Analysis of Swelling-Soil Column Infiltration Test[a]

Discussion by Jamshed U. Hasan[2] and Delwyn G. Fredlund,[3] M. ASCE

The writers wish to commend the author on an interesting analysis of the swelling-soil column infiltration test. In view of the dearth of research in this

[a]September, 1976, by Jacob Uzan (Proc. Paper 12367).
[2]Grad. Student, Univ. of Saskatchewan, Saskatoon, Canada.
[3]Prof. of Civ. Engrg., Univ. of Saskatchewan, Saskatoon, Canada.

area, the author's attempt is encouraging. The writers would like to suggest that the theory proposed by the author be considered within a more complete continuum mechanics context. The analysis suggested is a moving boundary transient flow analysis predicting the changes in pore-water and pore-air pressure with respect to time. The change in total stress is known by virtue of the test procedure. The analysis is a moving boundary, since part of the sample is swelling while part is compressing, as explained by the author.

All the required physical relationships for unsaturated soils are known for a rigorous analysis of the infiltration test (5, 6). The stress state of an unsaturated soil can be described by any two of a possible three stress state variables (7)

$\sigma - u_a$	$\sigma - u_w$	$\sigma - u_a$
and	and	and
$u_a - u_w$	$u_a - u_w$	$\sigma - u_w$

in which σ = total stress; u_a = pore-air pressure; and u_w = pore-water pressure.

The writers recommend the use of $\sigma - u_a$ and $u_a - u_w$ as independent stress state variables. Using a referential element of soil, the volumetric continuity requirement can be stated as (6)

$$\frac{\Delta V}{V} = \frac{\Delta V_w}{V} + \frac{\Delta V_a}{V} \quad \text{. . . . .} \quad (6)$$

in which V = overall volume of the referential element; V_w = volume of water in the element; and V_a = volume of air in the element.

Constitutive relationships linking the stress and deformation states have been proposed and tested by Fredlund and Morgenstern (8). The equation for the soil structure or overall volume is

$$\frac{\Delta V}{V} = m_1^s\, d(\sigma - u_a) + m_2^s\, d(u_a - u_w) \qquad (7)$$

in which m_1^s = compressibility of the soil structure when $d(u_a - u_w) = 0$; and m_2^s = compressibility of the soil structure when $d(\sigma - u_a) = 0$. The equation for the water phase is

$$\frac{\Delta V_w}{V} = m_1^w\, d(\sigma - u_a) + m_2^w\, d(u_a - u_w) \qquad (8)$$

in which m_1^w = slope of the water volume versus $(\sigma - u_a)$ plot when $d(u_a - u_w) = 0$; and m_2^w = slope of the water volume versus $(u_a - u_w)$ plot when $d(\sigma - u_a) = 0$.

The constitutive equation for the air phase will equal the difference between the soil structure and water phase equations. The moduli in the preceding equations will differ depending on whether the respective volumes are increasing or decreasing.

The general form of D'Arcy's law can be used for both the air and water phase flows:

$$m = -k\gamma \frac{\partial p}{\partial y} \qquad (9)$$

in which m = mass flow rate; γ = density of the fluid; k = coefficient of permeability; and $\partial p/\partial y$ = pressure gradient.

The preceding constitutive relations and flow laws can be substituted into the continuity requirement. This can be done independently for the air and water phases. The resulting two partial differential equations must be solved simultaneously for the appropriate boundary conditions. In addition to the initial pore-air and pore-water pressure boundary conditions, account must be kept of the moving boundary between the swelling and compressing portions of the soil column. The change in total stress with time is also a boundary condition. The partial differential equations can readily be solved using a finite difference technique. The writers would encourage the author to also pursue this more rigorous analysis of the swelling-soil column problem.

Appendix.—References

5. Fredlund, D. G., "Volume Change Behavior of Unsaturated Soils," thesis presented to the University of Alberta, at Edmonton, Canada, in 1973, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
6. Fredlund, D. G., "Engineering Approach to Soil Continua," *Second Symposium on Applications of Solid Mechanics*, McMaster University, Hamilton, Canada, 1974, pp. 46–59.
7. Fredlund, D. G., "Discussion on the Second Technical Session, Division Two (Flow and Shear Strength)," *Proceedings of the Third International Conference on Expansive Soils*, Vol. II, Jerusalem Academic Press, Haifa, Israel, 1973.
8. Fredlund, D. G., and Morgenstern, N. R., "Constitutive Relations for Volume Change in Unsaturated Soils," *Canadian Geotechnical Journal*, Vol. 13, No. 3, 1976, pp. 261-276.