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A Multistage Triaxial Test for Unsaturated Soils

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ABSTRACT: A triaxial testing procedure is presented for measuring the increase in shear strength resulting from soil suction in an unsaturated soil. Necessary modifications on a conventional triaxial cell are described. A simple graphical method is presented to interpret the test data in accordance with the shear strength equation for unsaturated soils.

KEYWORDS: soil tests, shear strength, triaxial tests, capillary pressures, pore air pressures, pore water pressures, high air entry disc, diffused air, multistage triaxial test

Triaxial testing is commonly used to quantify the shear strength of a soil, and procedures for such testing of saturated soils are well established [*1*]. However, triaxial testing procedures are needed to quantify the significance of soil suction with respect to an increase in shear strength for an unsaturated soil. The procedure outlined in this paper is applicable to relatively permeable unsaturated soils. If the procedure is applied to soils with low permeability, the length of time required to run the test may become excessive.

The proposed procedure can obtain a maximum amount of information from a limited number of tests and can eliminate the effect of variability in the soil from one test to the next. The test results can be readily applied to slope stability and earth pressure problems involving unsaturated soils [*2*].

The unsaturated soil shear strength theory [*3*] is used as a basis for the interpretation of the test data. The shear strength τ is written in terms of the stress state variables for an unsaturated soil and is an extension of the form of equation used for saturated soils.

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$$

where

c' = effective cohesion,
σ = total stress,
u_a = pore-air pressure,
ϕ' = effective angle of friction,
u_w = pore-water pressure,
$(u_a - u_w)$ = matric suction, and
ϕ^b = friction angle with respect to changes in $(u_a - u_w)$ when $(\sigma - u_a)$ is held constant.

Graphically the equation can be visualized on a three-dimensional plot (a modified Mohr-Coulomb envelope) by using the stress state variables as abscissas (Fig. 1). The equation can also be visualized as a two-dimensional graph with matric suction contoured as the third variable (Fig. 1).

Two unsaturated Hong Kong residual soils were tested to verify the proposed testing procedure. A couple of typical test results are presented in the discussion.

Apparatus

The laboratory testing program involved two conventional triaxial cells modified for unsaturated soil testing (Fig. 2). The plumbing layout for the triaxial apparatus control board is shown in Fig. 3. The modifications on the triaxial cells are as follows.

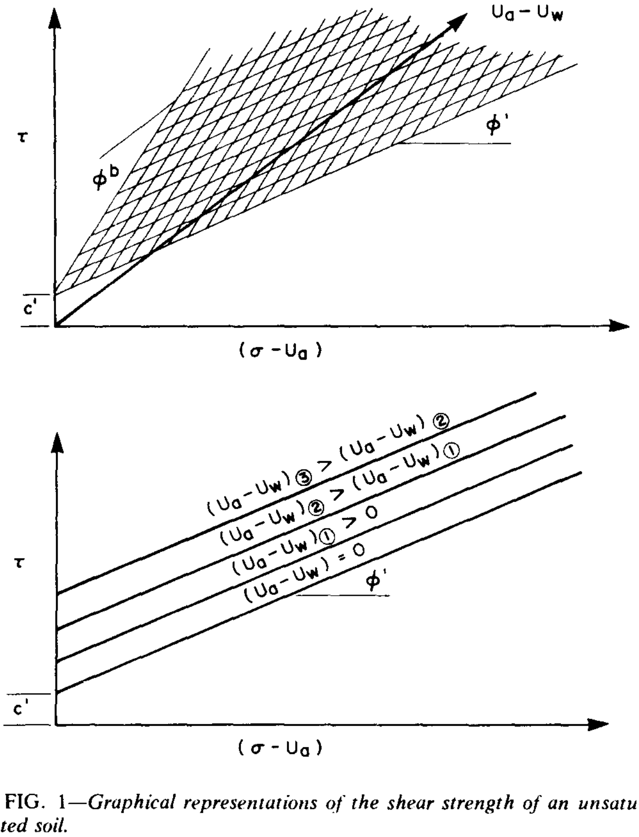

FIG. 1—*Graphical representations of the shear strength of an unsaturated soil.*

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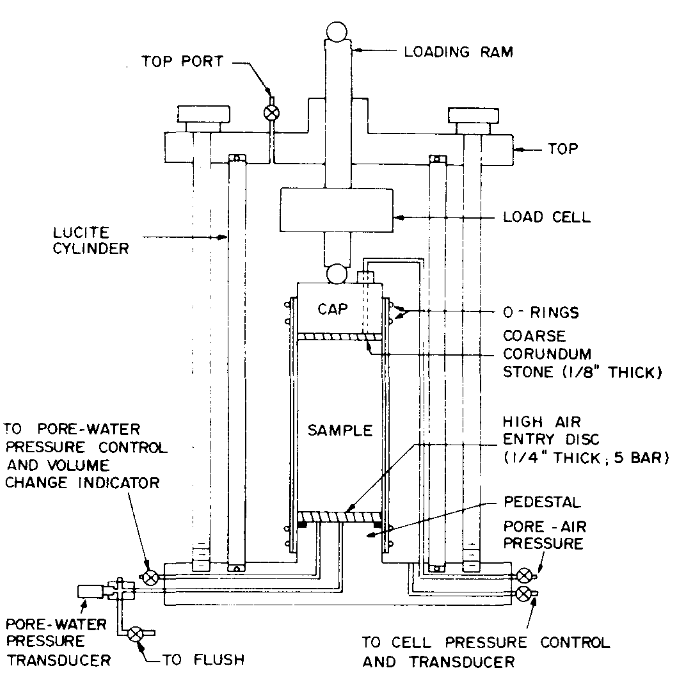


FIG. 2—*Modified triaxial cell for testing unsaturated soils (1 in. = 25.4 mm).*

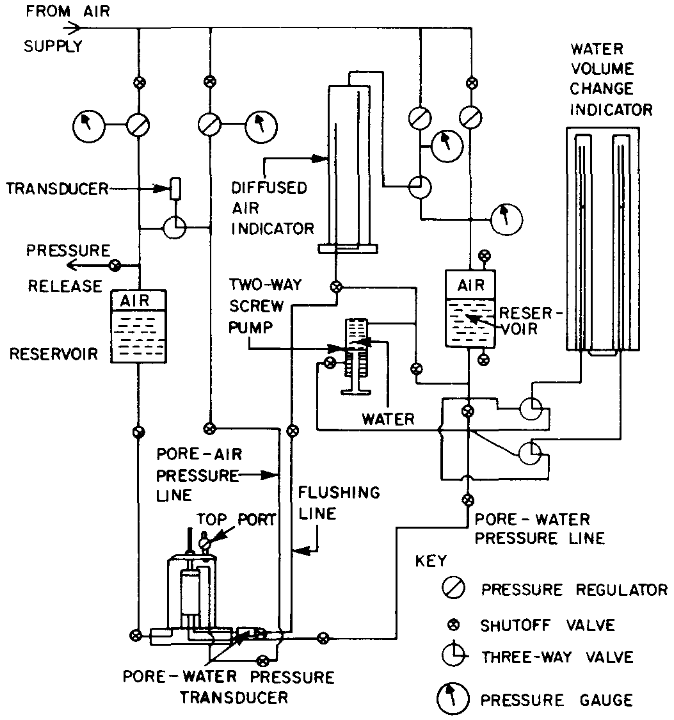


FIG. 3—*Schematic diagram of plumbing layout for triaxial apparatus control board.*

Pore-Water Pressure Control

Some means to measure or control the pore-air and pore-water pressures are necessary when unsaturated soils are tested. A porous, ceramic disc that allows the passage of water but prevents the flow of free air was sealed onto the base pedestal (Fig. 2) of each triaxial cell. The disc has small pores to allow the slow passage of water but resist air flow. Such a disc placed underneath the soil serves to separate the pore-air and pore-water pressures. So long as the difference between the pore-air and pore-water pressure does not exceed the "air entry value" [*4*] of the disc, there is a continuous column of water from the sample to the pore-water pressure transducer below the porous disc and the pore-water pressure can then be independently controlled (or measured). The ceramic discs used in the study blocked the entry of air at pressures below 505 kPa (5 bars) and were 0.635 cm (1/4 in.) thick.

Pore-Air Pressure Control

The addition of a controlled pore-air pressure line to a conventional triaxial cell is essential to control the pore-air pressure in the sample. The air pressure line was connected to a port on the loading cap (Fig. 2). Between the loading cap and the sample, a 0.32-cm (1/8-in.) thick, coarse corundum stone was used as the drainage element for air.

Flushing System

Although the ceramic disc did not allow the passage of free air, dissolved air could have diffused through the water in the disc and collect as free air bubbles at the base of the disc. The free air bubbles in an open system [*1*] will block the passage of water into the sample when the sample dilates or swells and in a closed system will cause misleading measurements of the pore-water pressure and total water volume change.

To cope with the diffused air problem, a flushing system was used. An extra pore-water drainage line with a control valve was connected to the base of the triaxial apparatus (Fig. 2). By closing the valve on the pore-water drainage line leading to the pore-water pressure control and opening the valve on the added drainage line, diffused air accumulated below the disc could be flushed out. For testing in a closed system, the volume of the diffused air can be measured with a diffused air indicator [*5*] (Fig. 3).

Testing Procedure

The testing procedure involved the control of the air and water pressures during the entire test rather than their measurement in a closed system [*1*]. Suction in the sample was maintained constant during the application of the deviator stress. Maintaining the pore-air and pore-water pressure is similar to performing a "slow" or drained test on a saturated soil. The axis-translation technique [*6*] was used to impose suctions greater than 100 kPa (1 atm).

To obtain maximum information from a limited number of tests and to eliminate the effect of soil variability, a multistage testing procedure was attempted [*7,8*]. The test procedure used is outlined as follows.

Reduction of Matric Suction

After the trimmed specimen was mounted in the triaxial cell (Fig. 2), two rubber membranes were placed around the specimen.

O-rings were placed over the membrane on the bottom pedestal. A spacer (two pieces of 3.2-mm [⅛-in.] plastic tubing) was placed between the membranes and the upper loading cap so that air within the sample could escape while water was added to the sample to reduce the suction.

The object of this part of the testing procedure was to relax the suction to a low value before Stage I of loading. The following procedure [9] was adopted for reducing the initial suction of the soil. First, the Lucite® cylinder was put in place around the sample and water was added in the cylinder up to a level about 0.5 cm (0.2 in.) below the top of the sample. The water provided support to the sample while additional water was subsequently added slowly to the sample through the air pressure line connected to the top loading cap. During this stage, the air pressure line was connected to a water reservoir to pass water into the top of the sample. Water could not readily be added to the bottom of the sample because of the low permeability of the ceramic disc and the danger of cracking the disc by upward pressure. The sample was then left for several hours to allow the water to distribute throughout the sample. The saturation process was continued until air could no longer be seen escaping from around the top of the sample (that is, from between the membrane and the top loading cap).

To use this procedure the Lucite cylinder must be in place around the sample while the "top" of the triaxial cell is detached (see Fig. 1). It is then possible to remove the spacer between the membranes and the upper loading cap, once the water has been added to the sample. At this point, the line connected to the top loading cap is disconnected from the water reservoir and connected directly to the air pressure line.

Several other procedures [9] were tried to reduce the initial suction in the sample but were inferior to the described procedure.

Application of Stresses

Once the sample had imbibed water, the top O-rings were placed around the loading cap. The stresses associated with the first stage of testing were applied and the sample was allowed to consolidate. A typical set of stresses for Stages I, II, III are given in Table 1. The associated stress state variables are given in Table 2.

The consolidation process after Stage II is similar to that experienced by a sample placed in a pressure plate apparatus [6]. Once no further water volume change can be detected from the sample, the sample is in equilibrium with the applied stresses.

TABLE 1—*Typical set of stresses for Stages I, II, and III.*

Stage	σ_3,[a] kPa	u_a, kPa	u_w, kPa
I	250	100	50
II	350	200	50
III	500	350	50

[a]Where σ_3 is a minor principal stress.

TABLE 2—*Associated stress state variables.*

Stage	$(\sigma_3 - u_a)$,[a] kPa	$(u_a - u_w)$, kPa
I	150	50
II	150	150
III	150	300

[a]Where σ_3 is a minor principal stress.

Loading

After consolidation is complete, the stresses are maintained while the sample is loaded at a constant strain rate. The choice of strain rate was based on the coefficient of consolidation observed during the consolidation of the sample [9]. A 95% degree of dissipation of the induced pore pressures in the sample was the prime criterion in determining the appropriate strain rates. A typical loading rate was 0.001% strain per minute.

A "cyclic loading" procedure was adopted in testing the sample. The deviator stress was applied until it was apparent that the stress was reaching a peak value. At this point, the vertical load was "backed-off" the sample (Fig. 4). A new set of stresses for Stage II was applied to the specimen, consolidation was again allowed, and the loading process was repeated as before. The procedure was further repeated for Stage III.

An alternate loading process, the "sustained loading" procedure, was attempted. The applied vertical load was sustained when the deviator stress reached a peak value. This procedure was later found to be inferior to "cyclic loading" because of problems of accumulated strain. Details will be presented later in this paper.

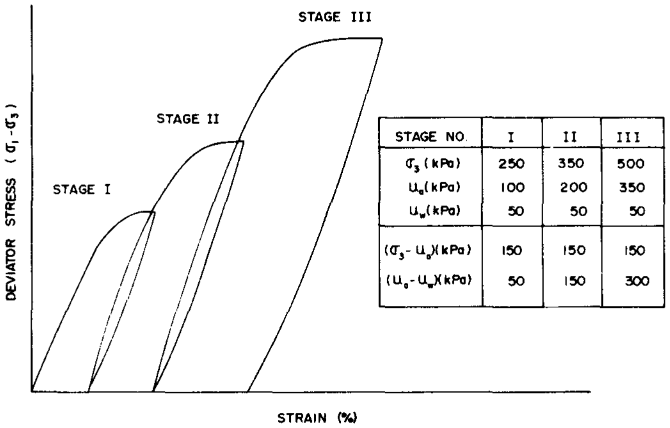
Data Interpretation

Test data can be readily interpreted in accordance with the unsaturated soil shear strength equation [3] with a graphical method. The stress circles corresponding to the failure conditions can be plotted on a two-dimensional graph with matric suction contoured as the third variable.

To interpret the data it is necessary to know the effective angle of friction ϕ'. Conventional triaxial tests were performed by Fugro (Hong Kong), Ltd., on saturated specimens to obtain ϕ'. The angle of friction for the decomposed granite was 33.4°, and the angle is 35.3° for the decomposed rhyolite. The details of these tests are presented in unpublished site investigation reports in the files of Fugro (Hong Kong), Ltd.

The ordinate intercepts (when $[\sigma - u_a]$ is equal to zero) of the various matric suction contours can be plotted versus matric suction to give the friction angle ϕ^b (Fig. 5). Details of a similar but analytical method can be found in other publications [3, 9].

The justification for the linear form of the unsaturated soil shear strength equation has been verified by experimental laboratory tests



STAGE NO.	I	II	III
σ_3 (kPa)	250	350	500
u_a(kPa)	100	200	350
u_w(kPa)	50	50	50
$(\sigma_3 - u_a)$(kPa)	150	150	150
$(u_a - u_w)$(kPa)	50	150	300

FIG. 4—*Ideal stress versus strain curves for a multistage test using the cyclic loading procedure (σ_1 = major principal stress and σ_3 = minor principal stress).*

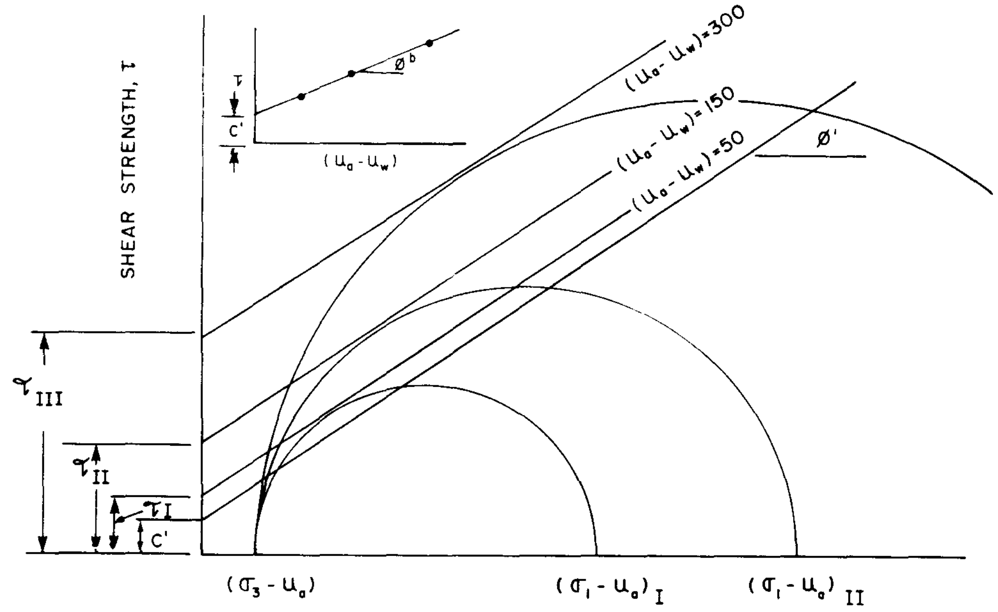
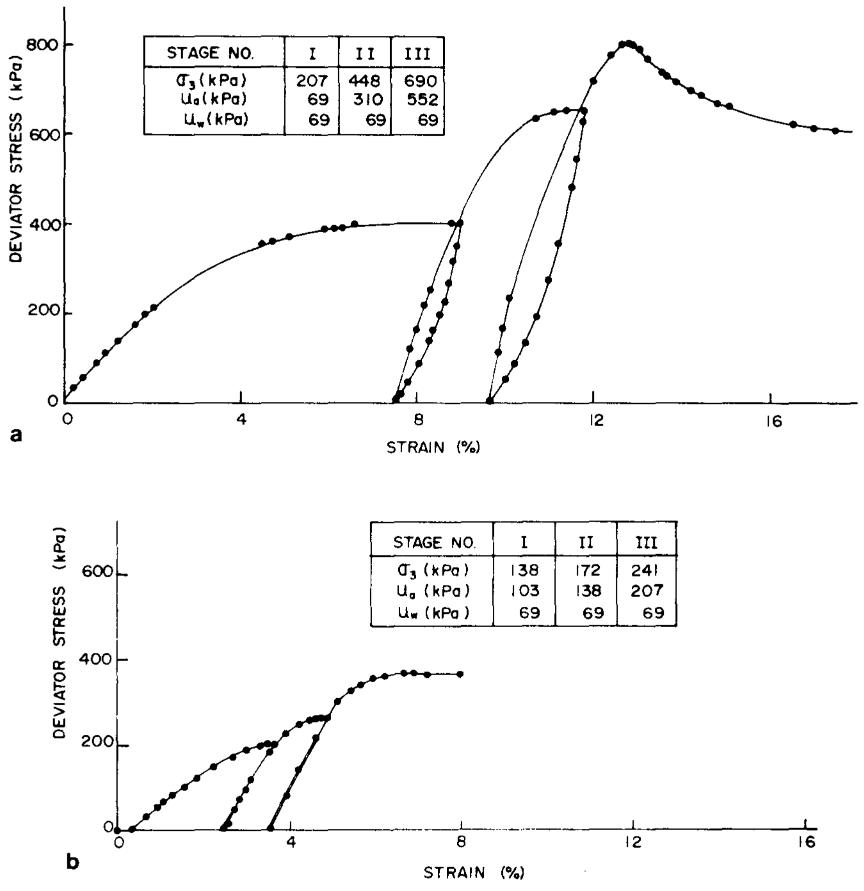


FIG. 5—*Interpretation of a multistage triaxial test to get the angle of friction with respect to matric suction* (*σ_1 = major principal stress and σ_3 = minor principal stress*).



STAGE NO.	I	II	III
σ_3 (kPa)	207	448	690
u_a (kPa)	69	310	552
u_w (kPa)	69	69	69

STAGE NO.	I	II	III
σ_3 (kPa)	138	172	241
u_a (kPa)	103	138	207
u_w (kPa)	69	69	69

FIG. 6—*Stress versus strain curves for decomposed granite:* (a) *Sample 5 and* (b) *Sample 22* (*σ_3 = minor principal stress*).

[*10, 11*]. The results of the reanalyses on published test data showed that the unsaturated soil shear strength theory [*3*] is satisfactory for engineering purposes.

Typical Test Results and Their Interpretation

The two most commonly found Hong Kong residual soils are decomposed granite and rhyolite. Seventeen 6.35-cm (2½-in.) diameter by approximately 13-cm (5-in.) long undisturbed samples were tested, ten of the samples being decomposed granite and seven being decomposed rhyolite. Samples 11B and 11C were decomposed rhyolite from the proposed Thorpe Manor site in Hong Kong. Samples 5, 22, and 16 were decomposed granite from two other sites in Hong Kong. Further details of the soils at these sites can be found in Refs *9* to *11*.

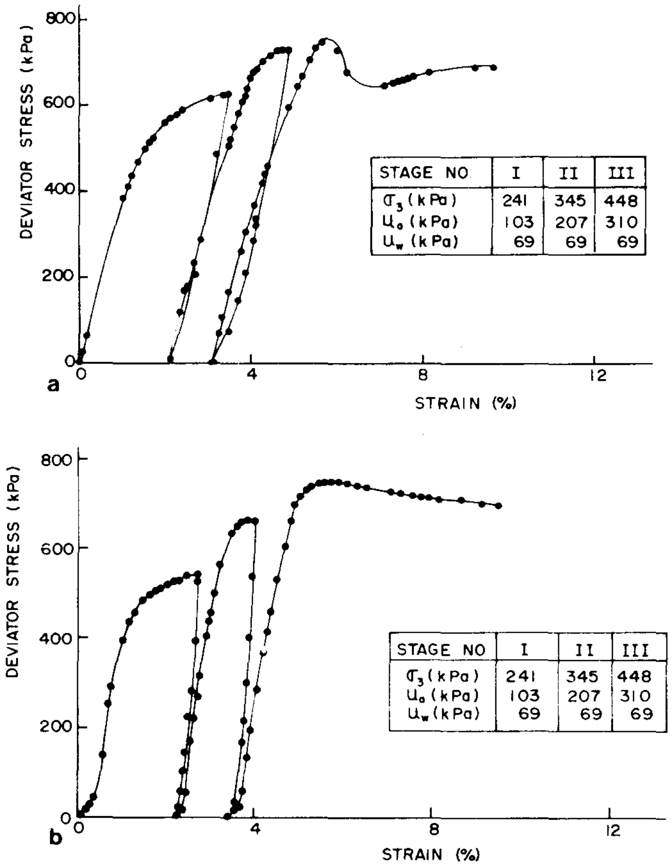
Deviator stress versus strain curves for two decomposed granite and two decomposed rhyolite samples are shown in Figs. 6 and 7. Further test data can be found in Refs *9* and *10*.

In general, the deviator stress of most of the samples dropped off significantly immediately after peaking in Stage III. The samples seemed to show signs of structural breakdown or collapse in the third stage of loading. The drop in strength in Stage III appeared to be related to the amount of strain accumulated while the sample was failing. Problems associated with excessive deformation in multistage triaxial tests have been noted by other researchers [*7, 12*]. It is suggested that the specimen should not be deformed excessively during the earlier stage of loading (that is, past its peak strength). When the sample is overly strained it will tend to develop a shear failure plane and undergo a strength decrease. The measured strength will then tend towards a lower ultimate shear strength. Different soils will have varying sensitivity to the accumulated strain.

The "cyclic loading" procedure is believed to be preferable to "sustained loading" in reducing the amount of strain accumulated in the sample. There are several advantages associated with releasing the applied deviator stress between stages of the test. It prevents further creep of the soil structure, which would occur under sustained loading. Part of the accumulated strain can be restored through elastic recovery (Figs. 6a to 7b). It has also been found that rapid unloading of the sample provides better strain recovery than slow unloading does [*9*]. If the load on the sample is sustained between stages, the sample may continue to deform by creep (Fig. 8). Although a time scale is not shown in Fig. 8, the creep phenomenon is reflected by a drop in deviator stress between the stages of loading.

The test data were analyzed in accordance with the unsaturated soil shear strength theory to establish the relationship between soil suction and shear strength. The unsaturated soil shear strength parameter ϕ^b is used to quantify the effect of soil suction on shear strength. Graphs were used to analyze the data. Plots showing the stress conditions at failure for each stage, as well as the interpretation of data for four of the samples tested, are shown in Figs. 9a to 10b. The relationship between matric suction and shear strength for each sample is also shown on each figure. The average friction angles ϕ^b based on data from all three stages of testing are indicated.

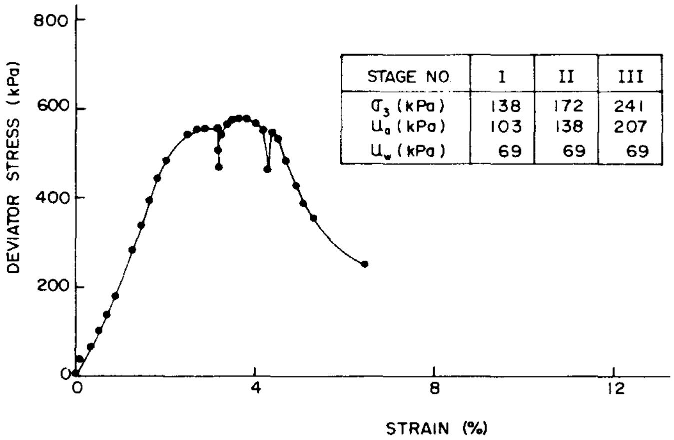
In theory, the ordinate intercepts of the various matric suction contours should fall in a straight line when plotted versus matric suction (Fig. 5). However, as a multistage test progresses, the soil structure of a sample will be disturbed to a certain degree. As a result, the measured peak strength for Stage III may actually be smaller than the peak strength for the sample under the same stress conditions in a single stage test. The loss in strength in the sample resulting from structural disturbance appears to be evident when the projected Stage III strength is compared with the measured Stage III strength (Fig. 11). As a result, a change in the friction angle ϕ^b from Stage I to Stage III can be observed (Fig. 12). An average friction angle ϕ^b based on results from all three stages of testing should give a conservative estimate of the significance of soil suction to the shear strength of an unsaturated soil. The value



STAGE NO	I	II	III
σ_3 (kPa)	241	345	448
u_a (kPa)	103	207	310
u_w (kPa)	69	69	69

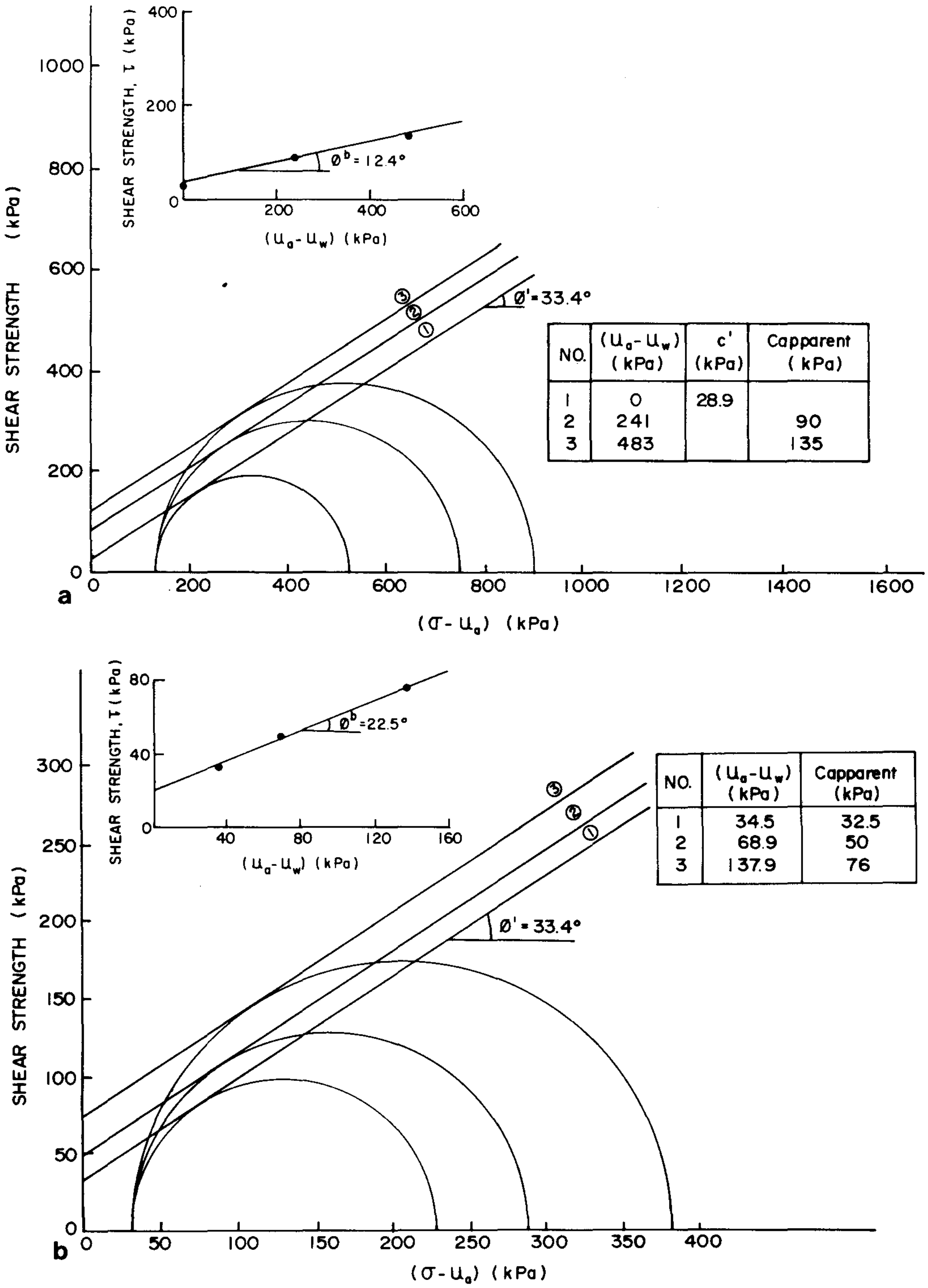
STAGE NO	I	II	III
σ_3 (kPa)	241	345	448
u_a (kPa)	103	207	310
u_w (kPa)	69	69	69

FIG. 7—*Stress versus strain curves for decomposed rhyolite;* (a) *Sample 11B and* (b) *Sample 11C (σ_3 = minor principal stress).*



STAGE NO	I	II	III
σ_3 (kPa)	138	172	241
u_a (kPa)	103	138	207
u_w (kPa)	69	69	69

FIG. 8—*Stress versus strain curve for decomposed granite (Sample 16) (σ_3 = minor principal stress).*



(a)

NO.	$(u_a - u_w)$ (kPa)	c' (kPa)	$c_{apparent}$ (kPa)
1	0	28.9	
2	241		90
3	483		135

(b)

NO.	$(u_a - u_w)$ (kPa)	$c_{apparent}$ (kPa)
1	34.5	32.5
2	68.9	50
3	137.9	76

FIG. 9—*Graphical analysis for decomposed granite:* (a) *Sample 5 and* (b) *Sample 22.*

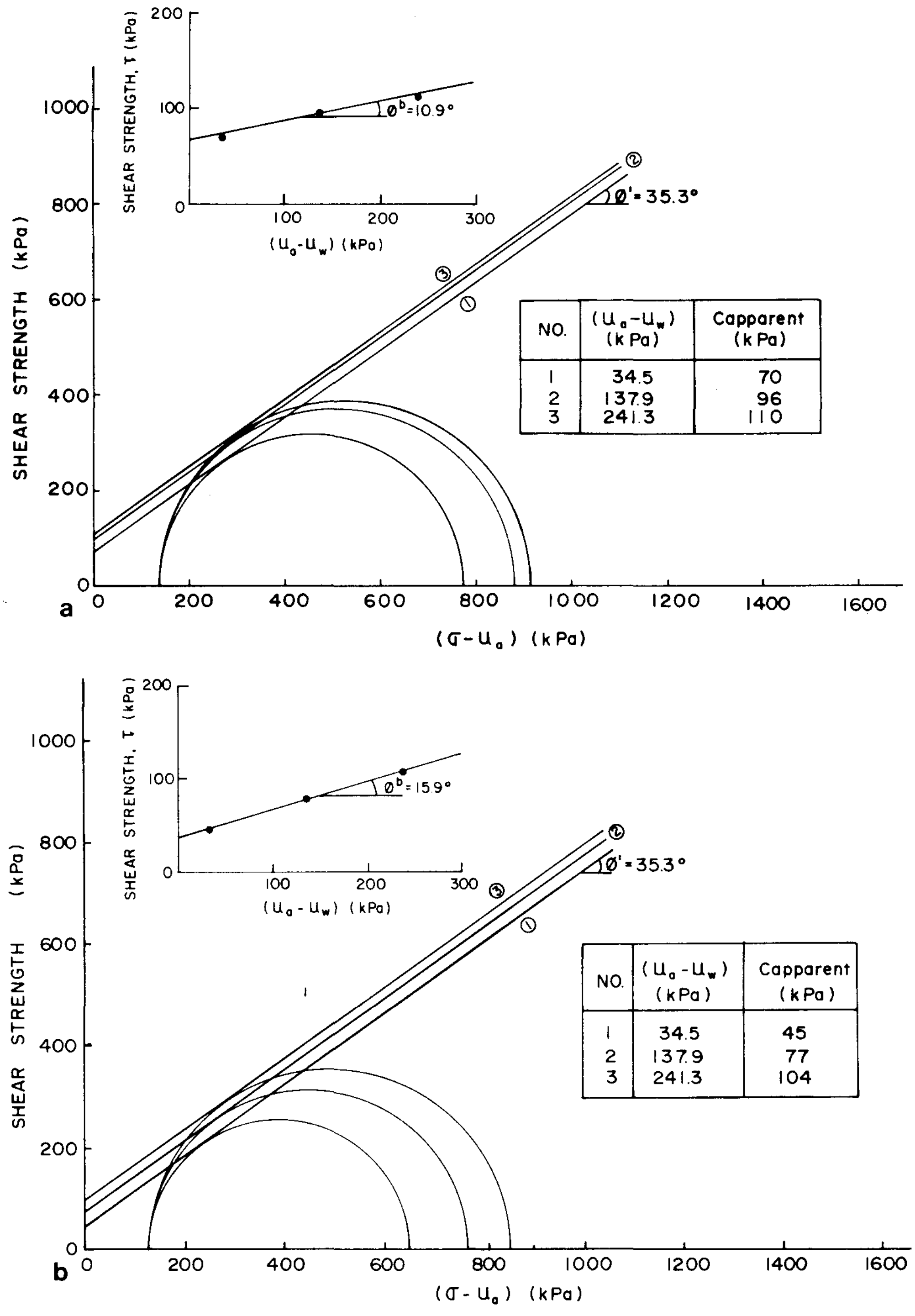


FIG. 10—*Graphical analysis for decomposed rhyolite:* (a) *Sample 11b and* (b) *Sample 11C.*

of the friction angle ϕ^b used in practice should be left to the discretion of the engineer.

Conclusion

A proposed triaxial testing procedure has been presented to quantify the effect of matric suction on the shear strength of an unsaturated soil. The proposed multistage triaxial testing procedure can be used to evaluate the friction angle ϕ^b from a single test.

The matric suction term in the shear strength equation for an unsaturated soil can be considered as contributing to the cohesion of the soil [*2*]:

$$C = c' + (u_a - u_w) \tan \phi^b$$

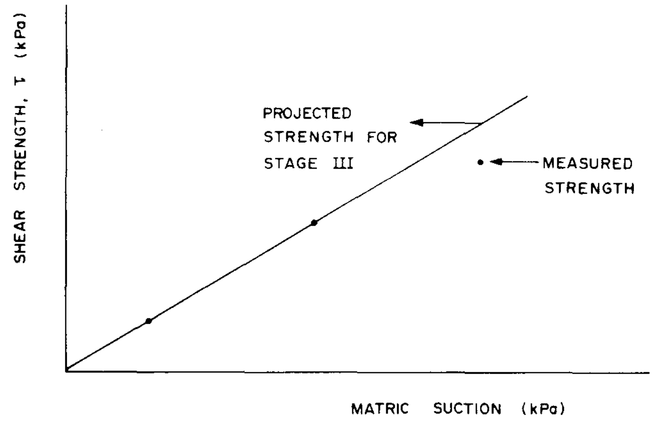


FIG. 11—*Comparison between the projected and measured strength for Stage III testing.*

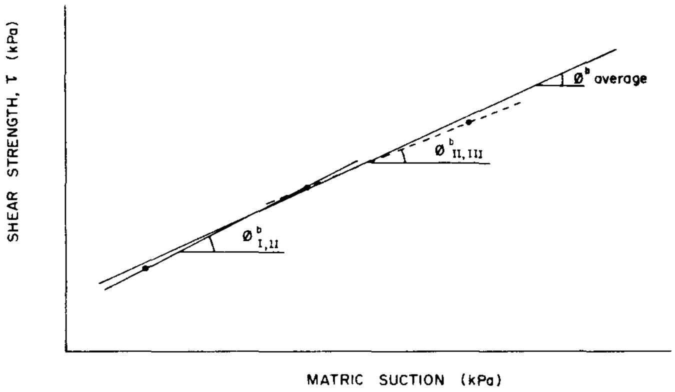


FIG. 12—*The change in ϕ^b in a multistage shear test.*

where C is the total or apparent cohesion of the soil. In other words, the suction in an unsaturated soil increases the cohesion of an unsaturated soil. In this way, conventional saturated soil shear strength concepts can be applied to practical problems involving unsaturated soils.

Acknowledgments

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