# Lateral earth pressures in expansive clay soils

D. E. PUFAHL, D. G. FREDLUND, AND H. RAHARDJO
*Department of Civil Engineering, University of Saskatchewan, Saskatoon, Sask., Canada S7N 0W0*

Received October 13, 1981
Accepted October 13, 1982

Lateral earth pressures produced by saturated clays with negative pore-water pressures and unsaturated expansive clays with positive matric suctions are considered from a theoretical standpoint. Simple earth pressure equations are formulated in terms of total stresses using the Mohr–Coulomb failure criteria and the assumptions consistent with the Rankine earth pressure theory.

Conventional practice is to separate the pressure that the soil exerts on the wall into the pressure produced by the soil structure (effective stress) and that produced by the water (neutral stress). Since two stress state variables are required to describe the behavior of unsaturated soils, the expressions can no longer be separated into two distinct components.

The change in lateral pressures resulting from decreases in pore-water pressure or increases in matric suction are quantified by considering a 6 m high wall for the active and passive cases. Tension cracks are shown to have little effect on the conditions shown.

The magnitude of the lateral pressure generated due to changes in matric suction under conditions where walls are restrained from moving depends upon the ratio ($K_T$) of horizontal to vertical stress and the matric suction of the backfill at the time that it is placed behind the wall. When structural members are cast directly against undisturbed clays, similar criteria govern the magnitude of the lateral pressures that may be generated due to changes in matric suction. The maximum pressure that can be developed in some cases will be equal to the passive pressure of the soil when it is saturated.

An expression for the critical height of a vertical bank for the saturated and unsaturated cases is developed. The role of tension cracks is shown by computing the critical height of a vertical slope for a typical soil both in the absence and in the presence of tension cracks.

In many instances it is apparent that methods devised to sustain the matric suction at or near its original value may result in a substantial savings in design or that they will ensure adequate performance standards for existing facilities.

*Keywords*: Lateral earth pressure, active pressure, passive pressure, unsaturated soils, expansive soils, tension cracks, retaining walls, unsupported excavations.

Les pressions horizontales des terres produites par des argiles saturées soumises à des pressions interstitielles négatives et par les argiles gonflantes non saturées et présentant des succions positives sont considérées d'un point de vue théorique. Des équations simples de pression des terres sont formulées en terme de contraintes totales en utilisant le critère de rupture de Mohr–Coulomb et les hypothèses conformes à la théorie de Rankine.

La pratique courante consiste à séparer la pression exercée par le sol sur le mur en une pression produite par la structure du sol (contrainte effective) et une pression produite par l'eau (contrainte hydrostatique). Deux variables d'état de contrainte étant nécessaires pour décrire le comportement des sols non saturés, les expressions ne peuvent plus être séparées en deux composantes distinctes.

Les changements de pression latérale résultant de diminution de la pression interstitielle ou d'augmentation de la succion sont quantifiées par analyse d'un mur de 6 m de haut, dans les cas actif et passif. On montre que les fissures de traction ont peu d'effet sur les conditions étudiées.

La grandeur de la pression horizontale produite par des changements de succion dans le cas où les murs ne peuvent se déplacer dépend du rapport ($K_T$) des contraintes horizontale et verticale et de la succion du remblai au moment où il est mis en place derrière le mur. Lorsque des éléments de structure sont coulés directement au contact d'argiles intactes, des critères semblables contrôlent la grandeur des pressions latérales qui peuvent être produites par des variations de succion. La pression maximum qui peut se développer dans certains cas sera égale à la pression passive dans le sol à l'état saturé.

Une expression de la hauteur critique d'un talus vertical est développée pour les cas saturé et non saturé. Le rôle des fissures de traction est illustré en calculant la hauteur critique d'un talus vertical dans un sol typique lorsque des fissures sont présentes et absentes.

Dans de nombreux cas il apparait que des méthodes visant à maintenir la succion à sa valeur originale, peuvent conduire à des économies importantes dans les projets, ou peuvent assurer un comportement convenable des ouvrages existants.

*Mots-clés*: pression latérale des tests, poussée, butée, sols non saturés, sols gonflants, fissures de traction, murs de soutènement, excavations non supportées.

Can. Geotech. J., **20**, 228–241 (1983) [Traduit par la revue]

## Introduction

The use of cohesive backfill behind earth retaining structures is avoided as much as possible because of the uncertainties associated with the behavior of these materials. This uncertainty is frequently reflected in the observed performance of these structures. For example, Ireland (1964) showed that 68% of the unsatisfactory retaining walls that were considered in one study either used clay as a backfill or were founded upon it.

Some of the problems encountered with these structures result from the tendency of the cohesive materials to creep under sustained shearing stress, others are often due to lack of effective drainage. Additional questions arise if the backfill is an unsaturated expansive clay that undergoes substantial changes in volume with changes in water content. Nevertheless, there are numerous situations where cohesive soils are used as backfill or where structural members are cast in place against them. It is apparent that the ability to predict the behavior of these soils under many of these circumstances is not completely developed.

Unsupported trenches or excavations with vertical or nearly vertical walls are frequently made in cohesive soils. If the area of the excavation is comparatively large a slope failure may not represent a serious hazard to men and equipment working at the base. On the other hand, when the excavation is narrow, as is the case for temporary trenches required for the installation of utilities, a failure often results in the death of workmen in the trench. It has been estimated that in excess of 200 deaths per year in the United States alone result from the failure of these trenches (Thompson and Tanenbaum 1977).

The purpose of this paper is to provide a simplistic treatment of anticipated earth pressures in cohesive soils under different conditions of saturation as well as to provide appropriate expressions for the critical height of vertical or nearly vertical cuts. The equations are formulated first for saturated conditions when the surface of the soil behind the wall is horizontal and where friction between the wall and the soil is negligible. In addition the groundwater conditions are considered to be static.

It is recognized that seepage patterns will develop that will influence the magnitude of the pressures on the wall and the height of the unsupported cuts that are computed from the equations provided here. Moreover, these flow patterns will be influenced by the configuration and location of sand drains that form an integral part of retaining wall design. However, it is virtually impossible to consider all groundwater conditions and to provide expressions for the earth pressures associated with them.

Since vertical tension cracks are ubiquitous in unsaturated cohesive soils it is of interest to assess their apparent effect on the pressure on walls and the critical height of vertical cuts. Since it is difficult to accurately estimate the depth of existing cracks based on present environmental and soil moisture conditions, the depth of crack can only be established from observations on previous excavations.

Finally, the importance of monitoring soil suction is considered and the increase in earth pressures and the decrease in critical height of a cut associated with decreases in suction are emphasized.

## General formulation

Much of the theoretical treatment of lateral earth pressures in cohesive soils where maximum pressures or resistances are the dominant unknowns have used the Mohr–Coulomb failure criteria and concepts of plastic equilibrium. A variety of methods is available in the literature to calculate these pressures. The major differences in the pressures that are computed using these methods are due mainly to the assumptions that are made regarding the angle or development of wall friction, the shape of the failure surface, the amount or type of movement of the wall, and the application of earth statics (Morgenstern and Eisenstein 1970). When water pressures are involved, the resulting pressures on the wall are usually separated into two components: those created by the soil structure (effective stresses) and those created by the water (neutral stresses). If the soil has an apparent cohesion intercept on the Mohr–Coulomb failure envelope the upper part of the "soil structure" in active failure conditions is apparently in tension. Frequently this portion of the effective pressure distribution is neglected when the total force on the wall is computed (Terzaghi and Peck 1967). This modification presupposes that the soil structure cannot adhere to the wall even though the total stress on the wall may be in compression.

Some difficulty arises when this same procedure is applied to unsaturated soils because a single effective stress equation cannot be used to describe the behavior of the soil structure (Fredlund and Morgenstern 1977). The two stress state variables that appear to be most satisfactory for most soil mechanics problems are $\sigma - u_a$ and $u_a - u_w$, where $\sigma$ = total applied stress, $u_a$ = pore-air pressure, and $u_w$ = pore-water pressure.

The $\sigma - u_a$ term is called the net total stress and $u_a - u_w$ is called the matric suction. Under these conditions the pressures acting on the wall can be computed in terms of total stresses, pore-air pressures, and matric suction. Since the pore-air pressure will invariably be equal to zero relative to the atmosphere, its inclusion in the equations is somewhat academic and so it will usually disappear in the final result.

If a comparison is to be made between pressures exerted by the same soil when it is saturated and when it is unsaturated, the resulting pressure on the wall for all degrees of saturation must be formulated in terms of total pressure. In addition it has been suggested that the effect of matric suction on a soil can be most easily visualized if it is considered as contributing to the apparent cohesion intercept on the Mohr–Coulomb failure envelope (Fredlund *et al.* 1978). In view of the latter proposal, the effect of pore-water pressure on saturated soils will be evaluated in the same manner so that a direct comparison may always be made.

The failure plane is assumed to be planar, the surface

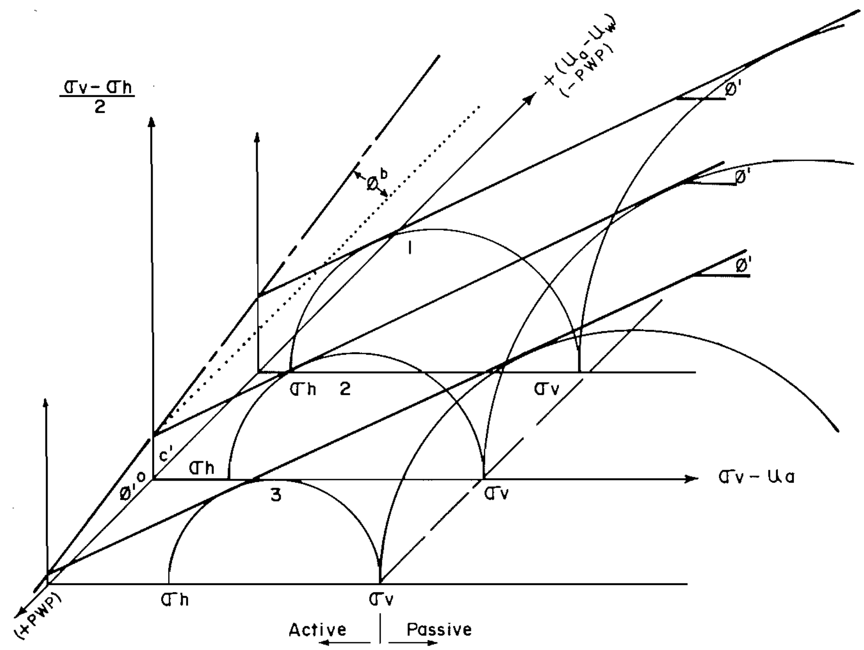


FIG. 1. Active and passive earth pressure for saturated and unsaturated soils (PWP is pore-water pressure).

of the backfill is horizontal, and wall friction is neglected; this set of conditions is in accordance with the classic Rankine earth pressure theory.

Finally all formulations will deal with drained loading. That is to say, pore-water pressures are due only to existing groundwater levels and environmental conditions, and not to changes in total applied stresses.

## Total horizontal earth pressures

The shearing resistance of an unsaturated soil was given by Fredlund *et al.* (1978) as:

$$[1] \quad \tau = (\sigma - u_a)\tan\phi' + [c' + (u_a - u_w)\tan\phi^b]$$

where $\phi'$ = the effective angle of shearing resistance of a saturated soil, $\phi^b$ = the angle of shearing resistance with respect to changes in matric suction, and $c'$ = the effective cohesion intercept.

Although $\phi^b$ may not be constant over a range of extremely large suction values, present experimental evidence indicates that it will be essentially constant over the range of suction values that are commonly encountered in practice (Ho and Fredlund 1982).

Equation [1] is illustrated by circle 1 in Fig. 1. Figure 1 is an extended Mohr–Coulomb failure envelope where $\sigma_v - u_a$ is the horizontal axis, $(\sigma_v - \sigma_h)/2$ is the vertical axis, and the third axis indicates the matric suction, $u_a - u_w$, as soil moves into the plane of the paper and a positive pore-water pressure as the soil moves out of the plane of the paper (e.g., circle 3).

As the soil moves from circle 1 (the unsaturated state) to circle 2 (the saturated state), $u_w$ approaches $u_a$ and [1] reverts to,

$$[2] \quad \tau = (\sigma - u_w)\tan\phi' + c'$$

which is the classic equation for the shear strength of a saturated soil (Terzaghi 1943).

Circle 2 in Fig. 1 indicates a saturated soil with zero pore-water pressure (somewhat hypothetical). From the geometry of this circle, the total active pressure at any depth will be:

$$[3] \quad \sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2c'}{\sqrt{N_\phi}}$$

where

$$N_\phi = \frac{1 + \sin\phi'}{1 - \sin\phi'} = \tan^2(45 + \phi'/2)$$

and $\sigma_v = \rho g z$ (total vertical stress), $\sigma_h$ = total horizontal stress, $\rho$ = total mass density of the soil (g/cm$^3$, Mg/m$^3$, t/m$^3$), $g$ = acceleration due to gravity (m/s$^2$), and $z$ = distance measured positively downward from the surface (m).

The pressure distribution in the soil during active failure is shown in Fig. 2.

$$[4] \quad Z_t = \frac{2\sqrt{N_\phi}\,c'}{\rho g}$$

The pressure distribution on the wall will be area abc if the tension zone is disregarded.

$$[5] \quad P_A = \left[\frac{\rho g H}{N_\phi} - \frac{2c'}{\sqrt{N_\phi}}\right]\frac{(H - z_t)}{2}$$

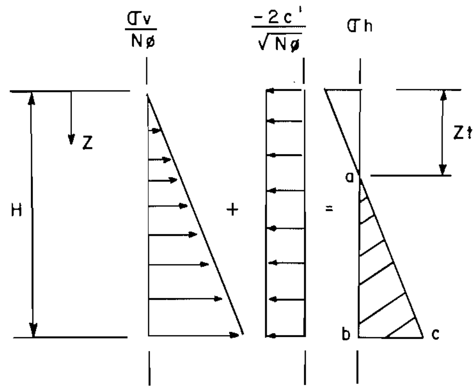


FIG. 2. Active earth pressure (saturated, pore-water pressure = 0).

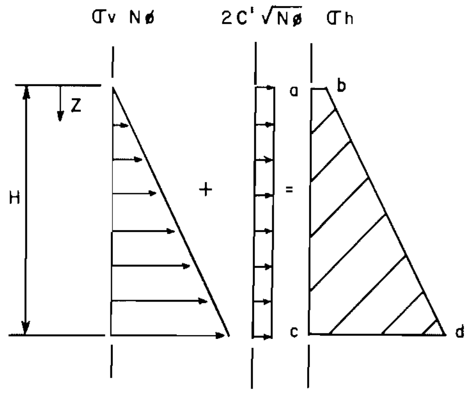


FIG. 3. Passive earth pressure (saturated, pore-water pressure = 0).

The total passive pressure at any depth will be,

$$[6] \quad \sigma_h = \sigma_v N_\phi + 2c'\sqrt{N_\phi}$$

The pressure distribution is shown in Fig. 3.

The total passive force will be:

$$[7] \quad P_p = \frac{\rho g h^2}{2} N_\phi + 2c'\sqrt{N_\phi}H$$

The same problem is considered with the water table at a depth, $D$, below the surface. The soil has a negative pore water pressure above the water table, but is still saturated.

The total active horizontal pressure will be predicted by:

$$[8] \quad \sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2}{\sqrt{N_\phi}}[c' - (z - D)\rho_w g \tan\phi']$$

The pressure distribution on the wall is shown in Fig. 4.

It is apparent from [8] and Fig. 1 that the water pressure acts to increase the apparent cohesion intercept above the water table where the pore-water pressures are negative and decreases the apparent cohesion intercept where the pore-water pressures are positive (circle 3, Fig. 1).

The depth, $Z_{ts}$, over which the soil is exerting an apparent tensile stress on the wall is

$$[9] \quad Z_{ts} = \frac{2\sqrt{N_\phi}[c' + D\rho_w g \tan\phi']}{(\rho_s - \rho_w)g + \rho_w g N_\phi}$$

The denominator in [9] can be rewritten so that

$$[10] \quad Z_{ts} = \frac{2\sqrt{N_\phi}[c' + D\rho_w g \tan\phi']}{\rho_s g + 2\rho_w g(\tan\phi')\sqrt{N_\phi}}$$

where $\rho_s$ = saturated mass density of the soil.

The total active force on the wall as indicated by area abc will be:

$$[11] \quad P_A = \left[\frac{\rho_s g H}{N_\phi} - \frac{2}{\sqrt{N_\phi}}\left\{c' - (H - D)\rho_w g \tan\phi'\right\}\right] \times \left\{\frac{(H - Z_{ts})}{2}\right\}$$

It is apparent from [10] and [11] that lowering the water table has a significant effect in reducing the total force on the wall.

The total passive pressure at any depth as shown in Fig. 5 will be:

$$[12] \quad \sigma_h = \sigma_v N_\phi + 2\sqrt{N_\phi}[c' - (z - D)\rho_w g \tan\phi']$$

The total passive force is equal to area abcd and is given by:

$$[13] \quad P_p = \frac{\rho_s g H^2 N_\phi}{2} + 2\sqrt{N_\phi}H\left[c' - \left(\frac{H}{2} - D\right)\rho_w g \tan\phi'\right]$$

## Unsaturated soil

From circle 1 in Fig. 1 the total active pressure will be:

$$[14] \quad \sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2}{\sqrt{N_\phi}} \times [c' + (u_a - u_w)\tan\phi^b - u_a \tan\phi']$$

A comparison of [8] and [14] indicates that the $(z - D)\rho_w g \tan\phi'$ term in [8] is equivalent to the $(u_a - u_w)\tan\phi^b$ term in [14]. A negative sign appears in [8] where a positive sign appears in [14]. However, this sign difference is resolved when appropriate substitutions of the variables are made into the respective equations. The pore-air pressure, $u_a$, in the unsaturated soil acts in the same way as a positive pore-water pressure in the saturated soil. In practice $u_a$ is equal to zero relative to the atmosphere and so its effect will not be important. It is included here so that a complete picture of the behavior of the material will be evident. As the soil in

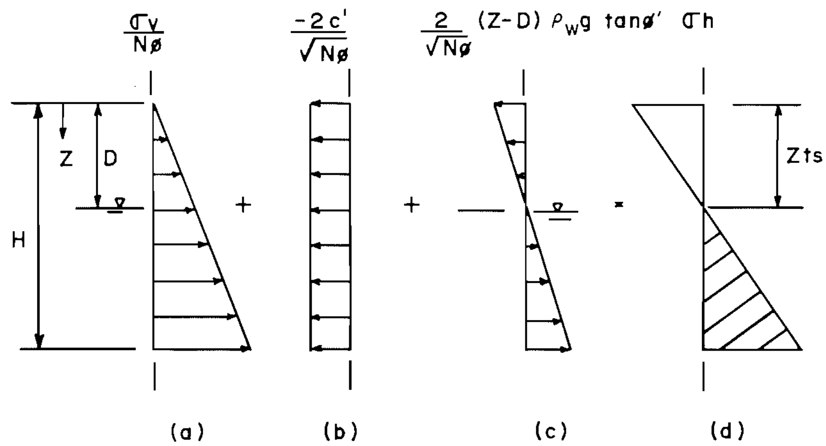


FIG. 4. Active earth pressure (saturated, water table below surface).

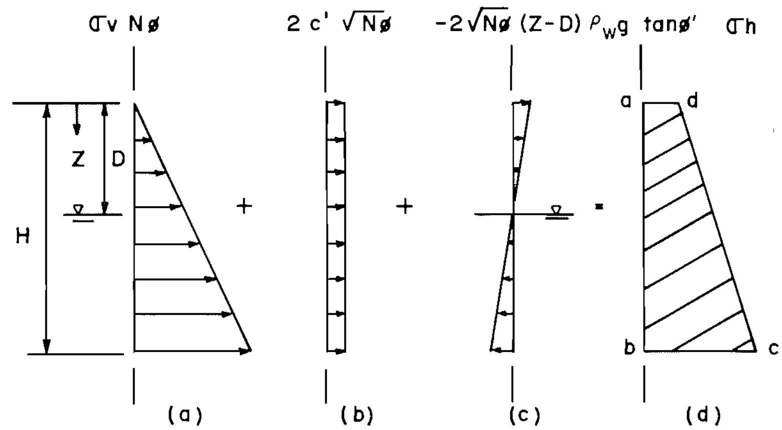


FIG. 5. Passive earth pressure (saturated, water table below surface).

[14] moves to the saturated state, $u_w$ approaches $u_a$ and [14] reverts to [8], where $u_w$ becomes $(z - D)\rho_w g$.

Equation [14] indicates that $u_a - u_w$ is constant with depth. Since $u_a - u_w$ will generally decrease with increasing depth, a suitable relationship should be used to account for this condition. Figure 6 shows a typical suction profile with depth, which is shown as the solid line between a and b; dbc represents the hydrostatic condition. A simple linear relationship may be used to adequately estimate the variation of the suction with depth for this proposed profile. Then for $z \leq D$,

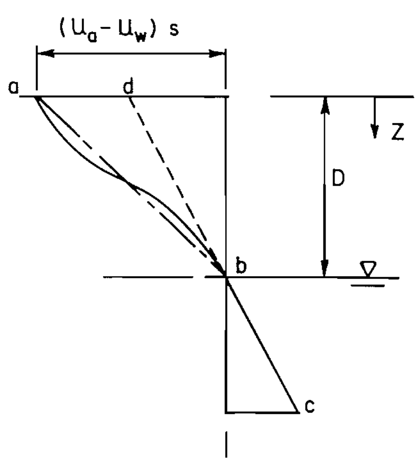


FIG. 6. Typical suction profile.

$$[15] \quad (u_a - u_w)_z = (u_a - u_w)_s \left(1 - \frac{z}{D}\right)$$

The total active pressure above $D$ will be:

$$[16] \quad \sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2}{\sqrt{N_\phi}} \times [c' + (u_a - u_w)_s(1 - z/D) \tan \phi^b]$$

The total active pressure below $D$ will again be given by [8].

The pressure diagrams resulting from [16] and [8] are shown in Fig. 7.

The distance $Z_{tu}$ over which tension apparently exists between the soil and the wall is predicted by [17]:

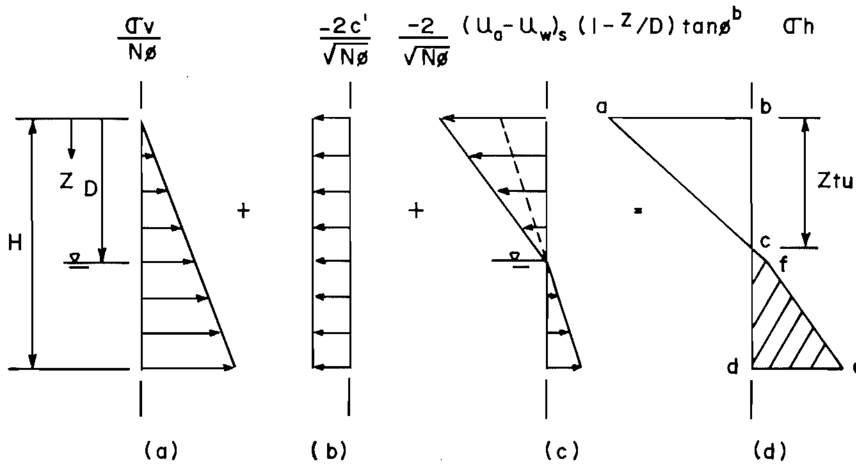


FIG. 7. Active earth pressure (unsaturated).

$$[17]\quad Z_{tu} = \frac{2\sqrt{N_\phi}[c' + (u_a - u_w)_s \tan\phi^b]}{\rho_u g + \dfrac{2\sqrt{N_\phi}}{D}(u_a - u_w)_s \tan\phi^b}; \qquad Z_{tu} < D$$

where $\rho_u$ = unsaturated mass density of the soil. When $Z_{tu} > D$,

$$[18]\quad Z_{tu} = \frac{2\sqrt{N_\phi}[c' + D\rho_w g \tan\phi'] + (\rho_s - \rho_u)gD}{\rho_s g + 2\sqrt{N_\phi}\rho_w g \tan\phi'}$$

The total force on the wall will be due to area cdef. The exact expression for this area is unduly complex and so, for practical purposes, it may be expressed as:

$$[19]\quad P_A = \left[\frac{\rho_u g D}{N_\phi} + \frac{\rho_s g(H-D)}{N_\phi} - \frac{2}{\sqrt{N_\phi}}\{c' - (H-D)\rho_w g \tan\phi'\}\right]\left\{\frac{H - Z_{tu}}{2}\right\}$$

This expression assumes that cfe is linear which is not rigorously correct; however, the error is not considered to be significant.

A comparison of [10] and [18] indicates that, because $Z_{tu}$ is greater than $Z_{ts}$, the magnitude of $P_A$ from [19] will be less than that from [11]. As soil moves from the unsaturated state, $u_w$ approaches $u_a$, [17] reverts to [10], and [19] reverts to [11].

The total passive pressure above depth $D$ will be:

$$[20]\quad \sigma_h = \sigma_v N_\phi + 2\sqrt{N_\phi}[c' + (u_a - u_w)_s(1 - z/D)\tan\phi^b]$$

The total passive pressure below $D$ will again be given by [12]. The pressure distribution is shown in Fig. 8.

$$[21]\quad P_p = N_\phi\left[\frac{\rho_u g D^2}{2} + \rho_u g D(H-D) + \rho_s g(H-D)^2\right] + 2\sqrt{N_\phi}\left[c'H + (u_a - u_w)_s(\tan\phi^b)\frac{D}{2} - \rho_w g \tan\phi' \frac{(H-D)^2}{2}\right]$$

## Earth pressures and tension cracks

The foregoing expressions for lateral earth pressures have assumed that the soil is intact throughout its entire depth. This will rarely be the case and vertical tension cracks either exist or will develop in the soil behind the wall. The following section provides equations for active earth pressures when tension cracks are present in the soil.

Figure 9 shows a retaining wall of height $H$ with tension cracks of depth $Z_c$ and with the water table at a depth $D$ below the surface.

If the soil above the water table and below depth $Z_c$ is assumed to be saturated, [8] may be modified to provide a

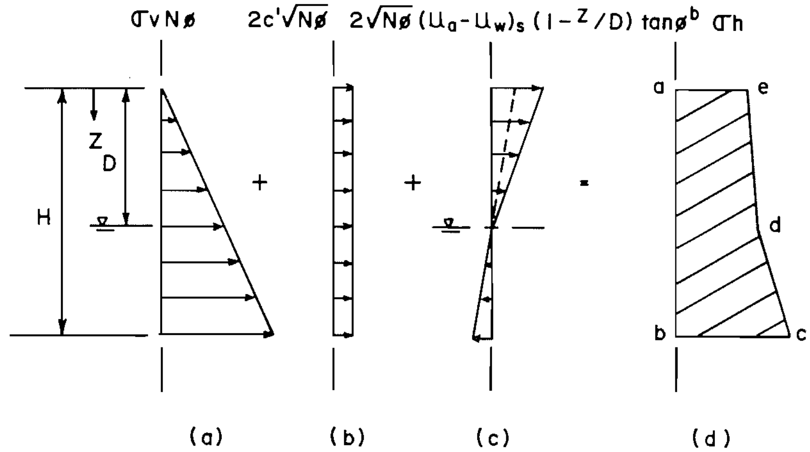


FIG. 8. Passive earth pressure (unsaturated).

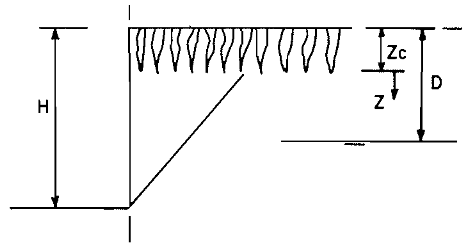


FIG. 9. Retaining wall with tension cracks in the backfill.

measure of the total pressure on the wall. The mass of the soil within depth $Z_c$ can be considered as a surcharge load, $q_s$. The horizontal component of the surcharge on the wall below depth $Z_c$ is $q_s/N_\phi$, where $q_s$ is equal to $\rho_u g Z_c$.

$$[22] \quad \sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2}{\sqrt{N_\phi}}[c' - (z - D)\rho_w g \tan\phi'] + q_s/N_\phi$$

The pressure diagrams from [22] are shown in Fig. 10.

The depth over which tension exists is $Z_{tsc}$, where

$$[23] \quad Z_{tsc} = \frac{2\sqrt{N_\phi}[c' + (D - Z_c)\rho_w g \tan\phi'] - q_s}{(\rho_s - \rho_w)g + \rho_w g N_\phi}$$

The total active force, $P_A$, on the wall is expressed by [24]:

$$[24] \quad P_A = \left[\frac{\rho_s g(H - Z_c)}{N_\phi} - \frac{2}{\sqrt{N_\phi}}\{c' - (H - D)\rho_w g \tan\phi'\} + \frac{q_s}{N_\phi}\right]\left\{\frac{H - Z_c - Z_{tsc}}{2}\right\}$$

When the soil is unsaturated above the water table [16] can be modified to account for the effect of tension cracks.

$$[25] \quad \sigma_h = \frac{\sigma_v}{N_\phi} - \frac{2}{\sqrt{N_\phi}}\left[c' + (u_a - u_w)_s\left(1 - \frac{Z}{D - Z_c}\right)\tan\phi^b\right] + \frac{q_s}{N_\phi}$$

where $(u_a - u_w)_s$ = matric suction at depth $Z_c$ below the surface. The total horizontal active pressure below the water table will be given by [22].

The pressure diagrams associated with [25] are essentially the same as shown in Fig. 10 except for parts $d$ and $e$.

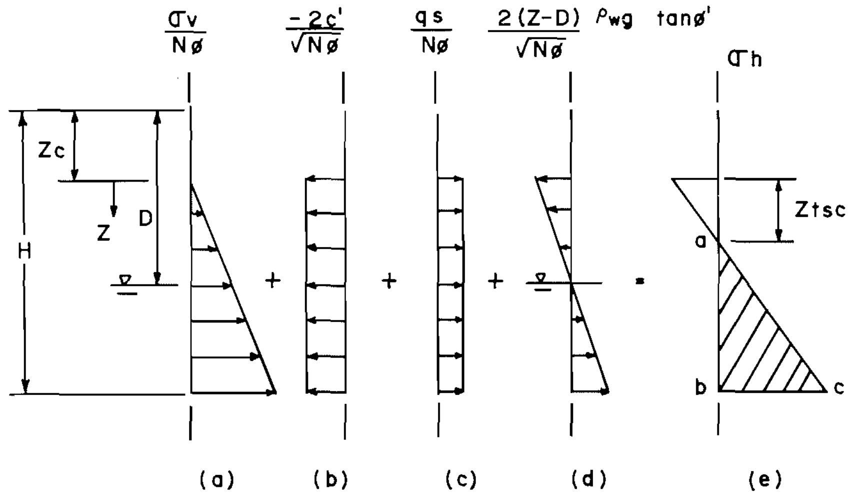


FIG. 10. Active earth pressure with tension cracks.

Figure 10 will be the same as Fig. 7*c* below depth $Z_c$. In this case the depth, $Z_{tuc}$, of tension is:

$$[26]\quad Z_{tuc} = \frac{2\sqrt{N_\phi}[c' + (u_a - u_w)_s \tan\phi^b] - q_s}{\rho_u g + 2\dfrac{\sqrt{N_\phi}}{(D - Z_c)}(u_a - u_w)_s \tan\phi^b}; \qquad Z_{tuc} < D - Z_c$$

When $Z_{tuc} > D - Z_c$,

$$[27]\quad Z_{tuc} = \frac{2\sqrt{N_\phi}[c' + (D - Z_c)\rho_w g \tan\phi'] + (\rho_s - \rho_u)g(D - Z_c) - q_s}{\rho_s g + 2\sqrt{N_\phi}\rho_w g \tan\phi'}$$

The total active force will be approximately equal to:

$$[28]\quad P_A = \left[\frac{\rho_u g(D - Z_c) + \rho_s g(H - D)}{N_\phi} - \frac{2}{\sqrt{N_\phi}}\{c' - (H - D)\rho_w g \tan\phi'\} + \frac{q_s}{N_\phi}\right]\left\{\frac{H - Z_c - Z_{tuc}}{2}\right\}$$

**Changes in pressure due to changes in water table**

Figure 11 provides a quantitative comparison of the lateral earth forces on a typical wall as the water table is lowered from the surface to the base of the wall. It also shows the change in pressure that occurs when the soil is unsaturated and the matric suction at the surface is 200 kPa. The water table is allowed to vary from 3 to 6 m while the matric suction at the surface is held constant. The average saturated density and the average unsaturated density of the soil are assumed to remain constant with decreases in pore-water pressure or increases in matric suction. Details of calculations are shown in the Appendix.

The effect of tension cracks in the backfill behind the wall is also considered for the active case. In this circumstance the suction is measured at the base of the tension crack. In reality the unsaturated soil will always be cracked so that the term unsaturated intact is hypothetical and is used here for comparison purposes only. The presence of tension cracks is insignificant in affecting the total force on the wall. Since the upper portion of the wall that was apparently in tension has been neglected in computing the total force on the wall, placing additional tension cracks in the soil has little effect except to reduce the total mass density of the soil.

Figure 11 shows the maximum active force, consistent with the assumptions involved in the formulation, that can be developed against the wall provided the wall is free to move away from the fill. If the wall is subjected to some restraint the horizontal pressures behind the wall will increase well beyond those shown in the figure.

In general, the magnitude of the lateral pressure developed against a fixed wall will depend upon the ratio, $K_T$, of horizontal to vertical pressures and on the matric suction at the time of placement.

Circle A in Fig. 12 denotes the total vertical and horizontal pressures at some depth *z* for fully active conditions. As the water content increases and the

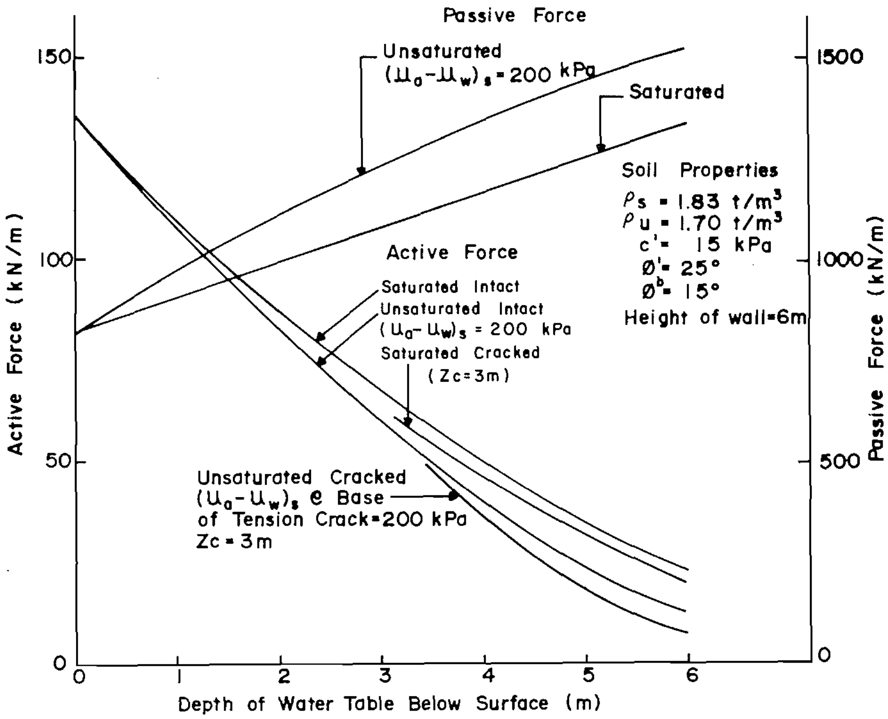


FIG. 11. Active and passive forces on a 6 m wall versus depth of water table (saturated and unsaturated).

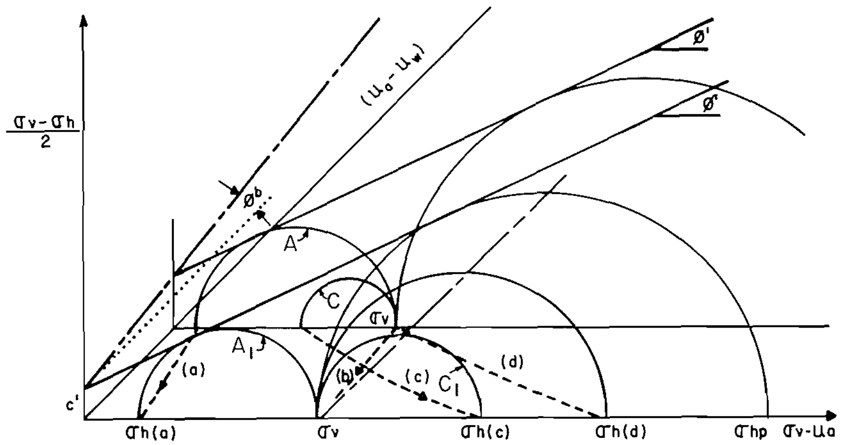


FIG. 12. Lateral pressures on walls subjected to limited movement.

suction decreases under these unrestricted movements, the horizontal pressures will move along stress path a and the vertical pressures along path b to circle $A_1$. The magnitude of the pressure increase in the horizontal direction can be noted in Fig. 11.

On the other hand, if backfill is placed behind a fixed wall at the same matric suction as in A but at a stress ratio $K_T$, as indicated in circle C, the horizontal earth pressure change will result in stress path c and circle $C_1$ will be the result. Similarly, if $K_T$ is equal to 1, then stress path d is proposed. The exact paths would have to be established experimentally and those shown here are only qualitative in nature.

There will undoubtedly be conditions in practice where backfill is placed against rigid walls or occasions when heavy structural members are cast against undisturbed clays with $K_T$ values of 1 or greater. In these circumstances, the horizontal stresses will move to $\sigma_{hp}$, which is the passive resistance of the soil. In these situations either the structural member will fail or the soil will fail in shear.

Figure 13 is a photograph of a compacted clay till

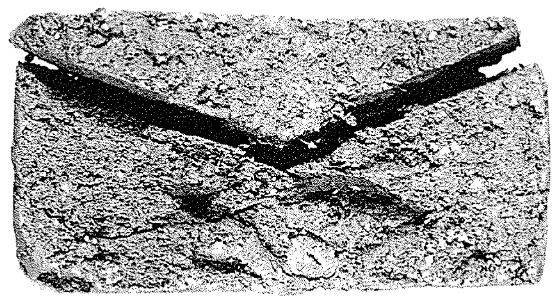

FIG. 13. Passive failure of compacted clay till due to one-dimensional swelling.

sample that was allowed to swell one-dimensionally while confined in a consolidation ring. It is evident that the soil has failed in the passive sense.

If a structural member is designed so that it is not free to move and, if it is expected to perform satisfactorily under the conditions outlined above, it must be designed to resist the full saturated passive earth pressure of the soil.

### Unsupported excavations

Frequently the slopes of temporary cuts or excavations in cohesive soils are allowed to stand unsupported in the vertical or nearly vertical position. If the slope is assumed to fail along a planar surface, the critical height $H_s$ can be determined by considering a balance of forces on the sliding wedge shown in Fig. 14.

$$[29] \quad H_s = \frac{4\sqrt{N_\phi}(c' + D\rho_w g \tan\phi')}{\rho_s g + 2\rho_w g (\tan\phi')\sqrt{N_\phi}}$$

(Note that $H_s$ is equal to $2Z_{ts}$ as computed from [10]).

If the soil is unsaturated above the water table, the maximum unsupported height ($H_u$) may be obtained most readily by summing forces parallel and perpendicular to the failure plane shown in Fig. 15.

$$[30] \quad S_1 = c'l_1 + (P_1 - u_a l_1)\tan\phi' + \frac{(u_a - u_w)_s l_1}{2}\tan\phi^b$$

$$[31] \quad S_2 = c'l_2 + \left[P_2 - \frac{(H_u - D)}{2}\rho_w g l_2\right]\tan\phi'$$

$$[32] \quad H_u = \frac{4\sqrt{N_\phi}\left[c' + \frac{m}{2}(u_a - u_w)_s \tan\phi^b\right]}{(1 - m^2)g[\rho_s + 2\tan\phi'\sqrt{N_\phi}\rho_w] + m(2 - m)\rho_u g}$$

where $m = D/H_u$. Equation [32] requires a trial and error solution because $H_u$ appears on both sides of the equal sign.

Although this is not readily evident, [32] reverts to [29] as $\rho_u$ approaches $\rho_s$ and $u_w$ approaches $u_a$.

### Effect of tension cracks on the unsupported height

If the upper zone of soil has been weakened by tension cracks of depth $Z_c$, the expression for the maximum unsupported height of bank, $H_{sc}$, when the soil is saturated upwards from the water table to the base of the crack and the water table is at a depth $D$ below the surface, may be obtained by summing forces in Fig. 16:

$$[33] \quad H_{sc} = \frac{4\sqrt{N_\phi}[c' + D\rho_w g \tan\phi'] - Z_c[(\rho_u - \rho_s)g + (N_\phi - 1)\rho_w g]}{\rho_s g + 2\rho_w g\sqrt{N_\phi}\tan\phi'}$$

If the soil is unsaturated above the water table, the expression for the critical height, $H_{uc}$, can be obtained by summation of forces shown in Fig. 17.

The expressions for $S_1$ and $S_2$ will again be given by [30] and [31]. However, $P_1$ and $P_2$ will be different. The end result is a rather complex expression for $H_{uc}$, as given in [34]:

$$[34] \quad H_{uc} = \frac{4\sqrt{N_\phi}\left[\left\{(1 - n)c' + \frac{(m - n)}{2}\tan\phi^b(u_a - u_w)_s\right\} - 2Z_c\rho_u g\right]}{(1 - m)^2\rho_s g + [m(2 - m) + n(n - 2)]\rho_u g - 2n^2\rho_u g + 2(1 - m)^2\rho_w g(\tan\phi')\sqrt{N_\phi}}$$

where $m = D/H_{uc}$, $n = Z_c/H$, and $D$ = the distance from the water table to the surface.

Since $H_{uc}$ appears on both sides of the equal sign, [34] again requires a trial and error solution and, because of its complexity, hand calculations are rather tedious. In addition, a variety of conditions can occur to reduce the critical height or the factor of safety for which the slope was designed. For example, if a sudden heavy rainstorm

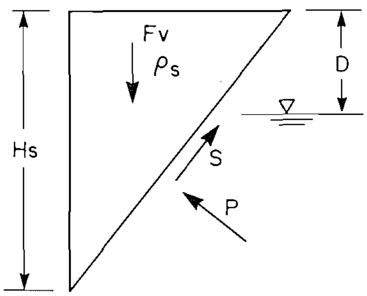


FIG. 14. Unsupported vertical slope (saturated, intact).

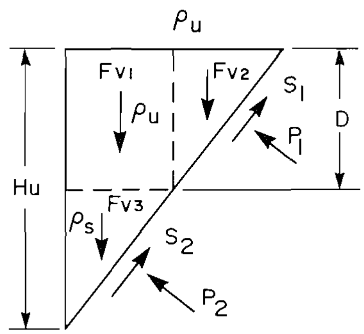


FIG. 15. Unsupported vertical slope (unsaturated, cracked).

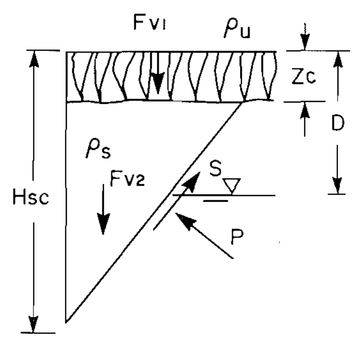


FIG. 16. Unsupported vertical slope (saturated, cracked).

occurs and fills the cracks with water, $\rho_u$ goes to $\rho_s$. In addition, a horizontal water force acting to the left must be included. Moreover, the negative pore-water pressure (or positive suctions) will be reduced throughout the depth, $D - Z_c$. In view of the variety of factors that can influence the critical height of the slope, it is better to formulate the expression in terms of a factor of safety equation and to use a computer program for the solution. This will allow more flexibility in the boundary conditions, the geometry of the problem, and the suction profile.

Figure 18 indicates the relationship between the critical height of an unsupported vertical slope, depth of water table, and depth of tension cracks. It is readily apparent that negative pore-water pressures or positive

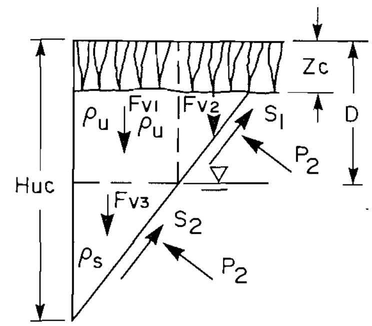


FIG. 17. Unsupported vertical slope (unsaturated, cracked).

matric suctions will increase the critical height substantially. On the other hand, tension cracks substantially reduce the critical height. It is evident that their presence must be considered when designing an open cut in cohesive soils.

## Conclusions

Expressions for active and passive earth pressures have been formulated in terms of total stresses for both the saturated and unsaturated states using the Mohr–Coulomb failure criterion and plastic equilibrium. The assumptions regarding the shape of the failure surface, the lack of wall friction, and the horizontal surface of the backfill are in accordance with the classic Rankine earth pressure theory.

The approach that has been taken is that the total lateral stresses on the wall govern the design and no attempt has been made to separate the stresses in the soil structure (effective stresses) from those in the water phase (neutral stresses). Water pressure or matric suction is considered to affect the magnitude of the apparent cohesion intercept. This is an obvious departure from classic analyses of lateral earth pressures in saturated soils. However, it was adopted here so that direct comparisons could be made between the saturated and unsaturated states. When total active forces were computed any portion of the wall apparently in tension was neglected.

The results indicate (Fig. 11) that, for saturated soils, negative pore-water pressure or, for unsaturated soils, matric suction acts to decrease the total active pressure on retaining walls or to increase the passive resistance on anchors or similar structures. It is apparent that for a given height of wall the total active force on the wall decreases slightly as the depth of tension crack increases.

The stress paths shown in Fig. 12 help to make it somewhat clearer why walls with cohesive backfills may undergo large outward movements during the life of the wall. As the soil dries a crack will open between the soil and the wall. Dust and debris will tend to collect in the opening, thus partially filling it with relatively incom-

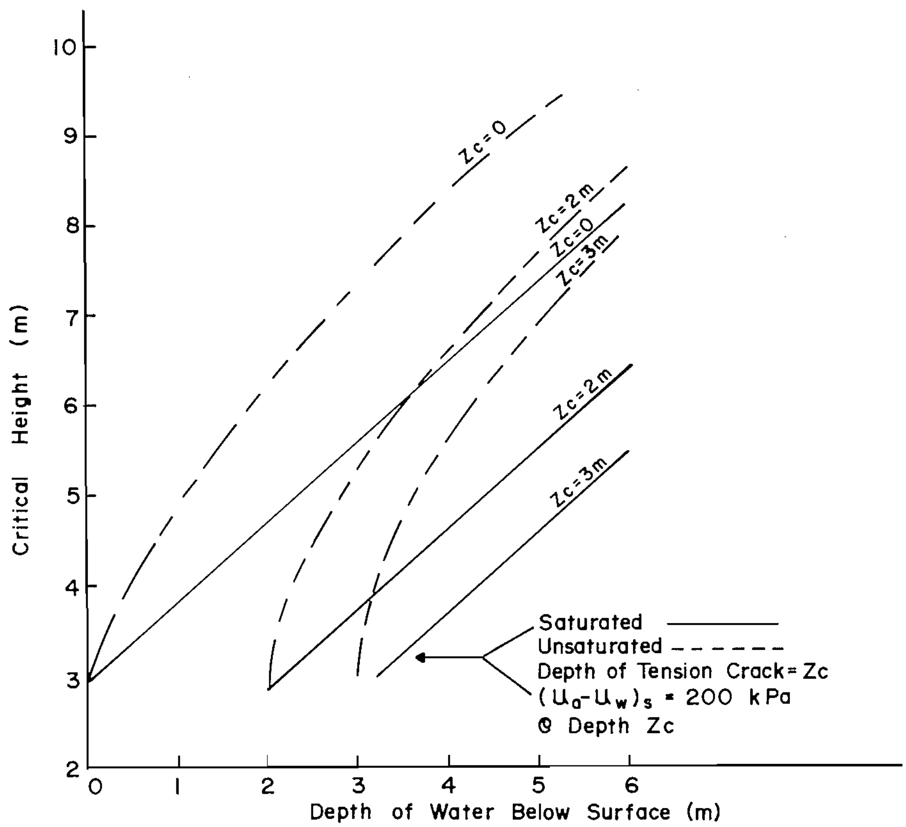


FIG. 18. Critical height of unsupported vertical slope with tension cracks versus depth of water table.

pressible material. As the matric suction decreases during wetter seasons the soil will swell, pushing out the wall until the resistance of the wall is in equilibrium with the total lateral force exerted by the soil. This cycle is repeated each time the soil experiences a decrease and then an increase in water content.

It is also apparent that very large pressures, those approaching the passive condition, will develop when a wall or other structural member is restrained from moving. The pressures developed will depend, for backfill, upon the ratio, $K_T$, of the horizontal to vertical stresses at the time of placement or, for undisturbed soils, on the *in-situ* $K_T$ ratio. If large lateral pressures are to be avoided because of volume changes in the soil, the backfill material should be placed at rather low densities and at water contents that are near optimum. This will result in lower values of $K_T$ and correspondingly lower values of pressure. When structural members are cast against undisturbed expansive clays, a compressible material placed between the structure and the soil will help to relieve some of the pressure generated during the lateral swelling of the clay.

The results in Fig. 18 indicate that the critical height of vertical slopes in cohesive soils will increase substantially as the water table is lowered or as the matric suction is increased. The effect of tension cracks is also displayed in this figure.

Since matric suction is so effective in increasing the critical height of a slope or increasing the safety factor of an existing slope, every attempt should be made to maintain the water content of these soils at a fairly constant value during the time that these excavations are expected to remain open.

Such obvious measures as covering the bank adjacent to the excavation with an impermeable membrane and diverting surface runoff during heavy rains would be beneficial. Tensiometers may be placed in the bank near the base of the tension cracks so that the magnitude of or changes in the suction may be monitored. These changes in suction may then be directly correlated with changes in the factor of safety of the unsupported bank and at least provide a rational means of predicting the continued safety of the excavation.

FREDLUND, D. G., and MORGENSTERN, N. R. 1977. Stress state variables for unsaturated soils. Journal of the Geotechnical Engineering Division, ASCE, **107**(GT5), pp. 447–466.

FREDLUND, D. G., MORGENSTERN, N. R., and WIDGER, R. A. 1978. The shear strength of unsaturated soils. Canadian Geotechnical Journal, **15**, pp. 313–321.
HO, D. Y. F., and FREDLUND, D. G. 1982. Increase in strength due to suction for two Hong Kong soils. Proceedings, Conference on Engineering and Construction in Tropical and Residual Soils, ASCE, Honolulu, pp. 263–295.
IRELAND, H. O. 1964. Design and construction of retaining walls. *In* Soil mechanics lecture series: design of structures to resist earth pressure. American Society of Civil Engineers, Chicago, IL.
MORGENSTERN, N. R., and EISENSTEIN, Z. 1970. Methods of estimating lateral loads and deformations. Proceedings, Lateral Stresses and Earth-retaining Structures, Specialty Conference, ASCE, Cornell University, Ithaca, NY, pp. 51–102.
TERZAGHI, K. 1943. Theoretical soil mechanics. John Wiley and Sons, New York, NY.
TERZAGHI, K., and PECK, R. B. 1967. Soil mechanics in engineering practice. John Wiley and Sons, New York, NY.
THOMPSON, L. J., and TANENBAUM, R. J. 1977. Survey of construction related trench caveins. Journal of the Construction Division, ASCE, **103**(CO3), pp. 501–512.

## Appendix

### *A.1 Retaining wall*

A retaining wall problem is analyzed to describe the use of all equations formulated in this paper for the active and passive forces. A cohesive backfill against a vertical wall of 6 m height is analyzed as shown in Fig. A.1. The calculations presented in this Appendix are only for a depth of water table, $D$, of 4.0 m. Conditions with various depths of water table have also been analyzed; the results are plotted in Fig. 11.

The active forces of an intact backfill can be computed according to [11] and [19] for the saturated and unsaturated conditions. The depth, $Z_{ts}$, of the tensile region in the saturated condition is obtained from [9]; it is $Z_{ts} = 3.24$ m.

The active force, $P_A$, for the saturated condition is computed from [11]. This gives $P_A = 50.1$ kN per metre of wall.

The depth, $Z_{tu}$, of the tensile region in the unsaturated condition is computed from [17]. This gives $Z_{tu} = 3.67$ m.

If the computed $Z_{tu}$ is greater than $D$, then $Z_{tu}$ should be calculated again according to [18]. In this problem, $Z_{tu}$ (i.e., 3.67 m) is less than $D$ (i.e., 4.0 m). The active force, $P_A$, for the unsaturated condition is computed from [19] and is $P_A = 39.9$ kN per metre of wall.

When tension cracks are present in the soil, the active earth forces are calculated using [24] and [28] for the saturated and unsaturated conditions, respectively. In this problem, the tension cracks are assumed to develop down to a depth of 3.0 m. The mass of the cracked soil is

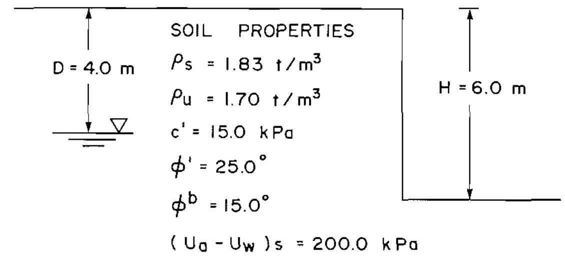


FIG. A.1. Retaining wall problem.

considered as a surcharge load, $q_s$, with a magnitude of $q_s = \rho_{tu} g Z_c = 50.02$ kPa.

The depth, $Z_{tsc}$, over which tension exists for the saturated condition is obtained from [23] and is $Z_{tsc} = 0.35$ m.

The active force of the cracked backfill for the saturated condition is calculated from [24] to be $P_A = 45.9$ kN per metre of wall.

The depth of tension for the unsaturated condition can be computed according to [26]. This gives $Z_{tuc} = 0.89$ m. Since $Z_{tuc}$ (0.89 m) is less than $D - Z_c$ (3.0 m), $Z_{tuc}$ can be substituted directly into [28] to calculate the active force for the unsaturated condition, giving $P_A = 36.0$ kN per metre of wall.

Similar calculations can be performed for conditions with different depths, $D$, to the water table. The computed active forces may then be plotted against depth of water table as shown in Fig. 11.

The passive force of an intact backfill is calculated from [13] for the saturated condition; this gives $P_p = 1164.6$ kN per metre of wall.

For the unsaturated condition, the passive force can be obtained from [21]; it is $P_p = 1438.6$ kN per metre of wall. The passive forces are computed for various depths of water table and plotted against depth of water table in Fig. 11.

### *A.2 Unsupported vertical slope*

An unsupported vertical slope as shown in Fig. A.2 is analyzed for the saturated and unsaturated conditions. In addition, the depths, $Z_c$, of the tension cracks are also

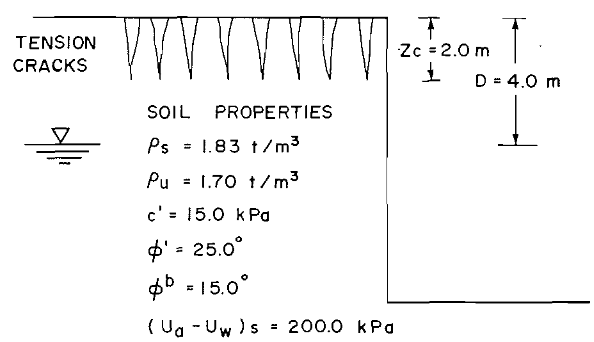


FIG. A.2. Unsupported vertical slope.

varied from 0 to 3.0 m. The calculations presented in this Appendix are only for a depth of water table, $D$, of 4.0 m and depths of tension cracks of 0 and 2.0 m. The results of analyses with various depths of water table are shown in Fig. 18.

The critical height of an intact slope can be computed according to [29] and [32] for the saturated and unsaturated conditions. Substituting the soil properties and the depth of water table, $D$, into [29] gives the critical height for the saturated condition, $H = 6.47$ m.

The critical height for the unsaturated condition is obtained from [32]. Rearranging this nonlinear equation and solving it as a quadratic equation give two values of $H_u$, $H_{u1} = 8.35$ m and $H_{u2} = -3.13$ m. The value of the critical height of the intact backfill, which is unsaturated, is $H_{u1} = 8.35$ m.

When tension cracks are present down to depth 2.0 m, the critical heights are calculated from [33] and [34] for the saturated and unsaturated conditions. Substituting the properties of soil and the depth of water table into [33] gives $H_{sc} = 4.63$ m.

The critical height for the unsaturated condition is given by [34], where: $m = D/H_{uc} = 4.0/H_{uc}$ and $n = Z_c/H = 2.0/6.0 = 1/3$. Again this is a nonlinear equation and solved as a quadratic equation to give $H_{uc} = 6.62$ m.

Similar calculations are carried out for various depths of water table and tension crack. The computed critical heights are plotted against depth of water table in Fig. 18.