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of  $\dot{\epsilon}_1 = 5 \times 10^{-4} \text{ min}^{-1}$ ) is by means of the individual paths for the separate elements A, B and C in Fig.5. This diagram shows four inter-related plots of the four variables  $p'$ ,  $q$ ,  $e$  and shear strain  $\epsilon_1 - \epsilon_3$ . It should be noted that element C close to the boundary where free drainage occurs behaves like a fully drained sample, whereas element A near the centre behaves initially like an undrained sample. Towards the end of the test, when sufficient time has elapsed for most of the excess pore pressures to have dissipated, the effective stress path for element A in Fig.5(a) turns back on itself and becomes tangential to the failure envelope formed by the critical state line.

## CONCLUSIONS

I have presented some predictions of non-homogeneous behaviour in triaxial specimens of clay. Only those have been considered which arise due to the effects of a finite coefficient of permeability, a finite loading rate and migration of pore water within and from the specimen. The effect of rough end platens has not been included here, although this has been considered separately by Carter.

It has been shown that the modified Cam-clay soil model, when used together with a coupled Biot-type consolidation analysis in a finite element computation, can make reasonable predictions of the non-homogeneous behaviour of a sample of normally consolidated clay in a triaxial compression test.

There is a need, which can now be met, of carrying out numerical experiments in parallel

D.Y.F. Ho and D.G. Fredlund (Oral discussion)

Discussion on the paper "SHEAR STRENGTH OF PARTIALLY SATURATED SOILS" by S.K. Gulhati and B.S. Satija. (Vol. 1, p. 609)

The discussers would first like to compliment Professor Gulhati and Dr. Satija for completing a very demanding and excellent unsaturated soil testing program. The discussers, however, have some concern over the manner in which the test data was interpreted. Namely, the interpretation presented by the author's paper does not render a smooth transition between the conventional saturated soil shear strength equation (Terzaghi, 1936) and the proposed equation for unsaturated soils.

The saturated soils shear strength equation as per Terzaghi (1936) is as follows:

$$\tau = c' + (\sigma - u_w) \tan \beta' \quad (1)$$

where  $\tau$  = shear strength of the soil

$c'$  = effective cohesion

$\sigma$  = total stress

$u_w$  = pore-water pressure

$\beta'$  = effective angle of internal friction

In a stress point form, the equation reads;

with laboratory experiments in order to:

- (i) prove the success of computational techniques,
- (ii) check the relevance of mathematical models,
- (iii) indicate the extent of the deficiencies of apparatus, and
- (iv) understand and interpret the results of centrifugal model tests.

Such work has applications not just for research purposes, but is now being used for design of engineering works. One important example which has been successfully tackled is the problem of a road embankment built on soft ground and the possible need for staged construction to avoid stability problems.

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- Carter, J.P. (1982). Some predictions of the non-homogeneous behaviour of clay in the triaxial test. Géotechnique (in press).
- Gibson, R.E. and Henkel, D.J. (1954). Influence of duration of tests at constant rate of strain on measured 'drained' strength. Géotechnique, 4, 6-15.

$$1/2(\sigma_1 - \sigma_3) = d' + [1/2(\sigma_1 + \sigma_3) - u_w] \tan \psi' \quad (2)$$

where  $\sigma_1$  = major principal stress

$\sigma_3$  = minor principal stress

$d'$  = the intercept when the stress point is zero

$\psi'$  = the angle between the stress point plane and the

$[1/2(\sigma_1 + \sigma_3) - u_w]$  axis

The proposed equation for the shear strength of unsaturated soils presented in the paper is stated as follows:

$$\frac{(\sigma_1 - \sigma_3)}{2} = a + (\sigma_3 - u_a)_f \tan \alpha + (u_a - u_w)_f \tan \beta \quad (3)$$

where  $\sigma_1$  = total major principal stress at failure

$\sigma_3$  = total confining stress at failure

$u_a$  = pore air pressure at failure

$a$  = apparent intercept at the  $(\sigma_1 - \sigma_3)$  axis of the three-dimensional plot between  $(\sigma_3 - u_a)_f$  and

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)$$

- $\alpha$  = coefficient with respect to changes in  $(\sigma_3 - u_w)_f$  when  $(u_a - u_w)$  is held constant
- $\beta$  = coefficient with respect to changes in  $(u_a - u_w)_f$  when  $(\sigma_3 - u_a)_f$  is held constant

The discussers would like to point out that the proposed equation for the shear strength of unsaturated soils is presented in a way which is neither in accordance with the Mohr-Coulomb failure hypothesis nor the stress point method (Figure 1). Moreover, there will not be a smooth transition between the proposed equation and Terzaghi's (1936) shear strength equation for saturated soils. When saturation is approached (i.e.,  $u_a = u_w$ ), the proposed equation does not revert back to Terzaghi's (1936) shear strength equation for saturated soils either in conventional or stress point form.

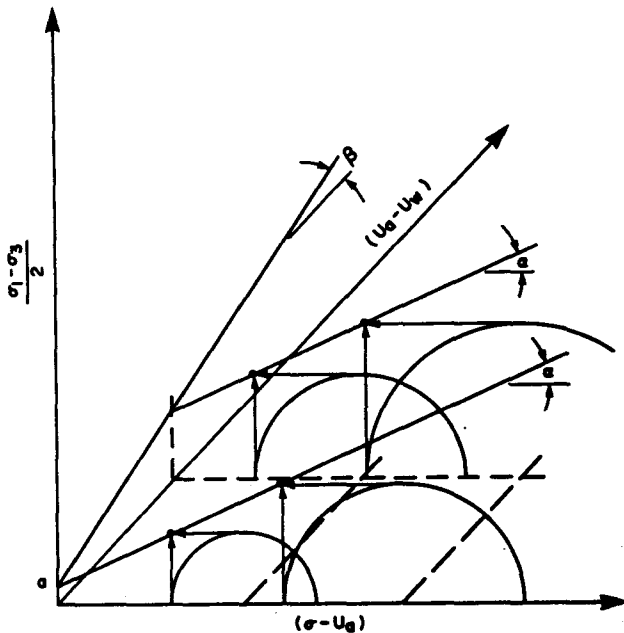


FIGURE 1 Satija's (1978) shear strength equation in three dimensional plot.

Fredlund et al (1978) have proposed shear strength equations for unsaturated soils utilizing the same independent stress state variables. When  $(\sigma - u_a)$  and  $(u_a - u_w)$  are used as the stress variables, the unsaturated soil shear strength equation as per Fredlund et al (1978) as follows:

$$\tau = c' + (\sigma - u_a) \tan \beta' + (u_a - u_w) \tan \beta^b \quad (4)$$

where  $u_a$  = pore air pressure

$\beta^b$  = the friction angle with respect to changes in  $(u_a - u_w)$  when  $(\sigma - u_a)$  is held constant

In stress point form, the equation becomes,

$$1/2(\sigma_1 - \sigma_3) = d' + 1/2(\sigma_1 + \sigma_3) \tan \psi' + (u_a - u_w) \tan \psi^b \quad (5)$$

where  $\sigma_1$  = the major principal stress (i.e., the total axial stress)

$\sigma_3$  = the minor principal stress (i.e., the total confining stress)

$\psi^b$  = the angle between the stress point plane and the  $(u_a - u_w)$  axis when  $[1/2(\sigma_1 + \sigma_3) - u_a]$  is held constant.

Graphically, the failure envelope will be a planar surface in either conventional or stress point form (Figure 2 and 3). It should be noted that there is a smooth transition from Fredlund et al's (1978) unsaturated soil shear strength equations to the conventional shear strength equations for saturated soils (Terzaghi, 1936) As saturation is approached, the pore air pressure,  $u_a$ ,

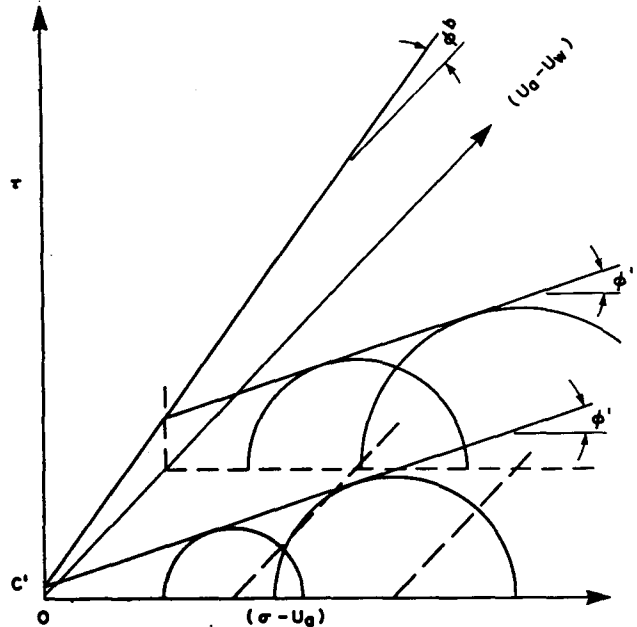


FIGURE 2 Three dimensional failure surface using stress variables  $(\sigma - u_a)$  and  $(u_a - u_w)$

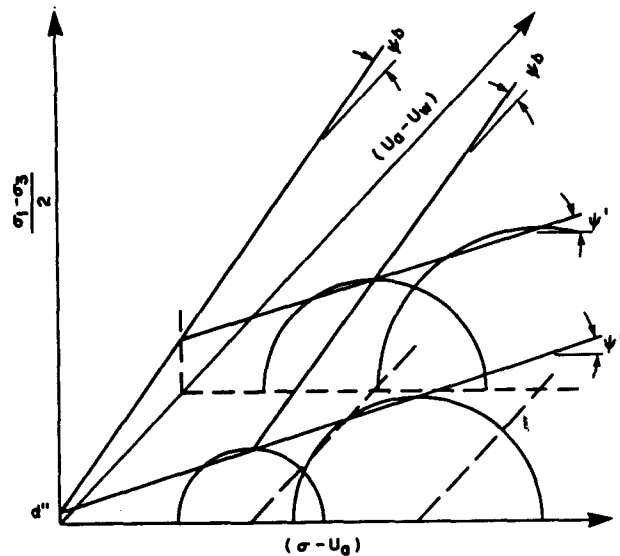


FIGURE 3 Fredlund et al (1978) three dimensional failure surface using stress variables  $(\sigma - u_a)$  and  $(u_a - u_w)$  in stress point form.

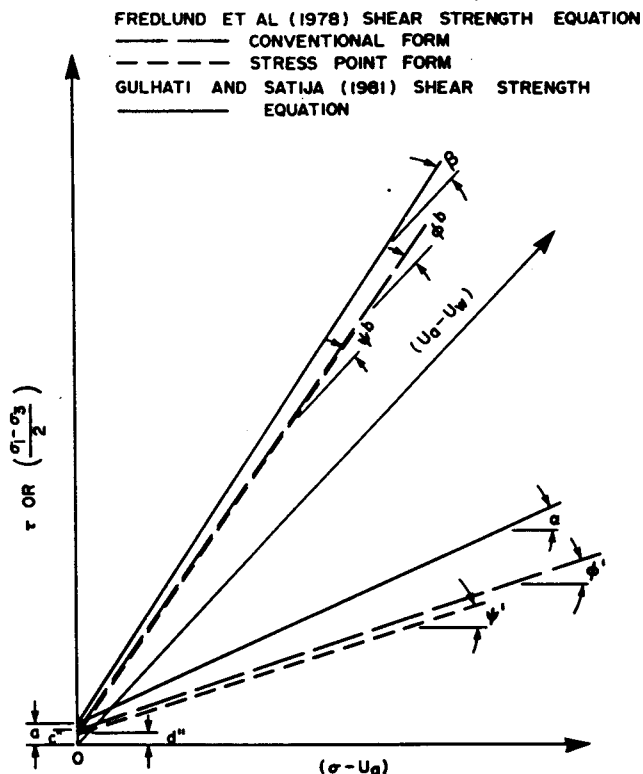


FIGURE 4 Comparison between unsaturated soil shear strength equations proposed by Fredlund et al (1978) and Satija (1978)

T. Kimura and K. Saitoh (Oral discussion)

ON THE COMPARISON OF CENTRIFUGE TESTS WITH LARGE-SCALE MODEL TESTS

In order to obtain practicable results through model tests simulating earth structures, it is essential that the law of similarity is maintained between the prototype and the model. For a problem in which the self-weight of soil is significant, conventional model tests do not allow us to model the prototype. One way of modeling this type of problem correctly is to use a centrifuge, and the usefulness of centrifuge modelling technique is now being widely recognized. It has to be pointed out, however, that the similarity law for centrifuge modelling is based upon the concept of continuum mechanics or the concept of stress in soil elements, not of soil particles. Some skepticism has been raised on this point, arguing that a slip band, in a model ground of sand for example, in a centrifugal field, say of  $Ng$ , would be  $N$  times wider than the actual one in the gravitational field leading to an unrealistic failure mode. But the Authors take a view that, as long as the same sand is used for a centrifuge test and a prototype, it would show a very similar behaviour since sand is essentially a frictional material.

In this short report the Authors attempt to put forward a comparison between centrifuge test results and those of large-scale model tests on

becomes equal to the pore water pressure,  $u_w$ . At this condition, Fredlund et al's (1978) unsaturated soil shear strength equations (equation 4 and 5) revert back to Terzaghi's conventional shear strength equations (equation 1 and 2) for saturated soils.

By comparing the unsaturated soil shear strength equations as per Fredlund et al (1978) with that presented by Gulhati and Satija (1981), it can be demonstrated that the proposed equation for the shear strength of unsaturated soils will predict too high a shear strength (Figure 4). In turn, the over-estimation of shear strength by Gulhati and Satija's equation would result in over-estimating factors of safety in slope stability analyses involving unsaturated soils (Ho, 1981).

While the discussers feel there is a superior procedure for interpreting the data of Gulhati and Satija, we want in no way to underestimate the value of the experimental test data.

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an identical problem of loaded slopes consisting of dense sand. Shields et al. (1977) performed a series of model tests on the bearing capacity of slopes of sand loaded on the top surface. The height of their slopes exceeded 2.0m, which was considered to be of the scale of actual slopes. Incidentally the Authors carried out very similar tests by using centrifuge to study the stability of tank pads underneath oil storage tanks.

A key sketch of the problem dealt with is given in Fig.1. The experiments were conducted for various combinations of four parameters; the breadth of a footing  $B$ , the height parameter  $\beta$ , the distance parameter  $\lambda$  and the angle of inclination of a slope  $\alpha$ . The sand used was Toyoura sand, of which effective particle diameter was 0.13 mm and uniformity coefficient was 1.38. The sand was poured into a container from a hopper through two sieves to form sand deposit with

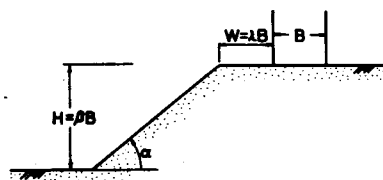


Fig.1 Key sketch of model