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The shear strength of unsaturated soil and its relationship to slope stability problems in Hong Kong

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Introduction

Unsaturated soils have not been commonly associated with slope stability problems since the negative pore-water pressure (or matrix suction) increases the soil's strength. In most countries of the world, natural slopes of saturated soils are in a stable condition but become unstable as a result of an increase in the elevation of the water table (i.e., pore-water pressures) or an increase in the shear stresses by surcharging at the top of a slope or cutting at the toe of a slope.

In Hong Kong, most of the natural slopes have soils that are desiccated from the surface to a considerable depth and the water table is far below ground surface. This results in negative pore-water pressures throughout a considerable depth of the soil profile. In fact, it is speculated that the negative pore-water pressures (or matrix suctions) may be a significant factor in the stability of slopes in Hong Kong. In other words, the slopes are stable because of the negative pore-water pressures and would become unstable if the negative pore-water pressures were eliminated.

In recent years, considerable attention has been given to the stability of slopes in residual soils. South America, for example, has experienced slope stability problems in residual soils which appear similar to the problems in Hong Kong[11]

Although it has been known that the soil's strength is increased by the negative pore-water pressure, its magnitude is not generally used in a stability analysis. This is mainly because there is reservation regarding the long-term reliability of matrix suction. As a result, limited laboratory shear strength testing has been performed on unsaturated soils and little attempt has been made to utilise the results for the computation of a factor of safety.

In 1979 the Geotechnical Division, University of Saskatchewan, Saskatoon, Canada, undertook a research programme associated with slope stability problems in Hong Kong. The purpose of this article is to present some of the findings to-date. The nature of the slope stability problem in Hong Kong has been previously well documented[7,10,13]. Two main questions were addressed in the initial investigation. First, what is the effect of different limit equilibrium methods of slices on the computed factor of safety for typical Hong Kong conditions? Second, what is the increase in strength with respect to an increase in matrix suction for two typical Hong Kong soils; namely, decomposed granite and decomposed volcanics? An associated research programme on insitu measurement of soil suction is underway in Hong Kong, but will not be discussed in this article. Also, other relevant factors such as the effect of plane strain testing, have not as yet been studied.

The subjects given consideration direct our attention to the key aspects of the slope stability problems in Hong Kong. It is important that engineers have an analytical tool and be able to render the problem amenable to a solution. It is also important to be able to quantify the shear strength of the soil and it is necessary to have a means of monitoring the effects of environmental changes (i.e., the matrix suction).

Comparison of methods of slope stability analysis

Slope stability analysis is commonly performed on slopes in Hong Kong. Generally, the Janbu's simplified method is used and effective shear strength parameters are used in the analysis (i.e., effective cohesion, c', and effective angle of firction, ϕ'). The computed factor of safety often gives values that are below one, (e.g., 0.9) even for slopes showing no sign of distress. The stability of the slope is commonly attributed to the effects of soil suction. But before discussing the quantitative significance of soil suction, the following question should be addressed, 'Could different limit equilibrium methods of slices yield different computed factors of safety?'

The commonly used methods of slices are: Ordinary or Fellenius Method, Simplified Bishop Method, Spencer Method, Janbu's Simplified Method, Janbu's Generalised Method, Corps of Engineers Method, Lowe and Karafiath Method, and Morgenstern-Price Method. The methods differ in the elements of statics used to compute the factor of safety and in the assumption regarding the interslice forces[5] The summation of forces in two orthogonal directions and the summation of moments, along with the failure criterion, still leave the slope stability analysis indeterminate. The methods mentioned above render the analysis determinate by making an assumption regarding the interslice forces. The shear resistance mobilised at the base of each slice, S_m, can be written in terms of the failure criterion of the soil.

$$S_m = [c' + (\sigma_n - u) \tan \phi'] \frac{\ell}{F} \quad \ldots\ldots\ldots (1)$$

where: σ_n = normal stress on the base of a slice,
u = pore-water pressure at the base of a slice,
ℓ = length of the base of a slice, and
F = factor of safety,

Each of the above mentioned methods of analysis can be visualised in terms of a general moment and force equilibrium factor of safety equation. The summation of moments about the centre of rotation gives the following factor of safety equation.

$$F_m = \frac{\Sigma[c'\ell + (P - u\ell)\tan\phi']R}{\Sigma Wx} \quad \ldots\ldots\ldots (2)$$

Where: P = normal force at the base of a slice,
R = radius,
W = total weight of a slice,
x = horizontal distance between the centroid of a slice and the centre of rotation, and
F_m = moment factor of safety.

The summation of forces horizontally gives the following equation.

$$F_f = \frac{\Sigma[c'\ell + (P - u\ell)\tan\phi']\cos\alpha}{\Sigma P \sin\alpha} \quad \ldots\ldots (3)$$

Where: α = angle of the base of a slice with respect to a horizontal line, and
F_f = force factor of safety.

The summation of forces vertically on each slice gives the normal force on the base of a slice in terms of other variables.

$$P = \frac{W - (X_R - X_L) - \dfrac{c'\ell\sin\alpha}{F} + \dfrac{u\ell\tan\phi'\sin\alpha}{F}}{m_\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where: X_R and X_L = the interslice shear force on the right and left sides of a slice, respectively, and

$$m_\alpha = \cos\alpha + \frac{\sin\alpha\tan\phi'}{F}$$

The difficulty in solving for the factor of safety arises in assessing the interslice shear forces (i.e., X_R and X_L). These forces can be ignored as in the simplified Bishop method which satisfies overall moment equilibrium. Likewise, they are initially ignored in Janbu's simplified method which satisifes overall force equilibrium. The interslice shears are later accounted for by an empirical factor, f_o. Or it is possible to make some other assumption regarding the interslice shear force distribution; the net result being one or the other methods of analysis. It is noteworthy that different assumptions regarding the interslice shear forces only affect the computation of the normal force at the base of a slice.

A typical slope for Hong Kong conditions[13] along with typical soil properties was used to assess the quantitative significance of various interslice shear force assumptions (Fig. 1). The frictional force mobilised along the failure surface was computed for various methods (Fig. 2). The results demonstrate only a small variation in the frictional resistance developed by the normal effective force. In addition, the magnitude of the overall frictional force mobilised is relatively small due to the steepness of the slope and the shallowness of the slip surface. Figure 3 shows the increase in factor of safety resulting from an increase in the cohesion of the soil. The increase in factor of safety is dramatic. Later in this paper, it will be demonstrated how matrix suction in the soil can be converted into an equivalent cohesion component.

The study of various method of analysis demonstrates that the factor of safety computed by different methods is of secondary significance. However, an increase in cohesion significantly increases the computed factor of safety. The Janbu's simplified method gave the highest factor of safety for the example problem.

Shear strength of an unsaturated soil

Fredlund et al[6] proposed an equation for the shear strength of an unsaturated soil. The shear strength, τ, was written in terms of the stress state variables for an unsaturated soil and was an extension of the form of equation used for saturated soils.

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad .(5)$$

where: u_a = the pore-air pressure,
u_w = the pore-water pressure, and
ϕ^b = the angle of internal friction with respect to matrix suction.

Graphically, the equation can be visualised on a three-dimensional plot (i.e., a modified Mohr-Coulomb envelope) using the stress state variables as abscissas (Fig. 4). The

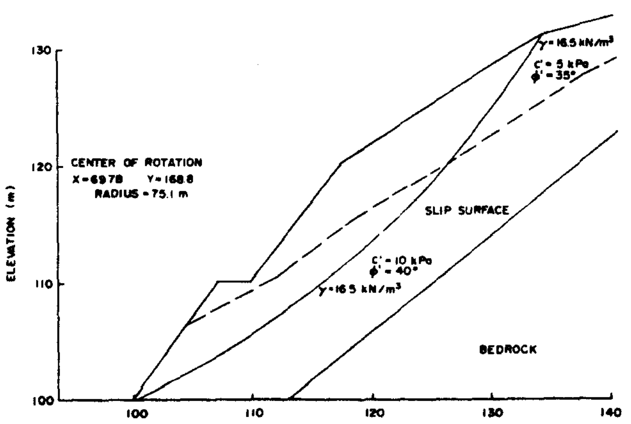

Fig. 1 Example of a typical slope in Hong Kong[13]

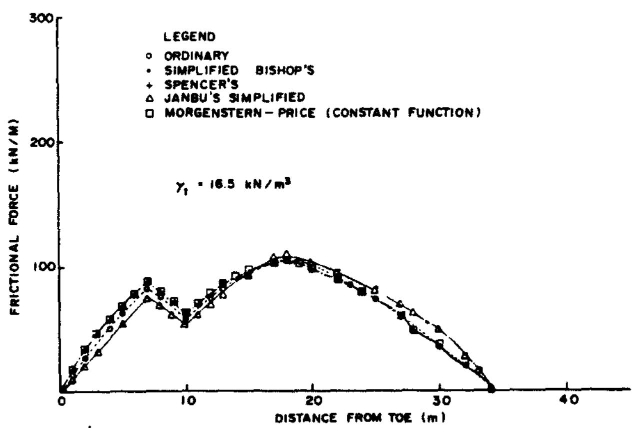

Fig. 2 Frictional force along failure surface for various slope stability methods (example problem)

equation can also be visualised as a two-dimensional graph with matrix suction contoured as the third variable (Fig. 4). The ordinate intercepts of the various matrix suction contours can be plotted versus matrix suction to give the friction angle, ϕ^b (Fig. 5). The form of the equation assumes that the failure surface is a plane; an assumption discussed in detail later in the article.

It should be noted that there is a smooth transition from the unsaturated to the saturated soil condition. As saturation is approached, the pore-air pressure, u_a, becomes equal to the pore-water pressure, u_w. At this condition, the shear strength equation reverts back to the conventional equation for a saturated soil.

The shear strength equation for an unsaturated soil can be incorporated into a slope stability analysis in the same manner as for a saturated soil. However, one of two approaches can be adopted.

1) First, it is possible to re-derive all the methods of slices factor of safety equations using the shear strength equation for the unsaturated soil. The mobilised shear force would be written as,

$$S_m = [c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b]\frac{\ell}{F} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

2) Second, it is possible to consider the matrix suction term as part of the cohesion of the soil. In other words, matrix suction increases the cohesion of the soil.[4] Therefore the slope stability equations do not need to be re-derived. Rather the cohesion of the soil, c, simply has two components (Fig. 6)

$$c = c' + (u_a\ u_w)\tan\phi^b \quad \ldots\ldots\ldots\ldots \text{(7)}$$

The mobilised shear force at the base of a slice can be written,

$$S_m = [c + (\sigma_n - u_a)\tan\phi']\frac{\ell}{F} \quad \ldots\ldots\ldots \text{(8)}$$

The latter approach is superior since it retains the same form of slope stability equation as for a saturated soil. It simply becomes necessary to know the increase in strength due to matrix suction (i.e., ϕ^b) and know (i.e., measure or estimate) the matrix suction in the soil.

Verification of the proposed shear strength equation

The credibility of the proposed procedure for computing the factor of safety in unsaturated soils hinges primarily on the validity of the form of the unsaturated soil shear

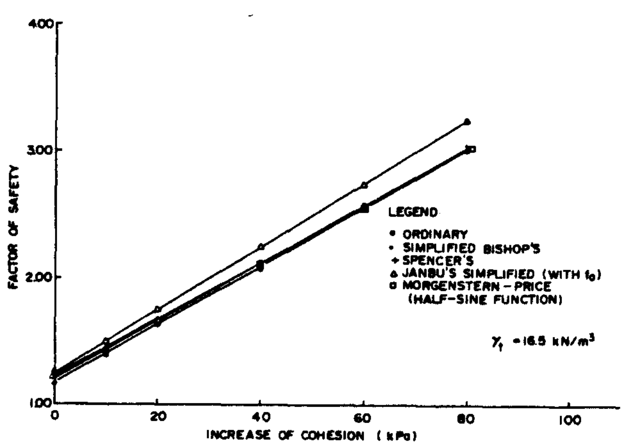

Fig. 3 Increase in factor of safety for an increase in cohesion for example slope

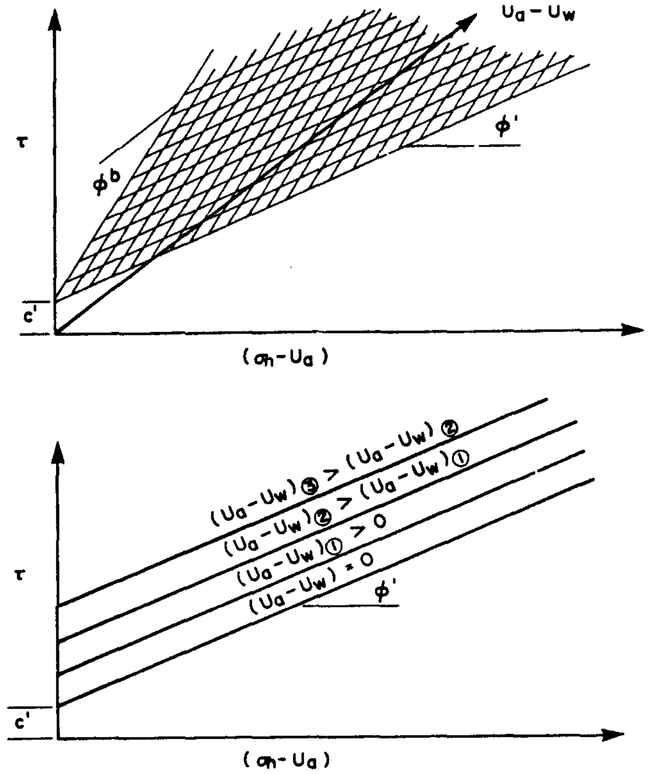

Fig. 4 Graphical representations of the shear strength of an unsaturated soil

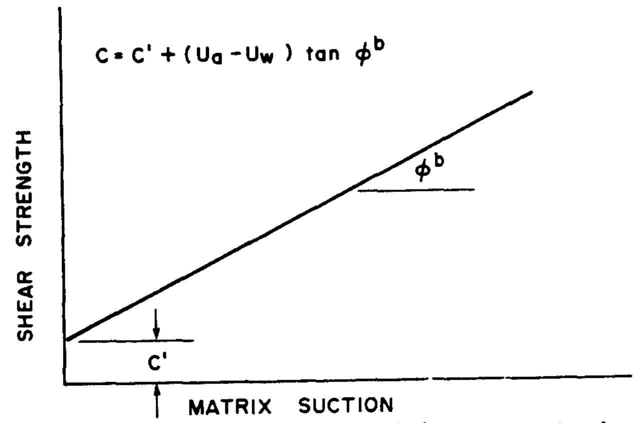

Fig. 5 Graphical representation of the increase in shear strength with matrix suction

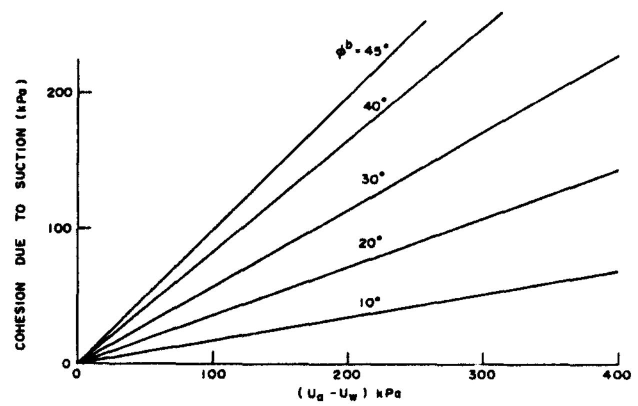

Fig. 6 Relationship between cohesion, matrix suction and the friction angle ϕb.

strength equation. The amount of laboratory testing of unsaturated soils where the stress state variables are known at failure is limited[4].

In 1960 Bishop et al[1] presented two sets of triaxial test data for unsaturated soils. One soil tested was a shale (22 per cent clay fraction) compacted at a water content of 18.6 per cent. The specimens were sheared at a constant water content and the pore-air and pore-water pressures were measured independently. Figure 7 shows a plot presenting the original test data. The data corresponding to the failure conditions can be reduced to stress state variables and used to plot a modified Mohr-Coulomb diagram (Fig. 8). If lines are drawn tangent to each of the Mohr circles at an angle, ϕ', it is possible to plot the ordinate intercepts versus the matrix suction at failure (Fig. 9)[6]. The resulting points fall along essentially a straight line. A regression analysis gives an angle to the horizontal of 18.1 degrees (i.e., ϕ^b). The correlation coefficient for the data points is 0.970. If the failure surface were ideally planar, all points should fall on a straight line. The scatter in the data is random, making it impossible to positively identify any warping of the plane.

Another set of tests were performed by Bishop et al[1] on a Boulder clay (18 per cent clay fraction) compacted at a water content of 11.6 per cent. The results are summarised in Table 1.

In 1978 Satija[12] performed an extensive laboratory testing programme on statically compacted Dhanauri clay. The triaxial testing apparatus was essentially the same as that used by Bishop.[1] The clay had a liquid limit of 48.5 per cent, a plastic limit of 25 per cent and had 25 per cent clay sizes. The soil was compacted at two densities but with the same initial water contents. Type A specimens had an average initial unit weight of 15.5 kN/m^3 and type B specimens had an average initial unit weight of 14.5 kN/m^3. Two types of triaxial tests were performed at each of the compaction conditions; namely, constant water content tests with pore-water pressure measurement (i.e., CW tests) and consolidated drained tests (i.e., CD tests)[2].

Although the samples for a particular series of tests were compacted at the same density and water content conditions, they were then conditioned to various initial stress states to form an equally spaced grid of tests on a three-dimensional surface. In this way, any warp in the surface would be readily observable. The data has been re-analysed using the procedure mentioned above and the results of the analysis are summarised in Table 2.

Each of the shear strength envelopes showed a planar type of failure surface. Figure 10 shows the plot of shear strength versus matrix suction for the CW tests on the high initial density soil. The correlation coefficients are high and reveals no tendency for warping of the failure surface.

At the Fourth International Conference on Expansive Soils (1980), Escario presented the results of a series of direct shear tests performed on compacted samples of Madrid Gray clay. The soil had a liquid limit of 81 per cent and a plastic limit of 38 per cent. The initial compaction conditions were a unit weight of 13.1 kN/m^3 and a water content of 29 per cent. The direct shear box was enclosed and had a high air entry disc sealed in the base to allow maintenance of a constant matrix suction during shear. Figure 11 shows the failure surfaces for various suction values. The failure conditions show a ϕ' varying from 19 degrees at low suctions to 25 degrees at high suctions and an average of 22.5 degrees. Figure 12 shows the matrix suction versus shear strength for the Madrid Gray clay. The ϕ^b value is 16.1 degrees when 22.5 degrees is used for ϕ' in the analysis.

In general, the test data to-date demonstrates that the failure surface for an unsaturated soil is essentially planar. At least, it appears to be sufficiently close to being planar for engineering purposes. This is particularly true when consideration is given to the conservative manner in which the data would need to be used in practice. It is suggested that the field stress levels be simulated as closely as possible in the laboratory.

Table 1 Triaxial tests on unsaturated soils[1] (Ref. 1)

Soil Type	c' (kPa)	ϕ'	ϕ^b	Correlation*
Compacted Shale w = 18.6%	15.8	24.8	18.1	0.970
Boulder Clay w = 11.6%	9.6	27.3	21.7	0.974

* Correlation coefficient is for the computation of ϕ^b

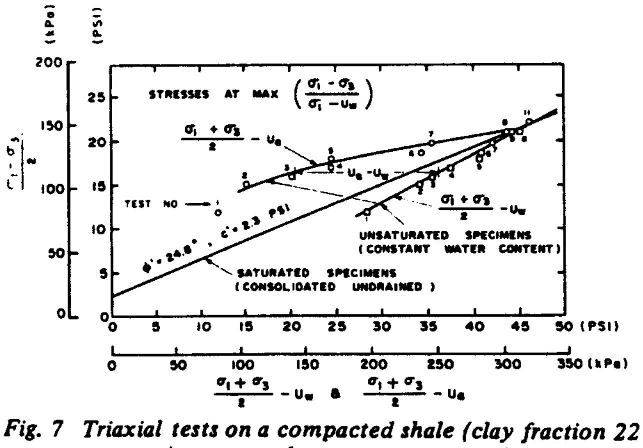

Fig. 7 Triaxial tests on a compacted shale (clay fraction 22 per cent) compacted at a water content of 18.6 per cent and sheared at constant water content[1]

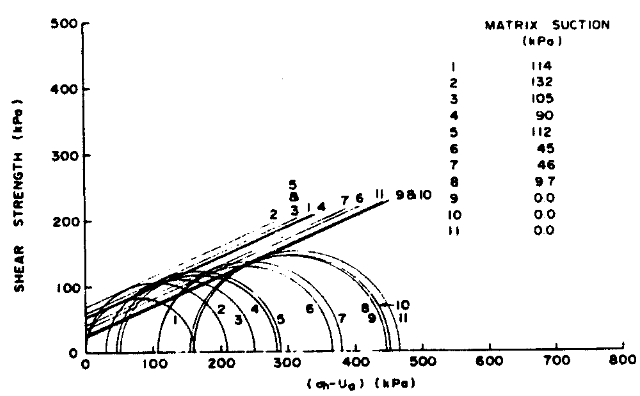

Fig. 8 Mohr-Coulomb failure circles for compacted shale

Table 2 Triaxial tests on unsaturated soils (Dhanauri Clay)[12]

Soil Type	c' (kPa)	ϕ'	ϕ^b	Correlation Coefficient *
CD Test $\gamma_d = 15.5\ kN/m^3$ w = 22.2%	37.3	28.5	16.2	0.974
CD Test $\gamma_d = 14.5\ kN/m^3$ w = 22.2%	20.3	29.0	12.6	0.963
CW Test $\gamma_d = 15.5\ kN/m^3$ w = 22.2%	15.5	28.5	22.6	0.992
CW Test $\gamma_d = 14.5\ kN/m^3$ w = 22.2%	11.3	29.0	16.5	0.971

* Refers to the best-fit line for ϕ^b

Proposed laboratory testing procedure

The laboratory testing programme was conducted on conventional equipment with two modifications (Fig. 13). First, it is necessary to seal a high air entry disc onto the base pedestal of the triaxial cell. The ceramic discs used in the study had an air entry value of 5 bars (505 kPa) and was 1/4 inches (0.635 cm) thick. It is imperative to seal the high air entry disc to the pedestal with epoxy in order to ensure the control (or measurement) or the pore-water pressure. There must also be a system provided for flushing de-aired water below the ceramic disc in order to remove diffused air. Second, a thin, coarse corundum stone was placed on the top of the sample. The port in the loading cap was connected to a controlled air pressure line.

The testing procedure selected involved the control or maintenance of the air and water pressures during the test rather than their measurement in a closed system[2]. Therefore, suction was maintained constant throughout the test. It is possible to maintain the pore-air and pore-water pressures since the coefficient of permeability of the soils under consideration is relatively high. Maintaining the pore-air and pore-water pressure is comparable to performing a 'slow' or drained test on a saturated soil.

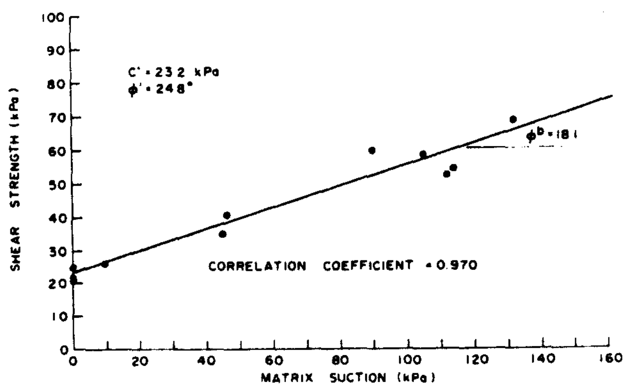

Fig. 9 Shear strength versus matrix suction for compacted shale

It is difficult to measure or control negative pore-water pressures that even approach -0.75 atmospheres due to cavitation. Therefore, an axis-translation technique was used which ensured that the pore-water pressure was always kept at a positive value[3].

An attempt was made to select the testing procedure in such a way as to obtain the maximum amount of information from a limited number of tests. It was desirous to devise a less demanding tests procedure than had previously been used; a procedure that could possibly be used in engineering practice. Utilising the fact that the failure surface was essentially planar, a multi-stage type of test procedure was attempted[8,14]. This helps eliminate the effect of variability in the soil from one test to the next.

The test procedure used is as follows:

1) The specimen was trimmed for testing. The specimens were 2.5 inches (6.35 cm) in diameter and approximately 5.5 inches (14.0 cm) long.

2) The sample was mounted in the triaxial apparatus and the rubber membrane was placed around the specimen.

3) O-rings were placed over the membrane on the bottom

4) A spacer was placed between the membrane and the loading cap in order that water could be added to reduce the suction of the soil to zero. In other words, an attempt was made to ensure that the suction applied to the specimen for Stage 1 was greater than the initial soil suction.

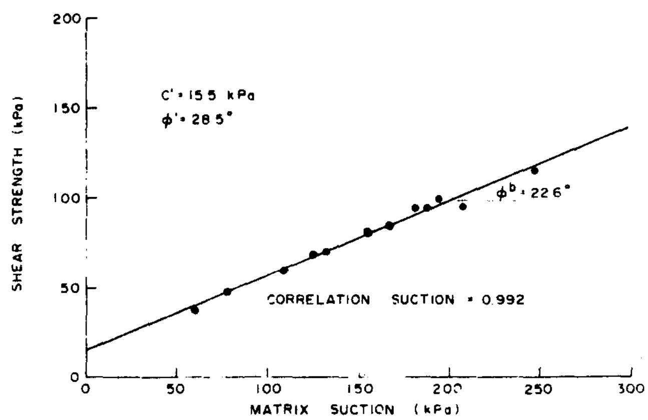

Fig. 10 Shear strength versus matrix suction for Dhanauri clay[12]

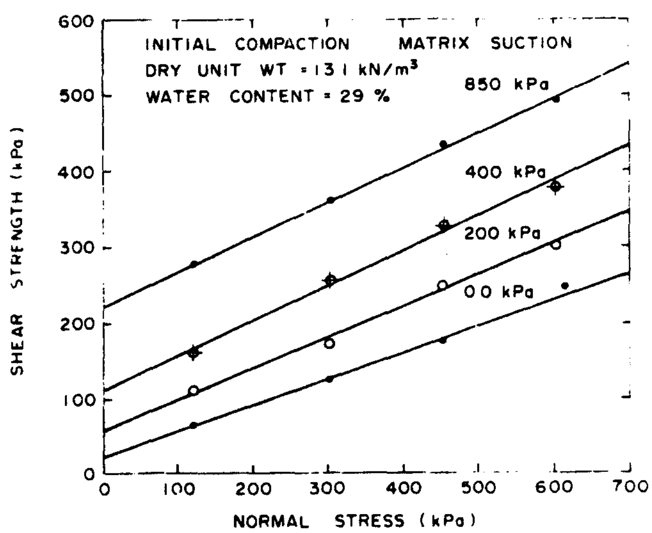

Fig. 11 Sear strength of Madrid Gray clay at various suction levels (Escario, 1980)

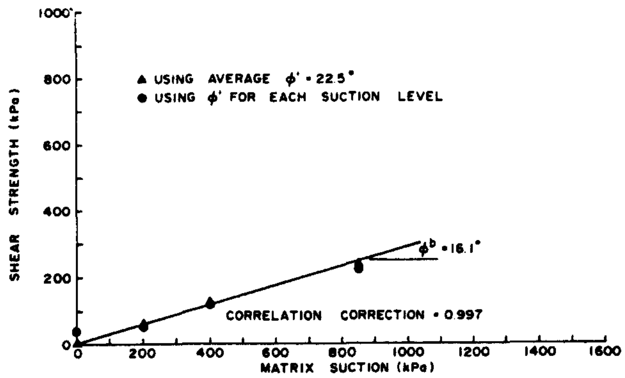

Fig. 12 Shear strength versus matrix suction for Madrid Gray clay (Escario, 1980)

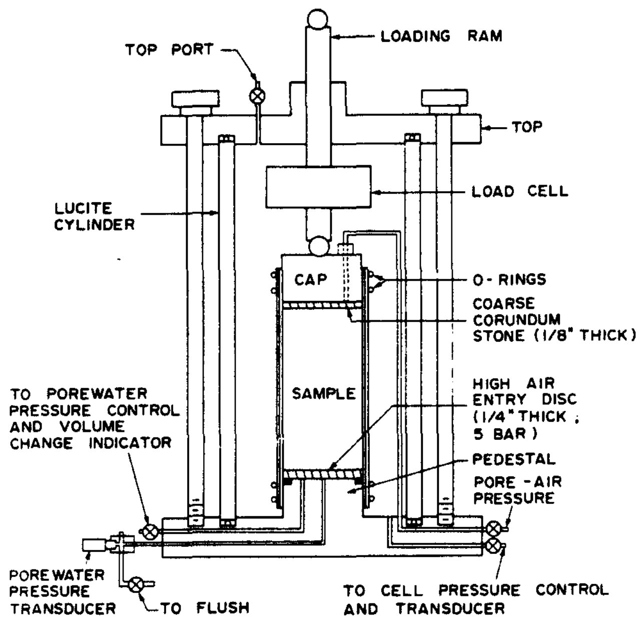

Fig. 13 Modified triaxial cell for testing unsaturated soils

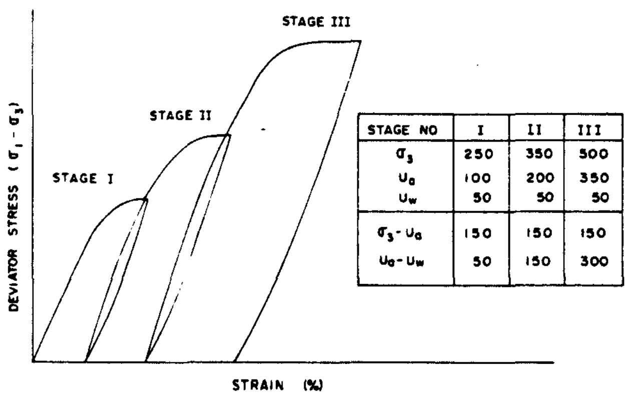

Fig. 14 Typical stress versus strain curves for a multi-stage test

5) After the wetting of the specimen appeared to be complete, the top O-rings were placed around the loading cap.

6) The stresses associated with the first stage of testing (i.e., Stage 1) were applied and consolidation was allowed to proceed. A typical set of stresses for Stages 1, 2, and 3 are as follows:

Stage No.	σ (kPa)	u_a (kPa)	u_w (kPa)
1	250	100	50
2	350	200	50
3	500	350	50

The associated stress state variables are:

Stage No.	$\sigma_3 - u_a$	$u_a - u_w$
1	150	50
2	150	150
3	150	300

The range of matrix suction values shown is quite large. These values were selected during the early stages of testing to assist in establishing the effect of matrix suction.

7) Once consolidation was complete, the stresses were maintained while the deviator stress was applied slowly. A typical strain was 0.001 per cent per minute.[12]

8) The deviator stress was applied until it was obvious that the stress was reaching a peak value. At this point, the deviator stress was reversed to zero (See Fig. 14).

9) A new set of stresses for Stage 2 was applied to the specimen, consolidation was again allowed to proceed and the deviator stress was applied as before.

10) The above procedure was further applied to Stage 3.

11) The data is most readily interpreted by plotting the peak stress conditions on a conventional Mohr-Coulomb type of plot and contouring the effect of matrix suction (See Fig. 15). An estimate of ϕ' is required for the interpretation of the data. Fortunately, the interpretation of the data for the effect of matrix suction, is not highly dependent on the estimate of ϕ'. Lines are drawn tangent to the failure stress circles to give a cohesion intercept for each suction level. The cohesion values are plotted versus matrix suction to give the angle ϕ^b (See Fig. 16).

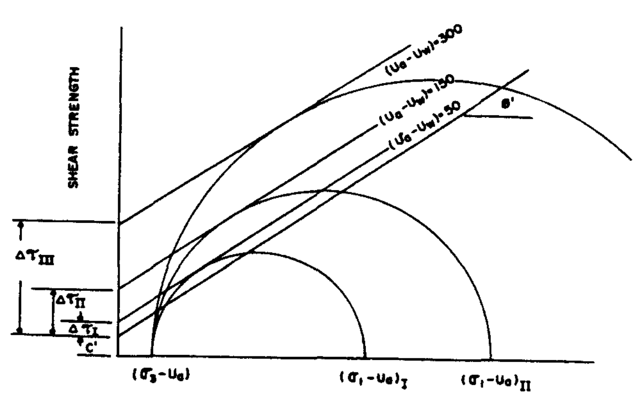

Fig. 15 Interpretation of a multi-stage triaxial test to get the angle of friction with respect to matrix suction

Presentation of data for Hong Kong soils

Tests were performed on undisturbed samples of decomposed granite and decomposed volcanic rock from Hong Kong. The testing programme is continuing and the analysis of the data is incomplete. However, several typical sets of data are presented to indicate the nature of the results.

Sample No. 5 consisted of decomposed granite. The stress versus strain curves for the three stages of loading are shown on Figure 17. The matrix suction was zero for the first stage of loading for this test. There is some evidence of structural breakdown or collapse during Stage 3 of loading. This is not surprising since the total strain in the sample is high. (Stage 1 should have been stopped sooner). The consolidation curves for Stage 2 of loading is shown on Figure 18. Although the decomposed granite has a relatively high permeability, the rate of consolidation is impeded by the high air entry disc. The impeded drainage must be taken into account in establishing the rate of applying the deviator stress. Figure 19 shows the plot of the stress conditions at failure for each stage, as well as the interpretation of the data. Figure 20 shows the plot of matrix suction versus shear strength for sample number 5. The best-fit angle, ϕ^b, for all three stages is 12.4 degrees while the angle for the first two stages would indicate an angle of approximately 15 degrees. The results of several multi-stage triaxial tests on decomposed granites are shown in Table 3.

Figures 21 to 23 show the test results for a specimen trimmed from a block sample of decomposed volcanics. Figure 21 shows the stress versus strain curves for the three stages of loading. Once again there is some drop-off in deviator stress immediately after peaking in Stage 3. Figure 22 shows the increase in strength due to matrix suction. The average angle, ϕ^b, for all three stages was 15.9 degrees while the first two stages show a value of approximately 18 degrees.

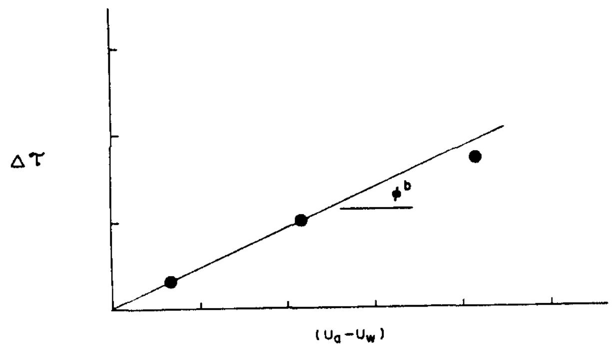

Fig. 16 Determination of the angle of friction with respect to matrix suction, ϕb.

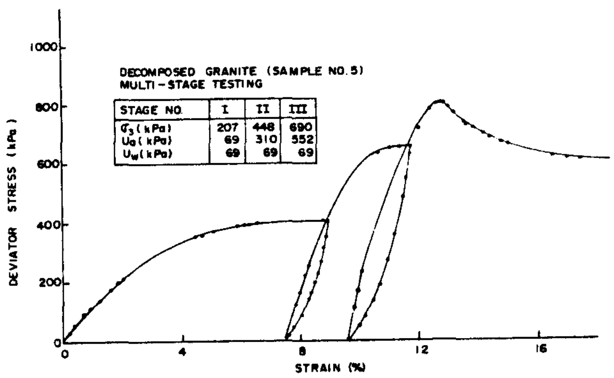

Fig. 17 Stress versus strain curves for decomposed granite (sample No. 5)

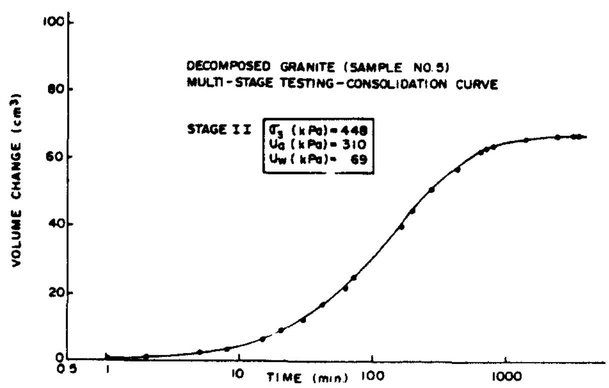

Fig. 18 Consolidation curve prior to stage 2 loading (sample No. 5)

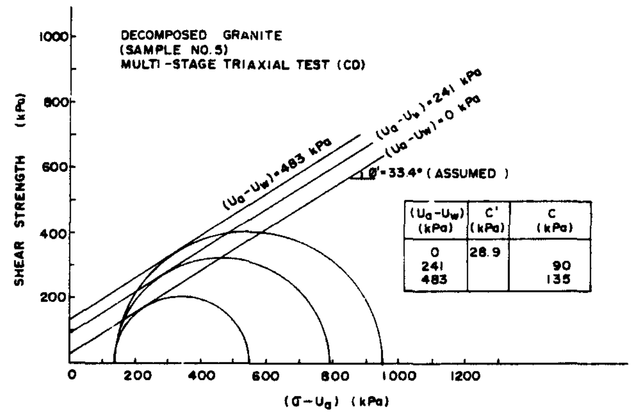

Fig. 19 Mohr failure circles for decomposed granite (sample No. 5)

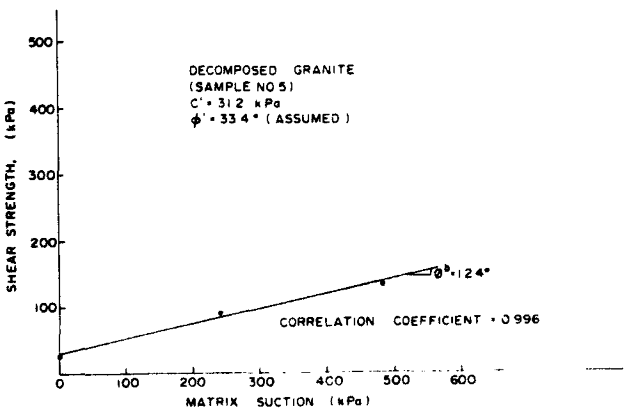

Fig. 20 Increase in shear strength with respect to matrix suction for decomposed granite (sample No. 5)

Table 3 Triaxial tests on Hong Kong soils

Sample No.	c' (kPa)	ϕ' *	ϕ^b
5	31.2	33.4	12.4
6	36.3	33.4	27.4
8	103.	33.4	14.6
10	44.	33.4	16.2
11A	33.	33.4	11.7

*Estimated

Significance of the triaxial test results

The testing programme on the Hong Kong soils should still be considered 'ad hoc.' There are some problems associated with the testing and the interpretation of the data. The effects of stress reversals and structural breakdown with strain need to be further examined. However, these difficulties appear to be of a secondary nature and should not mask the primary findings.

For example, an angle of friction with respect to matrix suction of 14 degrees results in an apparent cohesion of 25 kPa if the matrix suction is one atmosphere. From Figure 3, this cohesion can be translated into a 50 per cent

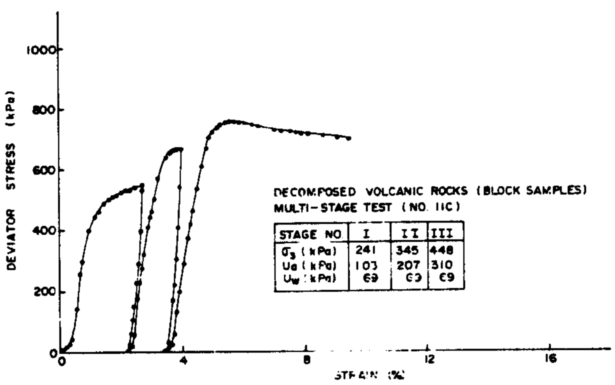

Fig. 21 Stress versus strain curve for decomposed volcanic (sample No. 11C)

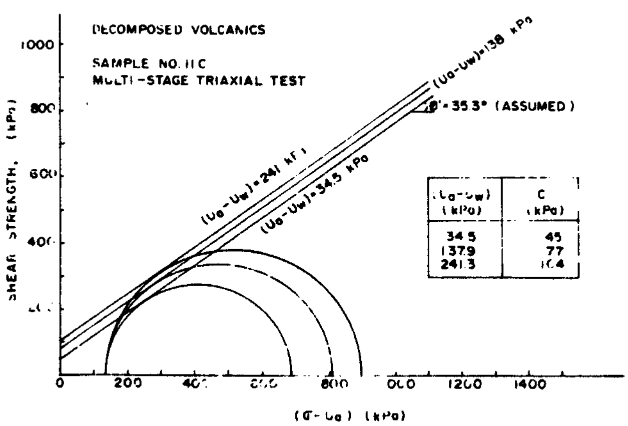

Fig. 22 Mohr failure circles for decomposed volcanics (sample No. 11C)

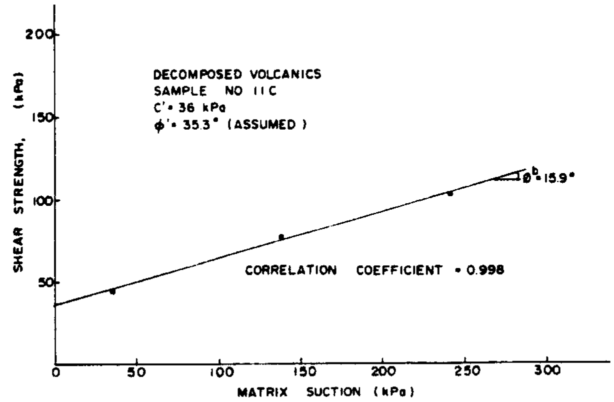

Fig. 23 Increase in strength with respect to matrix suction for decomposed volcanics (sample No. 11C)

increase in the factor of safety for the example problem shown in Figure 1.

At present, there is need for measurements of insitu negative pore-water pressures. Not only at one time and place, but at several locations on a continuing basis to gain insight into its change throughout the year. As well, adequate, theoretical moisture movement models need to be verified.

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