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Transient Flow Process In Unsaturated Soils Under Flux Boundary Conditions

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SYNOPSIS A transient flow model for unsaturated soil media has numerous applications in geotechnical engineering. The necessary physical relations are available for a rigorous formulat to describe transient flow process as under varying flux boundary conditions. Three partial differential equations are formulated and simultaneously solved to describe the heat flow, the po water pressure and pore-air pressure distributions within an unsaturated soil under flux boundary conditions. The solutions, in turn, allow for the computation of the volume-mass soil properties with time and space.

INTRODUCTION

The problem of fluid flow through an unsaturated porous media has been recognized as being important not only in geotechnical engineering but also in other related fields such as agricultural engineering, environmental engineering, mining engineering and petroleum engineering. There are a number of common problems where moisture movement in unsaturated soils must be understood in order for geotechnical engineers to provide satisfactory solutions. For example, the vertical ground movements experienced with expansive soils is the result of moisture flow in and out of the soil (14). Engineering structures founded on swelling and shrinking soils undergo significant ground movement and pose a serious problem to engineers around the world. Throughout the Prairie regions of Western Canada the glacio-lacustrine soil is often unsaturated to varying depths for part or all of the year. Construction on these soils often results in distress to the structure as the environmental conditions change with time. As well, recent resource developments within Canada as well as other countries of the world have brought to attention the need for predictive moisture movements associated with toxic mining waste disposal.

The engineering problems where the moisture flow through unsaturated soils play a primary role are difficult to analyse since satisfactory moisture flow formulations, which are coupled with realistic microclimatic boundary conditions, have not been available. The common seepage problems associated with saturated soils generally have either a head or a flux condition specified at the boundary. These boundary conditions can usually be related to a known water table or impervious boundary, respectively. In the case of unsaturated soils, the pore-water pressures are negative in response to the imposed climatic conditions at the ground surface. The microclimate can be characterized in terms of factors such as temperature, relative humidity, wind velocity, infiltration and evaporation or evapotranspiration. These factors must be quantified as boundary conditions and used in conjuction with an analytical formulation which considers moisture movement in both the liqui and vapor phases. An attempt is made in this paper to simulate a flux boundary condition a a step function going from the evaporation o evapotranspiration case to the infiltration case. An example problem is solved to demonstrate the solution of theoretical formulatic developed for the moisture flow in the liquid and vapor phases of an unsaturated soil.

TRANSIENT FLOW THEORY

One dimensional transient flow equations for saturated soils have basically involved differentiating the constitutive equation for the soil structure with respect to time and equating it to the divergence of the velocity of flow of water from an element (17). Two a three-dimensional seepage analyses for saturated soils have been either coupled or uncoupled from the equilibrium requirements. Unfortunately, a similar type of formulation for seepage analysis in unsaturated soils has been lacking. This has been the result of an incomplete understanding of the stress state, constitutive relations and other physical relations for an unsaturated soil.

Constitutive Relations

All the necessary constitutive relations required for the formulation of a moisture fl theory have been previously proposed (11, 12) $(\sigma_y - u_a)$ and $(u_a - u_w)$ are used as the stres state variables for an unsaturated soils wher σ_y - total stress in the y-direction and u_a - pore-air pressure and u_w - pore-water pressur

Darcy's law can be used to describe flow in the water phase (2) and Fick's law can be used to describe flow in the air phase (1). A modified Fick's law (7) can be used to describe the water vapor flow in the soil.

The assumption will be made that the air phase is continuous. This is generally the case when the degree of saturation is less than approximately 85%. It must be recognized that the coefficients of permeabilities with respect to the water and air phases which are functions of the volume-weight properties of the soil (3). Therefore, the permeability coefficients should be modified during the solution of the partial differential equations with respect to time. Since the permeability is a function of the degree of saturation, this leads to an introduction of a so-called 'gravity term' in the water phase equation. The revised water phase equation (10) is presented later.

Under non-isothermal condition the Fourier diffusion equation is used to describe conductive heat transfer in a soil mass. Fredlund (9) and Hasan and Fredlund (15) defined a pore pressure parameter, B_{aw}, to estimate the change in pore-water pressure resulting from a change in pore-air pressure. The change in pore-air pressure of interest in this paper is the result of the thermal gradient imposed on the soil. The B_{aw} pore pressure parameter is a function of the compressibility of the air-water mixture and the compressibility of the soil (9).

Transient Partial Differential Equations

Three partial differential equations are required for a rigorous non-isothermal analysis involving unsaturated soil (i.e., heat flow, water flow and air flow).

Heat Flow Equation

The Fourier diffusion equation expresses the heat flow rate in terms of the thermal conductivity and the temperature gradient. The thermal conductivity and heat capacity values used in an example problem are given in Table 1. (Shown later in text.)

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial y^2} \qquad [1]$$

where: θ = temperature,

α = thermal diffusivity factor = $\lambda / c\rho$

λ = thermal conductivity,

c = heat capacity,

ρ = bulk density

Water Phase Partial Differential Equation

The water phase partial differential equation is derived by considering the total flux of water through an element of unsaturated soil. The net flux of water is the result of imposed hydraulic and vapor pressure (or in other words, relative humidity) gradients at the surface of the boundary of the element. Equating the net flux of water to the time derivative of th water phase constitutive relationship, rearranging and simplifying gives the wate phase partial differential equation (10).

$$\frac{\partial u_w}{\partial t} = C_w \frac{\partial u_a}{\partial t} + c_v^w \frac{\partial^2 u_w}{\partial y^2} + c_{vv}^w \frac{\partial^2 p_v}{\partial y^2} + c_g \frac{\partial}{\partial}$$

where: $C_w = -(1 - m_2^w/m_1^w)/(m_2^w/m_1^w)$. m_1^w is a modulus which relates the volume of water the soil to the $(\sigma - u_a)$ variable on an arithmetic scale. m_2^w is a modulus which relates the volume of water in the soil to $(u_a - u_w)$ variable on an arithmetic scale. C_w is called the interactive constant associated with the water phase equation. Thi equation is further simplified by letting $R_w = m_2^w/m_1^w$. When the soil is saturated, R approaches unity.

$$c_v^w = \frac{1}{R_w} \frac{k_w}{g\rho_w} \frac{1}{m_1^w};$$

the coefficient of consolidation for the water (liquid) phase.

$$c_{vv}^w = \frac{1}{R_w} D_{vap} \left(\frac{\omega^*}{R\theta}\right) \frac{1}{g\rho_{wv}} \frac{1}{m_1^w};$$

the coefficient of consolidation for the water (vapor) phase.

$$c_g = \frac{1}{R_w} \frac{1}{m_1^w};$$

the coefficient associated with gravity term.

k_w = coefficient of permeability wit respect to water phase,

D_{vap} = molecular diffusivity of water in air,

ω^* = molecular weight of water vapor

ρ_w = density of liquid water

ρ_{wv} = density of water vapor, and

g = acceleration due to gravity.

The vapor pressure in equation [2] can be expressed as the product of the saturatior vapor pressure, p_v^s, and the relative humić h (7).

$$p_v = p_v^s h$$

The relative humidity, h, can in turn be expressed as a function of total potential, ϕ, molecular weight of water, specific volume of water vapor, ν, universal gas constant, R, and the absolute temperature, θ (16).

$$h = \exp\left[\frac{\phi\omega^*\nu}{R\theta}\right] \qquad [7]$$

$\phi = [(u_a - u_w) + \pi]$ (i.e., total potential),

π = osmotic suction; assumed to be constant equal to 102 kPa for this study.

Air Phase Partial Differential Equation

The air phase is compressible and flow occurs in response to a pressure gradient (i.e., the gravity term is negligible). The constitutive relationship for the air phase defined the volume of air in the element for any combination of the total, water and air pressure. Quantitatively, the air phase constitutive relationship is equal to the difference between the soil structure constitutive relationship and the water phase constitutive relationship. Equating the net mass flux of dry or moist air through the element to the time derivative of the air phase constitutive relationship, rearranging and simplifying, gives the modified air phase partial differential equation (5).

$$\frac{\partial u_a}{\partial t} = C_a \frac{\partial u_w}{\partial t} + C_\theta \frac{\partial \theta}{\partial t} + c_v^a \frac{\partial^2 u_a}{\partial y^2} \qquad [8]$$

where:

$$C_a = \frac{m_1^a/m_2^a}{(1 - m_2^a/m_1^a) + \dfrac{(1 - S)n}{m_1^a\ (\Delta u_a + u_{atm})}} \qquad [9]$$

m_1^a is a modulus which relates the volume of air in the soil to $(\sigma - u_a)$ variable on an arithmetic scale.

m_2^a is a modulus which relates the volume of air in the soil to the $(u_a - u_w)$ variable on an arithmetic scale.

C_a is the interactive pressure constant associated with the air phase equation. This equation is further simplified by letting $R_a = m_2^a/m_1^a$.

Δu_a = the difference between the pore-air pressure and standard atmospheric conditions.

$$C_\theta = \frac{1}{\theta}\left[\frac{(1-S)n\ (\Delta u_a + u_{atm})}{(1-R_a)(\Delta u_a + u_{atm})m_1^a + (1-S)n}\right]; \qquad [10]$$

the interactive thermal constant associated with the air phase equation.

S = degree of saturation,

n = porosity

$$c_v^a = \frac{DR\theta}{\omega}\left[\frac{1}{(1-R_a)(\Delta u_a + u_{atm})m_1^a + (1-S)n}\right]; \qquad [1$$

the coefficient of consolidation for the air phase.

D = transmission constant having the same units as coefficients of permeability (synonymous with k_a),

ω = molecular weight of liquid water.

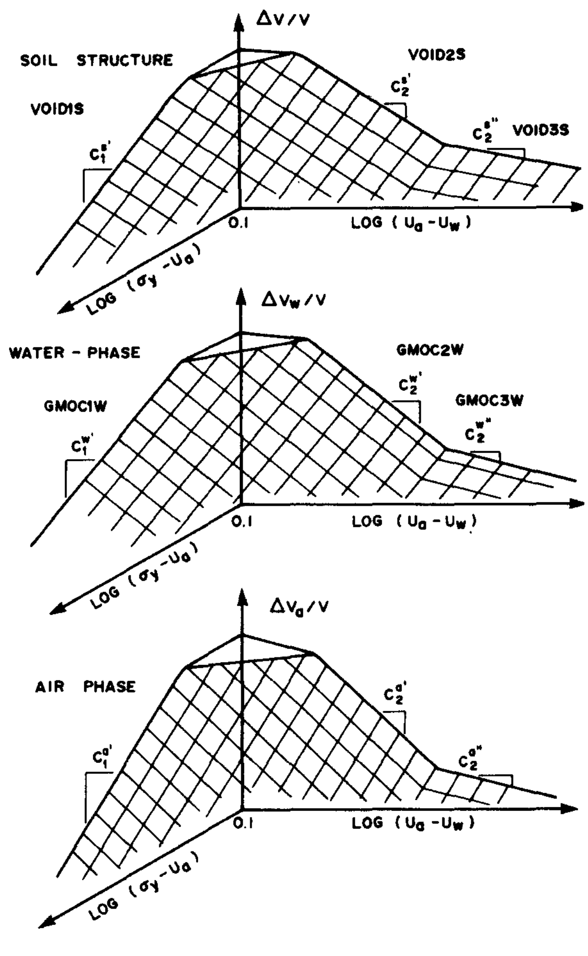

FIGURE 1 NONLINEAR CONSTITUTIVE SURFACES FOR THE VARIOUS PHASES OF AN UNSATURATED SOIL

The dissipation of the excess pressures of the pore-air and pore-water phases are obtained by solving equations [2] and [8] simultaneously. The magnitudes of the coefficients of consolidation for the water and air phases (i.e., c_v^w, c_{vv}^w and c_v^a) are primarily dependent upon the coefficients of permeability for the water and air phases.

The formulation of the above transient flow equations also assume that the volume change moduli (i.e., m_1^w, m_2^w, m_1^a and m_2^a) remain constant during the transient process. This is valid as long as the changes in stress state variables are relatively small. Under flux boundary conditions, the changes in stress state variables are large and the assumption of linear moduli is no longer valid. Therefore non-linear constitutive surfaces are assumed for the various phases of an unsaturated soil as shown in Figure 1. Figure 1 shows the change in i) volume of the overall element (i.e., soil structure), ii) volume of water and iii) volume of air versus $\log(\sigma_y - u_a)$ and $\log(u_a - u_w)$. It should be noted that the volume change versus $\log(u_a - u_w)$ plot is divided arbitrarily into three zones to account for nonlinearity of the constitutive surface.

NUMERICAL SOLUTIONS

The Fourier diffusion heat flow equation [1] is solved prior to solving the water phase and air phase equations using an explicit finite difference method. Figure 2 shows the finite difference designations for the example problem. The solution gives the change in temperature with space and time and must be solved at least one time step in advance of the water and air phase equations. The finite difference form of the heat flow equation is as below:

$$\frac{\theta_{(i,j)} - \theta_{(i,j-1)}}{\Delta t} = \alpha\left[\frac{\theta_{(i+1,j-1)} - 2\theta_{(i,j-1)} + \theta_{(i-1,j-1)}}{\Delta y^2}\right] \quad [12]$$

where: i = array used for depth increments,

and j = array used for time increments.

$$\theta_{(i,j)} = \theta_{(i,j-1)} + \beta_t[\theta_{(i+1,j-1)} - 2\theta_{(i,j-1)} + \theta_{(i-1,j-1)}] \quad [13]$$

where: $\beta_t = \alpha \frac{\Delta t}{\Delta y^2}$

A special numerical procedure is adopted to simultaneously solve the non-linear, water phase (i.e., equation [2]) and air phase partial differential (i.e., equation [8]) equations. These two equations are solved using an explicit forward difference technique. This procedure can be used since the non-linear partial differential equations are transformed into linear partial differenti equations. The following steps are used t solve for the pore-water pressure and the air pressure with time.

Write the transient flow equation for the phase in a finite difference form (equatio [13]).

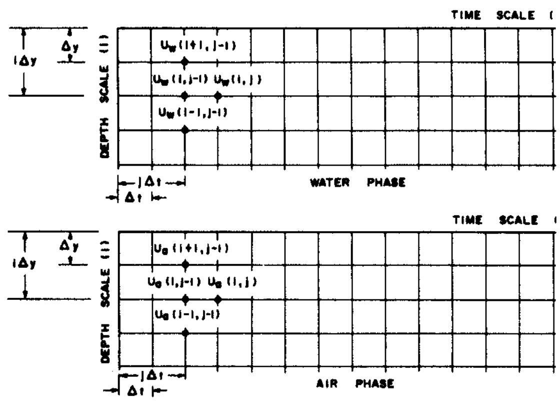

FIGURE 2 FINITE DIFFERENCE MESH FOR THE TRANSIENT FLOW EQUATIONS

$$\frac{u_{w(i,j)} - u_{w(i,j-1)}}{\Delta t} = C_w\left[\frac{u_{a(i,j)} - u_{a(i,j-1)}}{\Delta t}\right]$$

$$+ c_v^w\left[\frac{u_{w(i+1,j-1)} - 2u_{w(i,j-1)} + u_{w(i-1,j-1)}}{\Delta y^2}\right]$$

$$+ c_{vv}^w\left[\frac{p_{v(i+1,j-1)} - 2p_{v(i,j-1)} + p_{v(i-1,j-1)}}{\Delta y^2}\right]$$

$$+ c_g\left[\frac{k_{w(i+1,j-1)} - k_{w(i,j-1)}}{\Delta y}\right]$$

Write the transient flow equation for the phase in a finite difference form (equatio [15]).

$$\frac{u_{a(i,j)} - u_{a(i,j-1)}}{\Delta t} = C_a\left[\frac{u_{w(i,j)} - u_{w(i,}}{\Delta t}\right.$$

$$+ C_\theta\left[\frac{\theta_{(i,j)} - \theta_{(i,j-1)}}{\Delta t}\right]$$

$$+ c_v^a\left[\frac{u_{a(i+1,j-1)} - 2u_{a(i,j-1)} + u_{a(i-1,j-1)}}{\Delta y^2}\right.$$

Multiply equation [15] by C_w.

Solve the equation [14] and [15] simultaneously for pore-water pressure (u_w). Simplify and rearrange the resulting equation [16] such that the unknown pore-water pressure at the given time step (i.e., j^{th}) is on the left hand side and all known variables at the previous time step (i.e., $j-1^{th}$) are on the right hand side of the equation.

$$u_{w(i,j)} = u_{w(i,j-1)} + \frac{\beta_w g_1^w}{(1-C_a C_w)} + \frac{\beta_v p_1^v}{(1-C_a C_w)} + \frac{\beta_g k_1^w}{(1-C_a C_w)} + \beta_a f_1^a \left(\frac{C_w}{1-C_a C_w}\right) + \frac{C_w C_\theta}{(1-C_a C_w)} \theta_1 \qquad [16]$$

Multiply equation [14] by C_a.

Solve equation [14] and [15] simultaneously for the pore-air pressure (u_a). Simplify and rearrange the resulting equation [17] such that unknown pore-air pressure at the given time step (i.e., j^{th}) is on the left hand side and all known variables at the previous time step (i.e., $j-1^{th}$) are on the right hand side of the equation.

$$u_{a(i,j)} = u_{a(i,j-1)} + \left(\frac{C_a}{1-C_a C_w}\right)\beta_w g_1^w + \left(\frac{C_a}{1-C_a C_w}\right)\beta_v p_1^v + \left(\frac{C_a}{1-C_a C_w}\right)\beta_g k_1^w + \frac{\beta_a f_1^a}{(1-C_a C_w)} + \frac{C_\theta}{(1-C_a C_w)} \theta_1 \qquad [17]$$

where:

$$\beta_w = c_v^w\left(\frac{\Delta t}{\Delta y^2}\right), \quad \beta_v = c_{vv}^w\left(\frac{\Delta t}{\Delta y^2}\right), \quad \beta_a = c_v^a\left(\frac{\Delta t}{\Delta y^2}\right)$$

$$\beta_g = c_g\left(\frac{\Delta t}{\Delta y}\right),$$

$$g_1^w = u_{w(i+1,j-1)} - 2u_{w(i,j-1)} + u_{w(i-1,j-1)},$$

$$p_1^v = p_{v(i+1,j-1)} + 2p_{v(i,j-1)} + p_{v(i-1,j-1)},$$

$$f_1^a = u_{a(i+1,j-1)} - 2u_{a(i,j-1)} - u_{a(i-1,j-1)},$$

$$k_1^w = k_{w(i,j-1)} - k_{w(i,j-1)}, \text{ and}$$

$$\theta_1 = [\theta_{(i,j)}/\theta_{(i,j-1)} - 1]$$

Compute the pore-water and the pore-air pressures at the given time step from the known values at the previous time step.

For the non-isothermal case, compute the change in the pore-air pressure, Δu_a, between the two consecutive time steps (i.e., j^{th} and $j-1^{th}$).

The effect of a thermal gradient on the pore-water pressure, Δu_w, taken into account using the pore-pressure parameter, B_{aw} (i.e., $\Delta u_w = \Delta u_a/B_{aw}$). Correct the pore-water pressure, u at the given time (i.e., j^{th}) for increase in the pore-water pressure due to the thermal gradient.

When computations for both pore-water, pore-a pressures and degree of saturation of all dep steps are completed, revise the coefficients permeability for the water phase and the air phase using Corey's (3) relationship as below

$$k_{w(i,j)} = k_{ws}\left[\frac{S_{(i,j)}/100 - S_r}{1.0 - S_r}\right]^4 \qquad [1$$

$$k_{a(i,j)} = k_{as}\left[\frac{S_a - S_{(i,j)}/100}{S_a - S_r}\right]^4 \qquad [1$$

where:

- k_{ws} = water permeability of the soil at saturation
- k_a = coefficient of permeability of air phase
- k_{as} = air permeability of the soil at saturation
- S_a = limiting degree of saturation in decimal at which the air voids are continuous (e.g., 0.88)
- S_r = residual degree of saturation in decimal (e.g., 0.42)

Then march forward to the next time step. Repeat the above procedure up to the desired time interval. The solution of all the above example problems are obtained by setting the values of β_t, β_w, β_v and β_a terms ≤ 0.5 in order to satisfy the stability conditions of the partial differential equations.

TRANSLATION OF PORE PRESSURES TO CHANGES IN SOIL PROPERTIES

The solution of the three partial transient flow equations [i.e., 1, 2 and 8] give the temperature, pore-pressures and vapor pressur distribution with space and time in an unsatu rated soil, under complex environmental chang at the boundary. The pressure changes are, i turn, used to determine the change in void ratio, degree of saturation and moisture content. The volume change of an unsaturated soil can be converted into a prediction of swell or shrinkage by substituting the change in stress state variables, for a give period of time, into the respective constitutive relations.

Void Ratio (Swell or Shrinkage)

The change in volume of the element computed is equal to the change in void ratio, Δe. The total volume of the element, V, can be written as $(1 + e)$. The void ratio at various times during the transient process can be written:

$$e = e_i + \Delta e \qquad [20]$$

where: e = void ratio at any time,

e_i = void ratio at the beginning of the transient process, and

Δe = change in void ratio during the transient process.

The change in void ratio enables the computation of the amount of swell or shrinkage, ΔH, during the transient process.

$$\Delta H = H_i \left[\frac{\Delta e}{1 + e_i}\right] \qquad [21]$$

where: H_i = initial thickness of soil layer.

Water Content

The change in volume of water in a unit volume of soil can also be computed by substituting the change in stress state variables at any time, into the water phase constitutive relation. The gravimetric water content at any time during the transient process can then be written:

$$w = w_i + \Delta V_w \frac{\rho_w}{\rho_d} \qquad [22]$$

where: w = water content at any time,

w_i = water content (gravimetric) at the beginning of the transient process,

ΔV_w = change in volume of water, and

ρ_d = dry density of the soil.

Degree of Saturation

The degree of saturation is calculated using the following volume-weight relationship.

$$Se = wG_s \qquad [23]$$

where: G_s = specific gravity of the soil solids.

EXAMPLE PROBLEM

An example problem is used herein to demonstrate the solution of all the three partial differential transient flow equations [i.e., 1, 2 and 8] under flux boundary conditions. The example problem uses a non-linear constitutive surface and varying coefficients of permeability for the water and air phases in accordance with the equations [18] and [19].

The Mateer method (13) is generally used 1 compute the total evaporation or evapotra1 piration. This method is based on the en(budget approach and is applicable to Cana(conditions. Total evaporation or evapotr piration and infiltration values used in example problem are:

Evaporation (E) = 0.31 mm/day
Infiltration (I) = 0.26 mm/day

A computer program was written to calcula the total evaporation or infiltration. B on different input weather data, the prog computes the total loss (in the case of e ration) or gain (in the case of infiltrat of soil moisture in mm/day. The weather includes soil temperature, mean air tempe ture, percentage of possible sunshine, wi velocity, extraterrestial solar radiation

Flux Boundary Condition

A change in the microclimatic conditions the ground surface disturbs the equilibri condition in an unsaturated soil strata i following order.

i) a loss or gain in soil moisture and a negative or positive pore-water pressure gradient, and

ii) a decrease or increase in pore-water pressure at the ground surface.

As a result of change in pore-water press at the ground surface, the boundary condi require to solve the transient flow equat [i.e., 2 and 8] keep continuously changin with time. The revised pore-water pressu for each time increment is computed as fo

$$\text{Flux} = k_w i_w \quad \text{unit area}$$

$$\text{Flux} = \frac{k_w}{g\rho_w} \frac{\Delta u_{ws}}{\Delta y} \quad \text{unit area}$$

$$\Delta u_{ws}/\text{unit area} = \frac{\text{Flux}}{k_w} g\rho_w \Delta y$$

where: Δu_{ws} = change in pore-water press at the ground surface as a result of imposed flux bou condition. The value may for various time increment depending upon the microclimatic condition.

Flux can either be evaporation or evapotr piration (E) or infiltration (I).

Accordingly, the revised pore-water press at the next time increment will be equal

$$u_{w(0,2)} = u_{w(0,1)} \pm \Delta u_{ws}$$

Figure 3(a) and 3(b) show a schematic representation of the pore-water response under a flux boundary condition. Before commencement of either evaporation or infiltration process (at the ground surface), the negative pore-water pressure is assumed to vary linearly with depth and is shown in Figure 3(a) as $u_{w(0)}$.

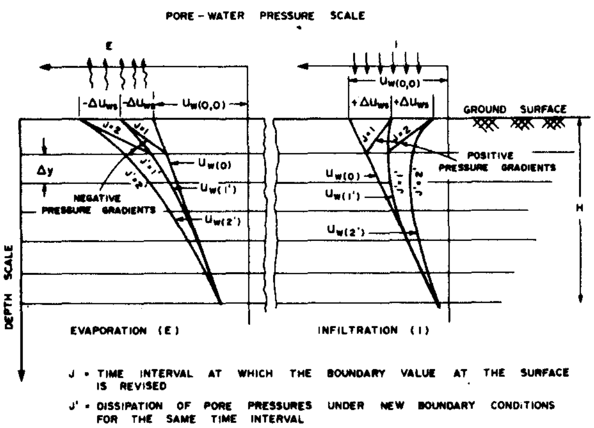

FIGURE 3(a) SCHEMATIC REPRESENTATION OF PORE-WATER PRESSURE CHANGE UNDER FLUX BOUNDARY CONDITIONS

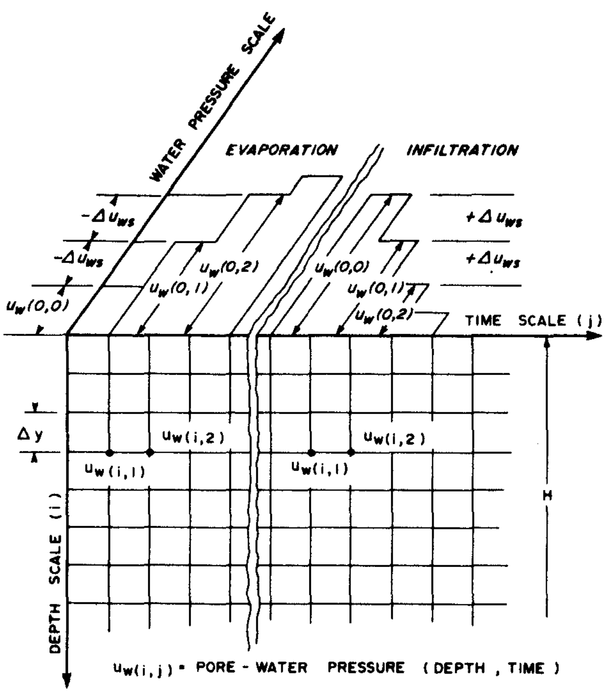

FIGURE 3(b) SCHEMATIC REPRESENTATION OF PORE-WATER PRESSURE CHANGE UNDER FLUX BOUNDARY CONDITIONS

The clay layer of thickness, H, is divided i a number of discrete layers of thickness, Δy As a result of flux boundary conditions, the pore-water pressure is continuously changing at the ground surface. Figure 3(a) shows a negative pressure gradient under evaporation (i.e., loss of soil moisture) and a positive gradient under infiltration (i.e., gain of s moisture). The revised pore-water pressure the ground surface is taken as the new bound pore-water pressure condition to solve the transient flow equations [i.e., 3 and 9] for the same time interval. The solution gives pore-water dissipation throughout the soil m and is shown schematically as $u_{w(2')}$ in Figu 3(a). Figure 3(b) shows a three-dimensional plot of pore-water pressure change at the ground surface as a result of either evaporation or infiltration periods.

In the example problem solved, a flux bounda condition comprising a step function of evap ration or evapotranspiration (0 - 6 hours), infiltration (6 - 12 hours) and evaporation evapotranspiration (12 - 24 hours) periods i day, are considered. The imposed flux bound condition is graphically represented in Figu 4.

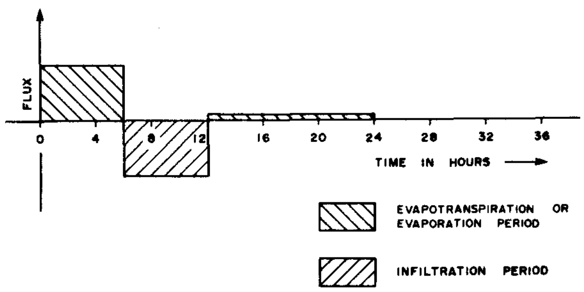

FIGURE 4 SCHEMATIC REPRESENTATION OF EVAPOTRANSPIRATION OR EVAPORATION AND INFILTRATION PERIODS

Figure 5 shows the flow chart of the compute program developed to solve flux boundary condition problems (6). The program initiali the boundary values of temperature, pore-wat pressure, pore-air pressure, degree of saturation, void ratio, water content, perme bility values of water and air, soil moduli, and time counter. The program scans the tim function of evapotranspiration/infiltration computes new values of pore-water pressure gradient at the ground surface (either positi or negative). The pore-pressures for the fi time step and all depth steps are then comput For each time step, the heat flow, water pha and air phase transient flow equations are solved and the new values of temperature, po pressures, degree of saturation, void ratio water content are computed for all the depth steps. The coefficients of permeabilities a revised before the next time step is conside The computer program prints out the output o logarithm of time basis.

TABLE 1:

SUMMARY OF CLASSIFICATION TESTS, COMPRESSIBILITY

AND PERMEABILITIES OF REGINA CLAY

Specific Gravity 2.8

Atterberg Limits

Liquid Limit	75.9
Plastic Limit	24.9
Shrinkage Limit	13.1
Plasticity Index	50.6

Grain Size

% Sand Sizes	8
% Silt Sizes	41
% Clay Sizes	51

Compressibility

VOID1S**	(Slope of void ratio versus $\log(\sigma_y - u_a)$ plot)	=	0.3
VOID2S	(Slope of void ratio versus $\log(u_a - u_w)$ plot)	=	0.3
VOID3S	(Slope of void ratio versus $\log(u_a - u_w)$ plot---range greater than 1100 kPa)	=	0.0
GMOC1W	(Slope of gravimetric moisture content versus $\log(\sigma_y - u_a)$ plot)	=	0.1
GMOC2W	(Slope of gravimetric moisture content versus $\log(u_a - u_w)$ plot)	=	0.0
GMOC3W	(Slope of gravimetric moisture content versus $\log(u_a - u_w)$ plot ---range greater than 1100 kPa)	=	0.0

$R_w = 0.7$, $R_a = -0.01$

Permeability**

k (saturated) = 1.0×10^{-9} m/sec

D_{vap} = 1.0×10^{-8} m^2/sec

Volume-Weight Properties

Bulk Density, ρ = 1.8 gm/cc

Porosity, n = 50%

Degree of Saturation, S = 70%

Thermal Properties

λ = Thermal Conductivity = 0.4574 cal/m sec°K

c = Heat Capacity = 30 cal/gm°K

Physico Chemical Properties

ω^* = Molecular weight of water vapor	=	18.015 gm/
ω = Molecular weight of moist air (80% of water vapor and 20% of air)	=	20.205 gm/
R = Universal gas constant	=	847.83 gm m/mol

** Estimated values from constant volume oedometer tests

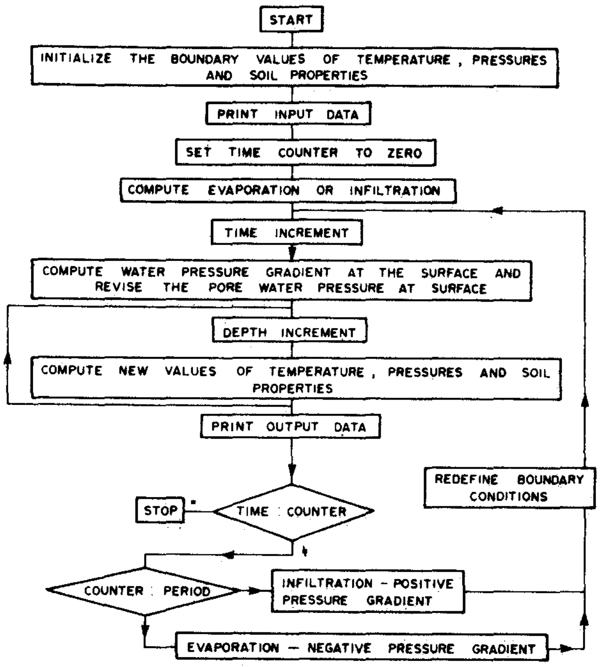

FIGURE 5 FLOW CHART SHOWING THE TRANSIENT FLOW ANALYSIS

One example problem is solved in this paper to demonstrate the solution to transient flow equation formulated under a flux boundary condition which in reality simulates a micro-climatic condition in the field. In an example problem solved, the soil properties used are typical of the clay from Regina, Saskatchewan, Canada, (8) (Table 1). For this study a one-metre thick layer of unsaturated Regina clay is considered. An isothermal flux boundary condition is assumed.

It is assumed that the subsoil system was initially in a known state of stress. The initial suction profile was assumed to vary linearly with depth. The pore-water pressure was -200 kPa at the ground surface. The pore-air pressure throughout the one-metre depth was atmospheric (i.e., 102 kPa). A microclimatic change was imposed at the boundary of the one-metre thick unsaturated soil layer.

RESULTS AND DISCUSSIONS

Environmental changes such as evaporation or evapotranspiration or infiltration create a change in initial pore-water pressure at the ground surface boundary. The unsaturated soil mass will eventually equilibrate to the new boundary conditions.

The solutions of all three conditions are presented by a family of curves in Figures 6 and 7. These figures show the distribution oi pore-water pressure and gravimetric moisture content with space and time.

Figure 6 shows the pore-water pressure distribution throughout the clay layer as the result of imposed flux boundary conditions at the surface. As the result of the evaporation or evapotranspiration period (i.e., 0 - 6 hours), the pore-water pressure decreased to a value c -735 kPa and subsequently increased to a valuє of -270 kPa at the end of the infiltration period (i.e., 6 - 12 hours). For the time period of 12 to 24 hours, the pore-water pressure again decreased to a value of -370 kI as the result of reduced evapotranspiration period. Twenty-four hours and thereafter, there was neither evapotranspiration nor infiltration. The final pore-water pressure (i.e., -370 kPa) dissipates throughout the soil mass with time and the soil system attair the original (i.e., -200 kPa at the ground surface) state of equilibrium condition at thє end of transient process. It is of interest t note that Figure 6 shows clearly the change ir pore-water pressure gradient from a positive gradient, at the end of infiltration period (i.e., 12 hours) to a negative gradient, at tł beginning of evaporation - second cycle perioć (i.e., 12 - 24 hours). Figure 6 demonstrates the way the mathematical model (developed) handles the flux boundary imposed at the surface boundary.

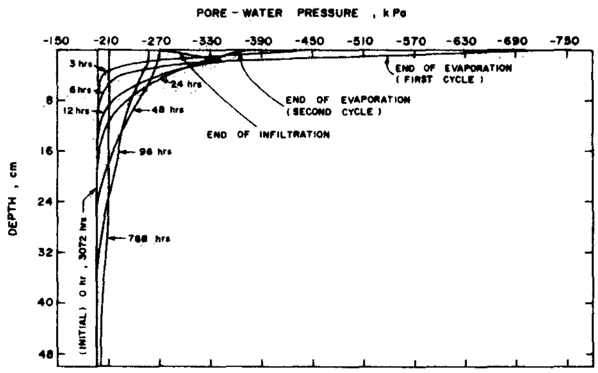

FIGURE 6 PORE-WATER PRESSURE DISTRIBUTION FOR AN EXAMPLE PROBLEM

Figure 7 shows the gravimetric water content distribution throughout the soil layer with space and time. The unsaturated soil was in a state of equilibrium condition (i.e., 24.50) before commencement of either evaporation or infiltration processes. As a result of 6 hour of continuous evaporation or evapotranspiratio the water content at the surface was reduced t 23.39% and increased again to 24.28% at the en of infiltration period. The second cycle of (reduced) evaporation or evapotranspiration caused the water content to reduce to a value of 24.02%. Twenty-four hours and thereafter, there is no pore-water pressure gradient at the surface, therefore the entire soil system tends to reach an original equilibrium state a shown in Figure 7.

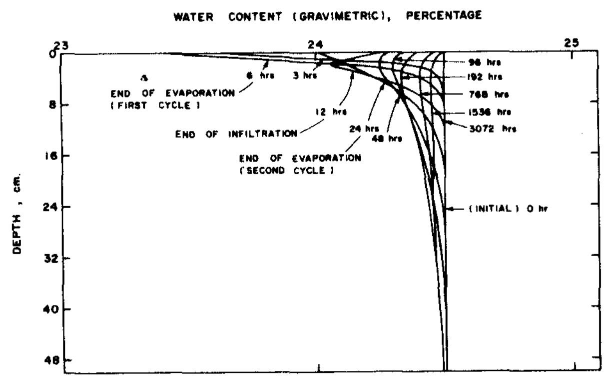

FIGURE 7 WATER CONTENT (GRAVIMETRIC) DISTRIBUTION FOR AN EXAMPLE PROBLEM

Figure 8 shows the total ground movement predicted versus log time. The total ground movement can either be heaving (i.e., + ve) or shrinking depending upon the imposed flux boundary condition. However, in the example problem solved as a result of evaporation, there is a decrease in water content value from its initial equilibrium value (i.e., 24.50). The reduction in water content will eventually result in total shrinking of the ground, as shown in Figure 8.

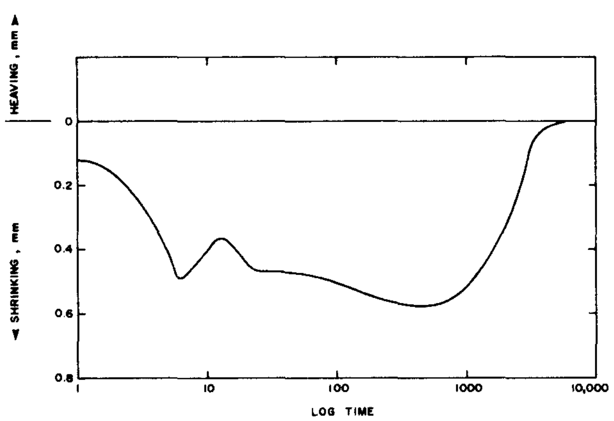

FIGURE 8 TOTAL GROUND MOVEMENT VERSUS LOG TIME (ENLARGED SCALE)

SUMMARY

The transient flow problem in unsaturated soil media is recognized as very important in geotechnical engineering. The necessary physical relations are available for a rigorous formulation to describe the transient flow process under varying flux boundary conditions. The heat flow equation is first solved using a forward finite difference technique. The partial differential equations (i.e., one the water phase and the other for the air are solved simultaneously using a special finite difference numerical procedure. Families of curves show pore-water pressu water content distribution throughout the layer as a result of microclimatic condit imposed at the surface boundary.

The distribution of water content and ove volume change enables the prediction of t heaving or shrinking throughout the trans process. The formulation and associated example describing a simulated realistic environmental boundary conditions superin show good promise for describing the beh of an unsaturated soil system under comp environmental changes.

ACKNOWLEDGEMENT

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