUNCTION, $\frac{X}{E}$	$Q_{passive}$ (kN)	K_p
7		34177	3.797
8		33470	3.719
9		34295	3.811
10		33327	3.703
11		36880	4.098

example, the interslice force function No. 3 in Figure 8.b results in lower passive resistance than the zero interslice function used in the computation of K_p values presented in Figure 7. Although the passive force is more critical, the reasonableness of function No. 3 (Figure 8.b) needs further investigation.

The selection of an interslice force function can also lead to an unreasonable distribution in the normal force at the base of a slice. Figure 9 compares two distributions of normal force (in the passive case) resulting from two types of interslice force function. A zero interslice function (Figure 9.a) assumes no interslice shear force developed along the slice boundary. This means that the vertical component of the lateral earth force is balanced only by the normal force of the last slice. The result gives an enormously large normal force at the last slice compared with the normal forces acting on other slices. The unreasonable normal force distribution can be avoided by using an interslice function which continuously distributes the shear force within the soil mass (Figure 9.b). This type of function produces a continuous distribution of normal force along the slip surface. Therefore, precautions should be exercised in choosing an appropriate interslice force function, especially for the passive case.

PROBLEMS OF NEGATIVE m VALUES

The computation of the passive force may involve negative m values which make the results become questionable. The shear force mobilized in the passive case has a downward direction which means



(a) Zero Interslice Function

(b) Triangular Interslice Function

Figure 9 Effect of the Interslice Force Function on the Normal Force Distribution (Passive Case) that S_m in equation [2] has a negative sign. This causes a negative sign to appear in the m_α equation for the passive case as shown in equation [15].

$$[15] \qquad m_\alpha = \cos \alpha - \frac{\sin \alpha \tan \phi'}{F}$$

Assuming a homogeneous soil (ϕ' constant) and F equals 1.0, m_α can become negative when the α angle at the base of the slice exceeds a limiting value. This limiting value of the α angle can be obtained by equating equation [15] to zero.

$$[16] \qquad \alpha_{limit} = 90.0° - \phi'$$

The limits for the α angle for various ϕ' angles with F equals to 1.0 are presented in Figure 10. In other words, m_α values can become negative in the passive case when the slip surface is too steep.

Negative m_α values occur when the α angle exceeds the limiting values shown in Figure 10. In other words, m_α values can become negative in the passive case when the slip surface is too steep. A negative m_α will produce a negative normal force which is impossible.

CONCLUSIONS

The general limit equilibrium method of slices, GLE, has been formulated for the lateral earth force analysis. The lateral earth force analysis is determinant if the slip surface is planar. In general, when the slip surface is curved the lateral earth force

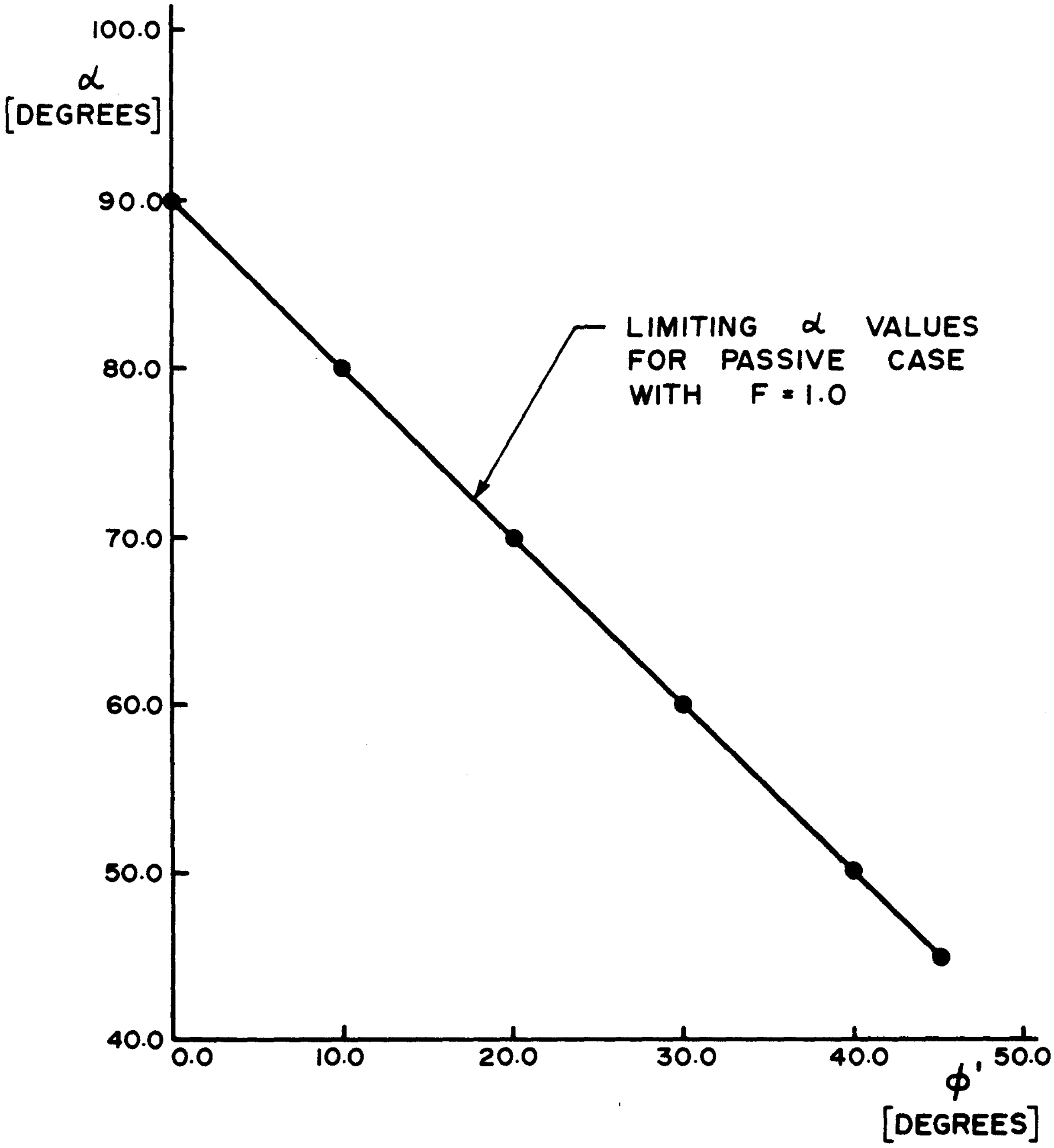


Figure 10 The Limiting α Values for the Passive Case

becomes an indeterminant problem. This indeterminancy is resolved in the GLE method by utilizing an assumption regarding the direction of the interslice forces.

The results obtained from the GLE method are in a close agreement with the results obtained from other theories. The critical slip surface in the passive case is more curved than the slip surface for the active case because of the larger shear stresses.

The interslice force function used in the analysis has a more significant influence on the passive force than on the active force values. However, an inappropriate assumption for the interslice force function could produce an unreasonable distribution of normal forces along the slip surface. Therefore, the theory of elasticity (e.g., the finite element method) is useful in estimating the form of the function.

The high curvature for the critical slip surface in the passive case can develop a steep slip surface. This steep slip surface may create a problem with negative m_α values or negative normal forces. The problem can be eliminated by limiting the magnitude of the α angle on the base of a slice (i.e., limiting the steepness of the slip surface).

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