Pore pressure parameters for unsaturated soils

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The concept of pore pressure parameters has been found convenient to visualize the pore pressure response of saturated soils for various applied stress changes. This paper derives pore pressure parameters that can be utilized in determining the pore pressure response of unsaturated soils. The expressions derived for the estimation of pore-air and pore-water pressures under undrained, isotropic loading are called the B_a and B_w pore pressure parameters. Measured pore pressures are compared with predicted values. Various factors influencing the proposed pore pressure parameters are discussed.

Pore pressure parameter expressions are also derived for the pore-air and pore-water pressures induced as a result of deviator stress changes during undrained loading (i.e., A_a and A_w).

Le concept de paramètres de pression interstitielle s'est avéré pratique pour représenter la génération de pression interstitielle résultant de différentes variations de contraintes dans les sols saturés. Dans cet article on établit des paramètres de pression interstitielle qui peuvent être utilisés pour l'étude des variations de pression interstitielle dans les sols non saturés. Les expressions auxquelles on arrive pour l'évaluation de la pression d'air et de la pression d'eau interstitielle en chargement non drainé isotrope sont appelées les paramètres de pression interstitielle B_a et B_w. Des pressions mesurées sont comparées aux valeurs prédites. Les différents facteurs qui affectent les valeurs des paramètres proposés sont discutés.

Des expressions des paramètres de pression interstitielle sont également établies pour la pression d'air et la pression d'eau produites par l'application d'un chargement déviatorique non drainé, soit A_a et A_w.

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Introduction

The behavior of an unsaturated soil tested under undrained loading conditions depends on the magnitude of the pore-air and pore-water pressures developed. The concept of pore pressure parameters has been found to be convenient (Skempton 1954; Bishop 1954) to obtain a clear visualization of how the pore pressures respond to various applied stress changes. Examples of their application are shear strength problems (Fredlund *et al.* 1978), volume change problems (Fredlund and Morgenstern 1976), and transient flow problems (Hasan and Fredlund 1977; Fredlund and Hasan 1979).

Hilf (1948) formulated an equation for the pore-air pressure developed in an unsaturated soil as a result of total stress changes under undrained conditions. Hilf utilized Boyle's and Henry's laws and assumed that the change in the pore-air pressure was equal to the change in the pore-water pressure. Skempton (1954) and Bishop (1954) presented the concept of pore pressure coefficients. In 1962, Bishop and Henkel proposed two pore pressure coefficients, B_a and B_w, to represent the induced pore-air and pore-water pressures, respectively, due to a change in applied load. Similar types of pore pressure parameters have recently been proposed for the tar sands (Harris and Sobkowicz 1978; Dusseault 1979).

In this paper, the B_a and B_w pore pressure parameters (i.e., B_a and B_w) are derived utilizing the stress state variables for an unsaturated soil (Fredlund and Morgenstern 1977) and recently proposed constitutive relations (Fredlund and Morgenstern 1976; Fredlund 1976). The proposed B_a and B_w pore pressure parameters have been previously suggested for the boundary conditions associated with the theory of consolidation for an unsaturated soil (Fredlund and Hasan 1979). Data from the research literature are used to substantiate the derived equations. Expressions are also derived for the A pore pressure parameters (i.e., A_a and A_w).

Derivation of the B Pore Pressure Parameters (Isotropic Loading)

The physical basis for the B pore pressure parameter is readily understood by considering a simple elastic model in which the soil skeleton and the pore-fluid phases are represented by compressible springs.

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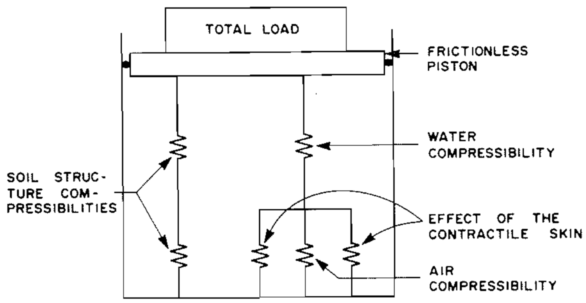

FIG. 1. Mechanical model for pore pressure generation.

Skempton (1954) and Bishop (1954) derived the following equation for the B pore pressure parameter:

$$[1] \quad B = \frac{\Delta u_w}{\Delta \sigma_3} = \frac{1}{1 + \dfrac{nC_w}{C_c}}$$

where Δu_w is change in pore-water pressure; $\Delta\sigma_3$ is isotropic change in total stress; n is porosity; C_w is compressibility of water; and C_c is compressibility of the soil skeleton.

In a saturated soil, the compressibility of the water is small relative to the soil skeleton compressibility and, therefore, the B pore pressure parameter approaches 1. The B parameter decreases with a decrease in the degree of saturation since air renders the pore fluid highly compressible. The Skempton (1954) and Bishop (1954) analysis did not take into account surface tension effects, which result in different pressures in the air and water phases.

For an unsaturated soil, it is possible to derive two pore pressure parameters to predict changes in the pore-air and pore-water pressures as a result of an isotropic total stress change. The two pore pressure parameters are defined as:

$$[2] \quad B_a = \Delta u_a/\Delta\sigma_3 \quad \text{and} \quad B_w = \Delta u_w/\Delta\sigma_3$$

where Δu_a is change in pore-air pressure.

Let us consider the piston and spring analogy shown in Fig. 1 in order to understand the mechanism of pore pressure generation in an unsaturated soil. The top loading cap will move downward upon loading by an amount dependent upon the compressibilities of the various phases and the applied load. The overall compression of a soil element and the corresponding induced pore-air and pore-water pressures are dependent on the compressibility of the pore-air, pore-water, and soil skeleton. In addition, the contractile skin (i.e., the air–water interface) has an effect on the relative magnitude of the pore-air and pore-water pressures. This interfacial effect is taken into account by using two independent stress variables to describe the stress state of the unsaturated soil and using these stress variables to write independent deformation equations for the air and water phases (Fredlund and Morgenstern 1976). Let us assume that the air pressure is uniform throughout the air phase and the escape of pore air and pore water is not permitted when the soil is loaded. The continuity requirement for an unsaturated soil element is:

$$[3] \quad \Delta V/V = (\Delta V_w/V) + (\Delta V_a/V)$$

where V is volume of the overall element (i.e., soil skeleton); V_w is volume of water in the element; and V_a is volume of air in the element.

The overall volume change of the air–water mixture can be written in terms of the compressibility of the pore fluid and the change in pore fluid pressure:

$$[4] \quad (\Delta V_w + \Delta V_a)/V = \beta_m n \Delta u_f$$

where β_m is compressibility of an air–water mixture; n is porosity of the soil; and Δu_f is change in the fluid pressure.

Fredlund (1976) derived the compressibility of an air–water mixture as:

$$[5] \quad \beta_m = S\beta_w + B_{aw}\left[\frac{(1-S) + HS}{(u_a + u_{atm})}\right]$$

where S is degree of saturation; β_w is compressibility of water; B_{aw} is change in pore-air pressure relative to a change in pore-water pressure; u_a is gauge pore-air pressure; u_{atm} is atmospheric air pressure (i.e., standard pressure); and H is Henry's solubility coefficient.

The fluid pressure has been referenced to the water phase in [5]. Substituting [5] into [4] and referencing the pore-fluid pressure to the water phase gives:

$$[6] \quad \frac{\Delta V_w + \Delta V_a}{V} = Sn\beta_w\Delta u_w + \left[\frac{(1-S)n + HSn}{(u_a + u_{atm})}\right]\Delta u_a$$

The volume change must equal that dictated by the constitutive relation for the soil skeleton (Fredlund and Morgenstern 1977). Equating [6] to the constitutive relation for the soil skeleton gives:

$$[7] \quad m_1^s\Delta(\sigma_3 - u_a) + m_2^s\Delta(u_a - u_w) = Sn\beta_w\Delta u_w + \left[\frac{(1-S)n + HSn}{(u_a + u_{atm})}\right]\Delta u_a$$

where m_1^s is compressibility of the soil structure when $\Delta(u_a - u_w)$ is zero; and m_2^s is compressibility of the soil structure when $\Delta(\sigma_3 - u_a)$ is zero.

Letting $R_s = m_2^s/m_1^s$, [7] can be solved for the change in pore-water pressure:

$$[8] \quad \Delta u_w = \left[\frac{R_s - 1) - \dfrac{(1 - S)n + HSn}{(u_a + u_{atm})m_1^s}}{R_s + Sn\beta_w/m_1^s}\right]\Delta u_a + \left[\frac{1}{R_s + Sn\beta_w/m_1^s}\right]\Delta\sigma_3$$

Equation [8] has two unknowns (i.e., u_w and u_a) and, therefore, a second independent equation is required. A second equation can be derived by considering the continuity of the air phase. Volume change due to the compression of the air must equal the volume change described by the constitutive relation for the air phase. The volume change of the air phase due to compression can be written as:

$$[9] \quad \frac{\Delta V_a}{V} = \left[\frac{(1 - S)n + HSn}{(u_a + u_{atm})}\right]\Delta u_a$$

Equation [9] is equated to the constitutive relation for the air phase (Fredlund and Morgenstern 1976):

$$[10] \quad m_1^a\Delta(\sigma_3 - u_a) + m_2^a\Delta(u_a - u_w) = \left[\frac{(1 - S)n + HSn}{(u_a + u_{atm})}\right]\Delta u_a$$

where m_1^a is air phase volume change modulus when $\Delta(u_a - u_w)$ is zero; and m_2^a is air phase volume change modulus when $\Delta(\sigma_3 - u_a)$ is zero.

Letting $R_a = m_2^a/m_1^a$, [10] can be solved for the change in pore-air pressure:

$$[11] \quad \Delta u_a = \left[\frac{R_a}{(R_a - 1) - \dfrac{(1 - S)n + HSn}{(u_a + u_{atm})m_1^a}}\right]\Delta u_w - \left[\frac{1}{(R_a - 1) - \dfrac{(1 - S)n + HSn}{(u_a + u_{atm})m_1^a}}\right]\Delta\sigma_3$$

Equations [8] and [11] must be solved simultaneously for the change in pore-water and pore-air pressures resulting from a change in the applied isotropic stress. These equations can be simplified by defining the following variables:

$$[12] \quad R_1 = \frac{(R_s - 1) - \dfrac{(1 - S)n + HSn}{(u_a + u_{atm})m_1^s}}{R_s + Sn\beta_w/m_1^s}$$

$$[13] \quad R_2 = 1/(R_s + Sn\beta_w/m_1^s)$$

$$[14] \quad R_3 = \frac{R_a}{(R_a - 1) - \dfrac{(1 - S)n + HSn}{(u_a + u_{atm})m_1^a}}$$

and

$$[15] \quad R_4 = \frac{1}{(R_a - 1) - \dfrac{(1 - S)n + HSn}{(u_a + u_{atm})m_1^a}}$$

Equations [8] and [11] then reduce to:

$$[16] \quad \Delta u_w = R_1\Delta u_a + R_2\Delta\sigma_3$$

and

$$[17] \quad \Delta u_a = R_3\Delta u_w - R_4\Delta\sigma_3$$

Equations [16] and [17] are combined and solved to give two pore pressure parameters for unsaturated soils:

$$[18] \quad B_w = \frac{\Delta u_w}{\Delta\sigma_3} = \frac{R_2 - R_1R_4}{1 - R_1R_3}$$

and

$$[19] \quad B_a = \frac{\Delta u_a}{\Delta\sigma_3} = \frac{R_2R_3 - R_4}{1 - R_1R_3}$$

Both [18] and [19] are nonlinear due to an absolute pore-air pressure term on the right side of the expressions. Therefore, the following iterative technique is used for the solution of these equations. Since the induced pore-air pressure is always less than the induced pore-water pressure, the value for the gauge air pressure, u_a, is initially assumed equal to zero in the computation of R_1, R_3, and R_4. These values are used in the initial calculation of the change in pore-air and pore-water pressures. For the second iteration, the pore-air pressure values are assumed to be equal to the previously computed pore-air pressures and are used in the calculation of the new R_1, R_3, and R_4 values. The procedure is continued until there is convergence between the assumed and computed pore-air pressures.

Derivation of the Pore Pressure Parameters (Three-dimensional Loading)

The B_a Pore Pressure Parameter

The volumetric deformation for the air phase is written assuming the soil behaves as an isotropic, homogeneous, elastic soil (Fredlund and Morgenstern 1976):

$$[20] \quad \frac{\Delta V_a}{V} = \frac{1}{3}[m_1^a(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u_a) + m_2^a 3\Delta(u_a - u_w)]$$

If triaxial loading conditions are assumed where σ_2 is equal to σ_3, [20] then becomes:

$$[21] \quad \frac{\Delta V_a}{V} = \frac{1}{3}[m_1^a\Delta(\sigma_1 + 2\sigma_3 - 3u_a) + m_2^a 3\Delta(u_a - u_w)]$$

The compression of the air phase (i.e., [9]) can be substituted into the air phase constitutive relation (i.e., [21]):

$$[22]\quad m_1^{a}\Delta(\sigma_1 + 2\sigma_3 - 3u_a) + m_2^{a}3\Delta(u_a - u_w) = 3\left[\frac{(1-S)n + HSn}{(u_a + u_{atm})}\right]\Delta u_a$$

Re-arranging [22], the change in pore-air pressure can be written as:

$$[23]\quad \Delta u_a = \frac{1}{(R_a - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{a}}} \times \left[R_a\Delta u_w - \Delta\sigma_3 - \frac{1}{3}\Delta(\sigma_1 - \sigma_3)\right]$$

For isotropic loading, the change in $(\sigma_1 - \sigma_3)$ is zero and [23] can be written as:

$$[24]\quad \Delta u_a = \frac{1}{(R_a - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{a}}} \times [R_a\Delta u_w - \Delta\sigma_3]$$

Dividing both sides of [24] by a $\Delta\sigma_3$ gives:

$$[25]\quad \frac{\Delta u_a}{\Delta\sigma_3} = B_a = \left[\frac{R_a}{(R_a - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{a}}}\right]B_w - \frac{1}{(R_a - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{a}}}$$

Equation [25] is the general equation for the pore-air pressure parameter, B_a. If we assume that R_a is equal to zero, the modulus m_2^{a} becomes equal to zero and the constitutive relationship reduces to:

$$[26]\quad \Delta V/V = m_1^{a}\Delta(\sigma_3 - u_a)$$

In this case, the B_a pore pressure parameter reduces to the form proposed by Hilf (1948). When the soil is completely saturated, the percent volume of free air [i.e., $(1 - S)n$] is zero, and B_a approaches 1.

The A$_a$ *Pore Pressure Parameter*

Substituting [25] into [23] and re-arranging gives:

$$[27]\quad \Delta u_a = B_a\Delta\sigma_3 - 3\frac{1}{\left[(R_a - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{a}}\right]}\Delta(\sigma_1 - \sigma_3)$$

Equation [27] can be reduced to:

$$[28]\quad \Delta u_a = B_a\Delta\sigma_3 + A_a\Delta(\sigma_1 - \sigma_3)$$

When the soil is fully saturated, B_a becomes equal to unity, and A_a for an ideal, elastic soil is equal to 1/3. The form of [28] is the same as that proposed by Skempton (1954) for a saturated soil.

The B$_w$ *Pore Pressure Parameter*

The constitutive relation for the soil structure in triaxial loading can be equated to the compressibility of the pore fluid (i.e., [6]):

$$[29]\quad m_1^{s}\Delta(\sigma_1 + 2\sigma_3 - 3u_a) + m_2^{s}\Delta(u_a - u_w) = S\beta_w n\Delta u_w + \left[\frac{(1-S)n + HSn}{(u_a + u_{atm})}\right]\Delta u_a$$

Equation [29] can be re-arranged as follows for isotropic loading:

$$[30]\quad \frac{\Delta u_w}{\Delta\sigma_3} = B_w = \left[\frac{(R_s - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{s}}}{R_s + Sn\beta_w/m_1^{s}}\right] \times B_a + \left[\frac{1}{R_s + Sn\beta_w/m_1^{s}}\right]$$

When the soil becomes fully saturated, R_a equals 1.0, and B_a equals 1.0. Therefore, [30] takes the form of Skempton's (1954) expression for the B pore pressure parameter:

$$[31]\quad B_w = 1/(1 + n\beta_w/m_1^{s})$$

The A$_w$ *Pore Pressure Parameter*

Equation [29] can be rewritten in the following form:

$$[32]\quad \Delta u_w = \left[\frac{(R_s - 1) - \dfrac{(1-S)n + HSn}{(u_a + u_{atm})m_1^{s}}}{R_s + Sn\beta_w/m_1^{s}}\right]\Delta u_a + \left[\frac{1}{R_s + Sn\beta_w/m_1^{s}}\right]\Delta\sigma_3 + 3\left[\frac{1}{R_s + Sn\beta_w/m_1^{s}}\right]\Delta(\sigma_1 - \sigma_3)$$

Substituting [30] into [32] gives:

$$[33]\quad \Delta u_w = B_w\Delta\sigma_3 + \frac{1}{3(R_s + Sn\beta_w/m_1^{s})}\Delta(\sigma_1 - \sigma_3)$$

Equation [33] can be written as:

$$[34]\quad \Delta u_w = B_w\Delta\sigma_3 + A_w\Delta(\sigma_1 - \sigma_3)$$

When the soil becomes fully saturated, B_w equals 1.0 and A_w equals 1/3 for an ideal, elastic soil and

[35] has a similar form to that proposed by Skempton (1954) for saturated soils:

$$[35] \quad \Delta u_w = \Delta\sigma_3 + A\Delta(\sigma_1 - \sigma_3)$$

Relationship of the *B* Pore Pressure Parameters to Hilf's Equation

Hilf (1948) presented the following equation for the estimation of the change in pore-air pressure in an unsaturated soil:

$$[36] \quad \Delta u_a = \frac{u_{ai}\Delta V/V}{(1-S)n + HSn - \Delta V/V}$$

where u_{ai} is initial pore-air pressure (absolute); $(1-S)n$ is volume of free air in the soil; HSn is volume of dissolved air in the soil; and $\Delta V/V$ is volume change from a one-dimensional oedometer test. Equation [36] can be re-arranged and written as follows:

$$[37] \quad \Delta V/V = \frac{\Delta u_a}{(u_{ai} + \Delta u_a)}[(1-S)n + HSn]$$

Hilf (1948) assumed that the interfacial tension between the fluid phases was negligible and that the change in pore-air pressure was equal to the change in pore-water pressure. Hilf also assumed that the volume change of a soil skeleton could be expressed as:

$$[38] \quad \Delta V/V = m_v\Delta(\sigma_1 - u_a)$$

where m_v is coefficient of compressibility for one-dimensional loading.

The modulus, m_v, was measured on the saturation plane where the pore-air pressure was equal to the pore-water pressure. Equations [38] and [37] can be equated and solved for the B_a pore pressure parameter:

$$[39] \quad B_a = \frac{1}{1 + \dfrac{(1-S)n + HSn}{m_v(u_{ai} + \Delta u_a)}}$$

Also inherent in this formulation is the assumption that $\Delta\sigma_1$ from one-dimensional loading is equal to $\Delta\sigma_3$. Let us define C as equal to $(u_{ai} + \Delta u_a)m_v/n$ and substitute it into [29]:

$$[40] \quad B_a = \frac{1}{1 + \dfrac{(1-S) + HS}{C}}$$

Equation [40] gives the pore-air pressure parameter for an unsaturated soil where the interfacial tension between the pore fluids is neglected and the stress – volume change behavior of the soil skeleton is described using a one-dimensional oedometer test.

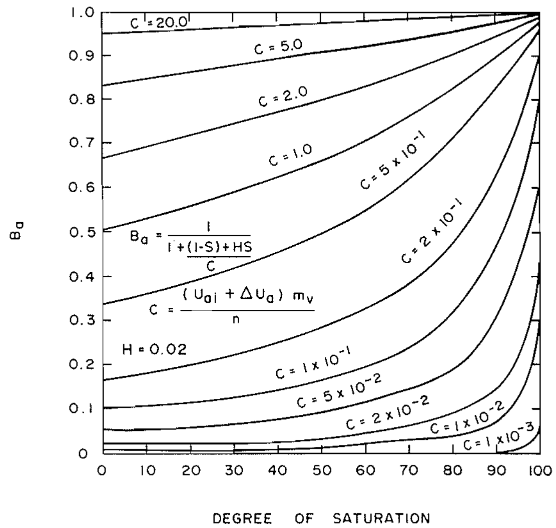

FIG. 2. Graphical solution of Hilf's equation.

Figure 2 presents the solution of Hilf's equation, [40], for various values of the degree of saturation, porosity, pore-air pressure, and compressibility of the soil. Since Hilf assumed that the change in pore-water pressure was equal to a change in pore-air pressure, the B_a pore pressure parameter is equal to B_w. In other words, the difference between a change in pore-water pressure and pore-air pressure could not be separated using Hilf's analysis.

Solution of the B_a and B_w Pore Pressure Parameter Equations

A computer programme, which utilizes an iterative technique, has been used to evaluate the B_a and B_w pore pressure parameters (i.e., [18] and [19]) for various initial conditions. Figures 3 and 4 show the B_w pore pressure parameter for two soil skeleton compressibilities. Figures 5 and 6 show comparable B_a pore pressure parameters for the same soil skeleton compressibilities. (Only two soil skeleton compressibilities have been arbitrarily selected to demonstrate the pore pressure parameter behavior.) All calculations are performed assuming there is sufficient time for free air to be dissolved in water. The magnitudes of B_a and B_w are significantly affected by the compressibility of the soil skeleton. As the compressibility of the soil skeleton increases, the pore pressure response increases and vice versa. The initial degree of saturation also has a marked influence on the amount of pore-air and pore-water pressures generated in an unsaturated soil. When the ratio of the soil skeleton modulus to the air phase modulus (i.e., m_1^s/m_1^a) is equal to unity, both B_a

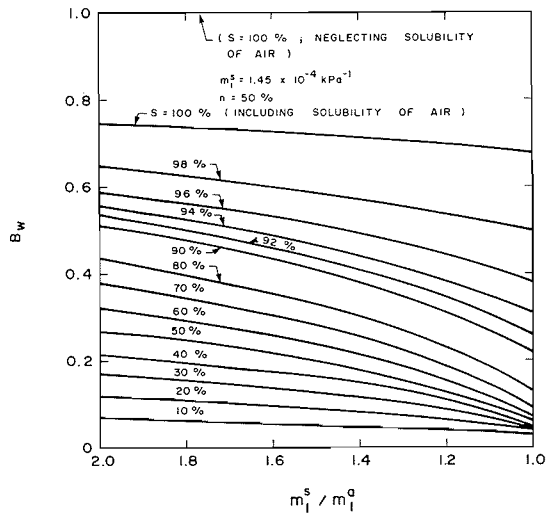

FIG. 3. B_w pore pressure parameter versus $m_1{}^s/m_1{}^a$ ($m_1{}^s$ = 1.45 × 10^{-4} kPa^{-1}).

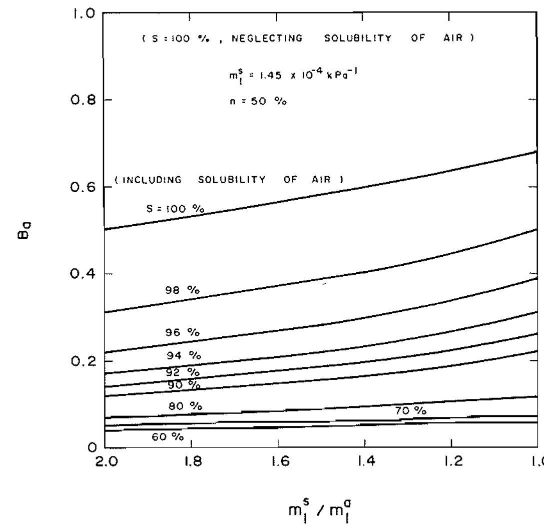

FIG. 5. B_a pore pressure parameter versus $m_1{}^s/m_1{}^a$ ($m_1{}^s$ = 1.45 × 10^{-4} kPa^{-1}).

and B_w are equal for all compressibilities of the soil skeleton and initial degrees of saturation. The values of B_a and B_w are the same as predicted by Hilf's equation when $m_1{}^s/m_1{}^a$ is equal to unity.

Figures 7 and 8 indicate the development of the B_a and B_w pore pressure parameters and the pore-air and pore-water pressures for two soils with arbitrarily selected different compressibilities. The soils are successively loaded under undrained loading conditions. These figures show that with increasing total stress a point is reached where B_a and B_w become essentially equal. As saturation is approached, B_a and B_w tend towards unity. The equations for B_a and B_w become discontinuous when the volume of free air is zero. At this point there is no air to be driven into solution and, therefore, the term *HSn* must be dropped from the equations to obtain B_a and B_w equal to 1 at complete saturation. Figures 7 and 8 show that the shape of the B_a and B_w pore pressure response curves are strongly influenced by the compressibility of the soil.

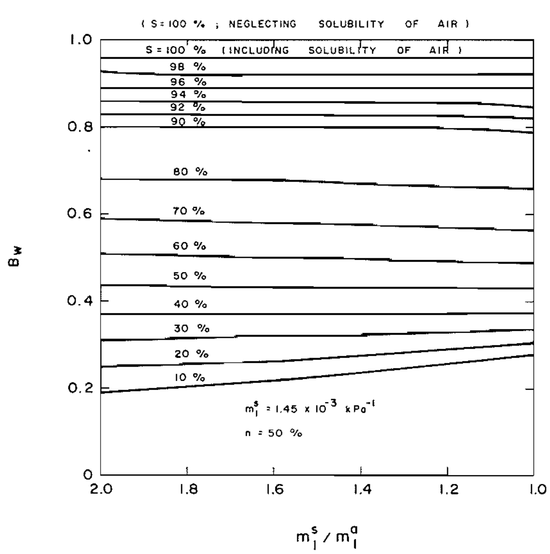

FIG. 4. B_w pore pressure parameter versus $m_1{}^s/m_1{}^a$ ($m_1{}^s$ = 1.45 × 10^{-3} kPa^{-1}).

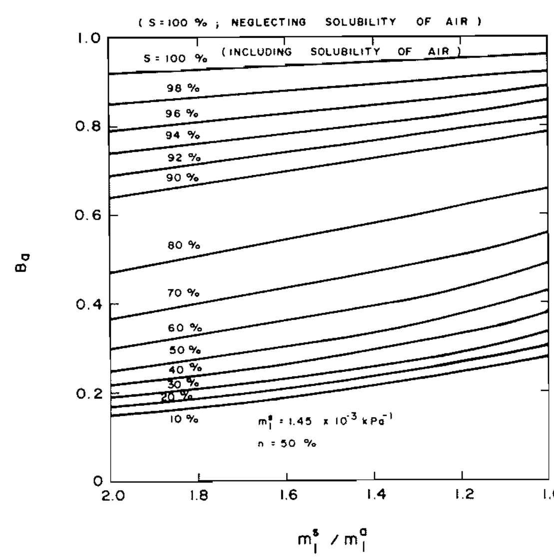

FIG. 6. B_a pore pressure parameter versus $m_1{}^s/m_1{}^a$ ($m_1{}^s$ = 1.45 × 10^{-3} kPa^{-1}).

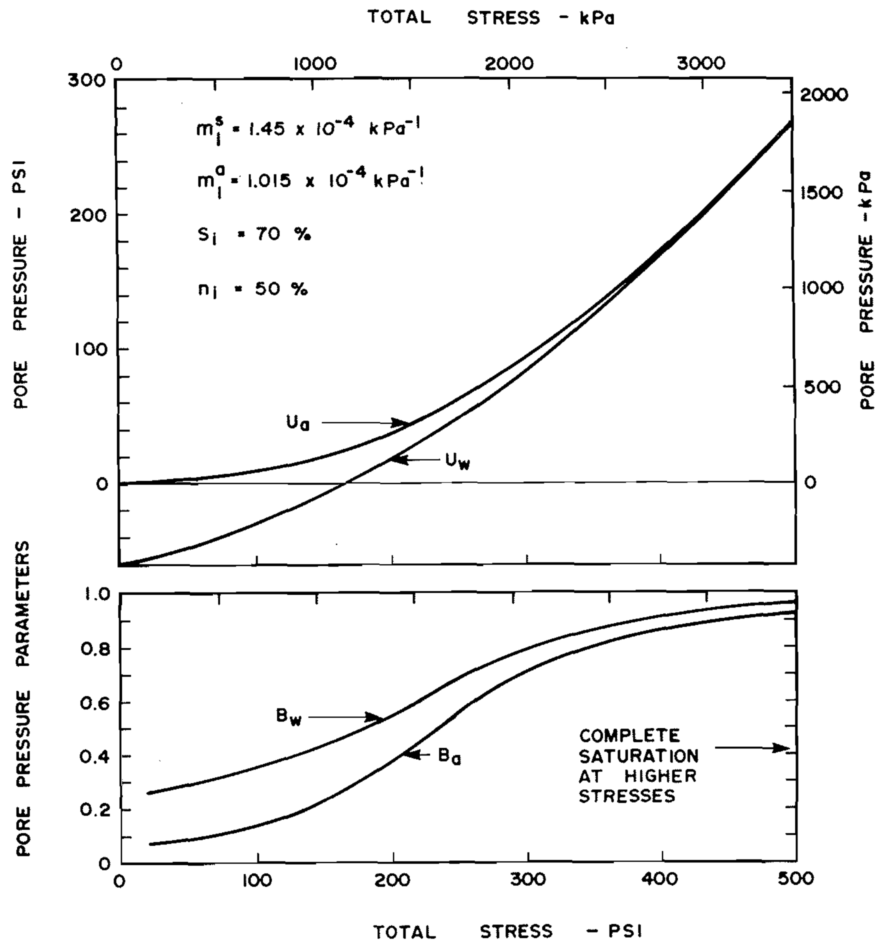

FIG. 7. Total vertical stress versus pore pressure parameters ($m_1^s = 1.45 \times 10^{-4}$ kPa^{-1}).

Experimental Results

Experimental data with respect to pore-air and pore-water pressures have been published by numerous researchers (Hilf 1948; Bishop and Henkel 1962; Gibbs 1963; Penman 1978; Hakimi *et al.* 1973). An attempt is made herein to compare the theoretical estimates of pore-air and pore-water pressures with some of the measured values. In 1948, Hilf published the results of piezometer measurements made in the compacted cores of several earth dams. The piezometers had coarse porous tips and measured the pore-water pressure. Figure 9 shows the readings from two piezometers installed in the Anderson Ranch dam. The published coefficient of permeability of the core material was 5.8×10^{-8} cm/s. The average soil skeleton compressibility modulus was 2.3×10^{-5} kPa^{-1}. A pore-air and (or) pore-water pressure versus total stress line is drawn using Hilf's method of analysis. Equations [18] and [19] for the B_a and B_w pore pressure parameters give the same results as the Hilf's analysis since the m_1^s/m_1^a ratio is assumed equal to unity. When the m_1^s/m_1^a ratio is assumed equal to 1.4, the pore-air and pore-water pressures are no longer the same and the pore-air pressure line lies above Hilf's solution.

Figure 10 shows pore-water pressures measured in an unsaturated soil by Gibbs (1963). A comparison of measured and predicted pore-water pressures is shown. It was necessary to assume soil moduli values in order to solve the B_a and B_w pore pressure parameter equations. The deviation between the measured and predicted values is due to a changing compressibility of the soil with increased loading. In the calculations, a constant soil skeleton compressibility was used throughout the range of loading. By decreasing the soil skeleton compressibility with increased loading it is possible to match the measured pore-water pressures.

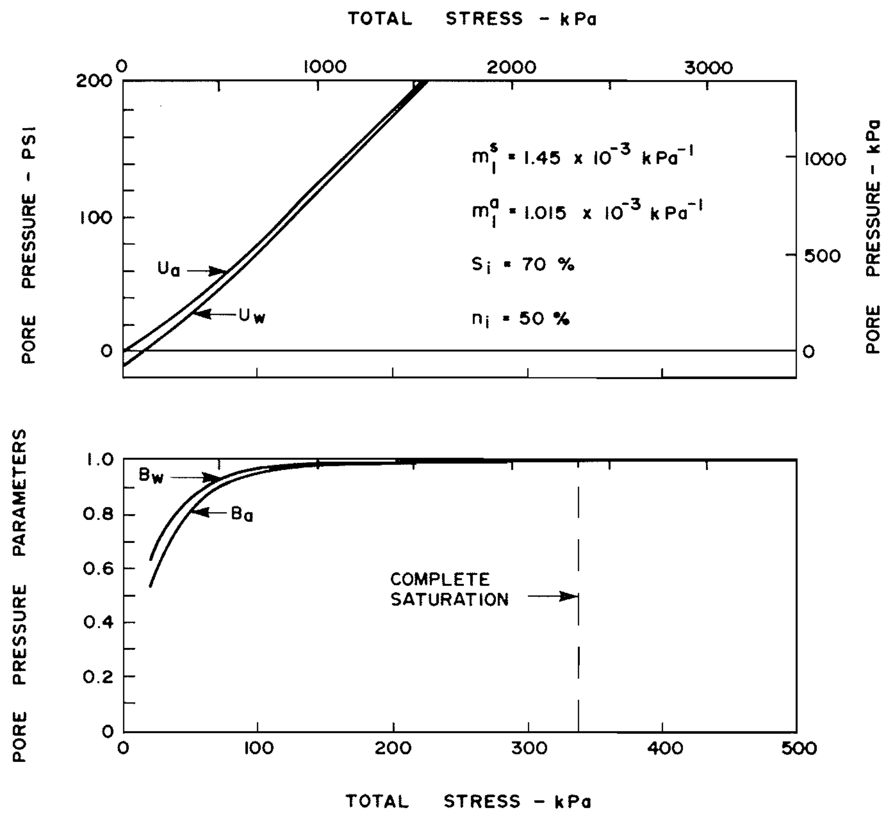

FIG. 8. Total vertical stress versus pore pressure parameters ($m_1^s = 1.45 \times 10^{-3}$ kPa^{-1}).

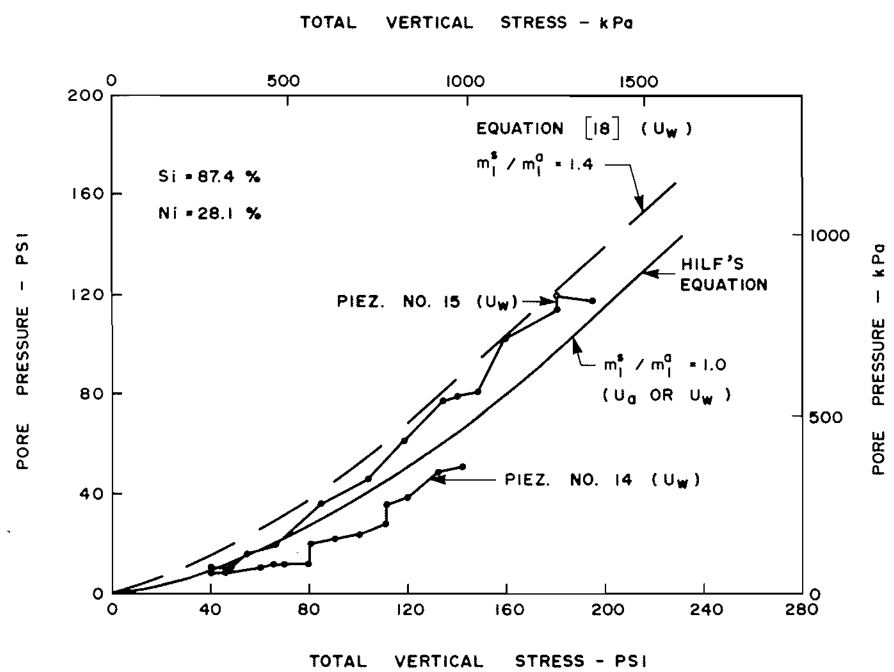

FIG. 9. Measured and predicted pore pressures (after Hilf 1948).

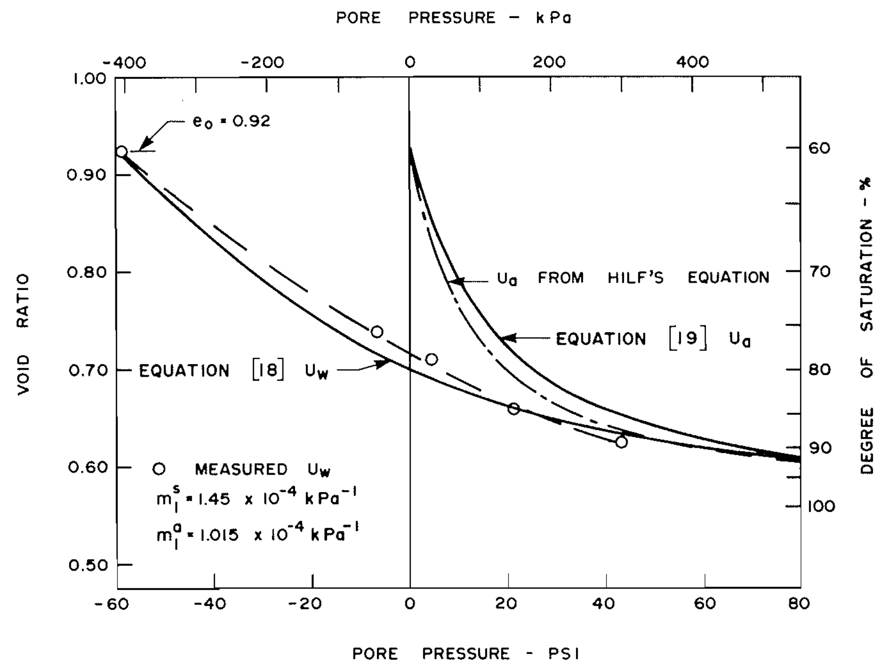

FIG. 10. Measured and predicted pore pressures (after Gibbs 1963).

Figure 11 shows pore-air and pore-water pressures measured by Bishop and Henkel (1962) in a triaxial cell, for two unsaturated soil samples. There is close agreement between the measured and predicted pore pressures. In this case, the compressibility of the soil was decreased at higher confining pressures to improve the agreement between the laboratory measurements and the theoretically evaluated results.

Discussion

The difference between the method of analysis presented by Hilf (1948) and that proposed in this paper is demonstrated in Fig. 12. In a cohesive unsaturated soil, the effect of matrix suction must be taken into account. In this case, the stress–deformation relationship is controlled by two compressibility moduli. The relative magnitudes of the pore-air and pore-water pressures depend primarily upon the relative compressibilities of the pore fluids and the soil skeleton. In cases where the soil has a highly rigid structure and the matrix suction is low, a reasonably good estimate of pore pressure would be

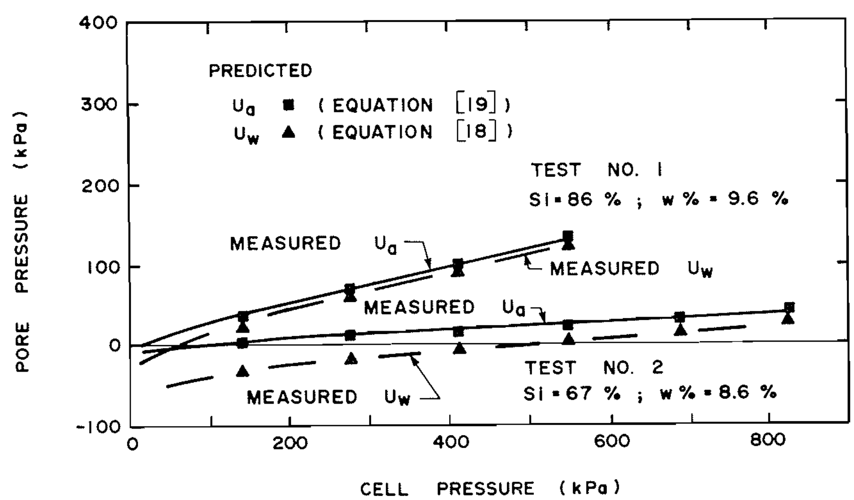

FIG. 11. Measured and predicted pore pressures (after Bishop and Henkel 1962).

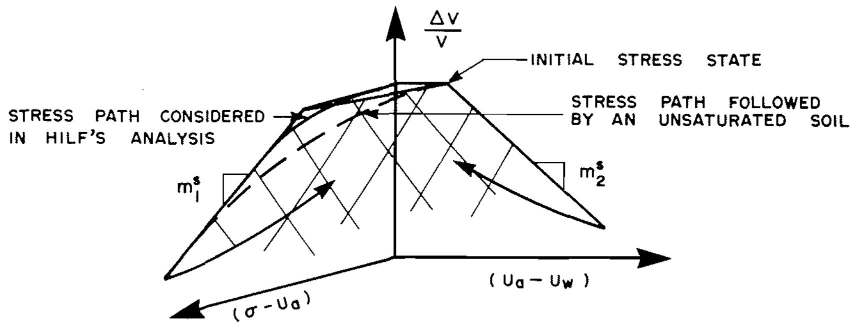

FIG. 12. Stress paths using Hilf's analysis and the proposed pore pressure equations.

anticipated using Hilf's equation. When the soil has greater compressibility and the initial matrix suction is significant, it is suggested that the proposed equations (i.e., [18] and [19]) be used to estimate the independent pore-air and pore-water pressures in response to applied loads.

Summary

(1) The equations for two pore pressure parameters, B_a and B_w, are derived for the pore-air and pore-water pressures in unsaturated soils resulting from undrained, isotropic loading.

(2) Two additional pore pressure parameters, A_a and A_w, are derived to account for pore pressure changes resulting from changes in deviator stresses applied during undrained loading of unsaturated soils.

(3) Predicted pore-air and pore-water pressures compare well with measured values. (It was necessary to estimate the soil compressibility moduli in some cases.)

(4) The pore pressure response of unsaturated soils is dependent upon the degree of saturation, porosity, total stress, and the relative magnitudes of the compressibility moduli of the various phases.

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