A COMPREHENSIVE LIMIT EQUILIBRIUM SOFTWARE PACKAGE FOR GEOTECHNICAL ENGINEERING

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ABSTRACT

SLOPE-II is a comprehensive computer program which calculates the factor of safety of an earth or rock slope using the limit equilibrium methods of analysis. The program was initiated in 1967 at the University of Saskatchewan and served as the first step in developing a family of computer programs to solve the problems most often addressed by the geotechnical engineers. Over the past 15 years, SLOPE-II has been extensively enhanced and a number of additional programs have been developed to compose a software system of some 8 programs and approximately 30,000 lines of Fortran code.

SLOPE-II forms the nucleus of the software system. The program provides 6 different methods of slope stability analysis and a method of lateral earth force analysis. Numerous features are available to enable the engineer to analyze the wide range of conditions normally encountered in engineering practice.

The remaining programs in the software system are divided into the categories of 'auxiliary dependent' and 'auxiliary independent' programs. The 'auxiliary dependent' programs are PROMSL (PROMpted SLope data input) and PLOT-1/2/3. PROMSL provides the user with an interactive online facility for either creating, modifying or verifying data files for input to SLOPE-II.

PLOT-1/2/3 are Calcomp plotting programs which help the user of SLOPE-II to visualize the results of the analysis through graphical representation. The 'auxiliary independent' programs are FINEL (FINite ELement stress analysis), SEEP (finite element SEEPage analysis) and HILF. The output from each of these programs is designed to be compatible for input to SLOPE-II.

At present, SLOPE-II and the 'auxiliary dependent' programs are being supported by Geo-Slope Programming Ltd., Calgary, Alberta. The programs are regularly enhanced, program fixes applied and new releases distributed. Orientation seminars are held periodically to assist users of SLOPE-II.

INTRODUCTION

Numerical methods in geotechnical engineering have been developed rapidly in the last two decades primarily due to the availability of digital computers. There are two types of numerical methods in geotechnical enigneering which use substantial amounts of computing time. These are the finite element methods and the limit equilibrium methods of analysis.

The stress versus strain relationship of soils is utilized in the finite element method to predict displacements and stresses. In the limit equilibrium methods, the statics of forces and moments are used to compute a number indicative of the overall stability of the structure. Geotechnical engineers are familiar with and frequently use limit equilibrium methods because of their simplicity in analyzing complex problems. As well, there are numerous classes of geotechnical problems which can be analyzed using essentially the same principles of limit equilibrium analysis.

While numerous finite element software packages have been developed, there has been a need for a comprehensive limit equilibrium software package for geotechnical engineering practice. SLOPE-II has developed over the past fifteen years into a comprehensive limit equilibrium software package. The program has extensive capabilities in analyzing a wide variety of complex problems. A number of computer programs either have been developed or are under development, in order to support the use of SLOPE-II and to provide better estimation of the variables used for input data.

LIMIT EQUILIBRIUM METHODS

There are three main categories of geotechnical problems which can be analyzed using the principles of limit equilibrium analysis (Janbu 1973). These are:

(1) Slope Stability
- Design of earthfill and rockfill dams, and assessing the stability of natural slopes.
- Back analysis of failed slopes in order to ascertain appropriate remedial action.

(2) Lateral Earth Force
- Active and passive earth forces.
- Design of retaining structures and tie-back anchoring systems.

(3) Bearing Capacity
- Shallow foundations.

Many geotechnical problems can fall into one of the above categories or a combination of categories. For example, a foundation construction on a slope requires analysis of slope stability and an analysis of a retaining structure. The availability of a comprehensive computer program which can be used to analyze different types of limit equilibrium problems is very useful.

The development of the limit equilibrium theory has produced numerous methods for slope stability analysis (Wright 1969). Janbu (1957) proposed a generalized procedure of slices which also included lateral earth pressure and bearing capacity calculations. The different methods of analysis are best understood through a comparison of the elements of statics satisfied in the derivation and the assumptions used to render the analysis determinate (Fredlund and Krahn 1977). An attempt has been made to unify the various methods of slope stability analysis under a general formulation called the general limit equilibrium method of slices, GLE (Fredlund, Krahn and Pufahl 1981). The GLE method has also been formulated for lateral earth force problems (Rahardjo 1982).

SOFTWARE SYSTEM

SLOPE-II is a comprehensive computer program which commenced development in 1967 under the name SLOPE. It was originally designed for analyzing slope stability problems using various limit equilibrium methods (Fredlund 1981). In 1978, the program was further tested, enhanced and removed to the private domain where it was called SLOPE-II. SLOPE-II is presently a proprietary product owned by Geo-Slope Programming Ltd., Calgary, Alberta. Enhancements and developments to SLOPE-II over the past fifteen years have produced a relatively large software system.

The software system consists of the following (see Figure 1):

(1) A main program called SLOPE-II
- Slope stability
- Lateral earth force

(2) Auxiliary dependent programs
- Data entry and verification: PROMSL
- Data presentation: PLOT-1/2/3 (Calcomp form)

(3) Auxiliary independent programs
- Finite Element Stress Analysis: FINEL.
- Finite Element Seepage Analysis: SEEP.
- Hilf's Analysis

Main Program (SLOPE-II)

SLOPE-II has been continuously enhanced and verified with the most recent extensions being the inclusion of lateral earth force calculations (Rahardjo 1982). At present, the SLOPE-II program has several different methods of slope stability analysis, along with active and pasive lateral earth force analyses. The methods available for slope stability and lateral earth force analyses are:

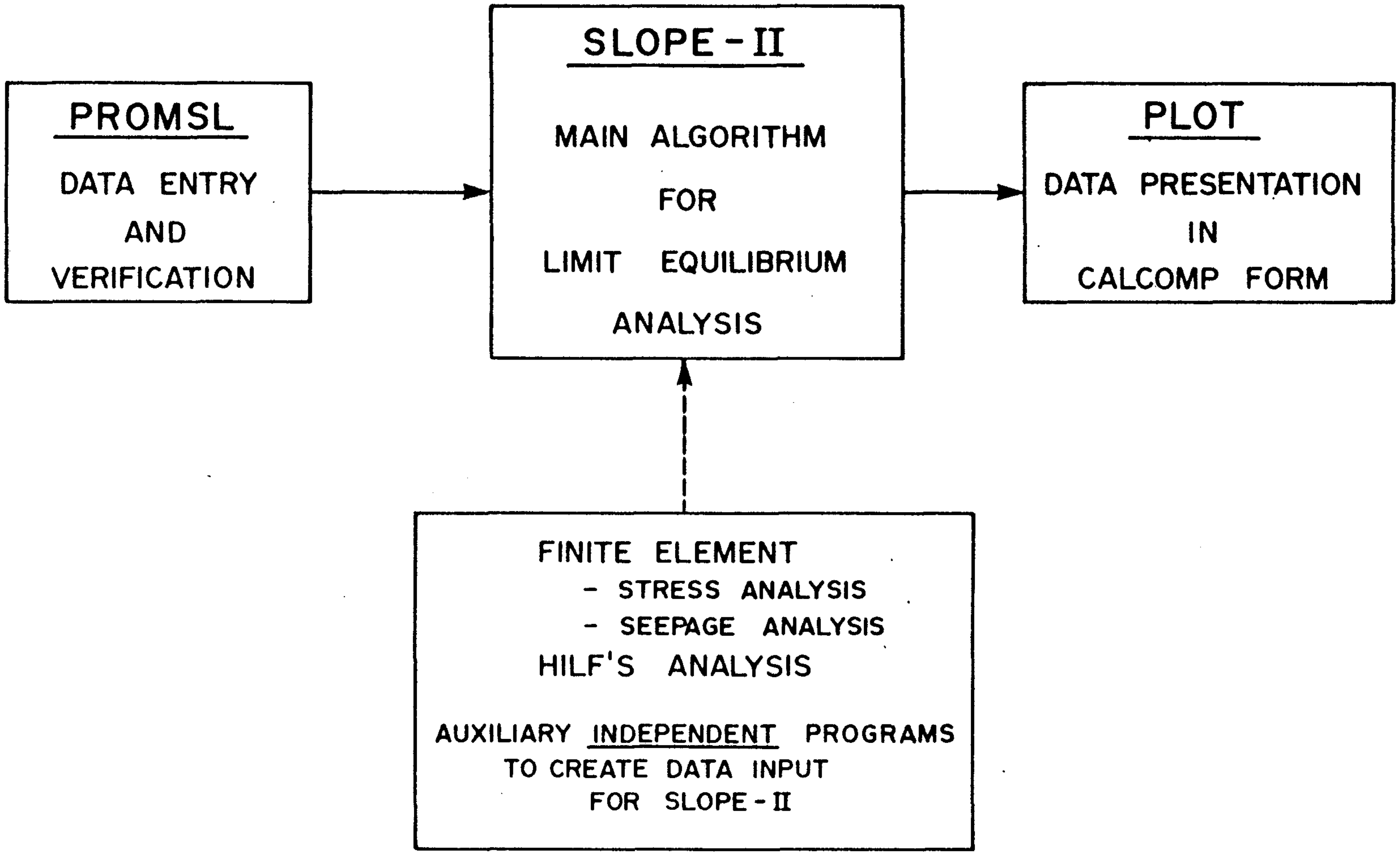


FIGURE 1 Structure of Software System

(1) Slope Stability
- the Fellenius or Ordinary method
- the Simplified Bishop method
- the Spencer method
- the Janbu's Simplified method
- the Janbu's Generalized method
- the Morgenstern-Price method,
- the GLE method, and
- (the Nonvellier method, Lowe-Karafiath method, Corps of Engineers and modified Swedish method which can be solved as special cases of the GLE solver.

(2) Lateral Earth Force
- the active earth force analysis
- the passive earth force analysis

The design of tie-back anchors and the bearing capacity of foundations are presently under study to determine how best to use SLOPE-II to solve these problems. They will make use of the general limit equilibrium solver, GLE.

The design and construction of all earthfill and rockfill dams require a great number of slope stability analysis. In general, the location of the critical failure plane is unknown. Therefore, various locations for the slip surface must be analyzed in order to determine the slip surface which gives the minimum factor of safety (Figure 2). The SLOPE-II program has the ability to search for the critical slip surface for both circular and composite slip surfaces. There are several methods of searching which can be independently specified by users (Figure 3.a, 3.b, 3.c) as well as an automatic search technique (Figure 3.d). Similarly, the maximum active force or the minimum passive resistance of the slope can be obtained using one of these searching methods (Figure 3.b).

SLOPE-II program can perform an analysis using complicated geometric boundaries and stratigraphic conditions. This provision is necessary in order to simulate the actual engineered problems. In addition, externally applied line loads, berms and surcharges, partial submergence of a slope or tension cracks and seismic loadings can be facilitated (Figure 4).

Pore-water pressure conditions play an important role in determining the stability of a slope. In order to closely simulate the field pore-water pressure conditions, six alternatives for pore-water pressure simulation are provided in SLOPE-II. These alternatives are particularly important for analyzing the stability of an earthfill dam at different stages of construction.

The pore-water pressures at the end-of-construction of a compacted fill are a function of the applied total stress (Figure 5.a). In other words, the pore-water pressure can be a linear or a non-linear function of the overburden pressure (Hilf 1948; Bishop 1957). The linear relationship can be obtained by using the pore-pressure coefficient, r_u, and the non-linear relationship can be obtained using Hilf's analysis.

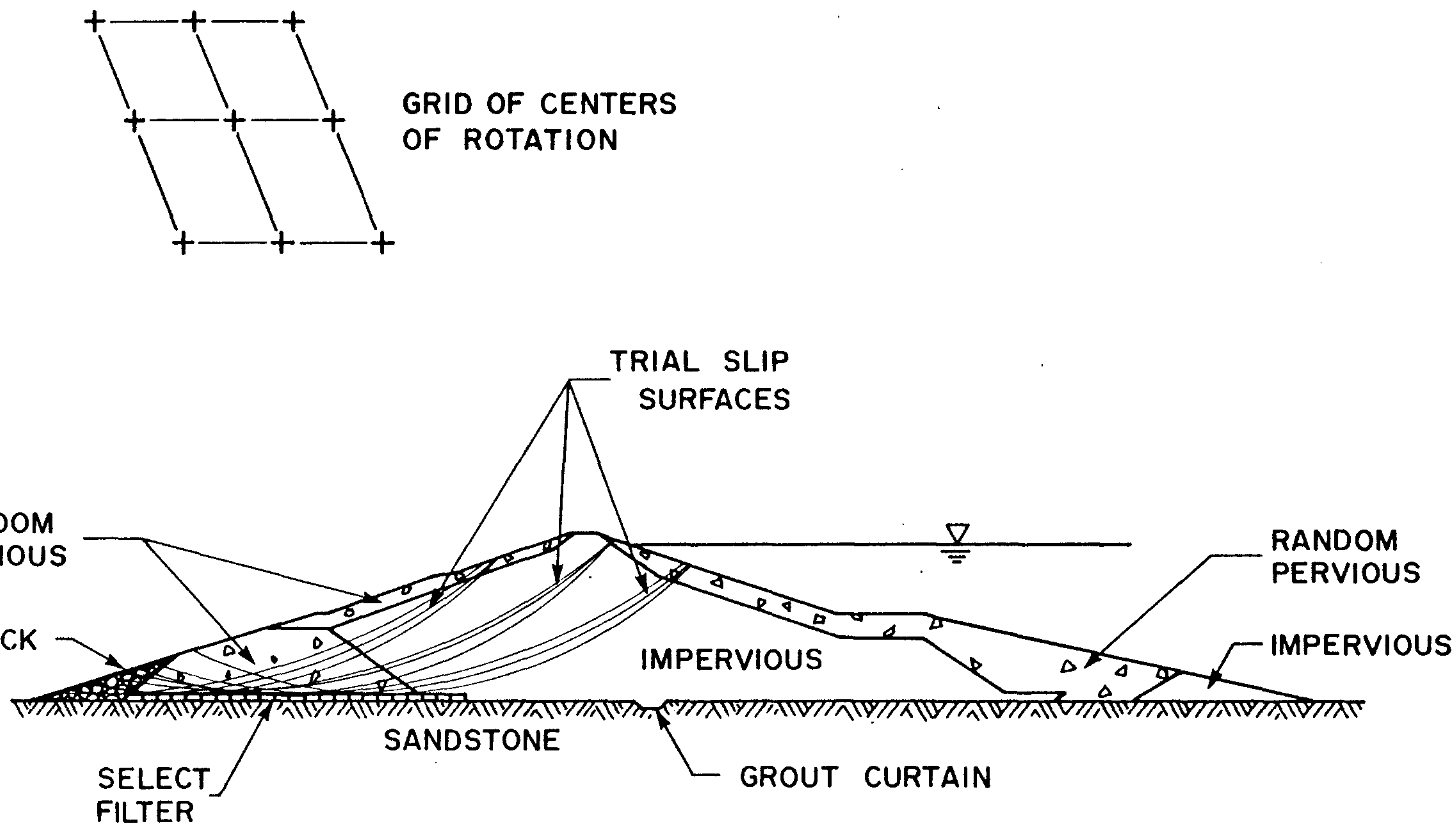
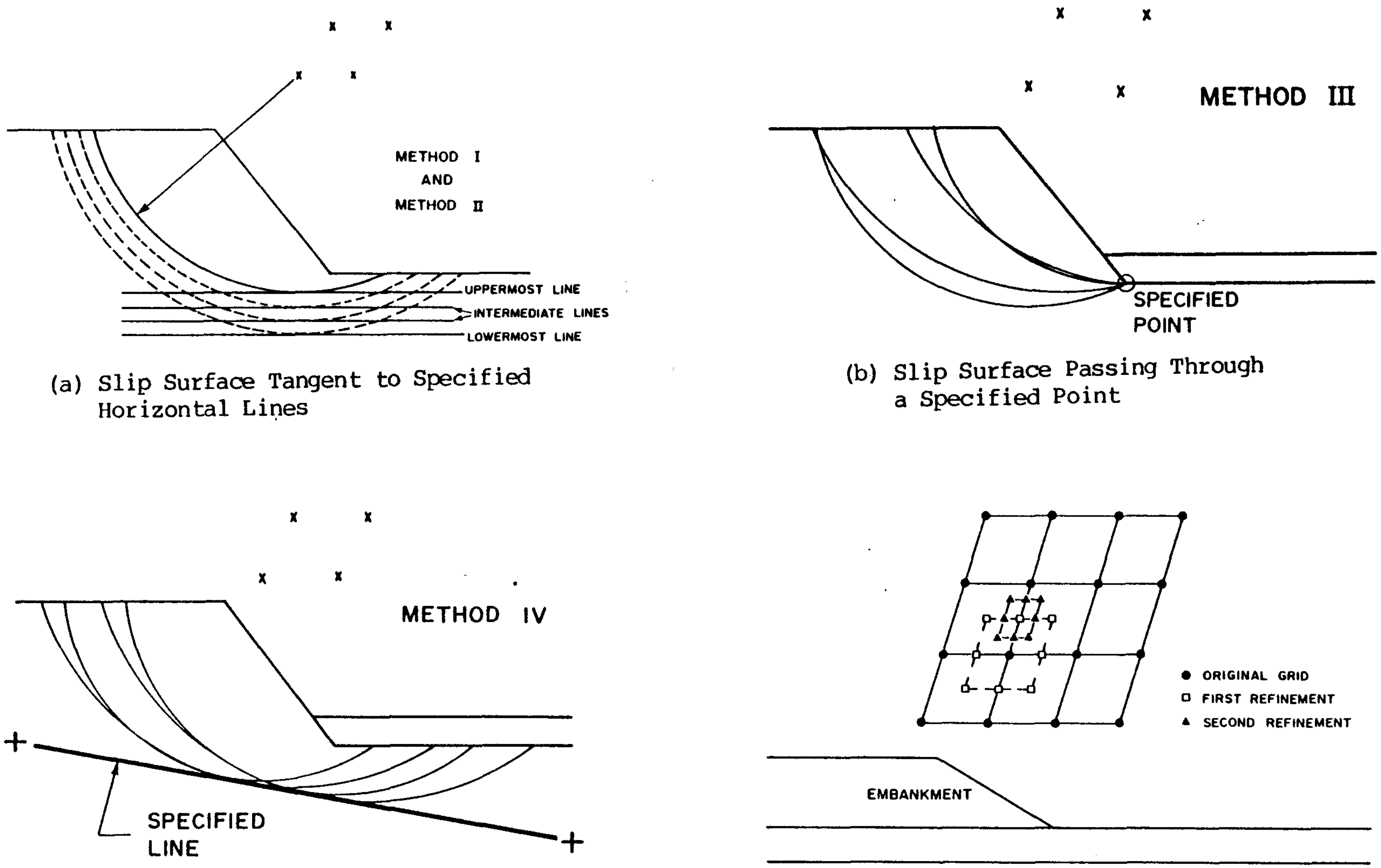


FIGURE 2 Typical Slope Stability Analysis for an Earthfill Dam



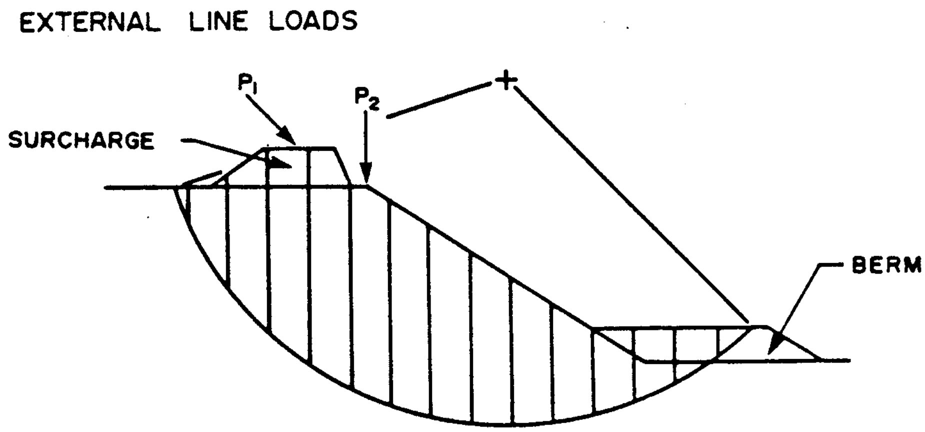
(a) Slip Surface Tangent to Specified Horizontal Lines

(b) Slip Surface Passing Through a Specified Point

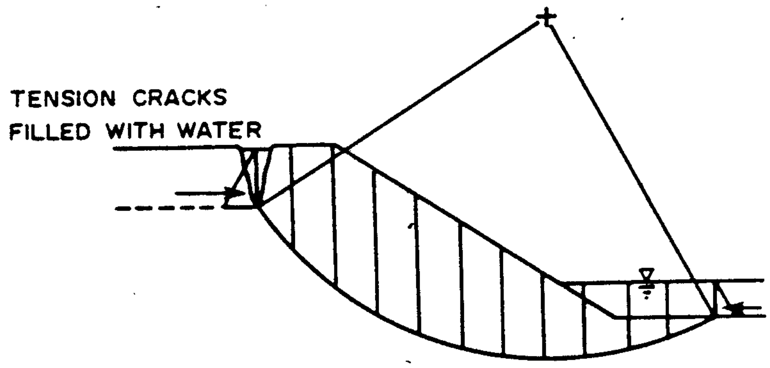
(c) Slip Surface Tangent to a Specified Line

(d) Automatic Search Technique

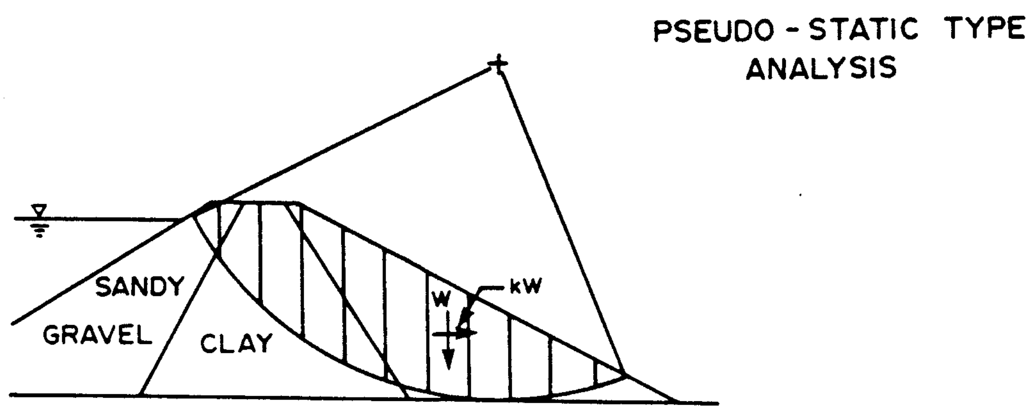
FIGURE 3 Methods of Searching for the Critical Slip Surface



(a) External Line Loads and Surcharges

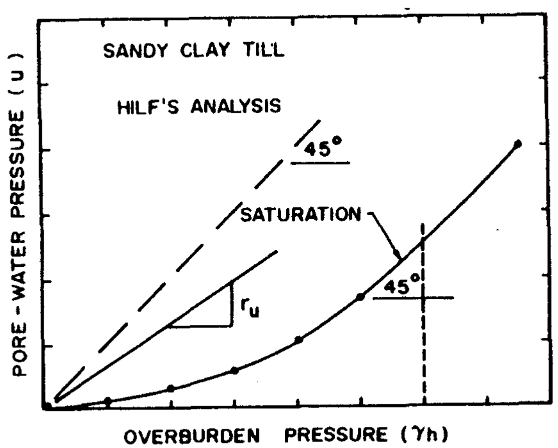


(b) Partial Submergence and Tension Cracks

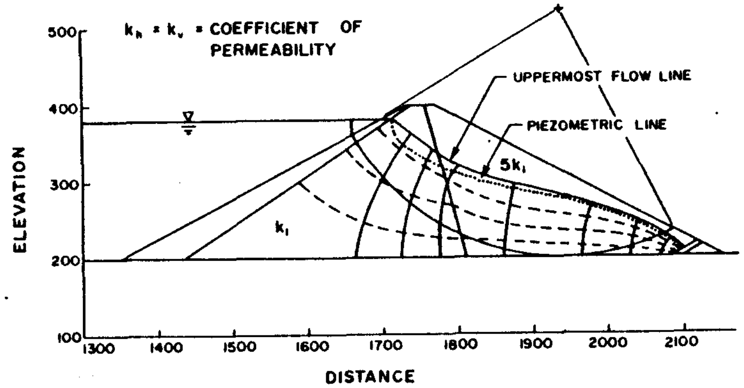


(c) Seismic Loading

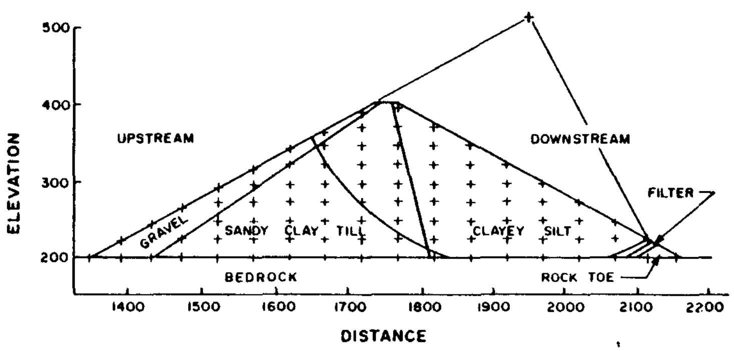
FIGURE 4 Several Features of SLOPE-II



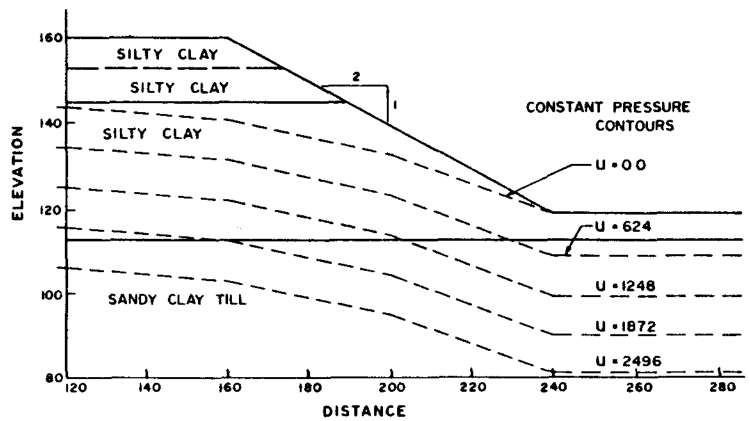
(a) Linear and Non-Linear (POR3)
Pore Pressure Coefficient



(b) Piezometric Lines (POR4)



(c) Grid of Pore-Water Heads (POR5)



(d) Contour of Pore-Water Pressure (POR6)

FIGURE 5 Several Procedures for Computing Pore-Water Pressure

A seepage analysis of a natural slope or a man-made embankment can be used to calculate pore-water pressures required for the stability analysis. The uppermost or phreatic flow line can generally be used as a piezometric line (Figure 5.b). This procedure is not rigorously correct, but generally gives slightly conservative results. Other alternatives are to use a grid of pore-water heads (Figure 5.c) or contours of pore-water pressure (Figure 5.d). All of these methods for estimating the pore-water pressure conditions can be obtained from flownets sketched "longhand" or from some supportive programs. These programs are categorized as "auxiliary independent" programs.

The main program, SLOPE-II, has approximately 10,000 lines of coding. The program is subdivided into a large number of subroutines; the longest being less than 1000 lines of coding. It is highly modularized and structured according to the functional features of a slope stability analysis. Thus the program can be reduced in size, if necessary, by sacrificing a few features (or subroutines). The program is written in Fortran in accordance with the specifications published by the American National Standards Institute (ANSI: American National Standards Institute, 1966a, 1966b). It has proven to be very portable and is currently operative on all major main frame manufacturers and several mini computer hardware vendors. The SLOPE-II program can be executed in either a batch or interactive mode.

Auxiliary Dependent Programs

The "auxiliary dependent" programs were specially designed to assist users in preparing the input data for the main program as well as visualizing the computer output. There are two types of programs in this category. These are PROMSL and PLOT-1/2/3.

PROMSL is an interactive program with approximately 6,500 lines of code. It provides three functions in assisting the user in creating a valid input data file for the SLOPE-II program. It displays an image of the coding form (i.e., variable names and column alignment) on the terminal and allows the user to create, to modify and to verify data files. These three functions of PROMSL can be described as follows:

(1) Data file creation

Users can create a SLOPE-II data file in accordance with the fixed format shown on the coding form or with a free format (Figure 6.a and 6.b). In the free format, all variables need only be separated using a comma.

Most of floating point input data can be input using up to 5 digits. It is also possible to request double precision on the floating point variables where up to 10 digits can be used to input the data (Figure 6.c and 6.d).

```
********
* HEAD *
********

KEY- TYPE OF         TRIAL                                                QUESTION
WORD ANALYSIS  DATE  NUMBER              COMMENTS                       12345678901
         1         2         3         4         5         6         7
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789
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HEAD21107-01-81  NO.1        EXCAVATION SILTY CLAY                      2122    11
```

(a) Fixed Format

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********
* HEAD *
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KEY- TYPE OF         TRIAL                                                QUESTION
WORD ANALYSIS  DATE  NUMBER              COMMENTS                       12345678901
         1         2         3         4         5         6         7
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789
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HEAD,211,07-01-81,  NO.1,   EXCAVATION SILTY CLAY,2,1,2,2,,,,1,1/
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(b) Free Format

```
HEAD21107-01-81  NO.1        EXCAVATION SILTY CLAY                      2122    11
GRID  0                                                                    1
  4.0170.0 20.0     2170.0 30.0     3 0.25  5.0     8140.0  0.0  5.0 .010 .000 62.4
GEOM  5                                                               00000000000
 50.0160.0174.0190.0240.0400.0400.0400.0400.0400.0400.0400.0400.0400.0400.0    0
160.0160.0153.0145.0120.0120.0120.0120.0120.0120.0120.0120.0120.0120.0120.0
```

(c) Condensed Form

```
HEAD21107-01-81  NO.1        EXCAVATION SILTY CLAY                      2122    11
GRID  0                                                                    1
    4.0000  170.0000   20.0000     2  170.0000   30.0000     3    0.2500
    5.0000     8  140.0000    0.0000    5.0000    0.0100    0.0000   62.4000
GEOM  5                                                               00000000000
   50.0000  160.0000  174.0000  190.0000  240.0000  400.0000  400.0000  400.0000
  400.0000  400.0000  400.0000  400.0000  400.0000  400.0000  400.0000     0
  160.0000  160.0000  153.0000  145.0000  120.0000  120.0000  120.0000  120.0000
  120.0000  120.0000  120.0000  120.0000  120.0000  120.0000  120.0000
```

(d) Expanded Form

FIGURE 6 Format Alternatives in Using PROMSL

(2) Data file modification

Slope stability analysis often requires comparisons of several computations using different properties of the soil, different external loadings, different geometries or different methods of analysis. All computations can be performed at one time using one input data file. In other words, one datafile will permit the performance of a parametric type study. An original set of data needs merely be followed by other additional sets of data which contain the variables that should be modified. These modifications for datafiles can be created through PROMSL.

(3) Data file verification

Changes can be made to an input data file by using the computer system editor. The edited data file can be verified by PROMSL to ensure its validity prior to execution by SLOPE-II. During the verification phase, "error" messages will appear if unacceptable data are encountered. The user is given the opportunity to re-enter the data. "Warning" messages are displayed if the data is suspect and "note" messages are shown to indicate when default values are assumed.

A digital plot of the geometry can also be plotted by PROMSL if it is requested by the user. In addition, statistics are printed out at the end of PROMSL on the number of lines of input data, the number of slip surfaces to be analyzed, the approximate number of lines of output, and the estimation of the execution time (cpu) that will be required by SLOPE-II.

The PLOT programs are Calcomp plotting programs which assist the user in analyzing the results from SLOPE-II graphically. The SLOPE-II program can produce specific output files which can be used as the data input files for the PLOT programs. There are three different plotting programs available. They consist of approximately 1,000 coding lines.

(1) PLOT-1

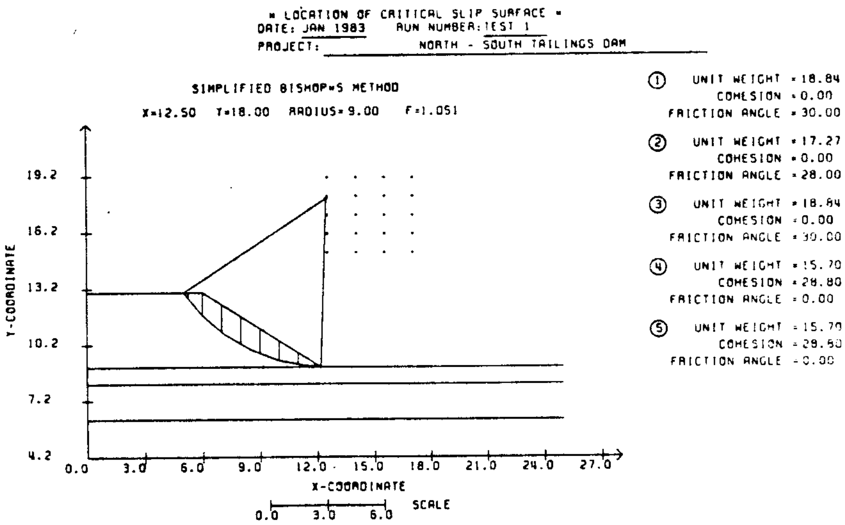
PLOT-1 allows the plotting of the geometry of the problem, the grid of centers of rotation used in the analysis and the slip surface corresponding to the minimum factor of safety (Figure 7.a).

(2) PLOT-2

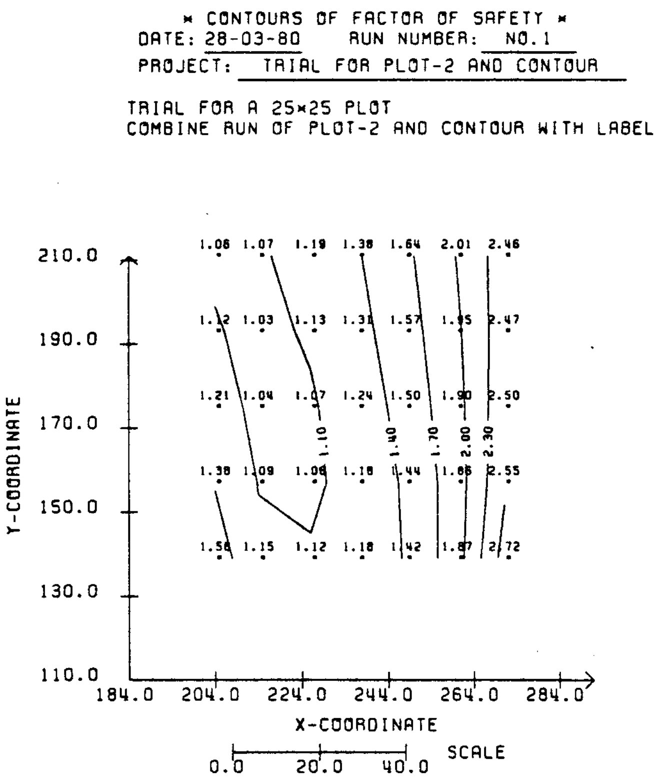
PLOT-2 is used to draw contour lines of the factor of safety (or lateral earth force magnitude) on the grid of centers of rotation used in the analysis (Figure 7.b).

(3) PLOT-3

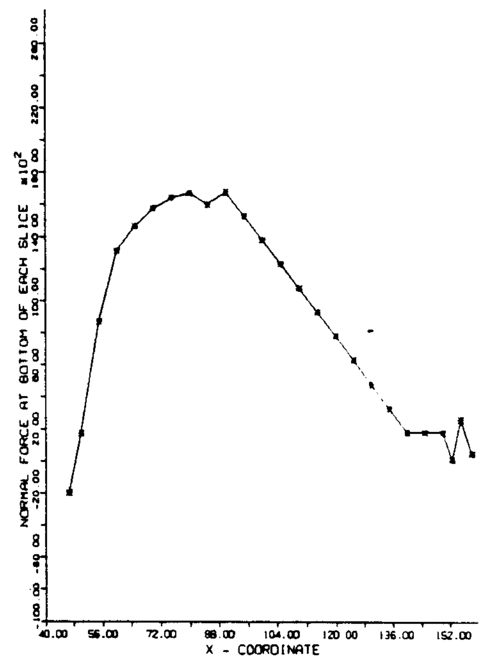
PLOT-3 enables the plotting of various force distributions across the slip surface analyzed. The forces plotted can be the normal, shear, pore-water, weight or interslice normals or shears. In addition, the force polygon for each slice can also be plotted using PLOT-3 (Figure 7.c). PLOT-3 is presently under development.



(a) Geometry of the Problem (PLOT-1)



(b) Contours of Factor of Safety (PLOT-2)



(c) Normal Force Distribution versus X-Coordinate (PLOT-3)

FIGURE 7 Calcomp Output from Plot Programs

Auxiliary Independent Programs

Auxiliary independent programs can be used to support SLOPE-II. Data from these programs can be used as input data for the SLOPE-II program (Figure 8).

(1) FINEL (Finite Element Stress Analysis)

FINEL is a finite element stress anlaysis program developed at the University of Saskatchewan, Saskatoon. The finite element stress program has been primarily used to obtain the direction of the force distribution within a potential sliding mass. This interslice force distribution is then used as a side force function to resolve the indeterminancy problems in the slope stability analysis (Wilson 1982; Fan 1983) and the lateral earth force computation (Rahardjo 1982). The results obtained from the finite element stress analysis can be used as the input data for the SIDE card in SLOPE-II (Figure 9.a).

The finite element stress analysis can also be performed using the A and B pore pressure parameters. In this case, the analysis produces a grid of pore-water heads representing the effect of applied loading conditions or stress changes. This pore-water heads can be input to the SLOPE-II program using the POR5 keyword data input (Figure 5.c).

(2) SEEP (Finite Element Seepage Analysis)

The analysis of seepage through an earthfill dam due to the potential head difference across the dam, excessive rainfall or rapid draw-down can be performed using the finite element method. SEEP is a finite element program developed at the University of Saskatchewan, Saskatoon, which accommodates steady state seepage through saturated and unsaturated zones (Papagiannakis 1982).

Figure 9.b shows the result of a seepage analysis using the SEEP program. It is possible to use the position of the piezometric line as input for the POR4 card in the SLOPE-II program (Figure 5.b). Another alternative is to use the grid of pore-water heads as the input for the POR5 card (Figure 5.c). If the pore-water heads are contoured, they become suitable input for the POR6 card in the SLOPE-II program (Figure 5.d). In other words, there are three ways in which the results of an independent seepage analysis can be used as input pore-water pressures for SLOPE-II.

(3) HILF

The pore-water pressure conditions can also be obtained from a non-linear relationship between total overburden pressure and pore-water pressure. This procedure was developed by Hilf (1948) at the United States Bureau of Reclamation (USBR) and is called Hilf's analysis.

HILF is a computer program which accommodates the pore-water pressure computation using the Hilf analysis. The results obtained

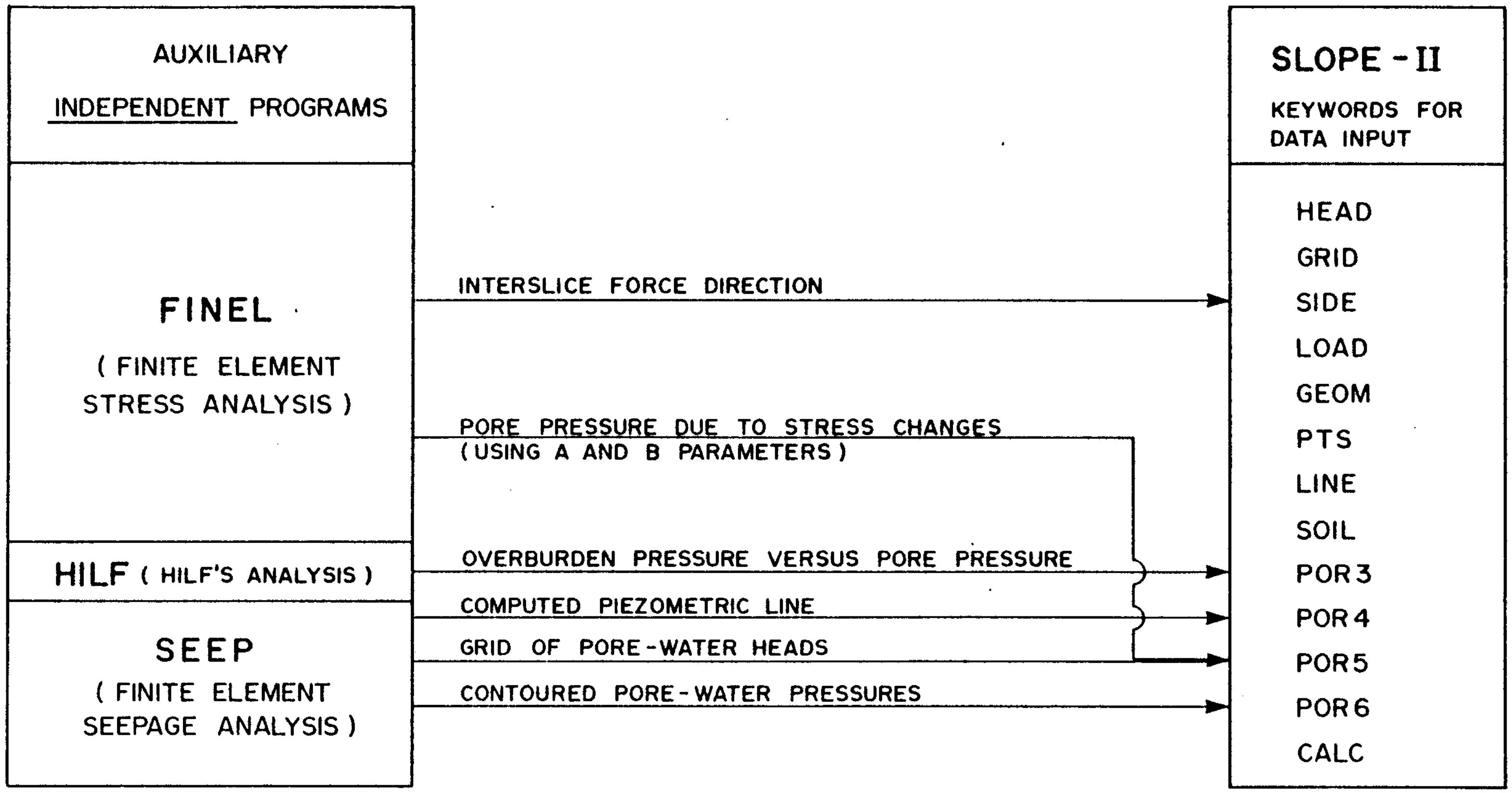
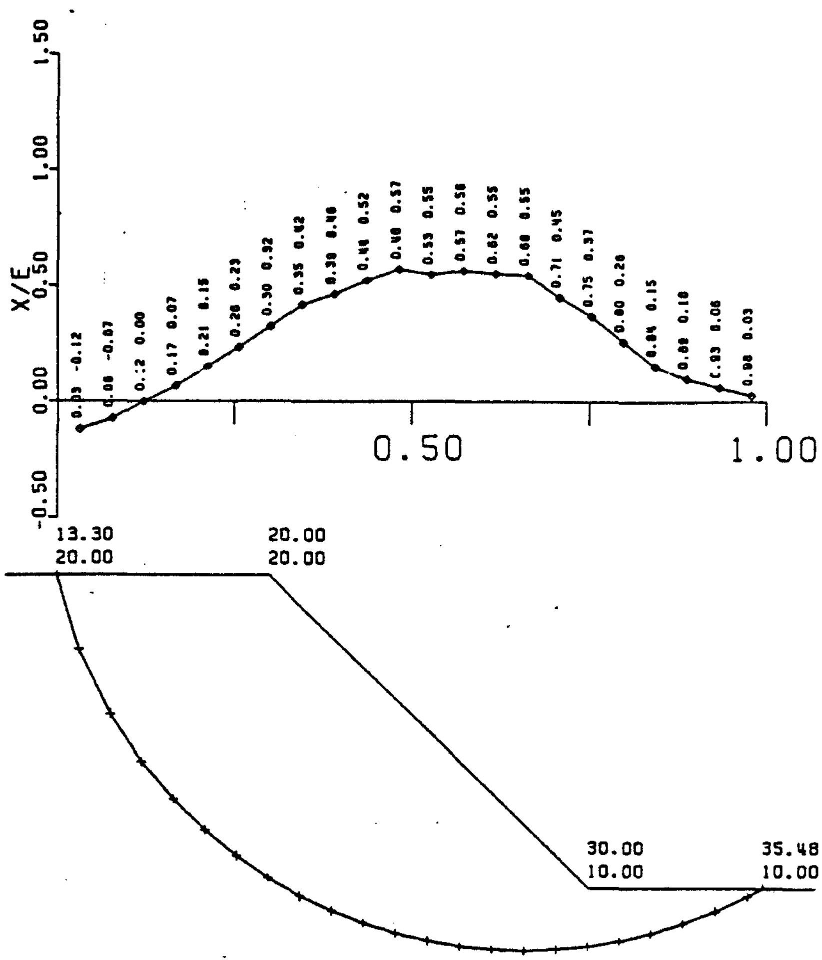
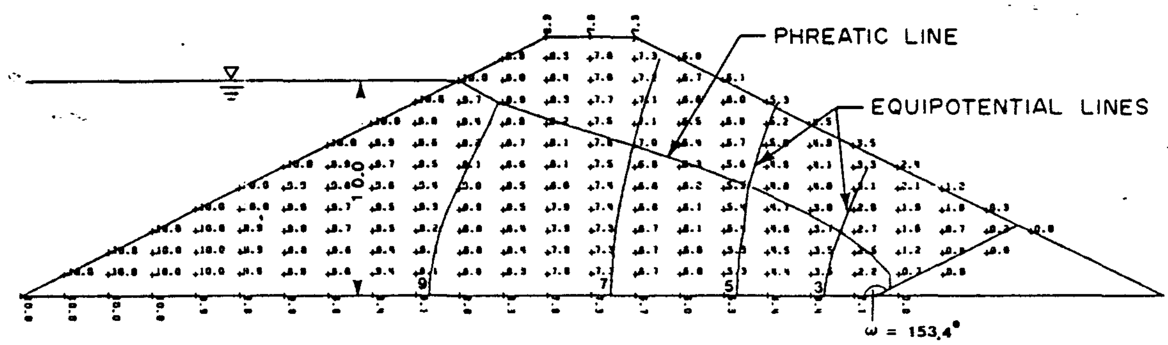


FIGURE 8 Programs which Assist in Generating Suitable Input Data to SLOPE-II



(a) Interslice Force Function Generated (After Wilson 1982)



(b) Results of Seepage Analysis for Dam (After Papagiannakis 1982)

FIGURE 9 Results of Finite Element Analyses from this program can be used as input data for the POR3 card of the SLOPE-II program (Figure 5.a).

SOFTWARE SUPPORT

In order to maintain the state-of-the-art status and quality of system, the following support is provided.

(1) Maintenance

SLOPE-II, PROMSL and PLOT programs are proprietary products of Geo-Slope Programming Ltd., Calgary, Alberta. These programs are distributed through lease arrangement with software bureaus and engineering companies.

The support of SLOPE-II, PROMSL and PLOT is performed based on ongoing research and the input provided by industry users. This input can be questions about the results obtained from these programs or specific enhancements requested by users. Each concern brought to the attention of Geo-Slope personnel is fully investigated and appropriate action initiated. Discrepancies in computer output are most often the result of incorrect input to the program. However, if errors are encountered in the program, fixes are performed.

All fixes and enhancements are documented chronologically and a complete system test is performed subsequent to each fix or enhancement. New releases containing all fixes and enhancements are issued annually unless critical errors have been encountered which require interim releases.

(2) Education

Assistance to users is provided through seminars and comprehensive user manuals. Special training is given by Geo-Slope to the personnel in each service bureau in order to educate them on the use of the programs. These personnel are responsible to forward technical questions or suggestions about the programs directly to Geo-Slope personnel.

PROMSL was designed as a self-teaching tool for the users in preparing the input data for the SLOPE-II program. Its documentation can be obtained directly from the terminal when the program is initiated.

(3) Research

The auxiliary independent programs are developed as part of the research program at the University of Saskatchewan, Saskatoon, and are not distributed. The ongoing research is geared towards recent developments in the theory and the ever-changing computer hardware environment.

FUTURE PERSPECTIVE

Rapid developments in microcomputers industry have resulted in a significant demand from users for implementing a reduced version of the SLOPE-II program onto microcomputers. Though attempts are being made to make SLOPE-II and/or PROMSL available on a microcomputer, it is anticipated that the SLOPE-II computer program will remain primarily on large mainframe computers with the microcomputers being used primarily as data entry devices.

New features that will likely be added to SLOPE-II in the near future are the computation of tie-back anchor and the bearing capacity analysis for foundation design.

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