DISCUSSION

Consolidation of Unsaturated Soils Including Swelling and Collapse Behavior by Lloret, A. and Alonso, E.E. (1980), Geotechnique 30, No. 4, 449-477.

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The authors should be commended on undertaking the development of a formulation for a difficult problem; namely, the analysis of the consolidation and swelling behavior of an unsaturated. In addition to the formulation, the partial differential equations were solved using the finite element numerical method. It appears, however, that their formulation has been derived without the awareness of similar formulations in other research publications (Hasan and Fredlund, 1977; Fredlund and Hasan, 1979; Dakshanamurthy and Fredlund, 1980). On one hand, this may seem unfortunate, but on the other, the similarity with an already published formulation is encouraging.

An attempt is made herein to briefly comment on significant similarities and differences in the formulations.

1. The starting pivocal point of both formulations is the same description of the stress state. Two independent stress state variables are used to describe the volume change behavior of an unsaturated soil. These are $(\sigma - p_a)$ and $(p_a - p_w)$ where σ = total stress, p_a = pore-air pressure, and p_w = pore-water pressure (Fredlund and Morgenstern, 1977).
2. The continuity requirement used in both formulations is the same. The requirement states that the soil structure volume change of a deformable, unsaturated soil must equal the sum

of the change in volume of the air and water phases due to compression or flow from the element (Fredlund, 1974).

The volumetric requirement can be satisfied through the use of either a referential or spatial type of element. The Lloret and Alonso (1980) formulation uses the latter type of element, while the Fredlund and Hasan (1979) formulation uses the former type of element. Either type of element will yield the same end result.

The continuity requirement also shows that it is necessary to independently satisfy both air and water phase continuity. This, in turn, gives rise to the need for two independent partial differential equations; one for the water phase and one for the air phase. Since the objective of the analysis is to predict the dissipation of the pore-water and pore-air pressures with time, it is necessary to have two equations.

3. Constitutive equations are required for two volume-weight properties. Lloret and Alonso (1980) use the procedure proposed by Matyas and Radhakrishna (1968) which independently relates void ratio and degree of saturation to both of the independent stress state variables (i.e., $(\sigma - p_a)$ and $(p_a - p_w)$). Spline functions are used to approximate the three-dimensional constitutive surfaces.

   Fredlund and Morgenstern (1976) proposed and experimentally verified constitutive equations for an unsaturated soil. The equations were written in the form of the volume change associated with the overall soil structure, the water phase, and the air phase. The general form of the constitutive equation for the strain, ε, for the soil structure is:

$$\varepsilon = \frac{1}{v}\frac{\partial v}{\partial(\sigma - p_a)} d(\sigma - p_a) + \frac{1}{v}\frac{\partial v}{\partial(p_a - p_w)} d(p_a - p_w) \qquad (1)$$

where v = unit volume.

$\frac{1}{v}\frac{\partial v}{\partial(\sigma - p_a)}$ = compressibility of the soil structure when $d(p_a - p_w)$ is zero.

$\frac{1}{v}\frac{\partial v}{\partial(p_a - p_w)}$ = compressibility of the soil structure when $d(\sigma - p_a)$ is zero.

The water phase constitutive relation describes the volume of water in the referential element, θ_w, in relation to the stress state.

$$\theta_w = \frac{1}{v}\frac{\partial v_w}{\partial(\sigma - p_a)} d(\sigma - p_a) + \frac{1}{v}\frac{\partial v_w}{\partial(p_a - p_w)} d(p_a - p_w) \qquad (2)$$

where v_w = volume of water in the element.

$\frac{1}{v}\frac{\partial v_w}{\partial(\sigma - p_a)}$ = slope of the water volume vs. $(\sigma - p_a)$ plot when $d(p_a - p_w)$ is zero.

$\frac{1}{v}\frac{\partial v_w}{\partial(p_a - p_w)}$ = slope of the water volume vs. $(p_a - p_w)$ plot when $d(\sigma - p_a)$ is zero.

The change in volume of air present in an element can be written as the difference between the soil structure volume change and the change in the volume of water present in the element. In other words, it is possible to also write a constitutive equation for the air phase, noting that only two of the three constitutive equations are independent.

The nonlinear compressibilities used in Equations (1) and (2) were assumed to be constants in the consolidation and swelling formulations proposed by Fredlund and Hasan (1979).

4. Darcy's flow law has been used to describe flow in the water phase in both the Lloret and Alonso (1980) formulation and the Fredlund and Hasan (1979) formulation. A similar flow law was used for the air phase in the Lloret and Alonso (loc. cit.) formulation, while the Fredlund and Hasan (loc. cit.) formulation used Fick's law. The two formulations are essentially the same with respect to flow laws.

   The coefficients of air and water permeability were assumed as constants in the Fredlund and Hasan (loc. cit.) formulation. During the solution of the partial differential equations by a numerical technique, the coefficients of permeability could be adjusted as necessary.

5. The two formulations differ on assumptions regarding the solution of air in the water phase. This effect is difficult to accurately model but should not significantly affect the results since the initial and final pore-air pressure will be equal for most practical problems.

6. The effect of external undrained loading has been modelled as an independent change in boundary condition in the Fredlund

and Hasan (1979) formulation. In other words, a change in load induces a change in the pore-air and pore-water pressures which can be described in terms of the pore pressure parameters (Hasan and Fredlund, 1980).

$$B_a = \frac{\Delta p_a}{\Delta \sigma} \tag{3}$$

$$B_w = \frac{\Delta p_w}{\Delta \sigma} \tag{4}$$

where B_a = pore-air pore pressure parameter,

B_w = pore-water pore pressure parameter.

The derivation of the pore pressure parameter requires the equations of compressibility for air-water mixtures (Fredlund, 1976).

The equations associated with the Fredlund and Hasan (1979) formulation are consistent in form with the equations used for the saturated soil consolidation theory and demonstrate a smooth transition from the unsaturated to the saturated state. The final partial differential equation for the pore-water phase is:

$$\partial p_w/\partial t = -C_w(\partial p_a/\partial t) + c_v^w(\partial^2 p_w/\partial y^2) \tag{5}$$

where C_w = an interactive constant associated with the water phase equation. It consists of a combination of the compressibility moduli for the unsaturated soil,

c_v^w = coefficient of consolidation for the water phase,

t = time,

y = vertical spatial coordinate.

The partial differential equation for the pore-air phase is:

$$\partial p_a/\partial t = -C_a(\partial p_w/\partial t) + c_v^a(\partial^2 p_a/\partial y^2) \quad (6)$$

where C_a = an interactive constant associated with the air phase equation. It consists of compressibility moduli, pore-air pressure and volume weight properties,

c_v^a = coefficient of consolidation with respect to the air phase. Both C_a and c_v^a contain a pore-air pressure term, and therefore, incorporate some nonlinearity.

Equations (5) and (6) must be solved simultaneously to give the dissipation of pore-air and pore-water pressure with time. The solution was obtained using a finite difference technique. The above equations are formulated for one-dimensional conditions; however, all the physical relations are available to extend the formulation to two-dimensional conditions (Fredlund, 1979; Dakshanamurthy and Fredlund, 1980).

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