The Prediction of Total Heave

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ABSTRACT

Numerous methods have been proposed in various geographic regions for the prediction of total heave. This paper provides a theoretical context within which the various methods can be visualized with respect to the stress path followed.

The effect of sampling disturbance is isolated as a significant factor affecting the prediction of heave. An empirical procedure is proposed to compensate for the effect of sampling disturbance. The determination of the insitu state of stress from the constant volume oedometer test is also described. This information is used in a method proposed in this paper for the prediction of total heave.

INTRODUCTION

Engineers are well aware of the severe distress that lightly loaded structures can suffer when placed on a swelling soil that is subjected to environmental changes. The prediction of heave of such structures has probably received more attention than any other analysis associated with swelling soils. As a result, numerous methods have been proposed and utilized for the prediction of total heave. Table 1 lists most of the proposed analytical procedures for predicting heave (McKeen, 1976) along with their country of origin. Each method for predicting heave has been used in a restricted geographical region and little attempt has been made to embrace the various procedures within one consistent theoretical context.

The objectives of this paper are as follows:

i) to formulate a general one-dimensional, theoretical frame-work that will embrace all methods of heave analysis and thereby allow their comparison.

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ii) to review some of the methods of heave analysis within the proposed theoretical context and demonstrate the stress paths followed.

iii) to propose another method for the prediction of heave. The method is based on one-dimensional oedometer tests and differs from previous procedures primarily in the manner of interpretation of the data.

Table 1

Proposed Methods of Predicting Heave

Name of Method	Reference		
	Year	Author	Country
METHODS UTILIZING THE OEDOMETER			
1. Double Oedometer Method	1957	Jennings, J.E. and Knight, K.	South Africa
	1969	Jennings, J.E.B.	
2. Salas and Serratosa Method	1957	Salas, J.A.J. and Serratos, J.M.	Spain
3. Volumeter Method	1961	DeBruijn, C.M.A.	South Africa
4. Mississippi Method	1962	Clisby, M.B.	U.S.A.
	1972	Teng, T.C., Matlox, R.M. and Clisby, M.B.	
	1973	Teng, T.C., Matlox, R.M. and Clisby, M.B.	
	1975	Teng, T.C., Matlox, R.M. and Clisby, M.B.	
5. Sampson, Schuster and Budge's Method	1965	Sampson, E., Schuster, R.L. and Budge, W.D.	U.S.A.
6. Noble Method	1966	Noble, C.A.	Canada
7. Sullivan and McCelland Method	1969	Sullivan, R.A. and McCelland, B.	U.S.A.
8. Holtz Method	1970	Holtz, W.G.	U.S.A.
9. Navy Method	1971	NAVFAC	U.S.A.
10. Direct Model Method (Texas Highway.Department)	1973	Smith, A.W.	U.S.A.
11. Simple Oedometer Method	1973	Jennings, J.E., Firth R.A., Ralph, T.K. and Nager, N.	South Africa
12. USBR Method	1973	Gibbs, H.J.	U.S.A.
OTHER METHODS			
1. McDowell Method	1956	McDowell, C.	U.S.A.
2. Van Der Merwe Method	1964	Van Der Merwe, D.H.	South Africa
3. Richards Method	1967	Richards, B.G.	Australia
4. Australian Method	1969	Aitchison, G.D. and Woodburn, J.A.	Australia
5. Corps of Engineers Method	1974	Johnson, L.D.	U.S.A.
	1977	Snethen, D.R., Johnson, L.D. and Patrick, D.M.	
	1977	Johnson, L.D.	
	1978	Johnson, L.D.	
	1978	Johnson, L.D. and Snethen, D.R.	

THEORETICAL CONSIDERATIONS

Two independent stress variables are required to describe the state of stress in an unsaturated soil (Fredlund and Morgenstern, 1977). The preferrable stress state variables are $(\sigma - u_a)$ and $(u_a - u_w)$ where σ = total stress, u_a = pore-air pressure, and u_w = pore-water pressure. $(\sigma - u_a)$ is herein called the "total stress term" and $(u_a - u_w)$ is called the "matrix suction term". Although other combinations are feasible for the stress state variables, this combination is most satisfactory since the effects of externally applied loads and the effects of environmental changes can readily by separated in terms of stress changes. The pore-air pressure in the prediction of heave analysis will be approximately atmospheric. The assumption is made that the osmotic suction of the soil remains constant or else is simulated between the laboratory and the insitu conditions. In either case it will not appear in the analysis.

The continuity requirement that must be satisifed for an unsaturated soil is as follows:

$$\frac{\Delta V}{V} = \theta_w + \theta_a \tag{1}$$

where: V = volume of a referential element.
θ_w = change in the amount of water (by volume) in the referential element.
θ_a = change in the amount of air (by volume) in the referential element.

The continuity requirement demonstrates that constitutive relations are required for at least two of the volume change variables. The change in the volume of the referential element (i.e., strain of the soil structure) can be written as the sum of the orthogonal linear strains (Fredlund, 1973).

$$\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{2}$$

where: $\varepsilon_x, \varepsilon_y, \varepsilon_z$ = strain in the x, y, and z directions.

The linear strains can be linked to the stress state variables in a semi-empirical manner to give suitable constitutive equations. The equations are an extension of the elasticity formulation used for saturated soils (Fredlund and Morgenstern, 1976).

$$\varepsilon_x = \frac{(\sigma_x - u_a)}{E_1} - \frac{\mu_1}{E_1}(\sigma_y + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_1} \tag{3a}$$

$$\varepsilon_y = \frac{(\sigma_y - u_a)}{E_1} - \frac{\mu_1}{E_1}(\sigma_x + \sigma_z - 2u_a) + \frac{(u_a - u_w)}{H_1} \tag{3b}$$

$$\varepsilon_z = \frac{(\sigma_z - u_a)}{E_1} - \frac{\mu_1}{E_1}(\sigma_x + \sigma_y - 2u_a) + \frac{(u_a - u_w)}{H_1} \tag{3c}$$

where: E_1 = Young's modulus with respect to $(\sigma - u_a)$,
μ_1 = Poisson's ratio and
H_1 = elastic modulus with respect to $(u_a - u_w)$.

The constitutive equation for the water phase can be written:

$$\theta_w = \frac{(\sigma_x + \sigma_y + \sigma_z - 3u_a)}{3H_1'} + \frac{(u_a - u_w)}{R_1} \tag{4}$$

where: H_1' = water phase modulus with respect to $(\sigma - u_a)$, and
R_1 = water phase modulus with respect to $(u_a - u_w)$.

The proposed constitutive equations have been experimentally tested for uniqueness near a point (Fredlund and Morgenstern, 1976) and for uniqueness when larger stress increments are used (Matyas and Radhakrishna, 1968; Barden et al, 1969). The results indicate uniqueness as long as the deformation conditions are monotonic. This means that the soil parameters must change for increases and decreases in their respective volumes.

EXAMPLE PROBLEM

Let us consider the construction of a light residential building on desiccated, unsaturated soil (Figure 1). Prior to construction the

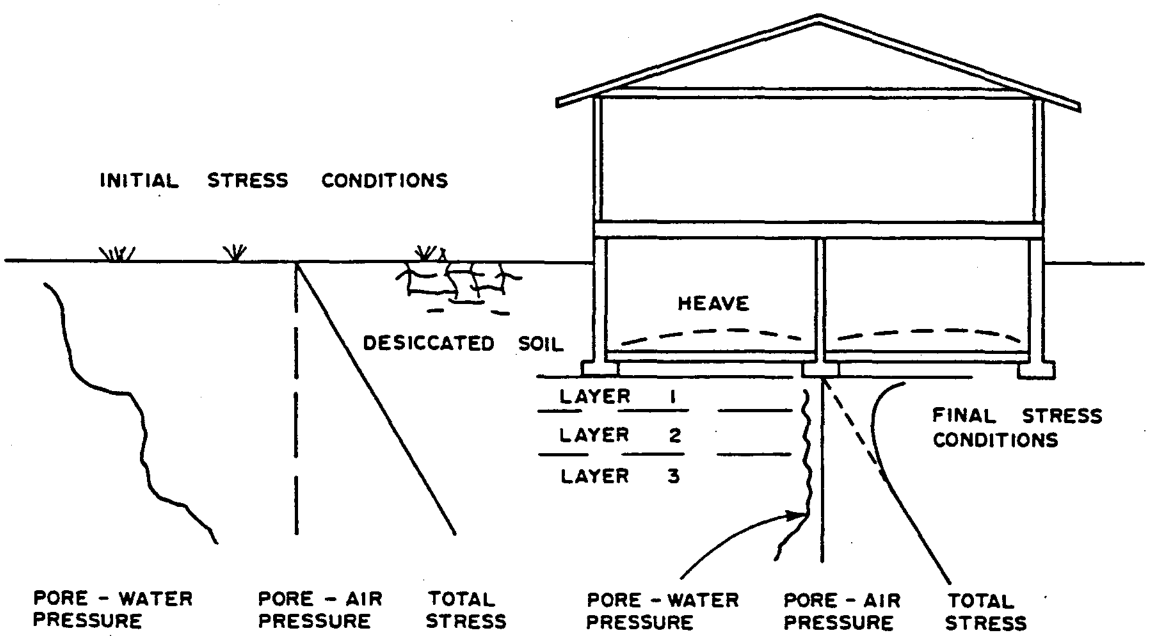

Figure 1 Initial and Final Stresses Below a Light Residential Building

pore-water will be in a state of tension due to evaporation and evapotranspiration at the ground surface. The unloading due to excavation will further induce tension in the water phase. The pore-air pressure will be approximately equal to atmospheric conditions. The construction of the house essentially curtails moisture movement away from the region below the basement floor slab. With time, water accumulates below the house resulting in a decrease in the pore-water tension and subsequent heaving of the house.

An analysis for the prediction of heave must utilize the constitutive equations to relate the initial and final stress states. In fact, all methods for the prediction of heave can be viewed in terms of i) the procedure used to evaluate the initial state of stress, ii) the manner in which the final state of stress is estimated and iii) the constitutive model used to relate the initial and final stress states.

DERIVATION OF THE ONE-DIMENSIONAL HEAVE ANALYSIS

Let us consider an element of an unsaturated soil from below the house basement. The strain in the y-direction (i.e., vertical direction) is given by equation (3b). For the one-dimensional case, the strains in the 'x' and 'y' directions are zero while their changes in total stress are equal. The $(\sigma_x - u_a)$ and $(\sigma_z - u_a)$ terms can be computed by equating equations (3a) and (3c). (5)

$$(\sigma_x - u_a) = (\sigma_z - u_a) = \frac{\mu_1}{(1 - \mu_1)} (\sigma_y - u_a) - \frac{E_1}{(1 - \mu_1)} (u_a - u_w)$$

Substituting equation (5) into equation (3b) gives,

$$\varepsilon_y = m_1^s \, d(\sigma_y - u_a) + m_2^s \, d(u_a - u_w) \tag{6}$$

where: $m_1^s = \frac{1}{E_1} - \frac{(1 + \mu_1)(1 - 2\mu_1)}{E_1(1 - \mu_1)}$; soil structure compressibility modulus associated with a change in $(\sigma_y - u_a)$,

$m_2^s = \frac{1 + \mu_1}{H_1(1 - \mu_1)}$; soil structure compressibility associated with a change in $(u_a - u_w)$.

It is often more convenient to use the change in void ratio, Δe, rather than strain. In this case,

$$\Delta e = \varepsilon_y \, (1 + e_o) \tag{7}$$

where: e_o = initial void ratio.

The amount of heave associated with each arbitrary layer (i.e., the i-th layer; Figure 1) is computed as the vertical strain, ε_y, multiplied by the thickness of the layer, h_i. The total heave for each layer,

Δh_i is,

$$\Delta h_i = h_i[m_1^s \, d(\sigma_y - u_a) + m_2^s \, d(u_a - u_w)] \tag{8}$$

The stress state variable changes must be computed for each layer from measured, assumed or computed initial and final stress boundary conditions. Appropriate moduli must be ascribed for each layer. The total heave is equal to the sum of the heaves computed for each layer.

$$\Delta h = \sum_{i=j}^{i=1} \Delta h_i \tag{9}$$

A change in the volume of water in the referential element can be computed on the basis of the assumptions associated with one-dimensional strain.

$$\theta_w = m_1^w \, d(\sigma_y - u_a) + m_2^w \, d(u_a - u_w) \tag{10}$$

where: $m_1^w = \frac{1 + \mu_1}{3H_1' \, (1 - \mu_1)}$; slope of the $(\sigma_y - u_a)$ versus water volume plot when $d(u_a - u_w)$ is zero.

$m_2^w = \frac{3(1 - \mu_1)H_1'H_1 - 2E_1R_1}{3(H_1'H_1R_1 \, (1 - \mu))}$; slope of the $(u_a - u_w)$ versus water volume plot when $d(\sigma_y - u_a)$ is zero.

The change in volume of water in the soil can be written as a change in water content as follows:

$$w = \frac{\theta_w \, (1 + e_o)}{G_s} \tag{11}$$

where: w = water content and
G_s = specific gravity

The degree of saturation, as well as all other volume-weight relations can now be computed.

A total of four compressibility moduli have been defined to completely describe the volume-weight soil properties under any set of stress conditions. However, in general this can be reduced. When predicting total heave, it is not necessary to compute change in water content. Therefore, only the m_1^s and m_2^s moduli are required. In addition, if changes in total stress are negligible, only the m_2^s modulus is required. It should also be noted that the final water contents can be estimated since the final degree of saturation will approach 100 percent (Fredlund, 1979).

$$\Delta w = S_f \Delta e / G_s - e_o \Delta S / G_s \tag{12}$$

where: S_f = final degree of saturation
ΔS = change in degree of saturation.

Equations (6) and (10) can be visualized as two three-dimensional surfaces (Figure 2). The surfaces differ for loading and unloading conditions, and for increasing and decreasing matrix suction.

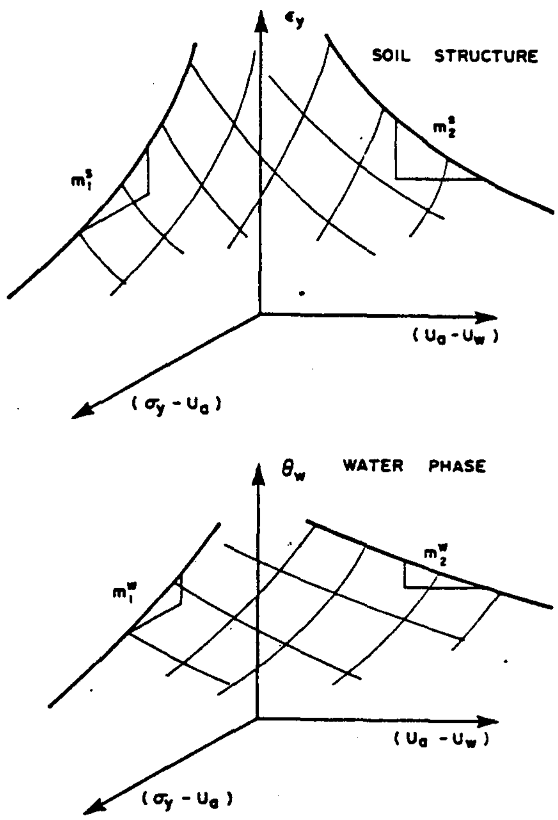

Figure 2 Constitutive Surfaces for an Unsaturated Soil

INITIAL AND FINAL BOUNDARY STRESS CONDITIONS

In order to perform a heave analysis, the initial and final stress conditions must be known. The initial total stress, σ_y, can be computed using conventional total stress theory. The initial pore-air pressure can be estimated as equal to atmospheric pressure. The initial pore-water pressure is highly negative and difficult to evaluate. Attempts have been made to measure the pore-water pressure insitu using devices such as tensiometers and psychrometers. Tensiometers have the disadvantage that their range is limited to one atmosphere negative. Psychrometers have the limitation that the soil must have a relatively high pore-water tension and be in an equilibrium temperature condition (Krahn, 1970). As well, the measured value is total suction and must be corrected for osmotic suction unless the soil moduli are measured in terms of total suction.

Axis-translation, null type tests can be performed in the laboratory on undisturbed samples. However, if these results are used to estimate the insitu pore-water pressure it is necessary to correct the measured value for changes in pore-water pressure due to unloading and sample disturbance.

One-dimensional oedometer tests can be performed on undisturbed samples to estimate the pore-water pressure. The test procedures used are variable and subjected to diverse interpretations. In particular, the effects of sample disturbance must be taken into consideration and it must be realized that measurements are being performed on the total stress plane.

The final total stress can be computed using the elastic stress theory. The final pore-air pressure is atmospheric. The final pore-water pressure conditions must be estimated. One of three possibilities provide the most logical assumption for the final pore-water pressure (Figure 3). First, it can be assumed that the water table will rise to the surface, creating a hydrostatic condition. This assumption produces the greatest heave prediction. Second, it can be assumed that the pore-water pressure approaches zero throughout its depth. This may be a realistic assumption; however, it is not an equilibrium condition. Third, it can be assumed that under long-term equilibrium conditions the pore-water pressure will remain slightly negative. This assumption produces the smallest prediction of heave. Any one of the above three assumptions produce relatively similar predictions of heave in most cases. This is due to the fact that most of the heave occurs in the uppermost soil layers where the change in matrix suction is largest and the soil modulus is largest. The choice of a final pore-water pressure boundary condition could vary from one geographic location to another depending upon the climate conditions.

VISUALIZATION OF PROCEDURES FOR HEAVE ANALYSIS

Each of the procedures for predicting heave mentioned in Table 1 can be visualized in terms of the stress paths followed on the constitutive surfaces. Only a few of the methods will be used to demonstrate the stress path followed.

Direct Model Method

The Direct Model Method is based on free-swell consolidation tests on undisturbed samples placed in an oedometer (Figure 4). The samples are subjected to the overburden pressure (or the load that will exist at the end of construction) and allowed free access to water. The predicted heaves are generally significantly below actual field heaves. The stress path followed by the test procedure is shown in Figure 4b. The conventional two-dimensional manner for plotting the test data is shown in Figure 4a. The under-estimation of heave appears to be primarily due to sampling disturbance.

Sullivan and McClelland Consolidometer Method

The prediction of heave is based on a constant volume oedometer

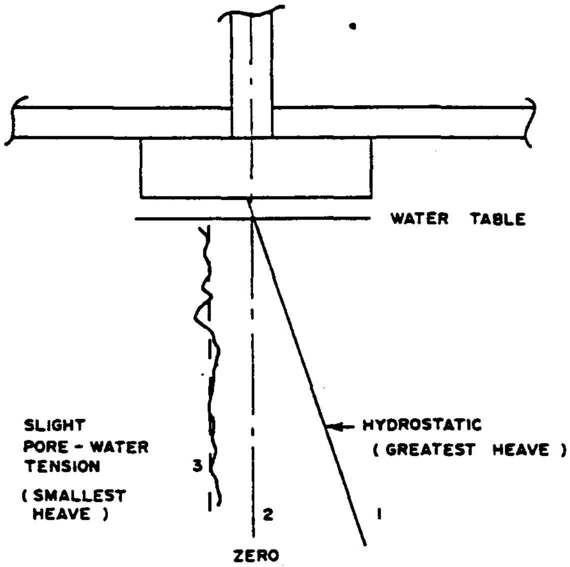

Figure 3 Final Pore-Water Pressure Boundary Conditions

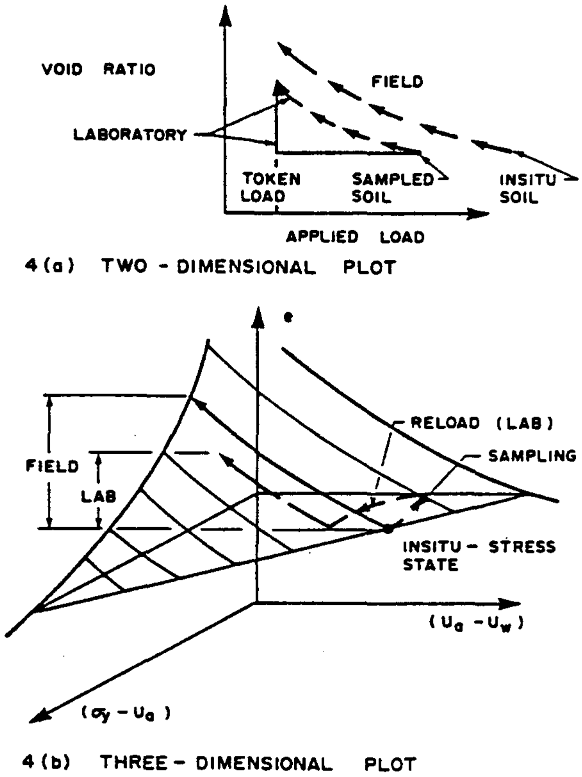

Figure 4 Stress Path for the Direct Model Method

test on an undisturbed sample initially subjected to the overburden pressure. Once the swelling pressure has been reached, the sample is rebounded. The stress path followed is shown in Figure 5. There are limited case histories published but in general the authors would expect this method to under-estimate heave since sample disturbance has not been taken into account.

Richard's Method

Richard's method is based on a laboratory suction test which yields a water content versus matrix suction curve. The suction measured at the 10 foot depth is used as the final equilibrium suction. A change in water content is computed which is empirically converted to a change in volume (Figure 6). The authors are not aware of substantiating case histories.

Jennings and Knights' Double Oedometer Method

This method is based on the two oedometer tests; namely, a free-swell oedometer test that is initially under a token load of 0.01 ton per square foot (1. kPa) and a natural water content oedometer test. The natural water content oedometer test data are adjusted vertically to match the free-swell test results at high applied loads. Various loading conditions and final pore-water pressures can be simulated in the analysis. In general, the analysis shows over-prediction of the actual heave (Jennings and Knight, 1957). The stress paths followed by the two tests are shown on Figure 7. The authors suggest that the

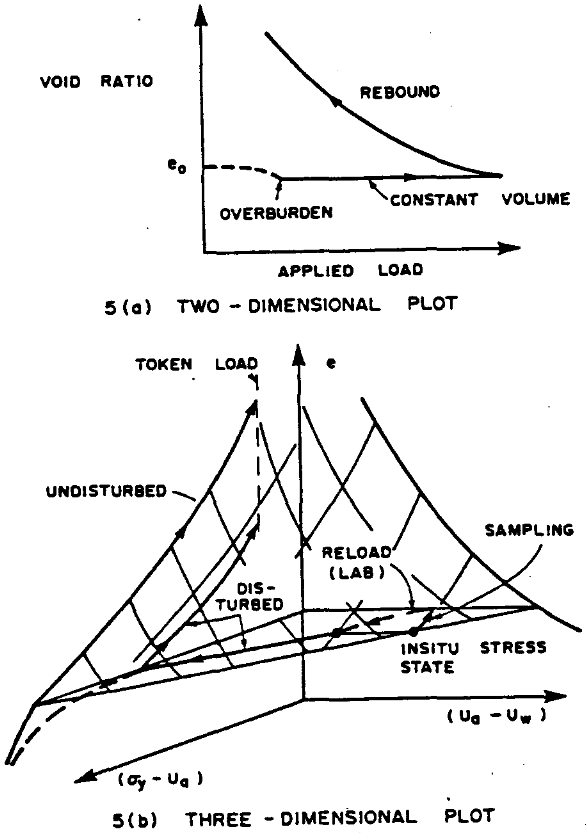

Figure 5 Stress Path for the Sullivan and McClelland Method

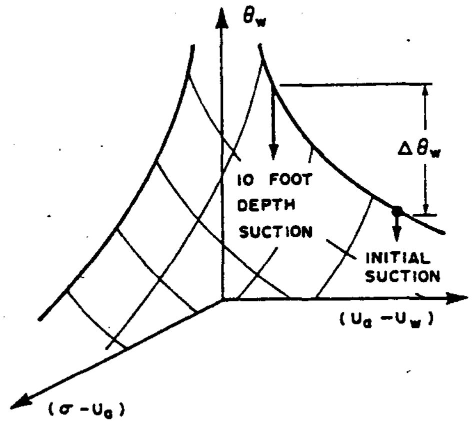

Figure 6 Stress Path for Richard's Method

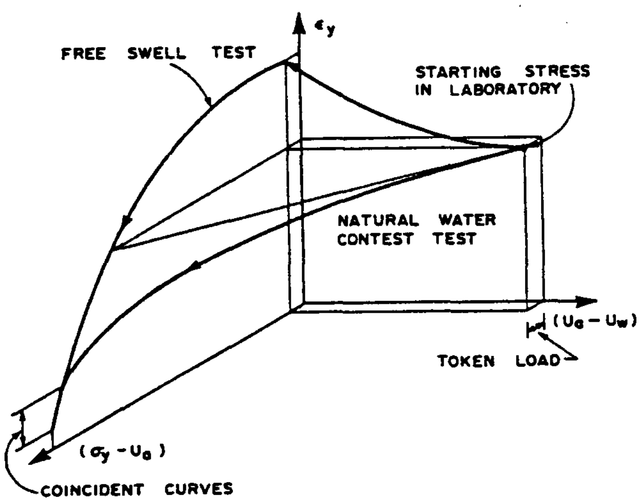

Figure 7 Stress Paths for Double Oedometer Test (Jennings and Knight, 1957)

predicted heave is generally satisfactory since the method of analyzing the data compensates for the effects of sample disturbance. In other words, the natural water content curve demonstrates the effect of sampling disturbance. The stress paths of more recent, up-date versions of the Double Oedometer method can be visualized on similar three-dimensional plots.

PROPOSED METHOD ON HEAVE ANALYSIS

The procedure proposed for the prediction of heave is based on constant volume oedometer tests performed on undisturbed samples. The data required from the laboratory test are the rebound modulus (and sometimes the loading modulus) and an estimate of the insitu pore-water pressure. The rebound modulus is relatively insensitive to sample disturbance (Schmertmann,1955). The main problem is to estimate the insitu pore-water pressure from the deformation characteristics measured in the laboratory. Prior to the interpretation of the laboratory results, the data should be corrected for the effects of compressibility of apparatus (Fredlund, 1969).

The constant volume oedometer tests are performed on samples that are at least 2-1/2 inch diameter, taken from the upper portion of the soil profile. The samples are placed in an oedometer with an initial load approximately equal to the existing overburden load. After initial dial readings are taken, the sample is immersed in water. As the sample attempts to swell, increased load is applied to the soil in order to maintain its initial volume. At some point the sample has no further tendency to swell under the applied load. This applied load is called the swelling pressure of the soil, P_s. The sample is then loaded on a logarithmic scale up to approximately sixteen tons per square foot (1700 kPa). The sample is subsequently rebounded in double increments, to a token load.

The data is commonly plotted as a total pressure versus void ratio curve but the actual stress path can be more easily visualized on a three-dimensional plot using the stress state variables as abscissas. Figure 8 shows an idealized interpretation of the constant volume oedometer results where sampling is assumed to not produce disturbance to the soil. However, sampling does cause disturbance and the actual stress path is shown in Figure 9. The net result of sample disturbance is a reduction in the swelling pressure, P_s. Therefore, the laboratory swelling pressure cannot be directly related to the insitu stress state.

No procedure has previously been proposed to correct the swelling pressure for the effect of sampling disturbance. However, the procedures commonly used to estimate the pre-consolidation pressure of a saturated soil are actually procedures which account for the effects of sample disturbance (Leonards, 1962). Figure 10 shows a modified form of the Casagrande's construction that is proposed to compensate for the effect of sample disturbance in desiccated soils.

The construction procedure involves first locating the point of maximum curvature on the void ratio versus total pressure curve. The point of concern is the maximum curvature immediately past the swelling

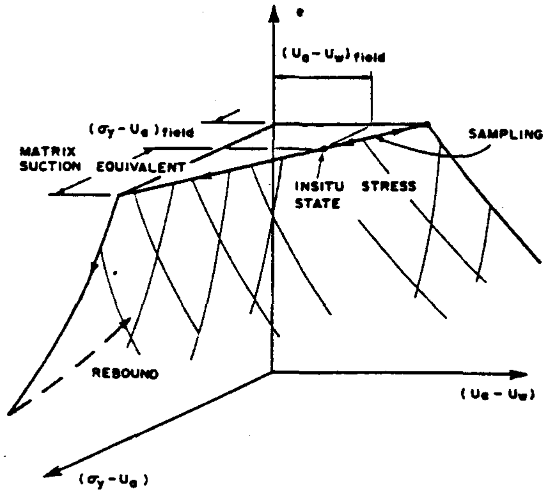

Figure 8 Ideal Interpretation of and Constant Volume Oedometer Test

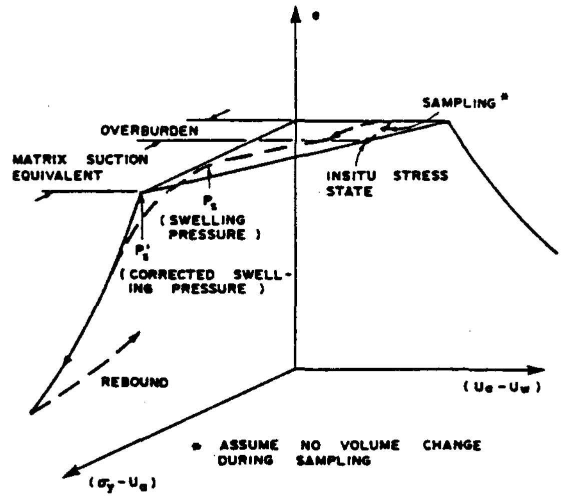

Figure 9 Actual Stress Path During Sampling and Constant Volume Oedometer Test

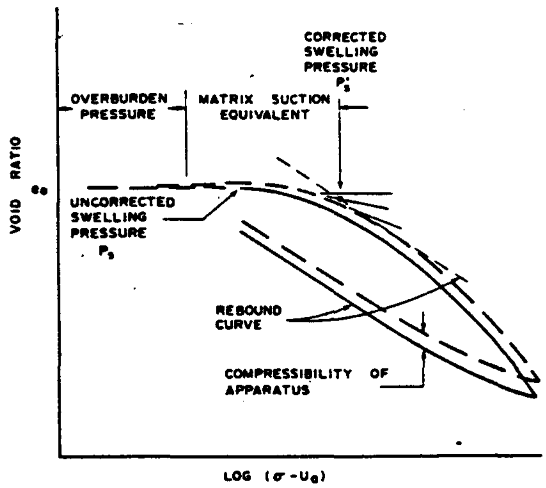

Figure 10 Construction to Compensate for Sample Disturbance

pressure. On some laboratory test results it is difficult to determine this point but in general it is readily discernable. Horizontal, tangential and bisector lines are drawn from this point. Then a line parallel to the rebound curve is drawn tangent to the recompression curve. The intersection of this line with the bisector gives an abscissa that approximates the undisturbed state of stress. This point is designated as the corrected swelling pressure, P_s'. Figure 11 shows uncorrected and corrected swelling pressure results obtained on samples of compacted Regina clay. Also shown are the results of a series of free-swell oedometer tests performed on the same soil (Fredlund, 1975). The swelling pressure from the corrected constant volume oedometer tests are slightly greater than those from the free-swell oedometer tests.

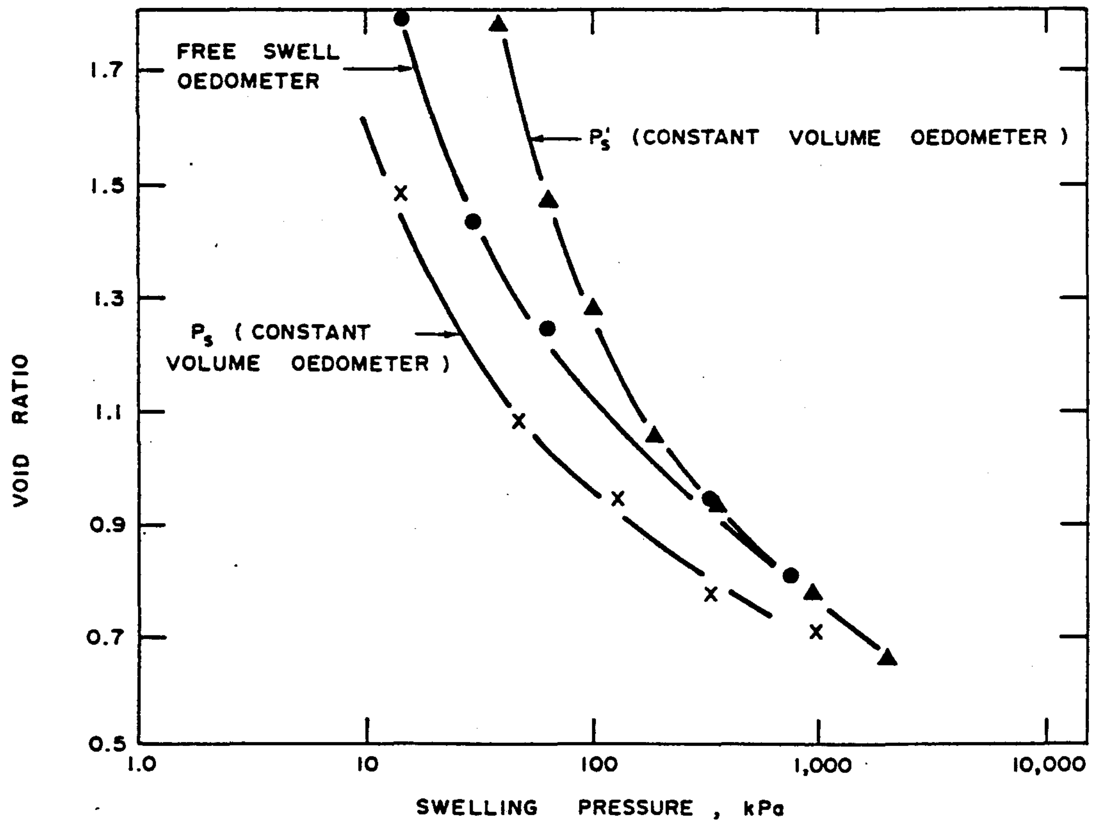

Figure 11 Void Ratio Versus Swelling Pressure for Compacted Regina Clay

The magnitude of the corrected swelling pressure, P_s', can be subdivided into two parts. These are the overburden pressure, and the matrix suction equivalent (i.e., $(u_a - u_w)_L$) on the total stress plane. The matrix suction equivalent is not equal in magnitude to the insitu matrix suction. However, since the soil modulus is measured on the total stress plane (i.e., $(\sigma - u_a)$ plane), the matrix suction equivalent can be used in an analysis to predict heave (Figure 12).

The corrections for sampling disturbance significantly affect the prediction of heave (Fredlund, 1975). The predicted heave using corrected swelling pressure values may be twice the predicted heaves when the correction is not applied. It is noteworthy that attempts to use constant volume oedometer tests without accounting for the effect of sampling disturbance corrections, result in heave predictions that are generally low. The use of free-swell oedometer tests has produced greater predicted heave values which appear to be more consistent with observed heave measurements. The reason being that the free-swell swelling pressure value incorporates hysteresis which gives the effect of overcoming sample disturbance. However, the authors do not advocate the use of the free-swell oedometer test due to the hysteretic effects incorporated in the measurement of swelling pressure.

The presentation of detail case histories to support the above procedure are beyond the scope of this paper. Numerous case histories

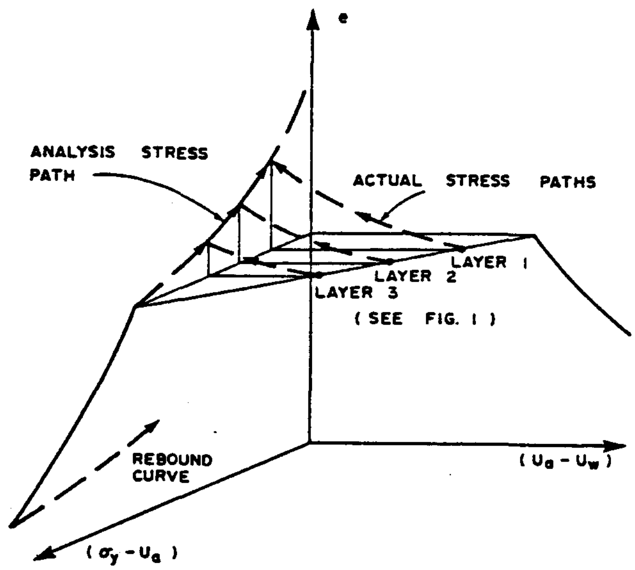

Figure 12 Comparison of the Actual Stress Path and the Stress Path Used in the Analysis

have been analyzed and in general the predicted heaves slightly exceed the measured heaves. The interpretation of the stress conditions statistically analyzed for undisturbed Regina clay (Fredlund et al, 1980) also support the proposed procedure for predicting heave.

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