Proceedings of the Fourth International Conference on Expansive Soils, Vol. 1, pp. 514-532, Denver, Colorado. 1980

Moisture and Air Flow in an Unsaturated Soil

V. Dakshanamurthy*
and
D. G. Fredlund**

ABSTRACT

A theoretical model is presented to predict moisture and air flow in an unsaturated soil under non-isothermal conditions. The model takes into account the complete constitutive relations for an unsaturated soil Two partial differential equations are derived; one for the water phase and the other for the air phase. These two equations are solved simultaneously and the solutions give the pore-air and pore-water pressures under transient flow conditions. In addition, a partial differential heat flow equation is solved and the corresponding pore-air and pore-water pressures are adjusted to account for temperature changes.

INTRODUCTION

The behavior of both natural and compacted soils near ground surface is often most easily understood in terms of changes in natural water content. Therefore, the prediction of moisture and air flow (and associated water vapor flow) under transient conditions is becoming increasingly important to geotechnical engineers concerned with the design of highway and airfield pavements. The prediction of moisture content for any given climate and topographic location involves many complex variables and research work in this area has largely remained semi-empirical with serious approximations.

Since the inception of modern soil mechanics, Terzaghi's one-dimensional consolidation theory for saturated soils has formed an extremely useful conceptual framework in geotechnical engineering. Unfortunately the study of the behavior of unsaturated soils has taken place in the absense of a similar theoretical framework. By the early 1950's there was considerable interest in the physics of moisture movement in porous media, both under isothermal and non-isothermal conditions. The complex nature of the pore-space in soil and the water held therein made it difficult to understand the force-field acting on the water. Philip and de Vries (1957) presented the following equation which describes moisture and heat transfer under combined moisture content and temperature gradients.

* Post-Doctoral Fellow, University of Saskatchewan, Saskatoon, Saskatchewan, Canada.
** Professor of Civil Engineering, University of Saskatchewan, Saskatoon, Saskatchewan, Canada.

$$Q = D_\theta \nabla\theta + D_T \nabla T + K_\theta \quad (1)$$

where Q = the net water flux, D_θ = the isothermal moisture diffusivity, $\nabla\theta$ = the moisture content gradient, D_T = the thermal diffusivity, and ∇T = the temperature gradient. The terms D_θ and D_T are made up of two components each; one for vapor flow and the other for liquid flow. The term K_θ is the gravity term.

Taylor and Cary (1964) proposed the following linear flow equation.

$$J_i = \sum_{k=1}^{N} L_{ik} X_k \quad (i=1,2, \ldots N) \quad (2)$$

where J_i = flux or flow of the i^{th} component of the system, L_{ik} = the phenomenological coefficient due to the k^{th} driving force affecting the i^{th} flux, X_k = the driving force of the k^{th} component and N = the number of driving forces. The components of a system can be hydraulic, matrix suction, chemical and temperature potentials. This equation is based on the law of irreversible thermodynamics for describing the movement of water through soil due to temperature gradient. In the realm of irreversible thermodynamics the flux of one component of a system may influence the flux of a second component of the system. For example, the use of equation (2) requires the calculation of the heat flux in addition to the net water flux in order to determine values of L_{ik}. For a closed soil-water system having an imposed temperature gradient and which has attained a steady-state flow condition, equation (2) was simplified by Taylor and Cary (loc. cit.).

$$q = -D_\theta \left[\frac{d\theta}{dx} + \beta^* \frac{d(\ln T)}{dx}\right] \quad (3)$$

where q = net water flux, x = distance along the column, T = absolute temperature, $\beta^* = -d\theta/d(\ln T)$ and $D_\theta = (D_{\theta liq} + D_{\theta vap})$ = the diffusivity of the liquid and vapor terms.

Cassel et al (1969) reviewed the theoretical models proposed for predicting the movement of water in soils in response to an imposed temperature gradient and concluded that the Philip and de Vries (loc. cit.) showed acceptable agreement of the predicted and observed water movement. Fick's law and the modified Taylor-Cary equation (3) both under-predicted the observed water movement.

Nachlinger and Lytton (1969) proposed two coupled differential equations representing an isothermal case and three coupled differential equations representing the non-isothermal case for flow through a porous media. Their approach is based on the principle of superimposed continuum mechanics, which postulates that each point of a mixture is occupied by a particle of soil, water and air. The driving forces associated with the liquid and gas phases are hydraulic and thermal gradients.

Researchers in soil science (Cassel et al 1969) and geotechnical engineering (Aitchison et al, 1965; Dempsey and Elzeftawy, 1976; Dempsey, 1978; and Sophocleous, 1978) have generally accepted the Philip and de Vries (loc. cit.) theory. This theory appeared to be most suitable for predicting moisture movement under thermal gradients in unsaturate soils. However, the adoption of the Philip and de Vries model in geotechnical engineering practice is somewhat undesirable for several reasons.

The Philip and de Vries' model predicts the transfer of moisture due to temperature and moisture content gradients. However, the flow water is fundamentally due to a total head gradient (i.e., elevation head plus pressure head), the same as for a saturated soil. Therefore it is not a moisture content gradient that produces flow. Neither is a suction gradient that produces flow since the air and water phases flow under independent total head gradients. A temperature gradient i the soil produces a change in the pore-air pressure which causes flow the air phase. In turn, the change in pore-air pressure produces a change in the pore-water pressure, inducing a gradient in the water phase. An attempt is made in the proposed formulation to adhere to th fundamental forces involved in the flow of air and water in an unsaturated soil. The proposed formulation also uses the complete constituti relations required for describing the volume change behavior of an uns urated soil (Fredlund and Morgenstern, 1976). These relations are applicable for changes in total stress, pore-air and pore-water pressu

An attempt is made in this paper to present a complete mathematic formulation to predict the transient flow of moisture under non-isothe mal conditions. The formulation is presented for the two-dimensional case. The quantity of water vapor flowing in and out of an unsaturate soil boundary is also computed. The total flow of moisture and water vapor computed allows for the determination of the net change in volum of the soil.

PHYSICAL REQUIREMENTS FOR THE FORMULATION

The state of stress in an unsaturated soil can be described by an two of a possible three stress state variables (Fredlund and Morgenste 1977). The stress variables selected to derive the transient flow equations in this paper are: $(\sigma - u_a)$ and $(u_a - u_w)$; where, σ = total stress, u_a = pore-air pressure, and u_w = pore-water pressure.

Continuity of an unsaturated soil element requires that the overa volume change of the element must be equal to sum of the volume change associated with the component phases (Fredlund, 1973).

$$\frac{\Delta V}{V} = \frac{\Delta V_w}{V} + \frac{\Delta V_a}{V} \quad (4$$

where: V = overall volume of referential soil element
V_w = volume of water in the referential soil element
V_a = volume of air in the referential soil element.

If any two of the volume changes are known, the third can be computed. In other words, it is necessary to have two constitutive equations to define volume change behavior in unsaturated soils.

The constitutive relationship for the soil structure and the water phase was proposed and verified by Fredlund and Morgenstern (1976). Assuming that the soil is isotropic, the respective constitutive relations for the two-dimensional case are presented in equations (5) and (6).

$$\frac{\Delta V}{V} = m_1^s\, d(\sigma_x - u_a) + m_1^s\, d(\sigma_y - u_a) + m_2^s\, d(u_a - u_w) \qquad (5)$$

$$\frac{\Delta V_w}{V} = m_1^w\, d(\sigma_x - u_a) + m_1^w\, d(\sigma_y - u_a) + m_2^w\, d(u_a - u_w) \qquad (6)$$

where: m_1^s = compressibility modulus of soil structure when $d(u_a - u_w)$ is zero.
m_2^s = compressibility modulus of soil structure when $d(\sigma_x - u_a)$ and $d(\sigma_y - u_a)$ are zero.
m_1^w = slope of the $(\sigma_x - u_a)$ versus volume of water plots when $d(u_a - u_w)$ is zero.
m_2^w = slope of the $(u_a - u_w)$ versus volume of water plot when $d(\sigma_x - u_a)$ and $(\sigma_y - u_a)$ are zero.

The constitutive relationship for the air phase is the difference between equations (5) and (6) because of the continuity requirement.

$$\frac{\Delta V_a}{V} = m_1^a\, d(\sigma_x - u_a) + m_1^a\, d(\sigma_y - u_a) + m_2^a\, d(u_a - u_w) \qquad (7)$$

where: $m_1^a = m_1^s - m_1^w$ and $m_2^a = m_2^s - m_2^w$

These equations provide a satisfactory description of the volume of water and air in a soil under monotonic deformation conditions. For conditions of stress reversal, the moduli need to be changed accordingly.

Flow of water in an unsaturated soil can be described by Darcy's law (Childs and Collis-George, 1950), in the same manner as for a saturated soil.

$$v_x = -k_x(w)\, \frac{\partial h_w}{\partial x} \qquad (8)$$

$$v_y = -k_y(w)\, \frac{\partial h_w}{\partial y} \qquad (9)$$

where: $h_w = \frac{u_w}{\gamma_w} + Y$

v_x and v_y = water velocities in 'x' and 'y' directions, respectively.
$k_x(w)$ and $k_y(w)$ = coefficients of permeability with respect to the water phase in the 'x' and 'y' directions, respectively. (Note: the soil is assumed to be isotropic with respect to the soil moduli but anistropic with respec to the coefficients of permeability).
x and y = depth in 'x' and 'y' directions.
h_w = total water head causing flow, and
Y = elevation head.

The flow of air phase is described by Fick's law (Blight, 1971).

$$m_{ax} = -D_x \frac{\partial p}{\partial x} \quad (1C$$

$$m_{ay} = -D_y \frac{\partial p}{\partial y} \quad (11$$

where: m_{ax} and m_{ay} = mass rate of air flow in 'x' and 'y' directions respectively.
D_x and D_y = transmission constants having the same units as coefficients of permeability in the 'x' and 'y' directions respectively.
p = absolute air pressure (i.e., $u_a + u_{atm}$)
u_{atm} = atmospheric air pressure.

DERIVATION OF THE TRANSIENT FLOW EQUATIONS

The two-dimensional transient flow equations for an unsaturated soil are derived using the conventional assumptions for Terzaghi's (1943) consolidation theory along with the following additions:

i) the air phase is continuous;
ii) the coefficient of permeability with respect to water and air and the volume change moduli remain constant during the transient processes; and
iii) the effects of air diffusing through water are ignored. The movement of water vapor is considered in an empirical manner. (The significance of these effects are presently under investigation and the results will be published in a future paper).

The above assumptions are not completely accurate for all cases. For example, the coefficient of permeability with respect to the water phase is a function of both water content and degree of saturation. However, the coefficient of permeability is assumed to be constant during the transient process. The coefficient of permeability with respect to the air phase is a function of water content and degree of saturation, but it is likewise assumed to be constant. The above approach can be justified since the partial differential equations are being solved using a finite difference technique which allows for the adjustment of the coefficients of permeabilities in the time scale.

The constitutive surfaces for the soil structure and water phases are non-linear. In the analysis the moduli are assumed linear for a small incremental change in stress state variable. However, these values could also be adjusted during the finite difference solution.

WATER PHASE PARTIAL DIFFERENTIAL EQUATION

Let us consider a referential soil element as shown in Figure 1. The water is assumed incompressible. In the transient flow process, water flows out of the element with time. The constitutive relationship for the water phase defines the volume of water in the element for any combination of total, air and water pressures.

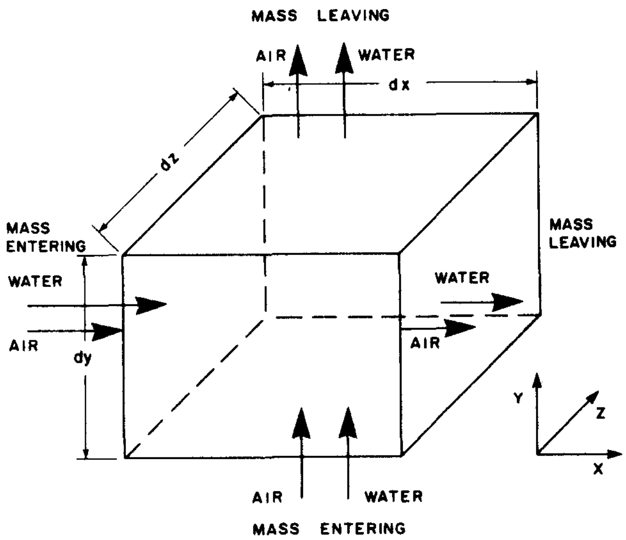

Figure 1 A Referential Element in the Soil Mass

Two-Dimensional Condition (Plane-Strain Case)

The volume of water entering and leaving the element in the 'x' and 'y' directions can be described by Darcy's law as follows:

$$V_{WEX} = -(k_x(w) \frac{\partial h_w}{\partial x})dy \tag{12}$$

$$V_{WEY} = -(k_y(w) \frac{\partial h_w}{\partial y})dx \tag{13}$$

$$V_{WLX} = -(k_x(w) \frac{\partial h_w}{\partial x} + \frac{\partial}{\partial x} (k_x(w) \frac{\partial h_w}{\partial x})dx)dy \tag{14}$$

$$V_{WLY} = -(k_y(w) \frac{\partial h_w}{\partial y} + \frac{\partial}{\partial y} (k_y(w) \frac{\partial h_w}{\partial y})dy)dx \tag{15}$$

V_{WEX} and V_{WEY} = volume of water entering the element in the 'x' and 'y' directions, respectively.

V_{WLX} and V_{WLY} = volume of water leaving the element in the 'x' and 'y' directions, respectively.

$\frac{\partial h_w}{\partial x}$ and $\frac{\partial h_w}{\partial y}$ = total water head gradients in the 'x' and 'y' directions, respectively.

The net flux of water per unit volume of the element is:

$$\frac{\partial(V_w/V)}{\partial t} = -\left[\frac{k_x(w)}{\gamma_w}\frac{\partial^2 u_w}{\partial x^2} + \frac{k_y(w)}{\gamma_w}\frac{\partial^2 u_w}{\partial y^2}\right] \tag{16}$$

Equation (16) can be equated to the time differential of the constitutive relationship for the water phase.

$$m_1^w \frac{\partial(\sigma_x - u_a)}{\partial t} + m_1^w \frac{\partial(\sigma_y - u_a)}{\partial t} + m_2^w \frac{\partial(u_a - u_w)}{\partial t} =$$

$$-\left(\frac{k_x(w)}{\gamma_w}\frac{\partial^2 u_w}{\partial x^2} + \frac{k_y(w)}{\gamma_w}\frac{\partial^2 u_w}{\partial y^2}\right) \tag{17}$$

The change in total stress in the 'x' and 'y' directions with respect to time can be set to zero. Simplifying and rearranging equation (17), the water phase partial differential equation can be written:

$$\frac{\partial u_w}{\partial t} = C_w \frac{\partial u_a}{\partial t} + c_{v_1}^w \frac{\partial^2 u_w}{\partial x^2} + c_{v_2}^w \frac{\partial^2 u_w}{\partial y^2} \tag{18}$$

where $C_w = -(1 - m_2^w/2m_1^w) / (m_2^w/2m_1^w)$ and is called the interactive constant associated with the water phase equation. This equation is further simplified by letting $R_w = m_2^w/2m_1^w$, When the soil is saturated $R_w = 1$.

$$c_{v_1}^w = \frac{1}{R_w}\frac{k_x(w)}{\gamma_w}\frac{1}{2m_1^w}$$; the coefficient of consolidation for the water phase with respect to the 'x' direction, and (19)

$$c_{v_2}^w = \frac{1}{R_w}\frac{k_y(w)}{\gamma_w}\frac{1}{2m_2^w}$$; the coefficient of consolidation for the water phase with respect to the 'y' direction. (20)

AIR PHASE PARTIAL DIFFERENTIAL EQUATION

The air phase is compressible and flow occurs in response to a pressure gradient. The constitutive relationship for the air phase defines the volume of air in the element for any combination of the total, air and water pressures.

Two-Dimensional Condition (Plane-Strain Case)

According to Fick's law, the mass of air entering the element in 'x' and 'y' directions are:

$$M_{AEX} = -D_x \frac{\partial p}{\partial x} dy \tag{21}$$

$$M_{AEY} = -D_y \frac{\partial p}{\partial y} dx \tag{22}$$

$$M_{ALX} = -(D_x \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} (D_x \frac{\partial p}{\partial x}) dx) dy \tag{23}$$

$$M_{ALY} = -(D_y \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} (D_y \frac{\partial p}{\partial y}) dy) dx \tag{24}$$

where: M_{AEX} and M_{AEY} = mass of air entering the element in the 'x' and 'y' directions, respectively.
M_{ALX} and M_{ALY} = mass of air leaving the element in the 'x' and 'y' directions, respectively.

The net flux of air through the element is:

$$\frac{\partial m}{\partial t} = -(D_x \frac{\partial^2 p}{\partial x^2} + D_y \frac{\partial^2 p}{\partial y^2}) \tag{25}$$

where: m = mass of air in the element.

The mass rate of change is written in terms of a volume rate of change by differentiating the relationship between mass and volume of air.

$$\frac{\partial(V_a/V)}{\partial t} = \frac{\partial(m/\gamma_a)}{\partial t} \tag{26}$$

For the isothermal condition, the density of air is:

$$\gamma_a = \frac{\omega}{R\theta} p \tag{27}$$

where: ω = molecular weight of air
R = universal gas constant
θ = absolute temperature

The mass of air is written in terms of the density of air, γ_a, the degree of saturation, S, and the porosity of the soil, n,

$$m = (1-S)n\gamma_a \tag{28}$$

Substituting equation (26) into equation (25) and representing the density of air, γ_a from equations (27) and (28) and simplifying,

$$\frac{\partial(V_a/V)}{\partial t} = -\frac{D_x R\theta}{\omega p}\frac{\partial^2 u_a}{\partial x^2} - \frac{D_y R\theta}{\omega p}\frac{\partial^2 u_a}{\partial y^2} + \frac{(1-S)n}{p}\frac{\partial u_a}{\partial t} \tag{29}$$

Equation (29) can be equated to the time differential of the air phase constitutive relationship.

$$m_1^a \frac{\partial(\sigma_x - u_a)}{\partial t} + m_1^a \frac{\partial(\sigma_y - u_a)}{\partial t} + m_2^a \frac{\partial(u_a - u_w)}{\partial t}$$

$$= -\left(\frac{D_x R\theta}{\omega p}\frac{\partial^2 u_a}{\partial x^2} + \frac{D_y R\theta}{\omega p}\frac{\partial^2 u_a}{\partial y^2} - \frac{(1-S)n}{p}\frac{\partial u_a}{\partial t}\right) \tag{30}$$

As in water phase equation, the change in total stress in the 'x' and 'y' directions with respect to time are set to zero. Simplifying and rearranging equation (30), the air phase partial differential equation can be written:

$$\frac{\partial u_a}{\partial t} = C_a \frac{\partial u_w}{\partial t} + c_{v_1}^a \frac{\partial^2 u_a}{\partial x^2} + c_{v_2}^a \frac{\partial^2 u_a}{\partial y^2} \tag{31}$$

where: $C_a = \dfrac{-m_2^a/2m_1^a}{(1-m_2^a/2m_1^a) + \dfrac{(1-S)n}{2m_1^a(u_a + u_{atm})}}$; the interactive constant associated with the air phase equation. (32)

This equation is further simplified by letting $R_a = m_2^a/2m_1^a$. When the soil is saturated $R_a = 1$.

$c_{v_1}^a = \dfrac{D_x R\theta}{\omega}\left[\dfrac{1}{(1-R_a)(u_a + u_{atm})2m_1^a + (1-S)n}\right]$; the coefficient of consolidation for the air phase with respect to the 'x' direction, and (33)

$c_{v_2}^a = \dfrac{D_y R\theta}{\omega}\left[\dfrac{1}{(1-R_a)(u_a + u_{atm})2m_1^a + (1-S)n}\right]$; the coefficient of consolidation for the air phase with respect to the 'y' direction. (34)

The dissipation of the excess pressure of the pore-air and pore-water phases are obtained by solving the equations (18) and (31) simultaneously as described in Appendix A.

HEAT FLOW EQUATION

The flow of heat through an unsaturated soil is of interest since it influences the flow of air (and water vapor) and water. The heat flow equation for the two-dimensional condition is as follows (Aldrich, 1956):

$$\frac{\partial \theta}{\partial t} = \alpha_1 \frac{\partial^2 \theta}{\partial x^2} + \alpha_2 \frac{\partial^2 \theta}{\partial y^2} \qquad (35)$$

where: θ = temperature
α_1 = thermal diffusivity factor in the 'x' direction = $\lambda_x / c\gamma$
α_2 = thermal diffusivity factor in the 'y' direction = $\lambda_y / c\gamma$
λ_x and λ_y = thermal conductivity in the 'x' and 'y' directions, respectively.
c = heat capacity
γ = soil density

The heat flow equation is solved using an explicit finite difference method (See Appendix A). The solution gives the change in temperature with time and space (Figure 2). The generated pore-air pressure due to thermal gradients in an unsaturated soil can be calculated at each node in the soil, using the ideal gas law equation. The change in pore-water pressure can be computed using a pore pressure coefficient. Accordingly, the adjusted values of the pore-air and pore-water pressures can be dissipated.

FLOW OF MOISTURE AND AIR INTO AND OUT OF AN UNSATURATED SOIL SYSTEM

At the soil boundary the quantity of moisture and air flowing in and out of an unsaturated soil is computed by multiplying the gradients by their respective permeability coefficients. The total flow of moisture and air computed enables the determination of the total change in volume, which can either be swelling or shrinking.

EXAMPLE PROBLEMS

A computer program has been developed to solve the partial differential equations for the one-dimensional case, (the computer program for the two-dimensional case is *ad hoc*). The example problems shown are for the one-dimensional case. Computer solutions are presented for various input values for (i) initial pore-water and pore-air pressures; (ii) volume change moduli for the soil structure and water phases; (iii) degree of saturation and porosity; and (iv) thermal conductivity and heat capacity, etc.

An example problem is solved to demonstrate the simultaneous solution of transient pore-air and pore-water flow equations, and the heat flow equation. Let us consider a ten-centimeter thick layer of compacted soil within an overall soil mass. The assumed soil propertie are as follows: S = 70%, n = 50%, m_1^a = 0.000116/kPa, m_1^s = 0.000145/kPa R_a = -0.01, R_w = 0.7, R_s = 0.7. It is assumed that the initial equilibrium condition is altered by a sudden environmental change such as a heavy rainfall. In addition, the effect of an appreciable temperature change is considered. Both changes disturb the state of equilibrium in the unsaturated soil mass. The sudden environmental change due to the heavy rainfall builds up an excess pore-water pressure at the bound ary of soil layer under consideration. The excess pore-water pressure causes simultaneous flow in the water phase and the air phase. Similarly, the temperature change results in an excess pore-air pressure, which causes simultaneous flow in the water phase and the air phase. The transient flow is assumed to continue until the soil element reache equilibrium with the designated boundary conditions. The solution to these transient flow conditions can be obtained by solving simultaneous equations (18), (31) and (35).

RESULTS AND DISCUSSIONS

The distribution of temperature, pore-water and pore-air pressures with space and time are obtained by solving equations (35), (18) and (31), respectively, and the results are presented by the family of curves shown in Figures 2 through 7.

Figures 2 and 3 show the temperature distribution within the soil layer with space and time. Figure 2 shows the results for an increase in temperature and Figure 3 shows the results for a decrease in temperature.

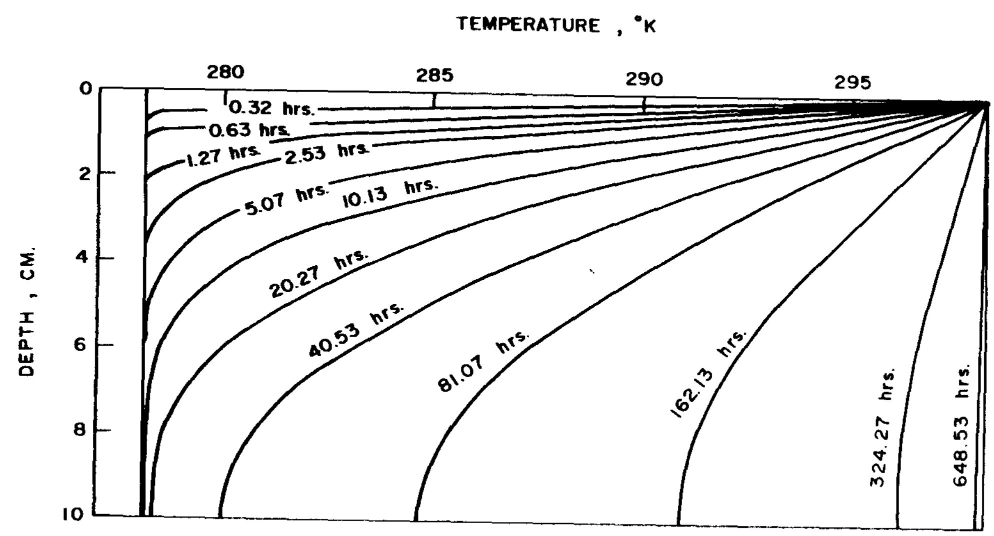

Figure 2 Temperature Isotherms for Example Problem (Temperature Increase)

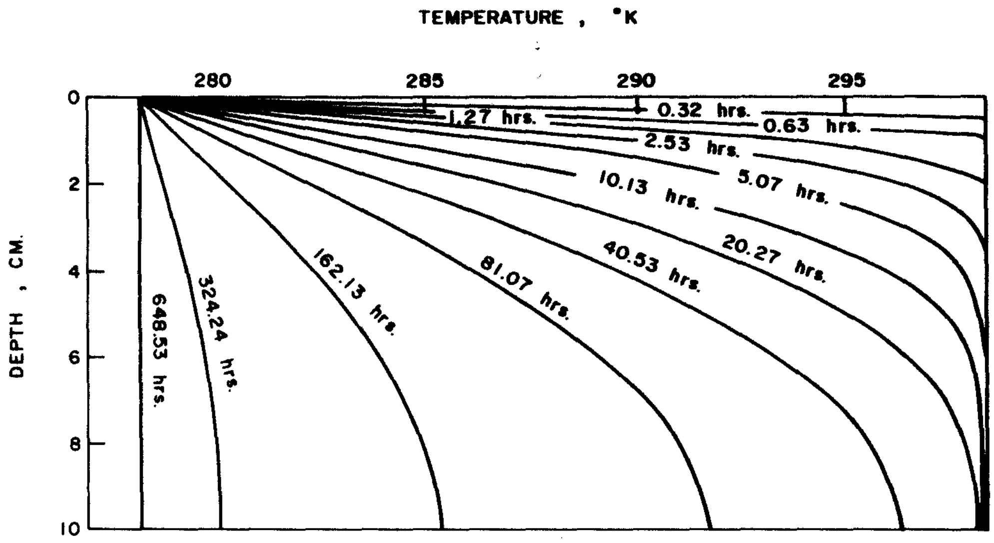

Figure 3 Temperature Isotherms for Example Problem
(Temperature Decrease)

Figure 4 shows the pore-water distribution throughout the soil layer due to change in the water pressure at the boundary. The initial (i.e., equilibrium) pore-water pressure was -184 kPa, and the boundary water pressure was changed to a constant value of -460 kPa. Figure 5 shows the reverse process to that presented in Figure 4. The initial pore-water pressure throughout the soil layer was -460 kPa and the water pressure at the boundary was changed to 0 kPa. This condition simulates the flooding of the surface of the soil layer.

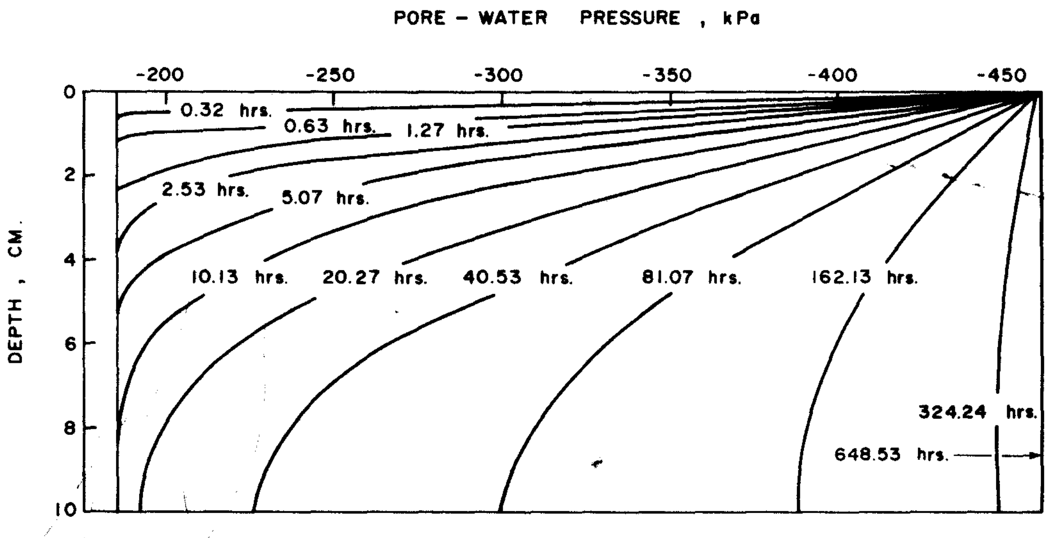

Figure 4 Pore-Water Pressure Decrease Due to a Decrease
in Water Pressure at Boundary (Example Problem)

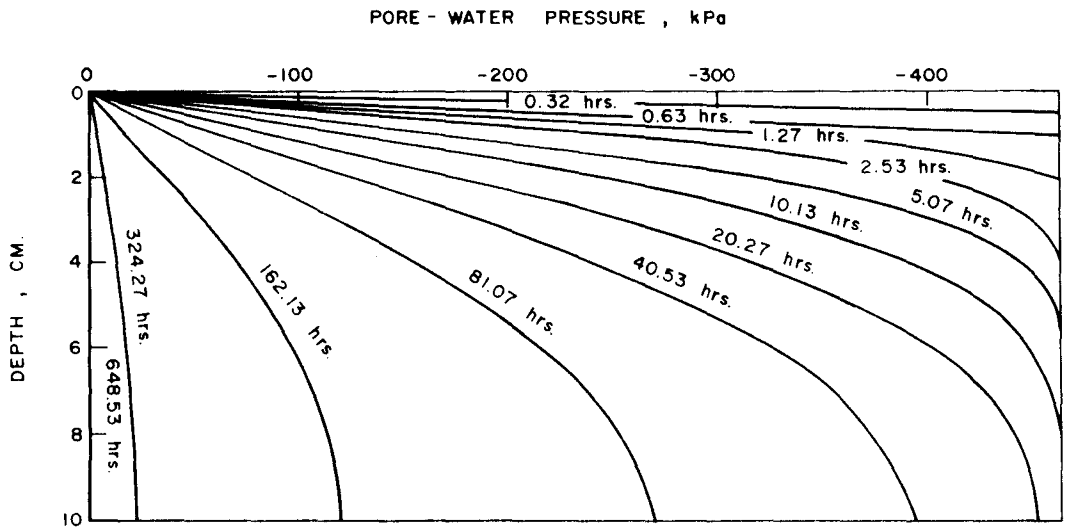

Figure 5 Pore-Water Pressure Increase Due to an Increase in Water Pressure at Boundary (Swelling)

Figures 6 and 7 show the increase and decrease of excess pore-air pressure generated due to the temperature gradients. It is assumed, in this case, that the pore-water pressure remain essentially unchanged due to the temperature gradient.

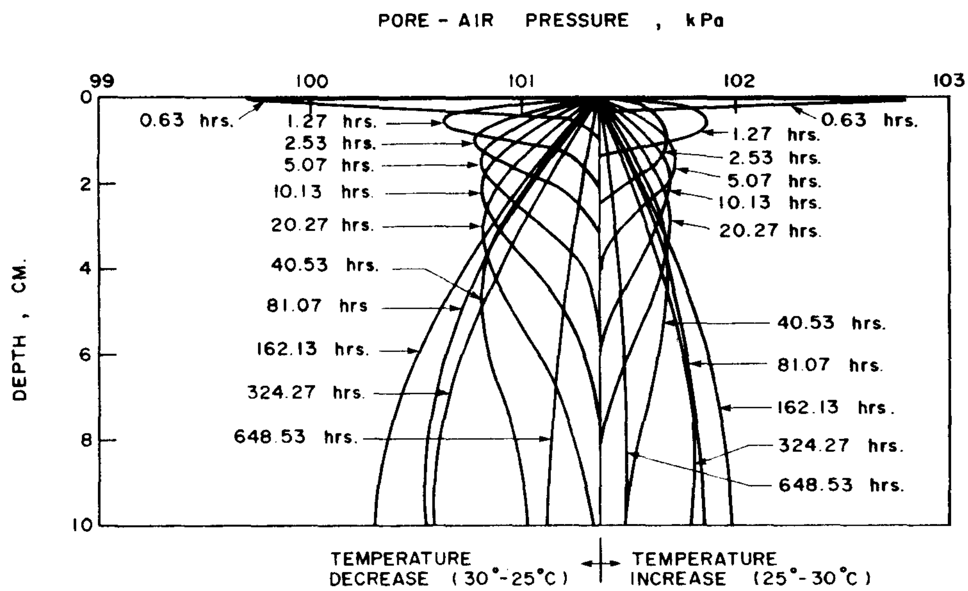

Figure 6 Increase and Decrease of Pore-Air Pressure Due to Temperature Change

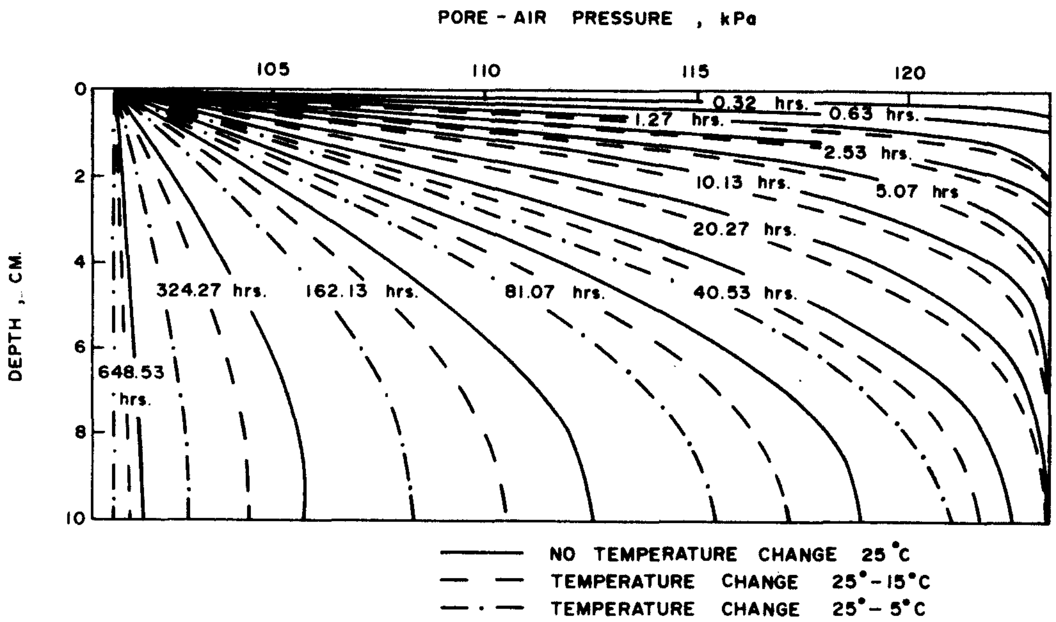

Figure 7 Decrease of Pore-Air Pressure Due to Change in Temperature and Hydraulic Conditions at Boundary

It is of interest to note that a smooth transition exists between transient flow equations (18) and (31) formulated in this research for an unsaturated soil and Terzaghi's conventional one-dimensional consolidation equation for a saturated soil.

Figure 8 shows the mass of air flowing in and out of an unsaturated soil at the boundary due to the temperature gradients. Figure 8 indicates that the flow of air can be either inward or outward, depending upon whether the temperature increases or decreases.

SUMMARY

All the necessary physical relationships are available to formulate a theoretical model to describe the transient flow processes in an unsaturated soil. The formulation presented herein is parallel in concept to that used in the conventional Terzaghi's type consolidation theory for a saturated soil system. The two partial differential equations (i.e., one for the water phase and the other for air phase) are solved simultaneously using a special numerical procedure (See Appendix A). It is of interest to note that there is a smooth transition between the proposed equations and those previously proposed for a completely dry soil or a fully saturated soil. At intermediate degrees of saturation there is varying amounts of pore pressure interaction between the air and water phases during the transient process. The heat flow equation is solved by a forward finite difference technique. This enables the computation of the generated pore-air due to the thermal gradients. The families of curves show pore-water and pore-air pressure

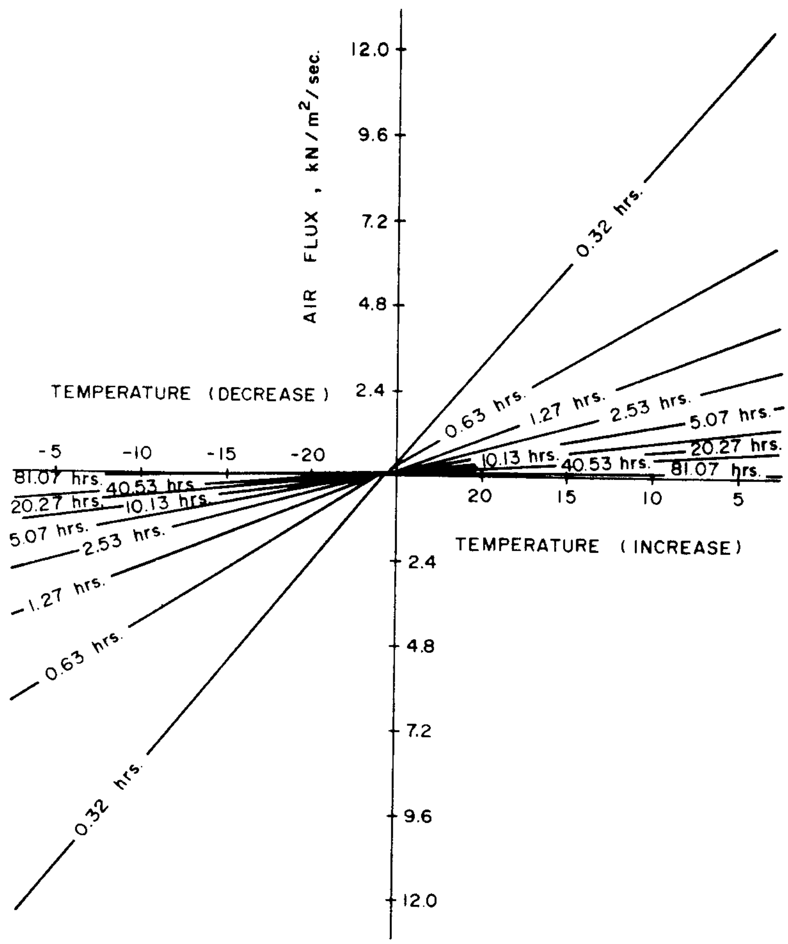

Figure 8 Air Flux at Soil Boundary Due to Temperature Change

dissipation as a result of hydraulic (i.e., air and water pressure) gradients and thermal gradients in an unsaturated soil.

At the soil-boundary, the mass of moisture and air flowing in and out of an unsaturated soil is computed. The movement of water and air into and out of an unsaturated soil results in an overall volume change of the soil (i.e., either swelling or shrinking). A complete understanding of this mechanism is of prime importance to geotechnical engineers interested in the design of shallow foundations, and highway and airfield pavements.

ACKNOWLEDGEMENTS

This study was supported under a research grant from the Saskatchewan Highways and Transportation Department, Government of Saskatchewan, Regina, Saskatchewan. Acknowledgement is made to Dr. R. Manohar, Professor of Mathematics, University of Saskatchewan, for his assistance during this study.

LIST OF REFERENCES

Aitchison, G.D., Russam, K. and Richards, B.G. (1965), "Engineering Concepts of Moisture Equilibria and Moisture Changes in Soils", Moisture Equilibria and Moisture Changes in Soils Beneath Covered Areas, Review Panel Presentation, Butterworth, Sydney, Australia, pp. 7-21.

Aldrich, H.P., Jr. (1956), "Frost Penetration Below Highway and Airfield Pavements", Highway Research Board Bulletin, No. 135, National Research Council, Washington, D.C., pp. 124-149.

Blight, G.E. (1971), "Flow of Air Through Soils", Journal of the Soil Mech. and Fdtn. Div., ASCE, April, pp. 607-624.

Cary, J.W. (1966), "Soil Moisture Transport Due to Thermal Gradients: Practical Aspects", Soil Science Soc. of America, Proceedings, Vol. 30, pp. 428-433.

Cassel, D.K. Nielsen, D.R. and Biggar, J.W. (1969), "Soil-Water Movement in Response to Imposed Temperature Gradients", Soil Science Soc. of America, Proceedings, Vol. 33, pp. 493-500.

Childs, E.C. and Collis-George, N. (1950), "The Permeability of Porous Material", Proc., Roy. Soc., 201A.

Dempsey, B.J. and Elzeftawy, A. (1976), "Mathematical Model for Predicting Moisture Movement in Pavement Systems", Transportation Research Record, No. 612, Washington, D.C., pp. 48-55.

Dempsey, B.J. (1978), "A Mathematical Model for Predicting Coupled Heat and Water Movement in Unsaturated Soil", International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 2, pp. 19-36.

Fredlund, D.G. (1973), "Volume Change Behavior of Unsaturated Soils", Ph.D. dissertation, University of Alberta, Edmonton, Alberta, Canada.

Fredlund, D.G. and Morgenstern, N.R. (1976), "Constitutive Relations for Volume Change in Unsaturated Soils", Canadian Geotechnical Journal, Vol. 13, No. 3, pp. 261-276.

Fredlund, D.G. and Morgenstern, N.R. (1977), "Stress State Variables for Unsaturated Soils", ASCE, Vol. 103, No. GT5, (May), pp. 447-446.

Fredlund, D.G. and Hasan, J.U. (1978), "One-dimensional Consolidation Theory: Unsaturated Soils", Canadian Geotechnical Journal, Vol. 16, No. 3, pp. 524-531.

Hasan, J.U. and Fredlund, D.G. (1977), "Consolidation Theory for Unsaturated Soils", Presented to the Symposium on Water Movement and Equilibrium in Swelling Soils, Organized by the Committee on Water in the Unsaturated Zone of the Section of Hydrology of the American Geophysical Union, San Francisco, U.S.A.

Nachlinger, R. and Lytton, R.L. (1969), "Continuum Theory of Moisture Movement and Swell in Expansive Clays", Research Report 118.2, Center for Highways Research, The University of Texas at Austin, U.S.A.

Philip, J.R. and de Vries, D.A. (1957), "Moisture Movement in Porous Materials Under Temperature Gradients", Transactions, American Geophysical Union, Vol. 38, No. 2.

Terzaghi, K. (1943), "Theoretical Soil Mechanics", New York, John Wiley and Sons, pp. 510.

Sophocleous, M.A. (1978), "Analysis of Heat and Water Transport in Unsaturated-Saturated Porous Media", Ph.D. dissertation, University of Alberta, Edmonton, Alberta, Canada.

APPENDIX A

The heat flow equation (35) is solved first, using an explicit forward difference technique. Then, a special numerical procedure is adopted to obtain the solution of both the water phase and air phase partial differential equations (18 and 31). These are solved simultaneously using an explicit forward difference technique. This procedure is advantageous in solving these two equations since the air phase equation is non-linear. The steps used to solve the transient flow equations, (18 and 31) are given below:

1. Write the transient flow equation for the water phase in a finite difference form. (Equation A.1)
2. Repeat step (1) for the transient flow equation for the air phase. (Equation A.2)
3. Solve the above two equations for pore-water pressure (u_w).
4. Repeat step (3) for the pore-air pressure (u_a).
5. Rearrange the equation for the pore-water pressure such that the unknown pore-water pressure at the next time step (i.e., $j+1^{th}$) is on the right hand side and all known variables at the previous time step (i.e., j^{th}) are on the left hand side of the equation.
6. Repeat step (5) for the pore-air pressure.
7. Compute the pore-water and the pore-air pressures at the next time step, from the known values at the previous time step.
8. When computations for both pore-water and pore-air pressures for all depth steps are completed, then march forward to the next time step.
9. Repeat steps (7) and (8).

Figure A.1 shows the finite difference designations for the examp problem. Following are the finite difference forms for the partial differential equations.

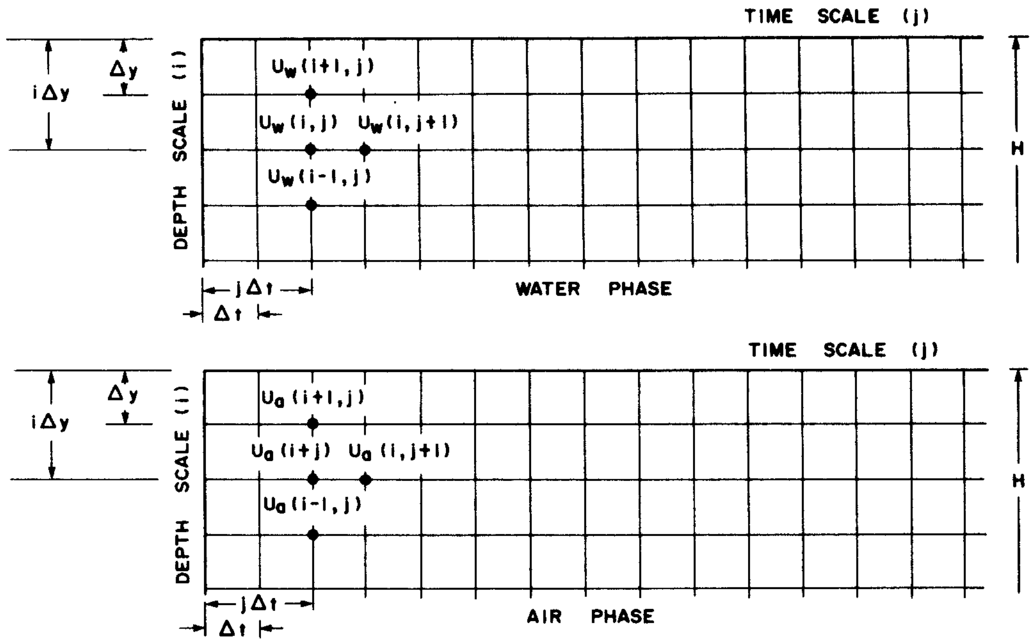

Figure A.1 Finite Difference Mesh for the Transient Flow Equations

Finite Difference Form of the Heat Flow Partial Differential Equation (One-dimensional case)

$$\frac{(\theta_{i,j+1} - \theta_{i,j})}{\Delta t} = \alpha \frac{(\theta_{(i+1,j)} - 2\theta_{(i,j)} + \theta_{(i-1,j)})}{\Delta y^2} \qquad (A.1)$$

Simplifying and rearranging:

$$\theta_{(i,j+1)} = \theta_{(i,j)} + \beta_t(\theta_{(i+1,j)} - 2\theta_{(i,j)} + \theta_{(i-1,j)}) \qquad (A.2)$$

where: $\beta_t = \alpha \frac{\Delta t}{\Delta y^2}$

The β_t term has been set to $\leq$ 0.5 for the solution of the example problem.

Finite Difference Form of the Water Phase Partial Differential Equation (One-dimensional case)

$$\frac{u_{w(i,j+1)} - u_{w(i,j)}}{\Delta t} = C_w \left[\frac{u_{a(i,j+1)} - u_{a(i,j)}}{\Delta t}\right]$$

$$+ c_v^w \left[\frac{u_{w(i+1,j)} - 2u_{w(i,j)} + u_{w(i-1,j)}}{\Delta y^2}\right] \qquad (A.3)$$

Finite Difference Form of the Air Phase Partial Differential Equation (One-dimensional case)

$$\frac{u_{a(i,j+1)} - u_{a(i,j)}}{\Delta t} = C_a \left[\frac{u_{w(i,j+1)} - u_{w(i,j)}}{\Delta t}\right]$$

$$+ c_v^a \left[\frac{u_{a(i+1,j)} - 2u_{a(i,j)} + u_{a(i-1,j)}}{\Delta y^2}\right] \qquad (A.4)$$

Solving equations (A.3) and (A.4) simultaneously, simplifying and rearranging gives,

$$u_{w(i,j+1)} = u_{w(i,j)} + \frac{\beta_w g_1^w}{(1 - C_a C_w)} + \left(\frac{C_w}{1 - C_a C_w}\right) \beta_a f_1^a \qquad (A.5)$$

$$u_{a(i,j+1)} = u_{a(i,j)} + \left(\frac{C_a}{1 - C_a C_w}\right) \beta_w g_1^w + \frac{\beta_a f_1^a}{(1 - C_a C_w)} \qquad (A.6)$$

where: $\beta_w = c_v^w \left(\frac{\Delta t}{\Delta y^2}\right)$, $\beta_a = c_v^a \left(\frac{\Delta t}{\Delta y^2}\right)$,

$$g_1^w = u_{w(i+1,j)} - 2u_{w(i,j)} + u_{w(i-1,j)}$$

and $$f_1^a = u_{a(i+1,j)} - 2u_{a(i,j)} - u_{a(i-1,j)}$$

The β_w and β_a terms have been set to ≤ 0.5 for the solution of the example problem.