A STEADY STATE MODEL FOR FLOW IN SATURATED-UNSATURATED SOILS

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ABSTRACT

A model is proposed describing continuous flow between saturated and unsaturated soil. The flow is assumed to be two-dimensional and under steady state conditions. In the unsaturated zone the coefficient of permeability is treated as a function of the negative pressure head. The nonlinear differential equation governing the flow is solved using an iterative finite element scheme. The flow equation for an element is derived using the Galerkin weighted residuals method. The finite element formulation is computer-implemented into a flexible computer program called SEEP. The presented example problems are compared to flow net solutions. The proposed flow model is compared to saturated-only flow models. The sensitivity of the solution is examined with respect to the function used to describe the relationship between the coefficient of permeability and the negative pressure head.

INTRODUCTION

There is an increasing number of engineering problems involving flow through saturated-unsaturated soils. Such problems range from seepage pressure calculations in earth structures to estimates of contaminant migration in groundwater systems. The solution of this variety of flow problems in engineering practice awaits the development of rigorous flow models simulating the complex problems at-hand.

In seepage analysis, engineers have traditionally relied on graphical and numerical methods when considering the flow of water in saturated soil. However, methods such as the flow net technique, (Casagrande, 1937), cannot adequately deal not even with relatively uncomplicated problems involving flow through saturated-unsaturated soils.

The proposed model describes continuous flow between saturated-unsaturated soil. Flow is assumed to be two-dimensional and under steady state conditions. The coefficient of permeability is treated as a function of the pore-water pressure when the pore-water pressure is negative. The nonlinear differential equation governing the flow is solved using an iterative finite element scheme.

The theoretical formulation is implemented in a computer program called SEEP. SEEP can handle arbitrary degrees of anisotropy and heterogeneity and includes plotting software for the graphical representation of the results. SEEP is available in the computer facility at the College of Engineering, University of Saskatchewan, Saskatoon, Canada.

BACKGROUND

Some of the earliest theoretical work in the area of flow through unsaturated soils was presented by Richards in 1931. The term "capillary conduction" was used to describe the moisture movement through unsaturated soils. It was first recognized by Richards, (1931), that "the essential difference between flow through a porous medium which is saturated and flow through medium which is unsaturated is that under the latter condition the pressure is described by capillary forces and the conductivity depends on the moisture content of the medium".

The research work by Richards, (1931), was truly commendable. Subsequent developments in the area of flow through unsaturated soils were delayed mainly because of a lack of understanding of the related phenomena. Furthermore, the application of various flow models to practical problems was difficult, since analytical tools for the solution of complex differential equations were not available.

In 1937, Casagrande published his classic paper entitled "Seepage through Dams", proposing graphical methods for the solution of seepage problems. The flow net technique considers the flow of water in the saturated zone only. To overcome the difficulty associated with flow through saturated-unsaturated soils, flow problems are separated into confined and unconfined. In the case of unconfined flow, the upper boundary of the flow region, referred to as the line of seepage, is unknown and should be determined when drawing a flow net. This is accomplished considering that the line of seepage is the zero pressure isobar and assuming that it is also the uppermost streamline, (Casagrande, 1937).

Research developments in the area of unsaturated soils allowed the development of relationships between the coefficient of permeability and the negative pressure head. In 1958, Gardner proposed a general relationship between the coefficient of permeability and the capillary pressure for unsaturated soils. The proposed relationship is given by the following equation:

$$k = \frac{a}{b + P_c^{\,n}} \tag{1}$$

where:

k = intrinsic or material permeability, (cm^2)

P_c = capillary pressure, ($dynes/cm^2$)

n = positive dimensionless constant

a,b = constants depending upon the system of units used.

In 1966, Brooks and Corey derived a mathematical relationship between the coefficient of permeability and the capillary pressure for unsaturated soils. The relationship has the following, general form:

$$\begin{aligned} k &= k_o \quad \text{for} \quad P_c \leq P_b \\ k &= k_o \left\{\frac{P_b}{P_c}\right\}^m \quad \text{for} \quad P_c \geq P_b \end{aligned} \tag{2}$$

where:

P_c = capillary pressure, (i.e., difference between pore-air and pore-water pressure), $dynes/cm^2$)

P_b = bubbling pressure, (i.e., the minimum capillary pressure at which continuous air phase exists in a porous medium).

k_o = coefficient of permeability at saturation, (cm^2).

k = coefficient of permeability at capillary pressure P_c.

m = positive dimensionless constant.

Equation (2) represents two straight lines in a plot of the logarithm of the coefficient of permeability versus the logarithm of capillary pressure. Similar results were experimentally obtained by Laliberte and Corey, (1967), as shown in Figure 1.

The availability of high speed digital computers made the use of numerical methods atractive for seepage analysis. Taylor and Brown, (1967), proposed a finite element model for seepage problems with a "free surface". This model considered the flow of water in the saturated zone only and as a result, "the principal problem is locating the free surface that has both zero flow normal to it and prescribed pressure". The proposed trial and error procedure for locating the free surface is cumbersome and often results in convergence problems.

Flow models considering continuous flow between saturated-unsaturated soil were first proposed by hydrogeologists and soil scientists. In 1971, Freeze presented a three-dimensional finite difference model for saturated-unsaturated transient flow in non-homogeneous and anisotropic geologic basins. The model "treats

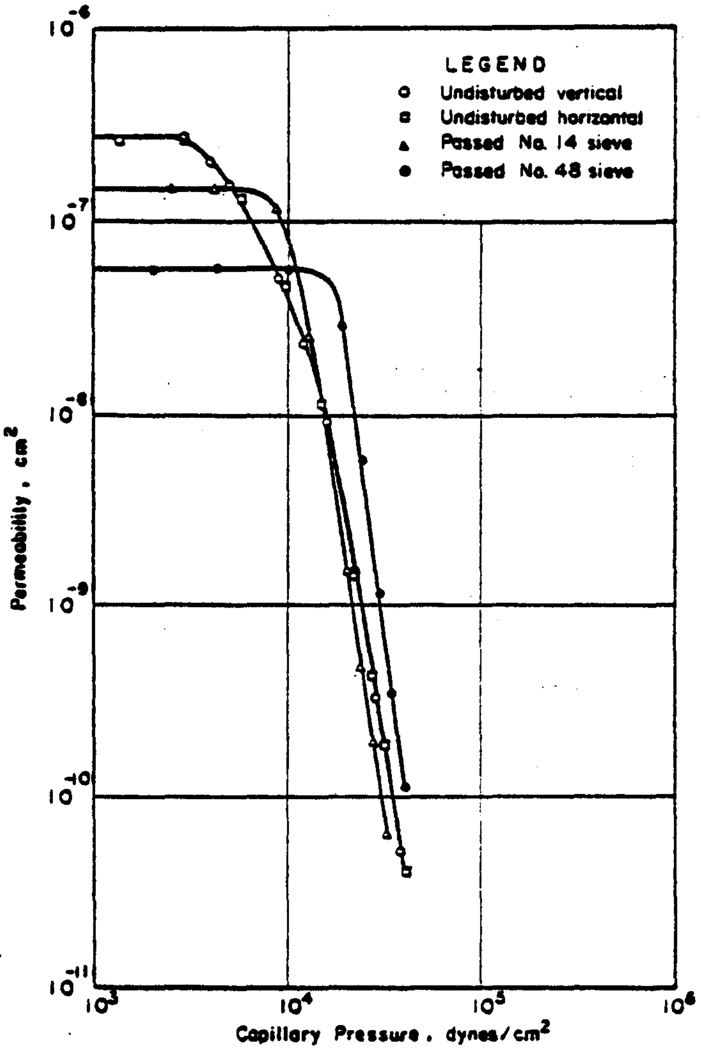


FIGURE 1 RELATIONSHIP BETWEEN CAPILLARY PRESSURE AND PERMEABILITY, (LALIBERTE AND COREY, 1967)

the complete subsurface region as a unified whole considering continuous flow between the saturated and unsaturated zones". The differential equation governing the flow is written as a function of the pore-water pressure head and is similar to the equation proposed by Richards, (1931). The solution is obtained using an iterative procedure and assuming a relationship between the coefficient of permeability and the pressure head.

A few months later, Freeze, (1971b), presented a paper entitled "Influence of the Unsaturated Domain in Seepage Trough Dams", comparing the traditional saturated - only models to the saturated-unsaturated approach. Criticizing the saturated-only models, Freeze (1971b) states that "boundary conditions that are satisfied on the free surface specify that the pressure head must be atmospheric and the surface must be a streamline. Whereas the first of these conditions is true, the second is not". With respect to the continuous saturated-unsaturated flow approach Freeze, (1971b), states "We can avoid the incorrect boundary conditions by solving the complete saturated-unsaturated boundary value problem".

Neuman, (1972), presented an iterative Galerkin finite element method for the solution of transient water flow problems in saturated -unsaturated porous media.

For the solution of steady state flow problems, engineers have traditionally relied on the flow net technique, (Casagrande, 1937). Flow models adopted later described flow in the saturated zone only, (eg., Taylor and Brown, 1967). This "loyalty" to the concepts of confined and unconfined flow along with a lack of understanding associated with unsaturated soil behavior have discouraged the use of models considering continuous flow in saturated-unsaturated systems. Recent developments in the area of unsaturated soils, (Fredlund and Morgenstern, 1976 and Daksanamurthy and Fredlund, 1980), offer the necessary background for using comprehensive flow models for saturated-unsaturated soils.

THEORY

The differential equation governing the flow is derived assuming that flow follows Darcy's law regardless of the degree of saturation of the soil, (Richards, 1931). Assuming steady-state flow conditions, the net flow quantity from an element of soil must be equal to zero. For the case where the direction of the maximum or minimum coefficient of permeability is parallel to either X or Y axis, the differential equation governing the flow can be written as follows:

$$\frac{\partial}{\partial x}\left(k_x(u_w)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y(u_w)\frac{\partial h}{\partial y}\right) = 0 \qquad (3)$$

where:

h = total head, (i.e., pressure head and elevation head), (m)

$k_x(u_w), k_y(u_w)$ = coefficient at permeability in the X and the Y direction, respectively, dependent upon pore-water pressure head, u_w.

The nonlinear differential equation, (Equation 3) can be solved using an iterative finite element scheme. In each iteration, the coefficient of permeability is assumed to be constant in an element with a value dependent upon the average pore-water pressure at its nodes.

An approximate value of the total head, h^a, is expressed as a function of the total head at the nodes of the element as follows:

$$h^a = \{L_1 L_2 L_3\} \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \end{Bmatrix} \qquad (4)$$

where:

L_i = the area coordinates defined in Figure 2.

h_i = total head at the nodes of the element.

THe flow equation for an element is derived using a Galerkin weighted residual method, (Zienkiewicz, 1977). Chosing the functions describing the total head distribution, (i.e., L_i) as weighting functions and integrating equation (3) over the area of an element, gives,

$$\int_A \{L\}^T [\frac{\partial}{\partial x}(k_x(u_w) \ \frac{\partial}{\partial x} \{L\}) + \frac{\partial}{\partial y}(k_y(u_w) \ \frac{\partial}{\partial y} \{L\})] dA \ \{h^n\} = 0 \qquad (5)$$

Integrating, equation (5) becomes,

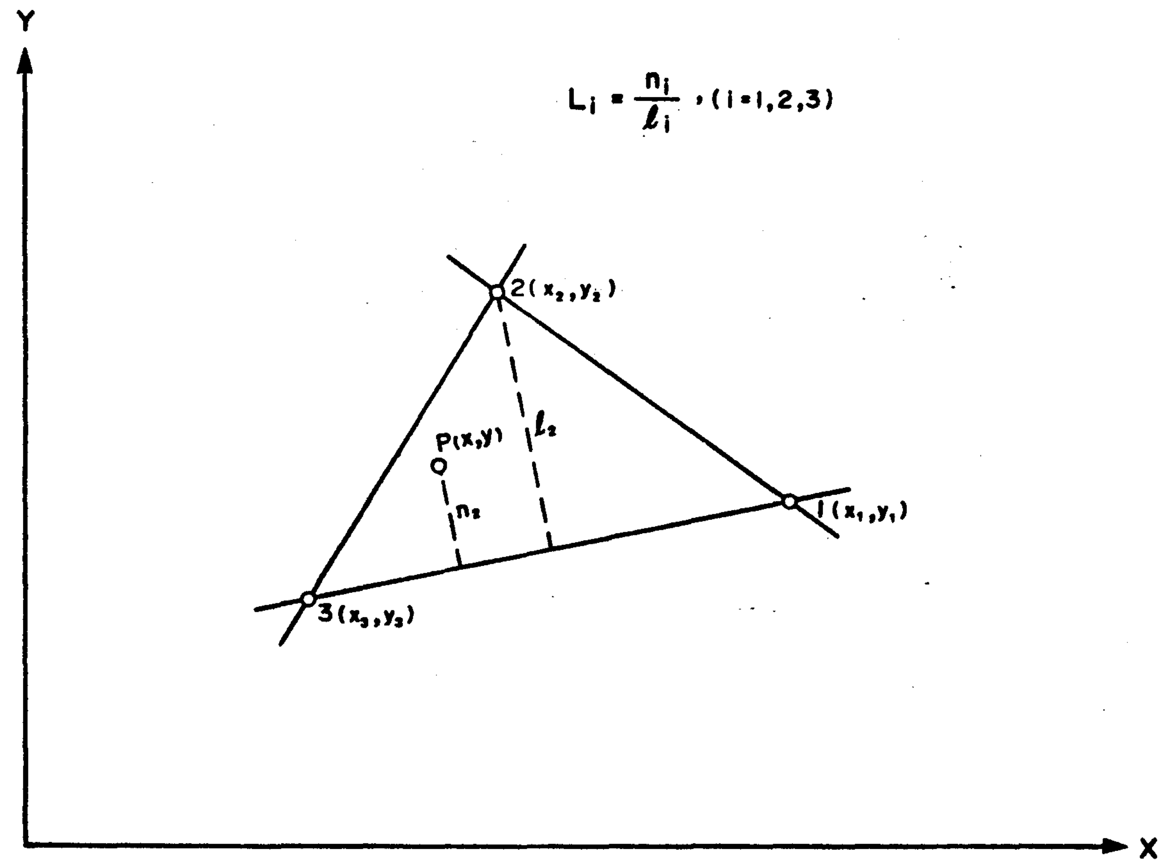


FIGURE 2 THE AREA COORDINATES

$$\int_A [k_x(u_w)\,\frac{\partial}{\partial x}\{L\}\frac{\partial}{\partial x}\{L\}^T + k_y(u_w)\frac{\partial}{\partial y}\{L\}\frac{\partial}{\partial y}\{L\}^T]\,dA\{h^n\} - \int_S \{L\}^T q\,dS = 0 \quad (6)$$

where,

A = area of the element

S = surface, (i.e., perimeter) of the element

q = flow across the sides of the element, (m^3/sec)

Equation (6) can be written in matrix form,

$$\int_A \begin{Bmatrix} \frac{\partial}{\partial x}\{L\} \\ \frac{\partial}{\partial y}\{L\} \end{Bmatrix}^T \begin{Bmatrix} k_x(u_w) & 0 \\ 0 & k_y(u_w) \end{Bmatrix} \begin{Bmatrix} \frac{\partial}{\partial x}\{L\} \\ \frac{\partial}{\partial y}\{L\} \end{Bmatrix} dA\ \{h^n\} - \int_S \{L\}^T q\,dS = 0 \qquad (7)$$

Equation (7) can be condensed and written as follows:

$$\int_A \{B\}^T\ \{k\}\ \{B\}\ dA\ \{h^n\} - \int_S \{L\}^T\ q\,dS = 0 \qquad (8)$$

where:

$$\{k\} = \begin{Bmatrix} k_x(u_w) & o \\ o & k_y(u_w) \end{Bmatrix}$$

$$\{B\} = \begin{Bmatrix} \frac{\partial}{\partial x}\{L\} \\ \frac{\partial}{\partial y}\{L\} \end{Bmatrix} = \frac{1}{2A} \begin{Bmatrix} y_2-y_3 & y_3-y_1 & y_1-y_2 \\ x_3-x_2 & x_1-x_3 & x_2-x_1 \end{Bmatrix}$$

x_i, y_i = Cartesian coordinates of the element nodes, (Figure 2). Since matrices {B} and {k} are constant throughout an element, equation (8) becomes,

$$\{B\}^T \{k\} \ \{B\} \ A \ \{h^n\} - \int_S \{L\}^T \, qdS = 0 \qquad (9)$$

Assembling equation (9) for all elements gives the global flow equations, (Equation 10).

$$\sum \{B\}^T \{k\}\{B\} \ A \ \{H^n\} - \int_S \{L\}^T \, qdS = 0 \qquad (10)$$

where, the "summation" is performed routinely, (Desai and Abel, 1972), $\{H^n\}$ is the column matrix including all nodes, and the surface integral is taken over the external perimeter of the geometry of the problem, (Zienkiewicz, 1977). When the boundary flow is zero, the second term in equation (10) drops to zero. When a flow value q is specified across a boundary, the surface integral in equation (10) distributes the surface flow into nodal flow at the corresponding boundary nodes, (Segerlind, 1976).

COMPUTER IMPLEMENTATION

The finite element formulation described above is implemented into the computer program called SEEP. SEEP allows up to 8 different soil materials with arbitrary anisotropy. Flow regions can also be solved, where the direction of the major or minor coefficient of permeability is at an angle to the horizontal, (i.e., X-axis). The boundary conditions that can be specified are either head or flow (i.e., positive or negative).

SEEP calculates element coefficients of permeability, water velocities and gradients and gives average nodal coefficients of permeability, water velocities and gradients.

Two different functions are programmed into SEEP for the relationship between the coefficient of permeability and the pressure head. First, a linear relationship between the logarithm of the coefficient of permeability and the negative pressure head, (Figure 3). Second, a linear relationship between the logarithm of the coefficient of permeability and the logarithm of pressure head, for pressure heads lower than -1.0 m, (Figure 4). Similar relationships were experimentally obtained by Richards, (1931) and Laliberte and Corey, (1967), respectively.

SEEP includes a method for determining the exit point of the phreatic line and adjusting the boundary conditions along free seepage boundaries. The method requires an initial guess for the exit point of the phreatic line. In subsequent iterations, the boundary conditions along the free surface are adjusted to satisfy the condition of negative pressure head for boundary nodes higher than the current exit point of the phreatic line.

SEEP includes some plotting capabilities for the graphical representation of the geometry of the problems and calculated nodal heads. The solution of various example problems presented in this paper are drawn using a computer program that plots and contours the calculated total head values and pressure isobar.

RESEARCH PROGRAM

The presented example problems give an indication of the capability of the proposed model to describe flow in saturated-unsaturated soils. The problems considered are separated into several categories. First, homogeneous and anisotropic dam cross-sections are considered with increasing degrees of anisotropy, (i.e., increasing ratios of horizontal over vertical coefficients of permeability). Second, heterogeneous dam cross-sections are considered with two soil materials of different

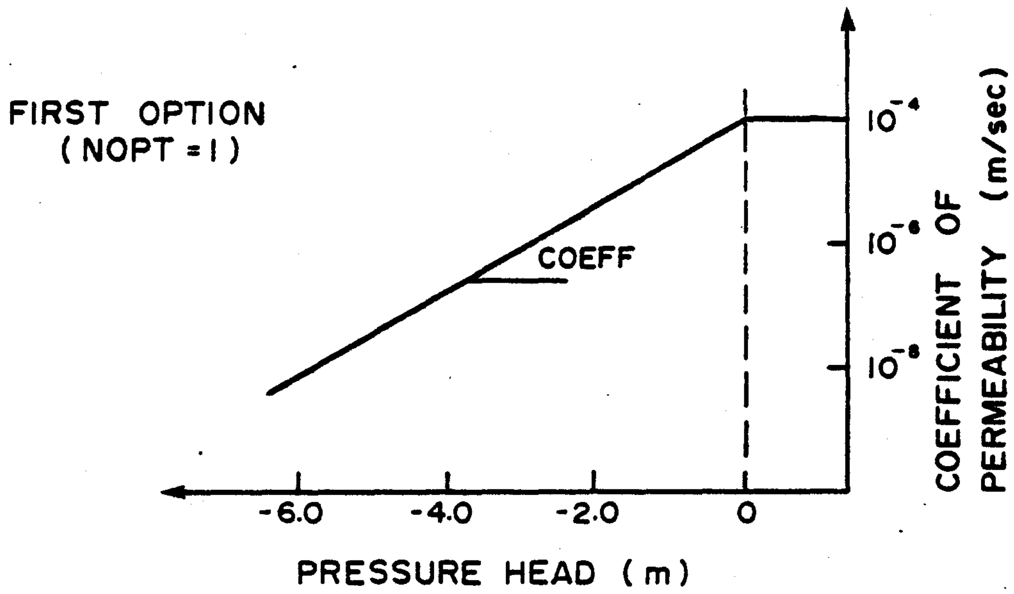


FIGURE 3 LINEAR RELATIONSHIP BETWEEN THE LOGARITHM OF THE COEFFICIENT OF PERMEABILITY AND THE NEGATIVE PRESSURE HEAD

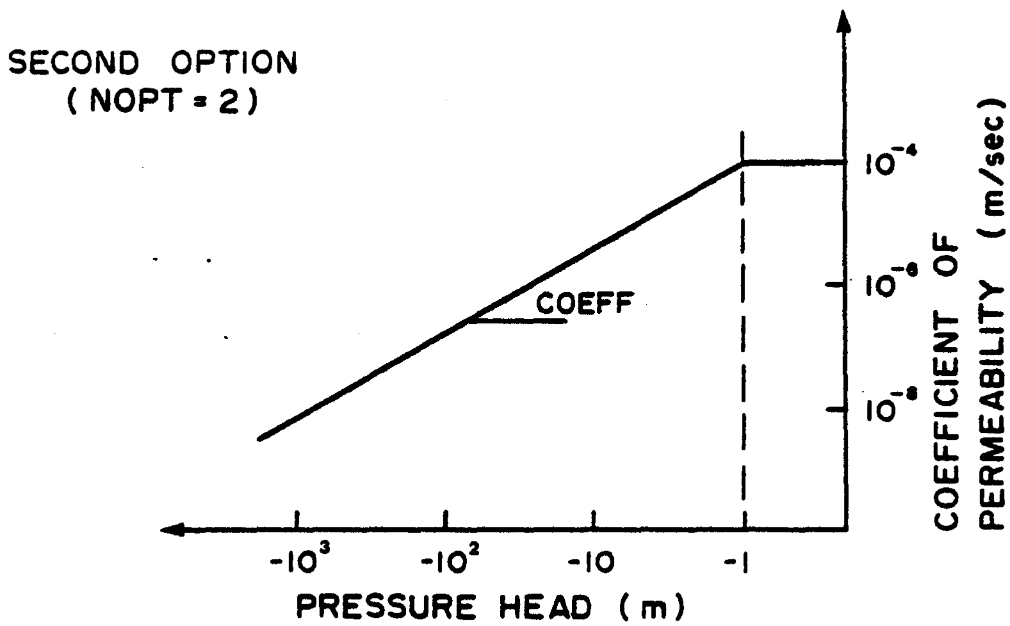


FIGURE 4 LINEAR RELATIONSHIP BETWEEN THE LOGARITHM OF THE COEFFICIENT OF PERMEABILITY AND THE LOGARITHM OF THE NEGATIVE PRESSURE HEAD

coefficients of permeability. Third, problems are solved, where the direction of the maximum coefficient of permeability is at an angle to the horizontal. Fourth, rainfall conditions are input as positive boundary flow along the upper boundary of dam cross-sections. Fifth, homogeneous and isotropic dam cross-sections are considered, where discharge takes place along a free surface.

The water discharge velocities computed for the elements of the discretized mediums allow the evaluation of seepage quantities through various cross-sections in the unsaturated zone. Furthermore, the calculation of seepage quantities through various cross-sections in the unsaturated zone allows the examination of the stream line condition, (i.e., zero flow across), for the zero pressure isobar, (Figure 5).

RESULTS

The example problems considered are shown in Figures 6 to 20. In all cases, a total head of 10.0 meters is specified at all nodes along the wetted upstream face of the dam cross-sections.

For the cases of dams with an horizontal toe drain, (Figures 6 to 17), zero pressure head, (i.e., atmospheric pressure) is specified at all nodes along the drain.

For the cases shown in Figures 6 to 15, and 18 to 20, the upper boundary is assumed impervious, (i.e., zero flow). For the problems shown in Figures 16 and 17, a boundary flow of 0.1×10^{-4} meters^3/sec and 0.2×10^{-4} meters^3/sec, respectively, is specified at the nodes along the upper boundary.

The problems shown in Figures 18 to 20 demonstrate the

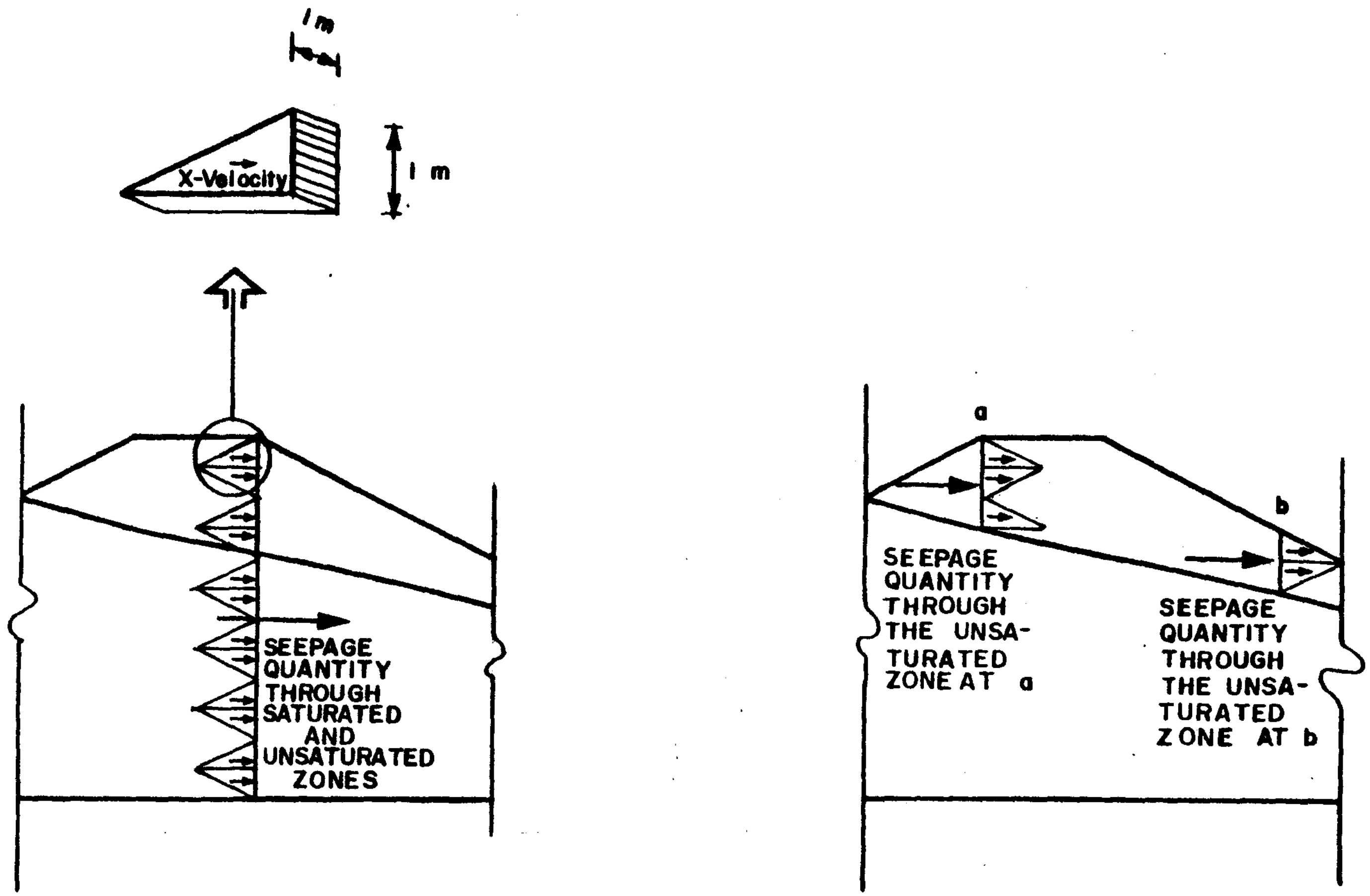


FIGURE 5 CALCULATION OF SEEPAGE QUANTITIES BY SUMMING WATER DISCHARGE VELOCITIES IN THE X-DIRECTION FOR ELEMENTS ALONG A VERTICAL LINE THROUGH A NODE

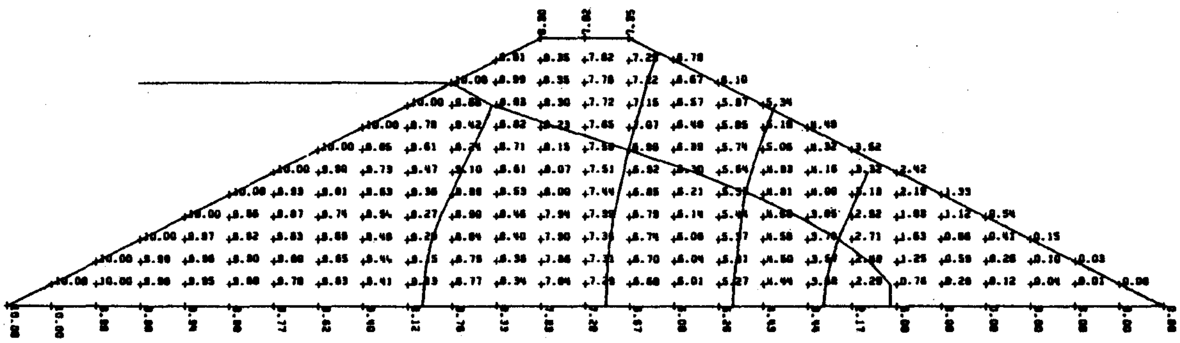


FIGURE 6 HOMOGENEOUS DAM WITH A HORIZONTAL DRAIN; NOPT = 1; 5TH ITERATION; CONVERGED

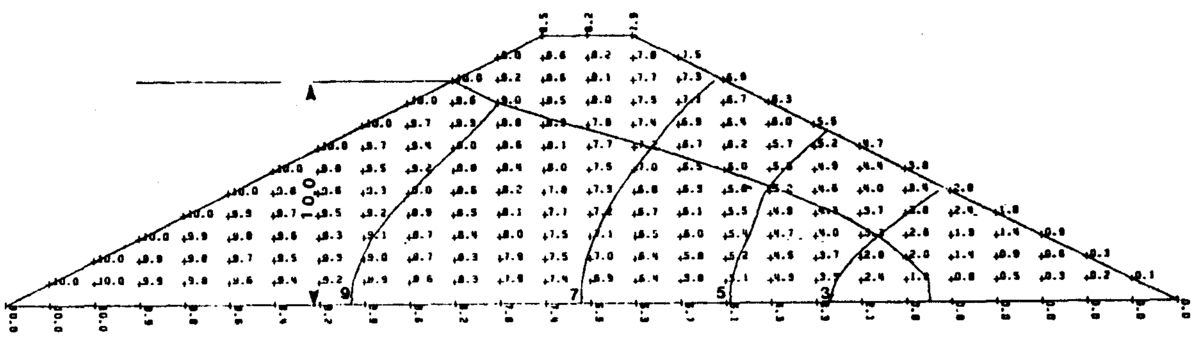


FIGURE 7 DAM WITH A HORIZONTAL DRAIN; ANISOTROPY, $k_x = 3 k_y$

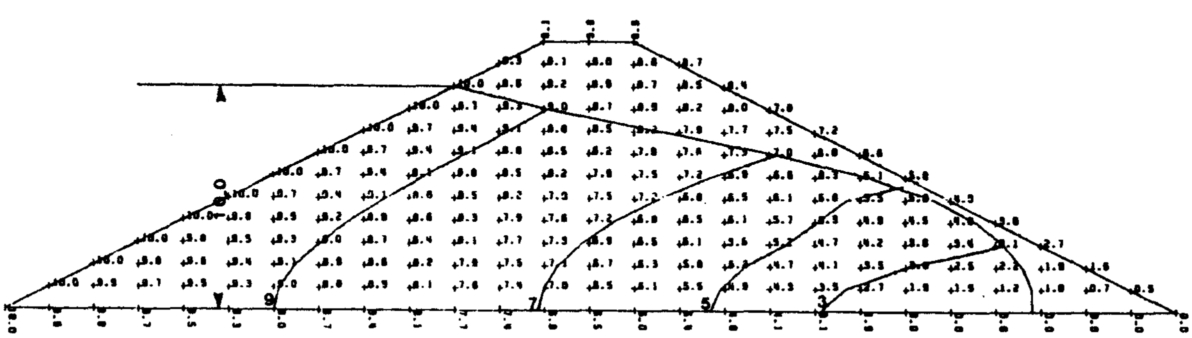


FIGURE 8 DAM WITH A HORIZONTAL DRAIN; ANISOTROPY, $k_x = 10 k_y$

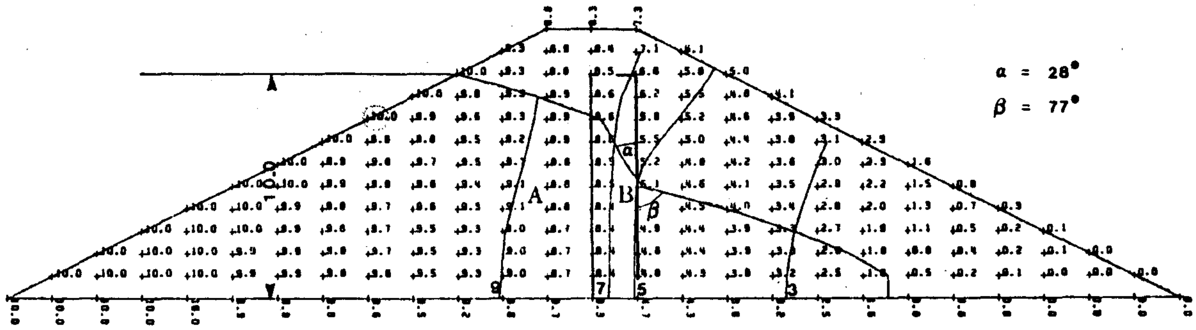


FIGURE 9 DAM WITH A CORE OF LOWER PERMEABILITY; $k_A = 10\ k_B$

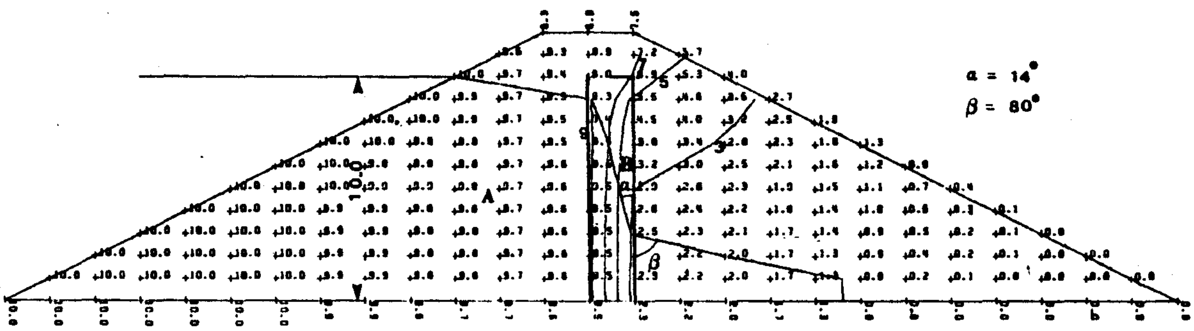


FIGURE 10 DAM WITH A CORE OF LOWER PERMEABILITY; $k_A = 10^2 k_B$

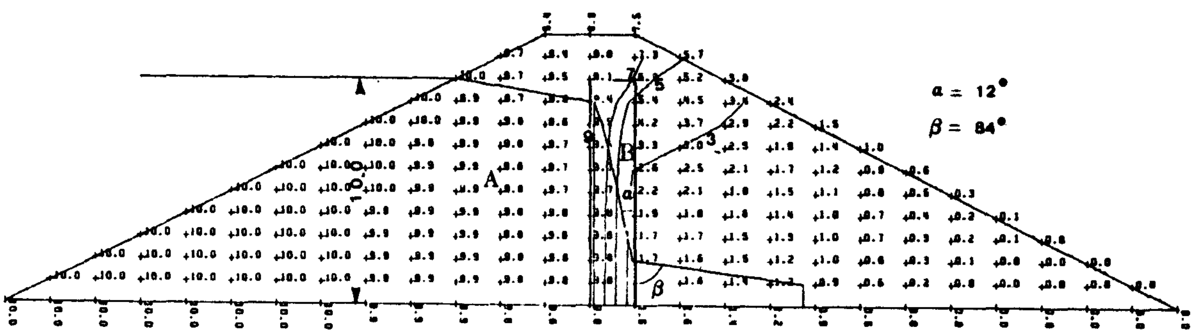


FIGURE 11 DAM WITH A CORE OF LOWER PERMEABILITY; $k_A = 10^3 k_B$

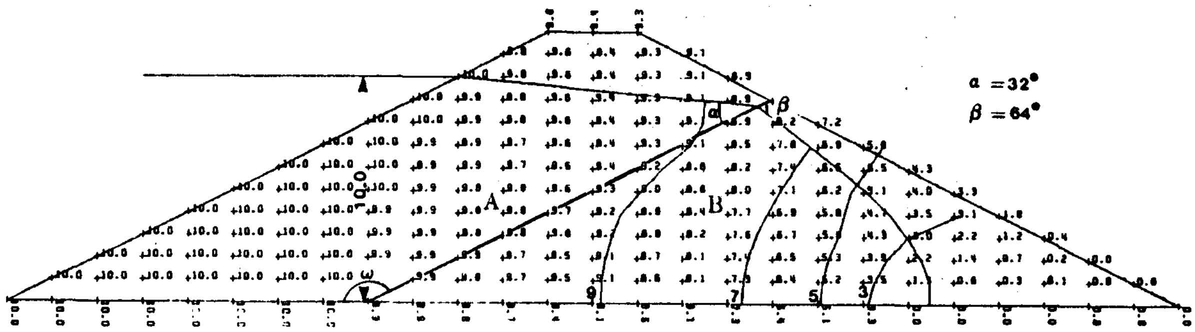


FIGURE 12 HETEROGENEOUS DAM WITH A HORIZONTAL DRAIN; PERMEABILITIES AT SATURATION: $k_A = 10^{-4}$ m/sec, $k_B = 10^{-5}$ m/sec

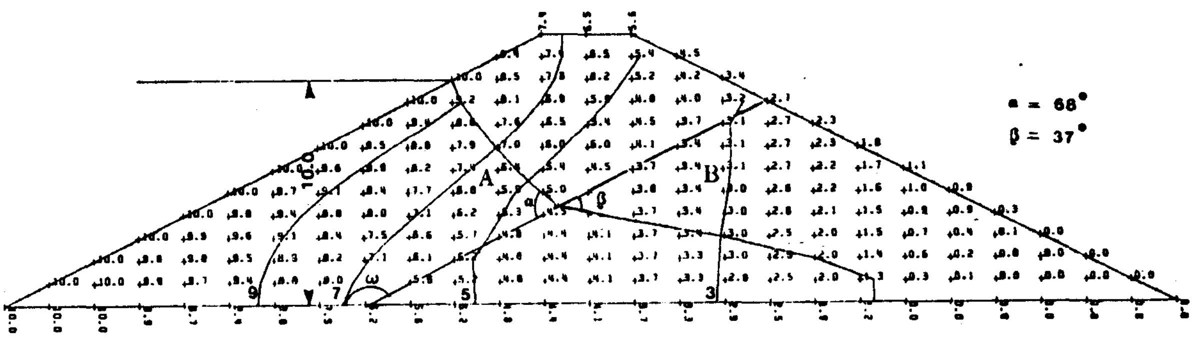


FIGURE 13 HETEROGENEOUS DAM WITH A HORIZONTAL DRAIN; PERMEABILITIES AT SATURATION: $k_A = 10^{-5}$ m/sec, $k_B = 10^{-4}$ m/sec

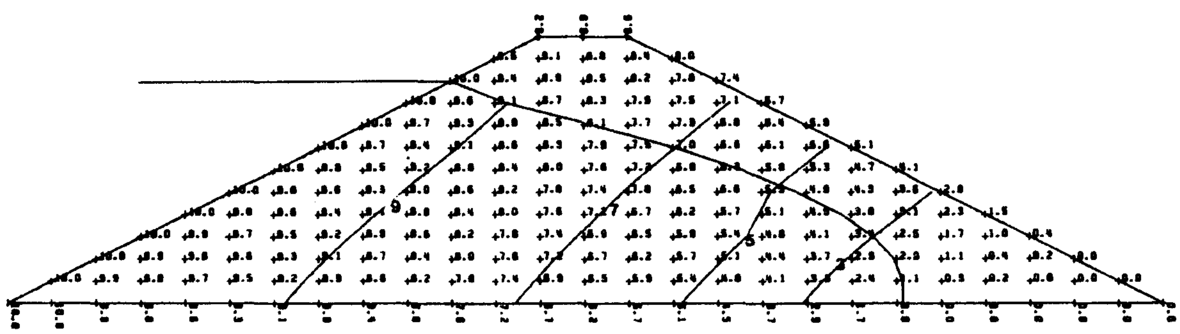


FIGURE 14 DAM WITH A HORIZONTAL DRAIN; THE DIRECTION OF THE MAJOR COEFFICIENT OF PERMEABILITY IS AT AN ANGLE 0.5 RAD TO THE X-AXIS

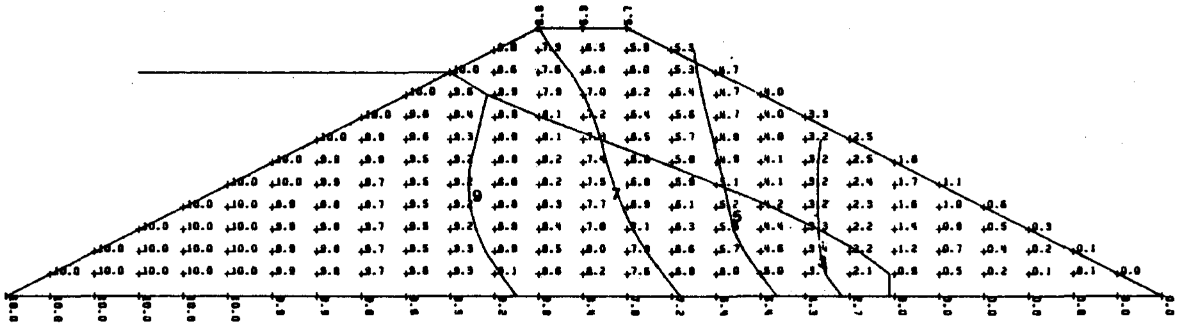


FIGURE 15 DAM WITH A HORIZONTAL DRAIN; THE DIRECTION OF THE MAJOR COEFFICIENT OF PERMEABILITY IS AT AN ANGLE -0.5 RAD TO THE X-AXIS

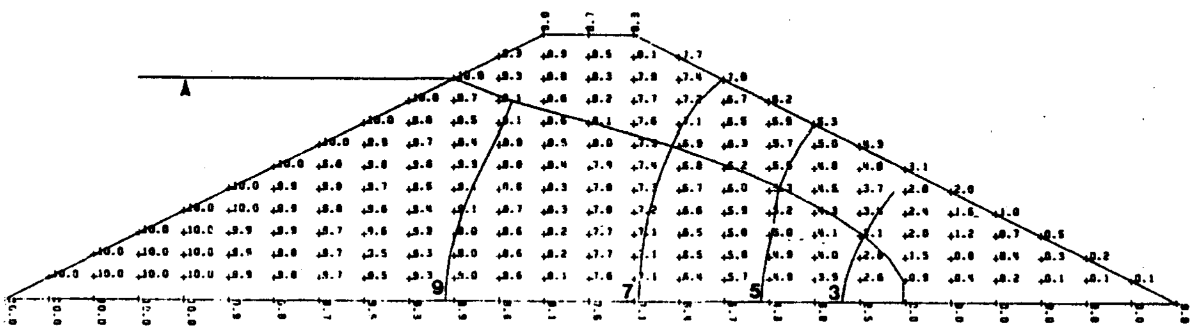


FIGURE 16 HOMOGENEOUS DAM UNDER RAINFALL; BOUNDARY FLUX = $0.1 * 10^{-4}$ M/SEC

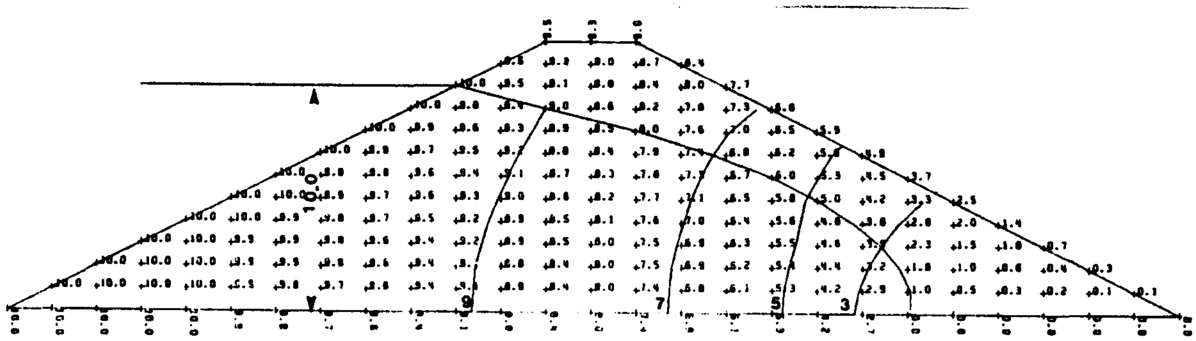


FIGURE 17 HOMOGENEOUS DAM UNDER RAINFALL; BOUNDARY FLUX = $0.2 * 10^{-4}$ M/SEC

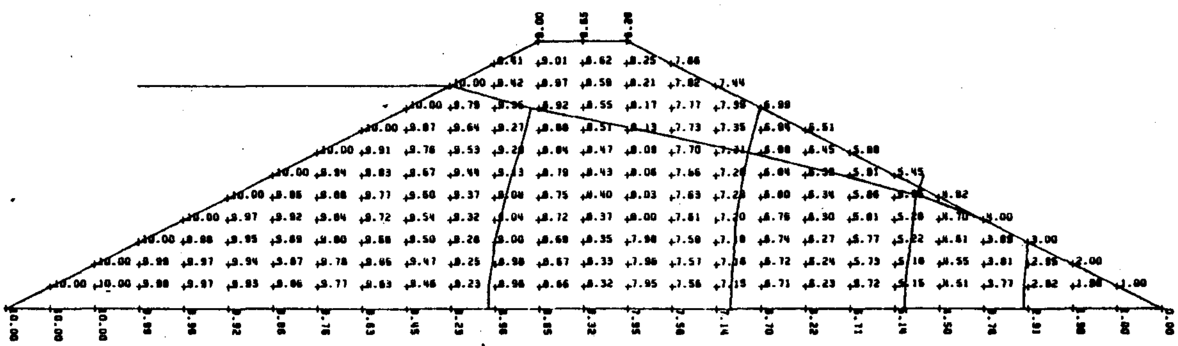


FIGURE 18 HOMOGENEOUS DAM WITH IMPERVIOUS LOWER BOUNDARY; NOPT = 1; 14TH ITERATION; CONVERGED

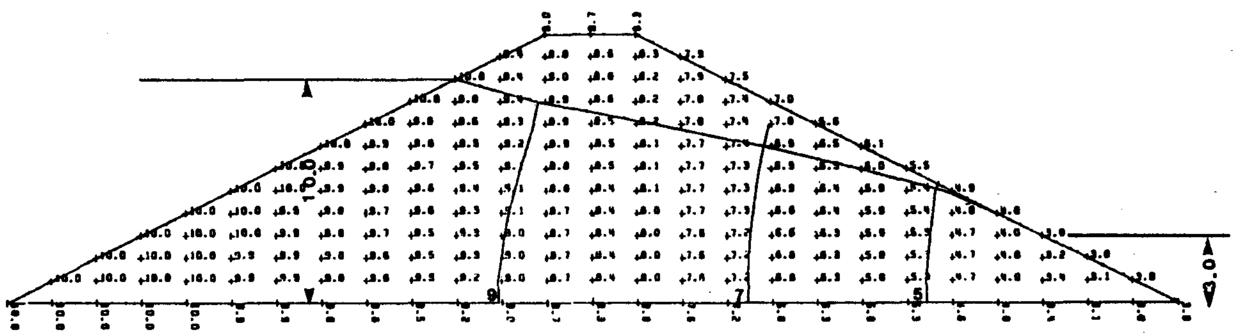


FIGURE 19 HOMOGENEOUS DAM WITH IMPERVIOUS LOWER BOUNDARY; 3.0 m OF DOWNSTREAM WATER

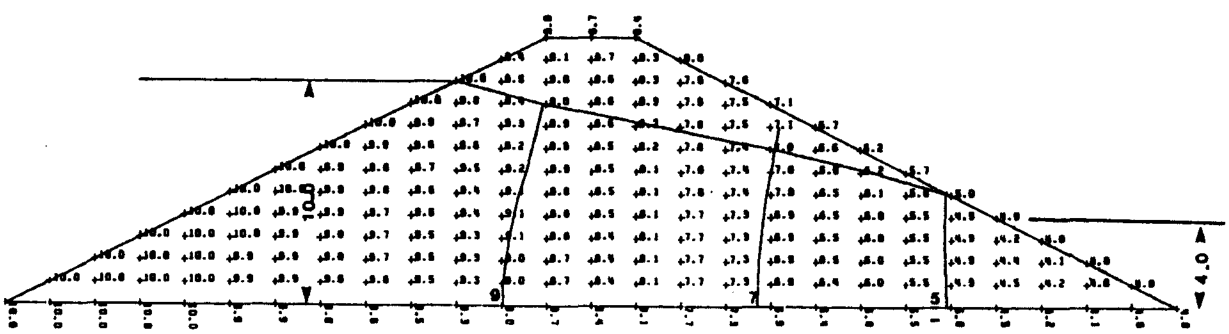


FIGURE 20 HOMOGENEOUS DAM WITH IMPERVIOUS LOWER BOUNDARY; 4.0 m OF DOWNSTREAM WATER

ability of the computer program SEEP to deal with flow problems involving free boundary surfaces. In the three cases shown, (Figures 18 to 20), the lower boundary of the dam is assumed to be impervious and the previously mentioned procedure for determining the exit point of the line of seepage is used.

For the example problems considered, the number of iterations required for convergence ranges from 6 to 14 with a tolerance of 1%, (i.e., acceptable difference between corresponding nodal heads in two successive iterations). However, it is observed that the number of iterations required for convergence depends upon the function used to express the relationship between the coefficient of permeability and the pressure head.

DISCUSSION

The results are first compared with flow net solutions by observing the requirements for the phreatic line and the equipotential lines, (Casagrande, 1937). Second, the proposed model is compared to models considering the flow in the saturated zone only by examining the validity of the assumptions employed by the latter with respect to the phreatic line, (Taylor and Brown, 1967). Third, the sensitivity and the stability of the solution are examined with respect to the function used to express the relationship between the coefficient of permeability and the pore-water pressure head.

For all problems with zero flow specified across the upper boundary, (Figures 6 to 15 and 18 to 20), the equipotential lines coincide the phreatic line of right angles. Furthermore, the elevation of the point of intersection of any equipotential line with the phreatic line is equal to the total head represented by this equipotential line. These observations offer evidence that the phreatic line, (i.e., zero pressure isobar) approximately coincides with the line of seepage, as defined by Casagrande, (1937). This latter condition will be further examined by comparing quantities of flow through various cross-sections.

The problems in Figures 16 and 17 demonstrate that when flow is applied externally, the phreatic line departs from the streamline condition.

For the problems shown in Figures 6 and 18, the quantities of flow obtained by summing element flow quantities are compared to the flow quantities calculated from flow net solutions. The results are summarized in Tables 1 and 2, where NOPT is the type of function used and COEFF is its slope in the unsaturated region, (Figures 3 and 4). Tables 1 and 2 show that the quantity of seepage is becoming lower for increasing COEFF values. This can be explained by a reduction in the conductivity of the unsaturated zone. It can also be seen that the quantity of seepage for NOPT = 1 and high COEFF values, (eg., 3.0 or 4.0), favourably agrees with the quantity of seepage obtained from the flow net solution.

Figures 7 and 8 point out the effect of anisotropy on the shape of the saturated zone. For higher ratios of horizontal to vertical coefficient of permeability, the saturated zone may coincide the downstream face of the dam. This undesirable situation

TABLE 1

Problem in Figure 6

Flow Quantitites in m^3/sec

	NOPT = 1		NOPT = 2	
COEFF	0.2	$2.645*10^{-4}$	1.0	$2.932*10^{-4}$
	0.6	$2.283*10^{-4}$	2.0	$2.731*10^{-4}$
	1.0	$2.230*10^{-4}$	3.0	$2.636*10^{-4}$
	2.0	$2.175*10^{-4}$	4.0	$2.580*10^{-4}$
	4.0	$2.129*10^{-4}$		
From Flow Net Solution = $2.08.10^{-4}$ to $2.43*10^{-4}$				

NOPT = Type of Function (Figure 3 or 4)
COEFF = Slope of Function in the Unsaturated Zone

TABLE 2

Problem in Figure 18

Flow Quantitites in m^3/sec

	NOPT = 1		NOPT = 2	
COEFF	0.2	$1.713*10^{-4}$	1.0	$1.814*10^{-4}$
	0.6	$1.636*10^{-4}$	2.0	$1.798*10^{-4}$
	1.0	$1.580*10^{-4}$	3.0	$1.793*10^{-4}$
	2.0	$1.518*10^{-4}$	4.0	$1.794*10^{-4}$
	4.0	$1.482*10^{-4}$		
From Flow Net Solution = $1.52*10^{-4}$ to $1.73*10^{-4}$				

NOPT = Type of Function (Figures 3 or 4)
COEFF = Slope of Function in the Unsaturated Zone

if not properly considered in the design stage can lead to the failure of the structure.

Figures 9 to 11 illustrate that the drop of the phreatic line is becoming sharper for decreasing coefficients of permeability of the core maerial. However, even for a three orders of magnitude difference in the coefficients of permeability, there is still some head drop taking place outside the core.

Figures 12 and 13 illustrate the deflection angles of the phreatic line at the boundary of the two materials with different permeabilities. Table 3 summarizes these angles and shows that the condition $\beta = 270^{o} - \alpha - \omega$ is not exactly fulfilled. The difference between the angle β and the term $270^{o} - \alpha - \omega$ again raises questions about the flow line condition of the zero pressure isobar.

TABLE 3

Deflection Angles of the Phreatic Line for Examining the Condition $\beta = 270^{o} - \alpha - \omega$

	α	β	$270^{o} - \alpha - \omega$
Figure 12	32^{o}	64^{o}	84^{o}
Figure 13	68^{o}	37^{o}	48^{o}

With $\omega = 135^{o}$, (Figures 12 and 13)

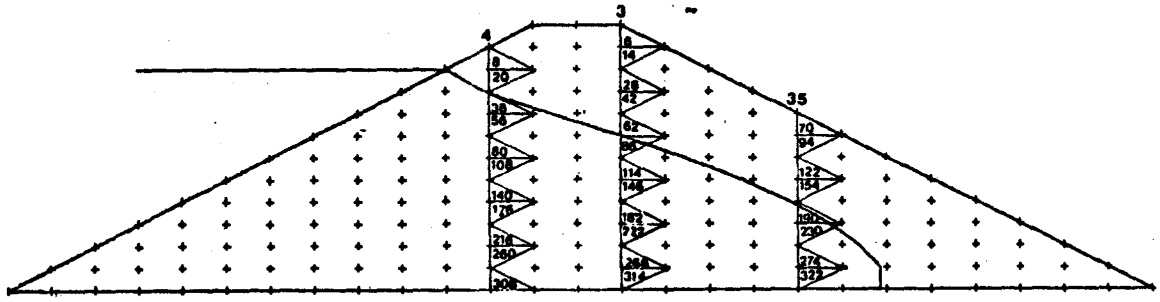
Figures 14 and 15 illustrate the distorted shape of the phreatic line and equipotential lines in flow regions where the direction of the maximum coefficient of permeability is at an angle to the horizontal.

In the cases where positive boundary flow is specified, (Figures 16 and 17), the location of the phreatic line is somewhat higher than in the case where zero flow is specified, (Figure 6).

Figures 18 to 20 illustrate the use of the previously mentioned procedure for revising the boundary conditions along a free surface. It can be seen that the presence of the downstream water causes the exit point of the phreatic line to rise. This shows that the exit point of the phreatic line in no case coincides the free surface at the downstream water level.

The limitations of the traditional saturated-only flow models, in terms of computing effort and convergence difficulty, are well known, (Taylor and Brown, 1967). Furthermore, the assumption such models employ with respect to the line of seepage can be questioned. For example, is the zero pressure isobar, (i.e., phreatic line) a stream line?

The concept of zero flow across the phreatic line is examined by comparing the quantitites of flow across different cross-sections, (Figure 5). Figure 21 summarizes the flow quantity calculations across three sections for the example problem shown in Figure 6. It can be seen that, although total seepage quantitites through the saturated-unsaturated sections are approximately equal, the quantities of flow across the sections



	NODE 4		NODE 3		NODE 35	
UNSATURATED ZONE	ELEMENT		ELEMENT		ELEMENT	
	8	$0.1451*10^{-4}$	6	$0.4384*10^{-5}$	70	$0.9052*10^{-5}$
	20	$0.2101*10^{-4}$	14	$0.4869*10^{-5}$	94	$0.1170*10^{-4}$
			26	$0.9768*10^{-5}$	122	$0.2128*10^{-4}$
			42	$0.1298*10^{-4}$	154	$0.2758*10^{-4}$
	SUM	$0.3642*10^{-4}$	SUM	$0.5585*10^{-4}$	SUM	$0.6961*10^{-4}$
SATURATED ZONE	ELEMENT		ELEMENT		ELEMENT	
	36	$0.2963*10^{-4}$	86	$0.3025*10^{-4}$	190	$0.4916*10^{-4}$
	56	$0.2963*10^{-4}$	114	$0.3165*10^{-4}$	230	$0.4916*10^{-4}$
	80	$0.2700*10^{-4}$	146	$0.3165*10^{-4}$	274	$0.5948*10^{-4}$
	108	$0.2700*10^{-4}$	182	$0.3273*10^{-4}$	322	$0.5948*10^{-4}$
	140	$0.2556*10^{-4}$	222	$0.3273*10^{-4}$		
	176	$0.2556*10^{-4}$	266	$0.3332*10^{-4}$		
	216	$0.2492*10^{-4}$	314	$0.3332*10^{-4}$		
	260	$0.2492*10^{-4}$				
	308	$0.2475*10^{-4}$				
	SUM	$2.3890*10^{-4}$	SUM	$2.256*10^{-4}$	SUM	$2.1728*10^{-4}$
	TOTAL	$2.7539*10^{-4}$	TOTAL	$2.815*10^{-4}$	TOTAL	$2.8689*10^{-4}$

FIGURE 21 ADD ELEMENT X-VELOCITIES ALONG VERTICAL LINES PASSING THROUGH NODES 4, 3 AND 35. (PROBLEM SHOWN IN FIGURE 6).

in the unsaturated zone are not equal. This implies that there is flow across the phreatic line. As a result, the zero pressure line is not a streamline.

The assumption of zero flow across the phreatic line is not only unnecessary when considering continuous flow in saturated-unsaturated soil, but it is also incorrect. This conclusion agrees favourably with the research work by Freeze, (1971b).

The sensitivity of the finite element solution with respect to the function used to express the relationship between the coefficient of permeability and the pressure head was part of an extensive parametric study by Papagianakis, (1982). The two problems shown in Figures 6 and 18 are solved using various slopes of the two types of linear relationships in the unsaturated zone, (Figures 3 and 4). The solution in all cases converged in less than 20 iterations for an 1% tolerance (i.e., acceptable difference between corresponding heads in two successive iterations). It was also observed that the number of iterations required for convergence increased for steeper functions.

The calculated total head values and the location of the phreatic line are relatively unchanged under a wide range of functional relationships. The water conductivity of the unsaturated zone however, decreases for increasing COEFF values. Therefore, the relative quantity of flow in the saturated and the unsaturated zone depends upon the function used. As expected, the solution obtained for a low conductivity in the unsaturated zone (i.e., high COEFF values) agrees favourably with Casagrande's solutions, (1937).

CONCLUSIONS

The main conclusions of the research can be summarized as follows:

1. There is no need to retain the arbitrary categorization of flow problems as confined and unconfined systems.
2. There is continuous flow between the saturated and the unsaturated zone.
3. It is incorrect to assume that the zero pressure isobar is the upper streamline and that there is no flow across this isobar.

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