EVALUATION OF THE UNIVERSITY OF SASKATCHEWAN SLOPE STABILITY PROGRAM

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Introduction

Considerable effort has been devoted towards developing a comprehensive and flexible user-oriented slope stability computer program (1,2) at the University of Saskatchewan. The program is now being used by consulting firms, government agencies and universities in Western Canada. Through this extensive usage, cases and conditions have been encountered which were not anticipated during the initial stage of development. Furthermore, requests have been received to introduce features capable of handling special problems. Modifications and additional capabilities have been added to the program in order to make it able to handle virtually all two-dimensional situations.

It is essential to evaluate any computer program in its developmental stages in order to instill confidence in it and to provide recommendations on its efficient use. This has been done for the program under consideration. This paper outlines the features of the SLOPE program in its latest stage of development and presents an evaluation of the program.

Program features

The SLOPE program is capable of computing the factor of safety by six different methods. These are:

1. The Ordinary, Fellenius or Swedish Method.
2. The Simplified Bishop Method.
3. Spencer's Method (Taylor's Modified Swedish Method or the Corps of Engineer's Method is a special case of Spencer's Method and therefore they can also be solved)
4. Janbu's Simplified Method
5. Janbu's Rigorous Method
6. The Morgenstern-Price Method

This feature has made it possible to conveniently compare all the methods (3). The factor of safety equations for all the methods are written in the same form and in this way the same basic variables must be computed for each method. The difference between the methods arises from the manner in which the equations are solved.

A limited number of "Keywords" related to various aspects of a stability problem are utilized to provide a flexible means of data input. A stability problem is completely defined by means of:

1. a HEADing card. On this card the method of analysis is specified by means of a code number and a series of questions are answered with codes to control the computer output and calculations. These questions are listed on Figure 1.
2. a GRID card stating the slice width, centers of rotation, radii, number of trial slip surfaces, and the earthquake coefficient that should be considered.
3. SIDE cards which can be used to define the side force assumptions for Spencer's, Janbu's Rigorous, and Morgenstern-Price Methods.
4. GEOMetry cards to describe the soil stratigraphy. The boundaries between soil layers are entered by means of a series of lines which are defined by X- and Y- coordinates.

HEADING CARD

TYPE OF ANALYSIS	DATE	RUN NUMBER	COMMENT	QUESTIONS
H E A D				4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7	8 9 10 11 12 13 14 15	16 17 18 19 20 21 22 23	24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65	72 73 74 75 76 77 78 79 80

TYPE OF ANALYSIS

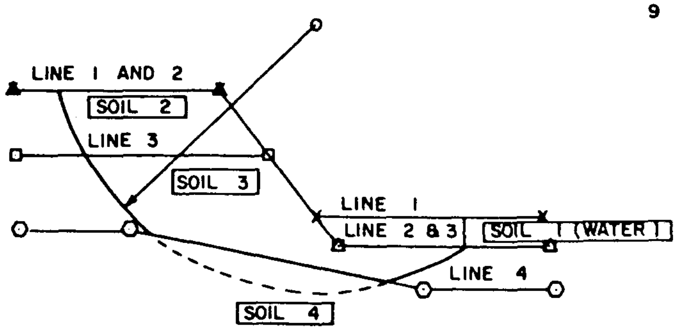
210 - ORDINARY METHOD OF SLICES
211 - SIMPLIFIED BISHOP'S METHOD
212 - SPENCER'S METHOD
213 - JANBU'S SIMPLIFIED METHOD
214 - JANBU'S GENERALIZED METHOD
215 - MORGENSTERN - PRICE METHOD

QUESTIONS

1 SHOULD INPUT DATA BE PRINTED OUT ?
2 SHOULD ALL FACTORS OF SAFETY BE PRINT?
3 SHOULD MINIMUM FACTORS OF SAFETY BE ?
4 SHOULD ALL FORCES ON EACH SLICE BE OUT ?
5 SHOULD THE DATA FOR THE CRITICAL SLIP BE PUNCHED ?
6 SHOULD MINIMUM FACTORS OF SAFETY BE
7 SHOULD THE PREVIOUS PORE PRESSURE DAISREGARDED ?
8 SHOULD NEGATIVE PORE PRESSURES BE SIERO ?
9 SHOULD A DIGITAL PLOT OF THE GEOMETRY_IP SURFACE BE PRINTED ?

N, ZERO OR BLANK MEAN "NO

Y OR 1 MEAN "YES"



NOTE : SOIL LAYER NO. LIES IMMEDIATELY BELOW CORRESPONDING LINE NO.

FIGURE 1 AN EXAMPLE CODING FORM

An alternate method can be used where the X- and Y-coordinates of points (PTS) are defined and they are then listed in the order they fall on a particular line. This list of points is identified by the "Keyword", LINE.

5. SOIL cards to describe the density, cohesion, angle of internal friction and r_u value for each stratigraphic unit.

6. POR3, POR4, or POR5 cards to describe the porewater pressures. POR3 is used to identify pore pressures as computed from Hilf's analysis (i.e. a nonlinear relationship between pore pressure and overburden pressure), POR4 is used to identify piezometric lines and POR5 to identify a grid with designated pore pressure heads. The pore pressure grid can be computed either from a flow net, or a finite difference or finite element seepage analysis.

7. a LOAD card to describe the magnitude and location of external loads.

8. a CALCulate card to signify the commencement of calculations for the factors of safety.

These "Keywords" signify the types of data being read in. With this technique, different conditions can be re-analyzed by altering any one particular key-worded data set while the rest remains unchanged. For example, if one or more of the soil properties were to be changed, the problem could be re-analyzed by inserting a SOIL card with the appropriate soil property changes after the first CALC card and then inserting another CALC card after the SOIL cards. This avoids re-submitting the card deck and associated computer costs.

Self-explanatory coding forms are associated with each of the "Keywords". Figure 1 shows a typical example.

Presently the program is capable of handling:

1. Fifteen soil types or stratigraphic units

2. Pore pressure input by piezometric lines (maximum of 15); by the pore pressure parameter r_u (one for each soil type); by a grid of points (maximum 999) as calculated, for example, by a finite element program.

3. External line loads (maximum of ten). These line loads do not affect the normal force at the base of each slice.

4. Surcharge loading which affects the normal force at the base of each slice.

5. Earthquake loading.

6. Partial submergence and tension cracks.

7. One thousand slices.

8. A grid of rotation centers which is 10 by 10 with 20 radii for each center.

9. Fifteen points to describe each geometric and piezometric line.

10. Composite or circular failure surfaces for all six methods of analysis. The composite surface may be of any specified shape. If the soil beneath a particular soil interface is designated as bedrock, the slip surface follows the soil interface wherever the slip circle falls within the bedrock designated zone. In this way, it is possible to analyze a grid of rotation centers and find the minimum safety factor even for composite sliding surfaces. There is no other program known to have this capability.

Other programs which have been developed and documented at the University of Saskatchewan, can be used in conjunction with the SLOPE program. These are:

1. Calcomp Plot Programs for plotting the geometry and failure surface, and the grid of safety factors.

2. A program to compute the nonlinear relationship between pore pressure and overburden pressure by Hilf's Analysis.

3. Finite element and finite difference programs to compute the pore pressures for a steady state seepage condition.

The SLOPE program has been coded according to the American National Standards Institute standard FORTRAN making the program portable; that is it can be moved from machine to machine with a minimum of programming effort. The only subroutine which is presently non-portable is the digital plot subroutine. This subroutine, however, can be easily blanked out or modified to suit a particular operating system.

Comparison with case histories

Various case histories were selected from the literature and analyzed with the SLOPE program. They were chosen so that all the methods of analysis could be evaluated and so that a wide range of problems would be examined. The sections analyzed, together with the references from which they were taken, are shown on Figures 2 to 12 inclusive.

The Congress Street Open Cut in Chicago (Fig. 2), the Brightling Sea Slide (Fig. 3) and the Seven Sisters Slide (Fig. 4) are examples of stability analyses where the undrained shear strength is used for one of the soil layers. An effective stress analysis using Bishop's Method was used for the Northolt (Fig. 5), Selset (Fig. 6) and Green Greek Slides (Fig. 7). For the Northolt and Selset sides the pore pressures were defined by a piezometric line while for the Green Greek Slide the pore pressure parameter, r_u, was used. The Saskatchewan Dam (Fig. 8), the Northolt Slide (Fig. 9), the Sudbury Slide (Fig. 10), the Folkstone Warren Slide (Fig. 11) and the University of Alberta River Bank Slide (Fig. 12) are all examples of failures with a composite sliding surface.

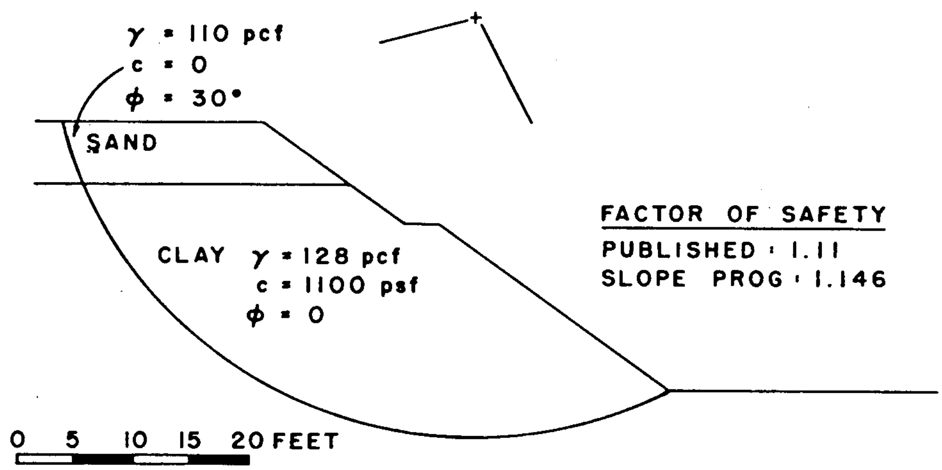


FIGURE 2 THE CONGRESS STREET OPEN CUT IN CHICAGO [IRELAND (1954)]

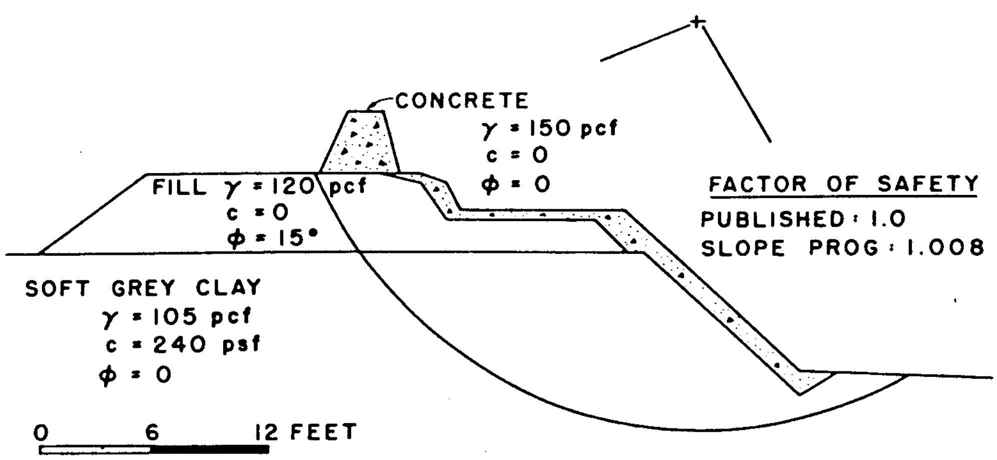


FIGURE 3 THE BRIGHTLINGSEA SLIDE [SKEMPTON AND GOLDER (1948)]

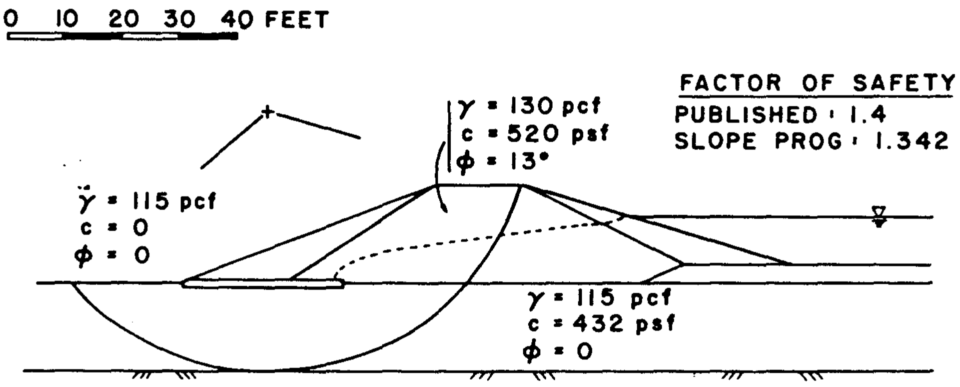


FIGURE 4 THE SEVEN SISTERS SLIDE
[PETERSON et al, (1957)]

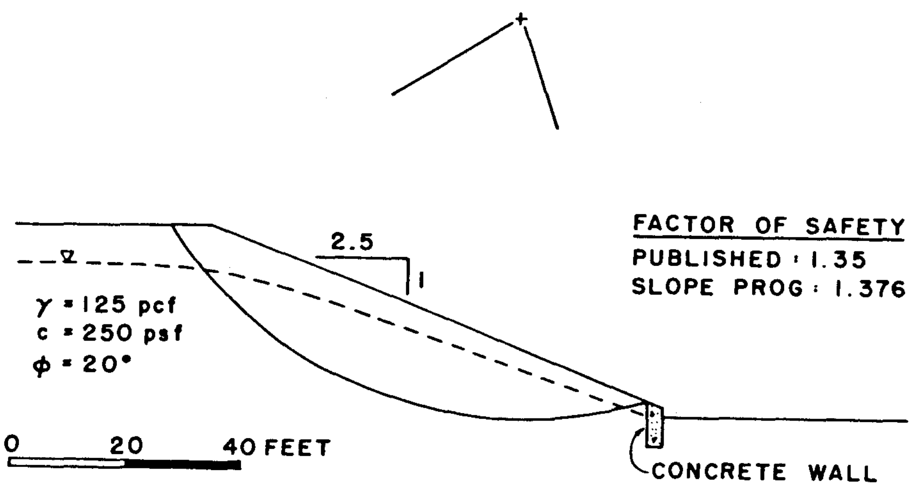


FIGURE 5 THE NORTHOLT SLIDE
[HENKEL (1957)]

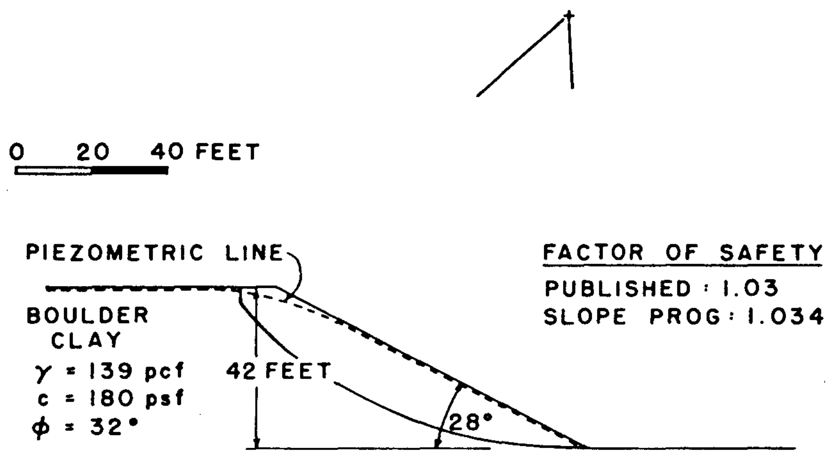


FIGURE 6 THE SELSET LANDSLIDE
[SKEMPTON AND BROWN (1961)]

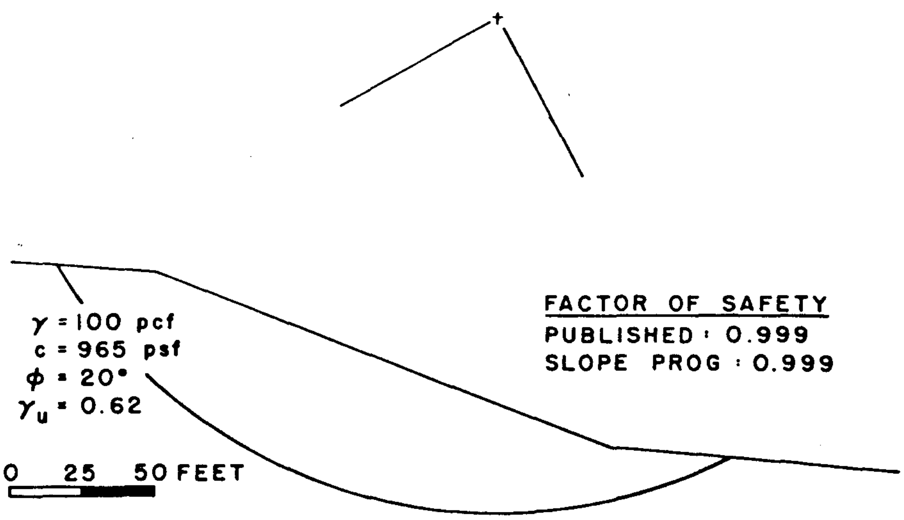


FIGURE 7 THE GREEN GREEK SLIDE
[CRAWFORD AND EDEN (1967)]

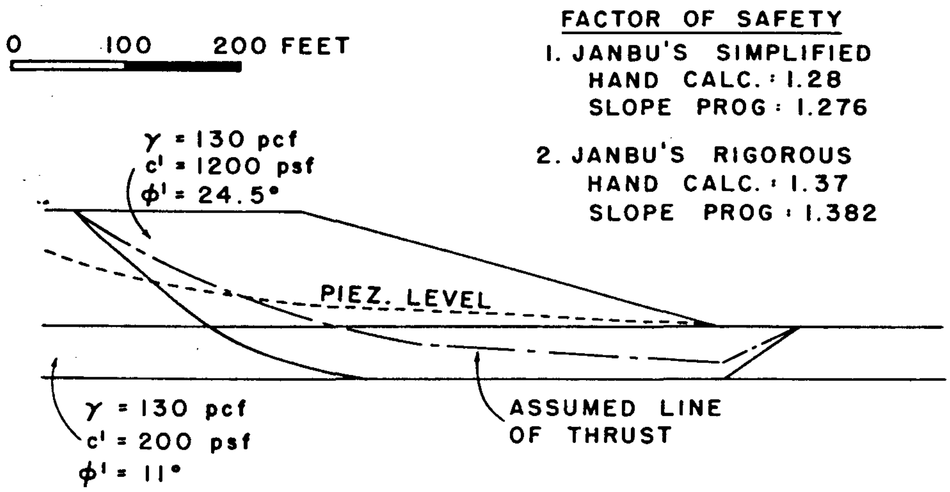


FIGURE 8 SASKATCHEWAN DAM [FREDLUND (1974); HAND CALCULATIONS BY P.F.R.A.]

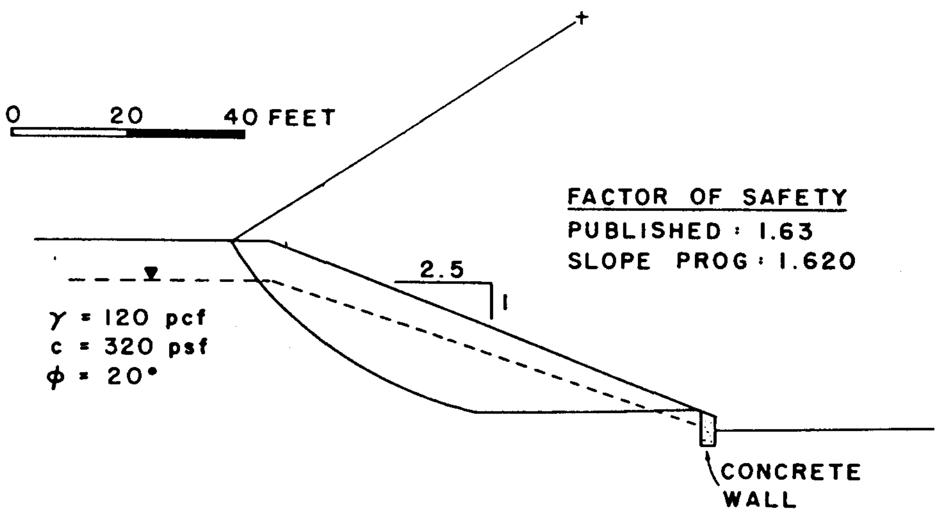


FIGURE 9 THE NORTHOLT SLIDE [SKEMPTON AND HUTCHINSON (1969)]

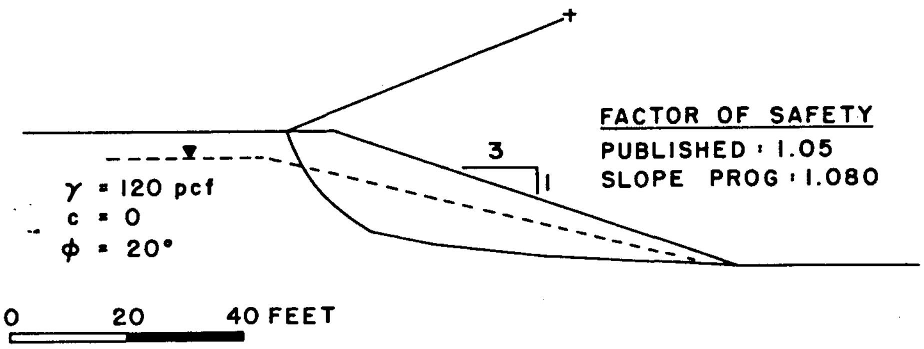


FIGURE 10 THE SUDBURY SLIDE
[SKEMPTON AND HUTCHINSON (1969)]

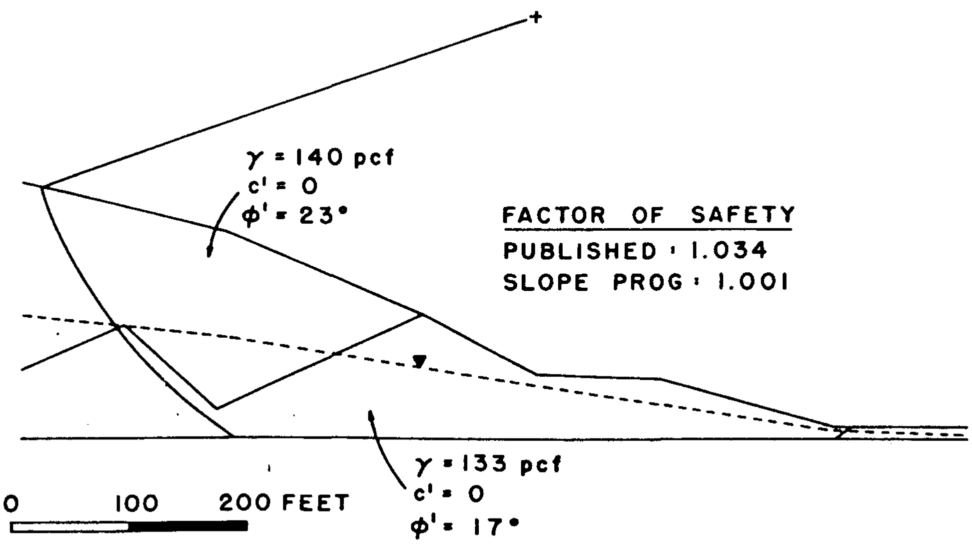


FIGURE 11 THE FOLKSTONE WARREN SLIDE
[HUTCHINSON (1969)]

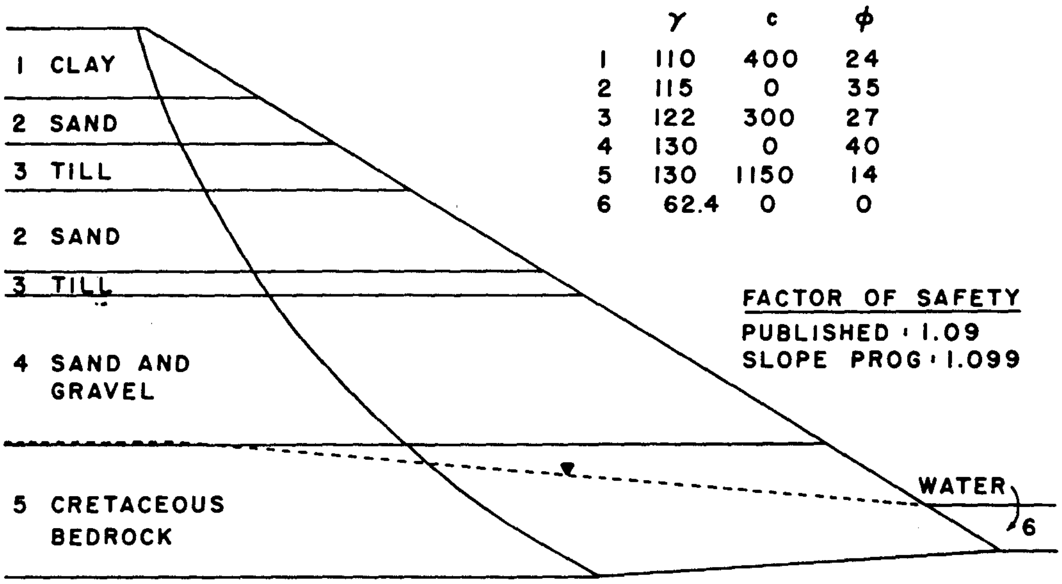


FIGURE 12 THE UNIVERSITY OF ALBERTA RIVER BANK SLIDE [THOMSON (1970)]

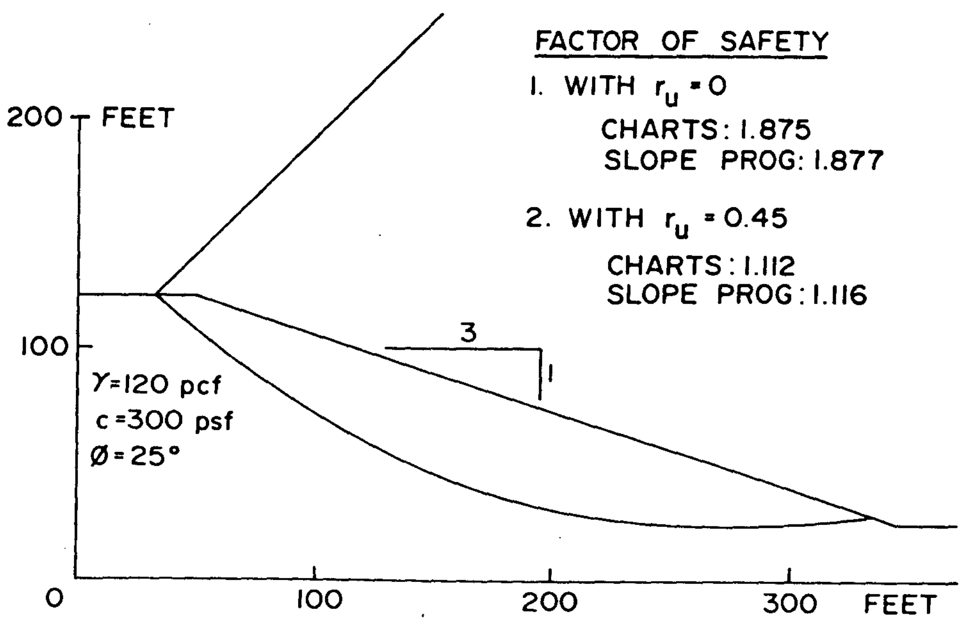


FIG. 13 COMPARISON OF SLOPE PROGRAM WITH THE BISHOP-MORGENSTERN STABILITY CHARTS

The University of Alberta River Bank Slide has eight soil layers (two and three occur twice) and is an example where the toe of the slide is partly submerged.

Each of the Figures 2 to 12 shows the factor of safety given in the publications and the safety factor computed by SLOPE. Table I gives a summary of the results.

The maximum difference between the published factor of safety and the factor of safety computed by SLOPE is 5.8 percent for the Seven Sisters Slide. One probable reason for this is that the published safety factor is only given to the nearest 10 percent. For all the other eleven case histories, the two safety factors agree within ± 3.6 percent.

Comparison with stability charts and example problems

Safety factors were computed for an example problem from Bishop and Morgenstern's (14) stability charts and then analyzed with the SLOPE program. Figure 13 shows the section analyzed. Two pore pressure conditions were considered, one with r_u = 0.0 and the other with r_u = 0.45. The minimum safety factors were determined by computing the safety factors for a large number of trial slip circles.

The safety factors determined by the charts and by the SLOPE program are shown on Figure 13. The answers differ by less than one-half of one percent for both pore pressure conditions.

Spencer (15) and Morgenstern and Price (16) presented examples in their original publications on their methods of analysis.

TABLE I

COMPARISON OF SLOPE PROGRAM WITH CASE HISTORIES

SLIDE	METHOD OF ANALYSIS	PUBLISHED FACTOR OF SAFETY	SLOPE PROGRAM	REMARKS	REF.
Congress St. Slide, Chicago	Ordinary	1.11	1.146	Circular	(4)
Brightlingsea	Ordinary	1.0	1.008	Circular	(5)
Seven Sisters	Ordinary	1.4	1.342	Circular	(6)
Northolt	Bishop's	1.35	1.376	Circular	(7)
Selset	Bishop's	1.03	1.034	Circular	(8)
Green Greek	Bishop's	0.999	0.999	Circular	(9)
Sask. Dam	Janbu's Simplified	1.28	1.276	Composite	(10)
Sask. Dam	Janbu's Rigorous	1.37	1.382	Composite	(10)
Northolt	Morgenstern-Price	1.63	1.620	Composite	(10)
Sudbury	Morgenstern-Price	1.05	1.080	Composite	(11)
Folkestone Warren	Morgenstern-Price	1.034	1.001	Composite	(12)
U of A River Bank	Morgenstern-Price	1.09	1.099	Composite	(13)

These example slopes are shown in Figures 14 and 15 and were analyzed with the SLOPE program. The published and SLOPE safety factors are shown together with the sections on Figures 14 and 15. For these example problems, the two safety factors differ by less than two percent.

Comparison with the University of Alberta Morgenstern-Price program

Morgenstern and Price (17) developed a computer program at the time their method was first published. This program is in use in many parts of the world and is presently available through several sources (18).

The Newton-Raphson numerical technique was used by Morgenstern and Price (17) to solve the equilibrium equations. Fredlund (1) used a "Best Fit Regression" method to solve the Morgenstern-Price equations, a method similar to that used by Spencer (15). Therefore it became of special interest to compare the University of Alberta Morgenstern-Price Program with the SLOPE program.

Figure 16 shows an example problem involving both circular and composite failure surfaces. The section was analyzed for six combinations of geometry, soil properties and water combinations for two different side force functions. These results are presented on Table II.

Although the two computer programs use different techniques for solving the equations and different methods of inputing the geometry and side force functions, the safety factors obtained are essentially the same.

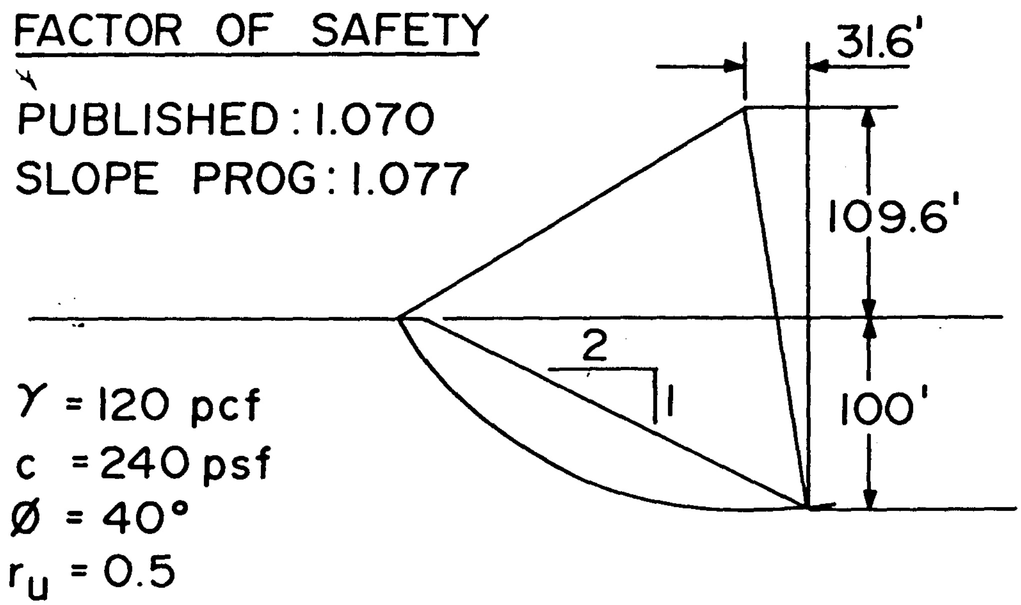


FIG. 14 SPENCER'S EXAMPLE PROBLEM

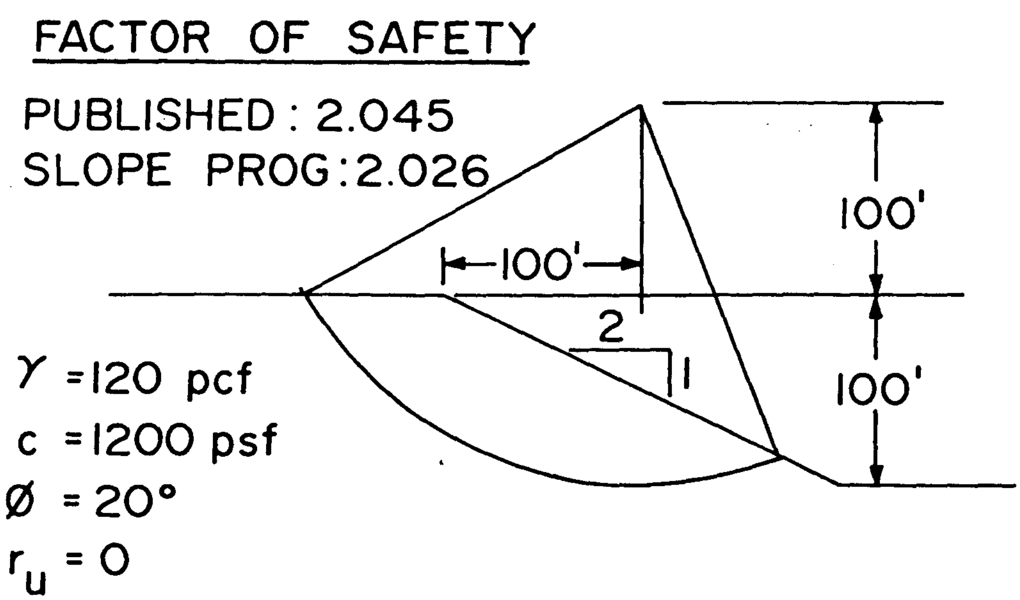


FIG. 15 MORGENSTERN - PRICE'S EXAMPLE PROBLEM

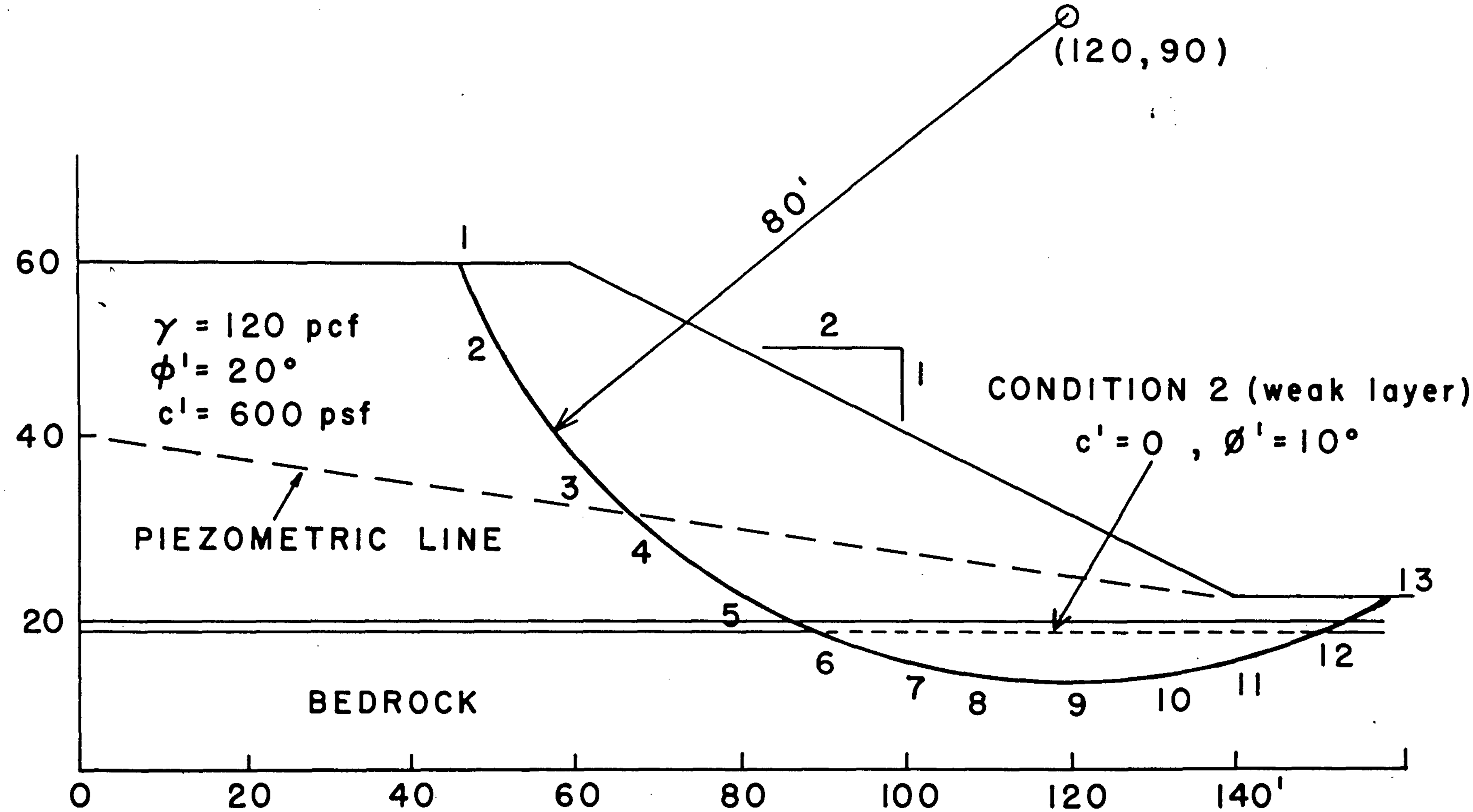


FIGURE 16 EXAMPLE PROBLEM

TABLE II

COMPARISON OF SLOPE PROGRAM WITH THE UNIVERSITY OF ALBERTA MORGENSTERN-PRICE PROGRAM*

Case No.	Example Problem	University of Alberta Program				University of Saskatchewan SLOPE Program			
		Side Force Function				Side Force Function			
		Constant		Half Sine		Constant		Half Sine	
		F	λ	F	λ	F	λ	F	λ
1	Simple 2:1 slope, 40 feet high, Ø' = 20°, c' = 600 PSF	2.085	0.257	2.085	0.314	2.076	0.254	2.076	0.318
2	Same as 1.) with a thin, weak layer with Ø' = 10°, c' = 0	1.394	0.182	1.386	0.218	1.378	0.159	1.370	0.187
3	Same as 1.) except with r_u = 0.25	1.772	0.351	1.770	0.432	1.765	0.244	1.764	0.304
4	Same as 2.) except with r_u = 0.25 for both materials	1.137	0.334	1.117	0.441	1.124	0.116	1.008	0.130
5	Same as 1.) except with a piezometric line	1.838	0.270	1.837	0.331	1.833	0.234	1.832	0.290
6	Same as 2.) except with a piezometric line for both materials	1.265	0.159	Not Converging		1.250	0.097	1.245	0.101

* See Reference (18)

** Tolerance on both programs is 0.001

The average lamda values computed by the programs differ by approximately nine percent; however, this difference does not significantly affect the final factor of safety.

Effect of slice width

The example problem shown on Figure 16 was analyzed for various slice widths. Figures 17 and 18 show the variation of factor of safety with slice width for a circular and composite failure surface. The slice width is expressed as a percent of the slide length as shown on Figures 17 and 18.

For the circular failure surface, the safety factor is essentially not affected by the slice width until it becomes greater than 10 percent of the slide length. For the composite failure surface, however, the safety factor oscillates about the actual safety factor by approximately $\pm$ 5 percent for large slice widths. This oscillation is reduced to about $\pm$ 1 percent when the slice width becomes less than 2 percent of the slide length.

Computing costs increase substantially as the slice width decreases (number of slices increases) especially for the Spencer and Morgenstern-Price methods as shown in the next section. Therefore, a significant savings in computing cost can be achieved for composite sliding surfaces by increasing the slice width if one is satisfied with a safety factor correct to within $\pm$ 5 percent.

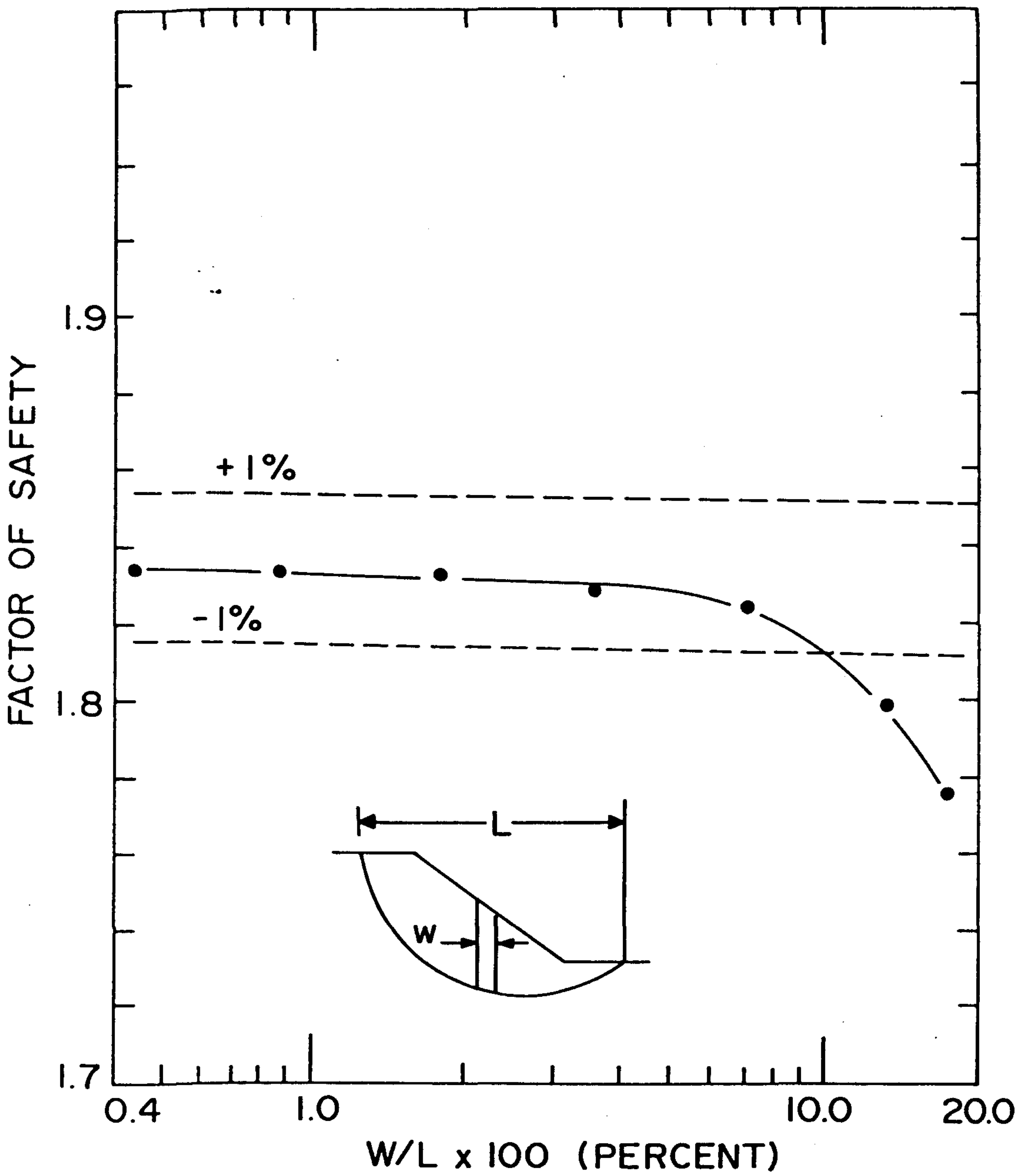


FIG. 17 EFFECT OF WIDTH OF SLICE ON FACTOR OF SAFETY FOR A CIRCULAR FAILURE SURFACE

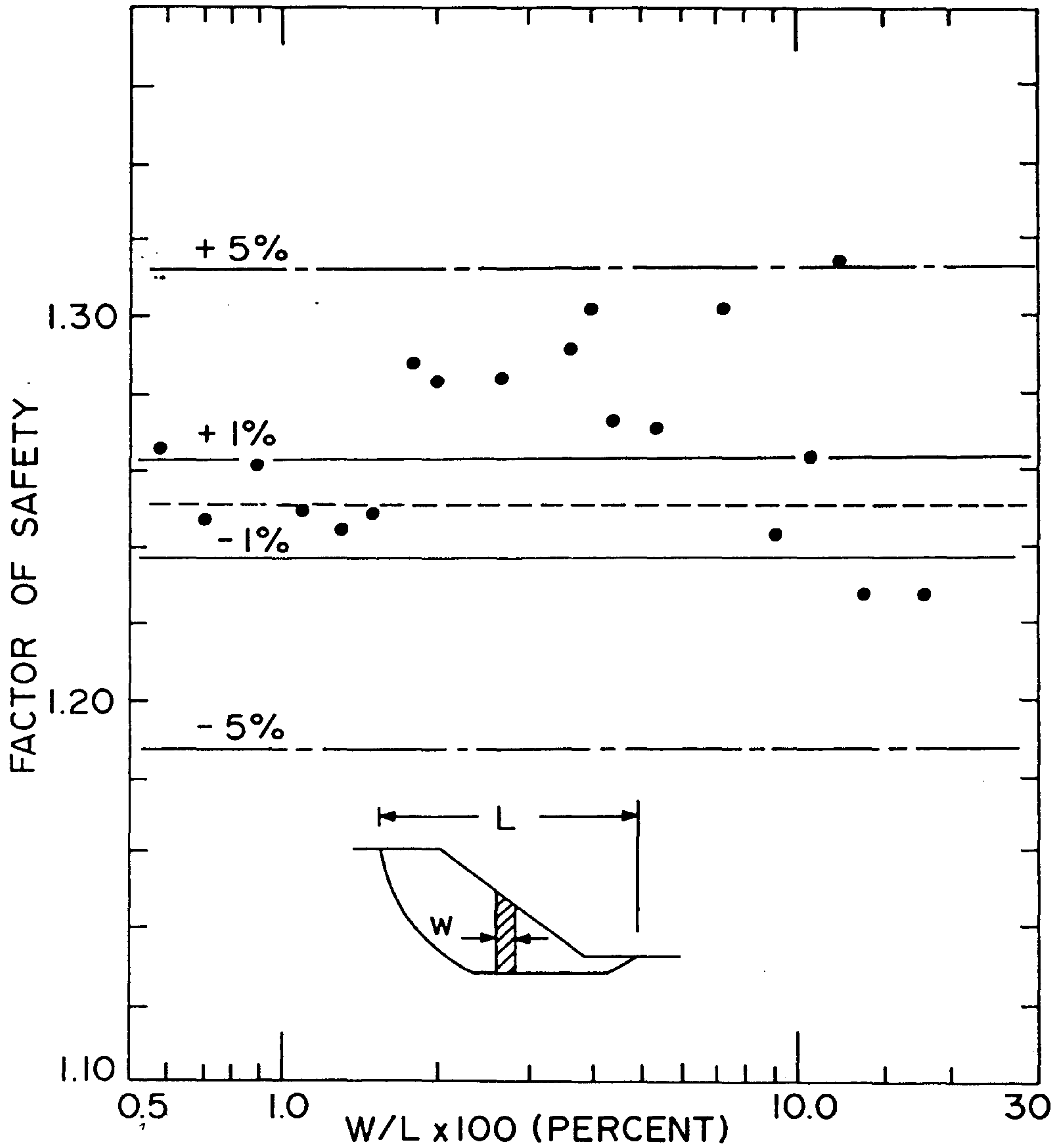


FIG. 18 EFFECT OF WIDTH OF SLICE ON FACTOR OF SAFETY FOR A COMPOSITE FAILURE SURFACE

Computing Time

The computing time required for the various conditions and methods was examined by analyzing a simple section with a height of 100 feet and a slope of 2 horizontal to 1 vertical. An IBM 370/158 computer was used.

Figure 19 shows the CPU time required per stability analysis for all the methods. The cost is similar for the Ordinary, Bishop and Janbu Methods while the cost for the Spencer and Morgenstern-Price Methods is much higher than the former methods. Savings in computing cost can be achieved by finding the critical failure surface by, for example, Bishop's method and then, if desired, analyzing the critical slip surface by the Morgenstern-Price method.

The Morgenstern-Price and Spencer Methods show a marked increase in computing time required as the slice width becomes smaller as illustrated in Figure 20.

The effect of the number of stability analysis trials performed on the total CPU time is shown in Figure 21. The total CPU time may be broken down into the time required for setting up the first trial and the time required for each stability trial thereafter. The basic cost of formulation is constant and after that the increase in total cost is directly proportional to the number of stability analysis trials performed.

Concluding remarks

The study described in this paper has shown that the factors of safety computed with the University of Saskatchewan SLOPE program are essentially the same as those published in the literature for case histories, example problems and stability charts.

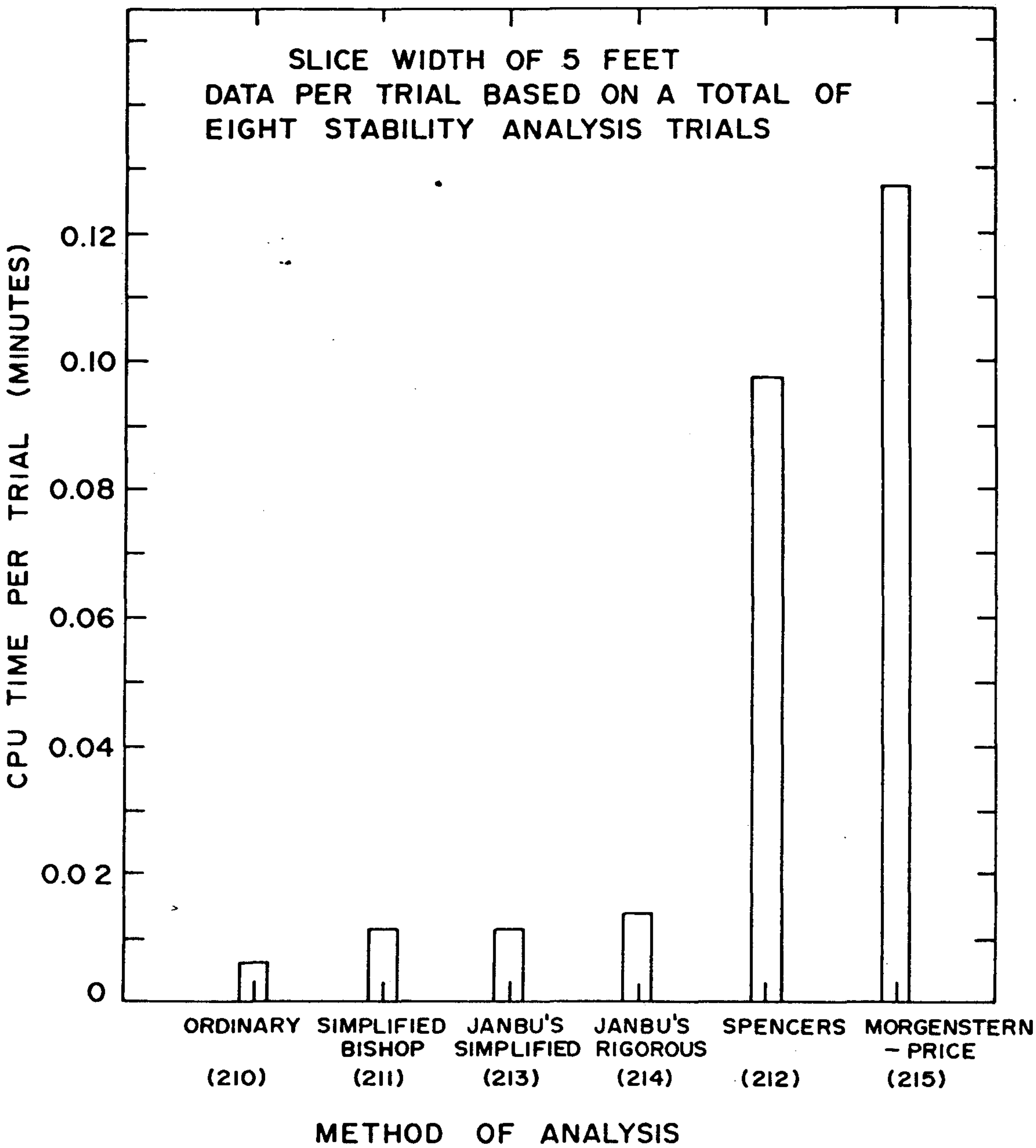


FIG. 19 TIME PER STABILITY ANALYSIS TRIAL FOR ALL METHODS OF ANALYSIS

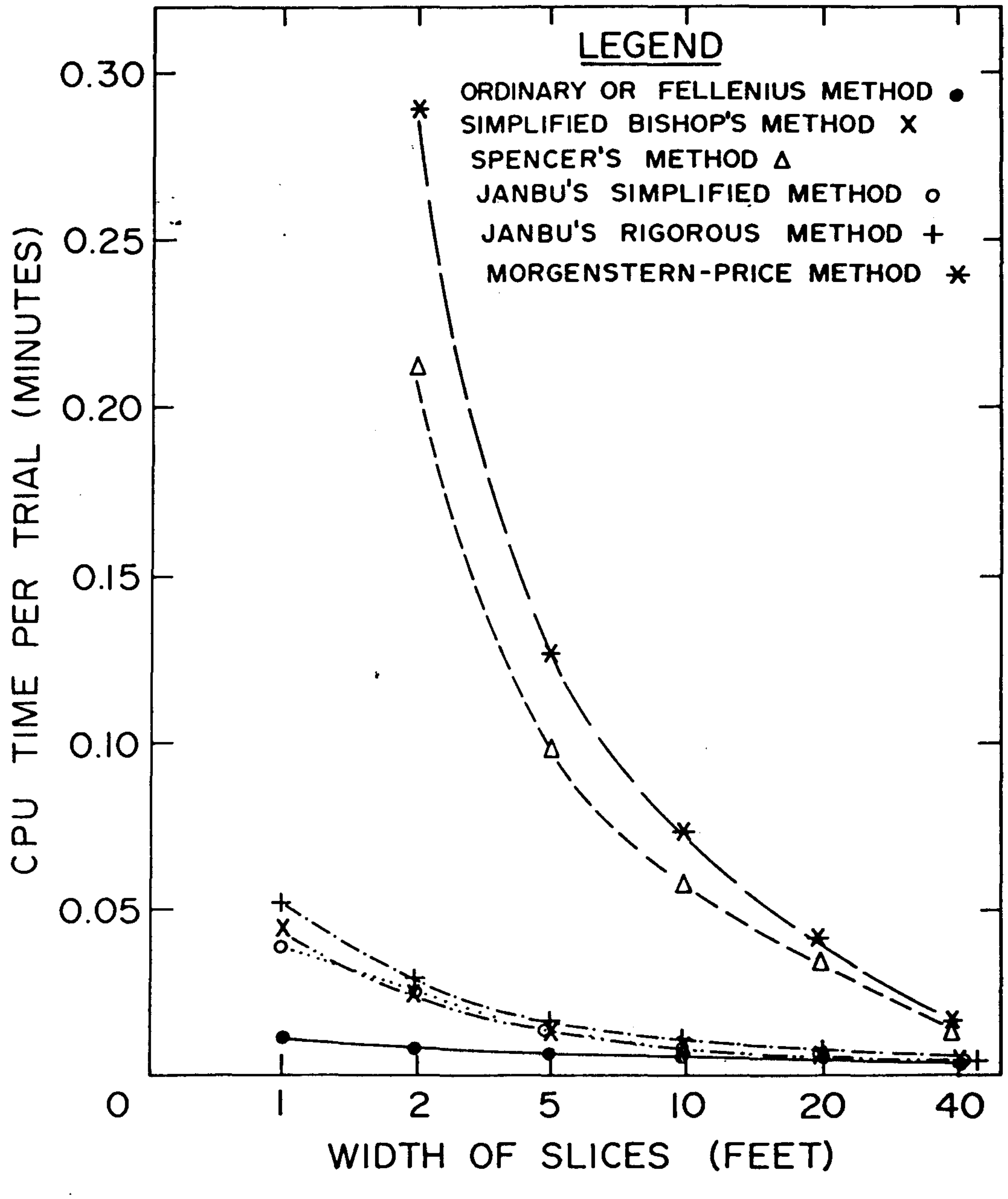


FIG. **20** EFFECT OF SLICE WIDTH ON TIME PER STABILITY ANALYSIS TRIAL

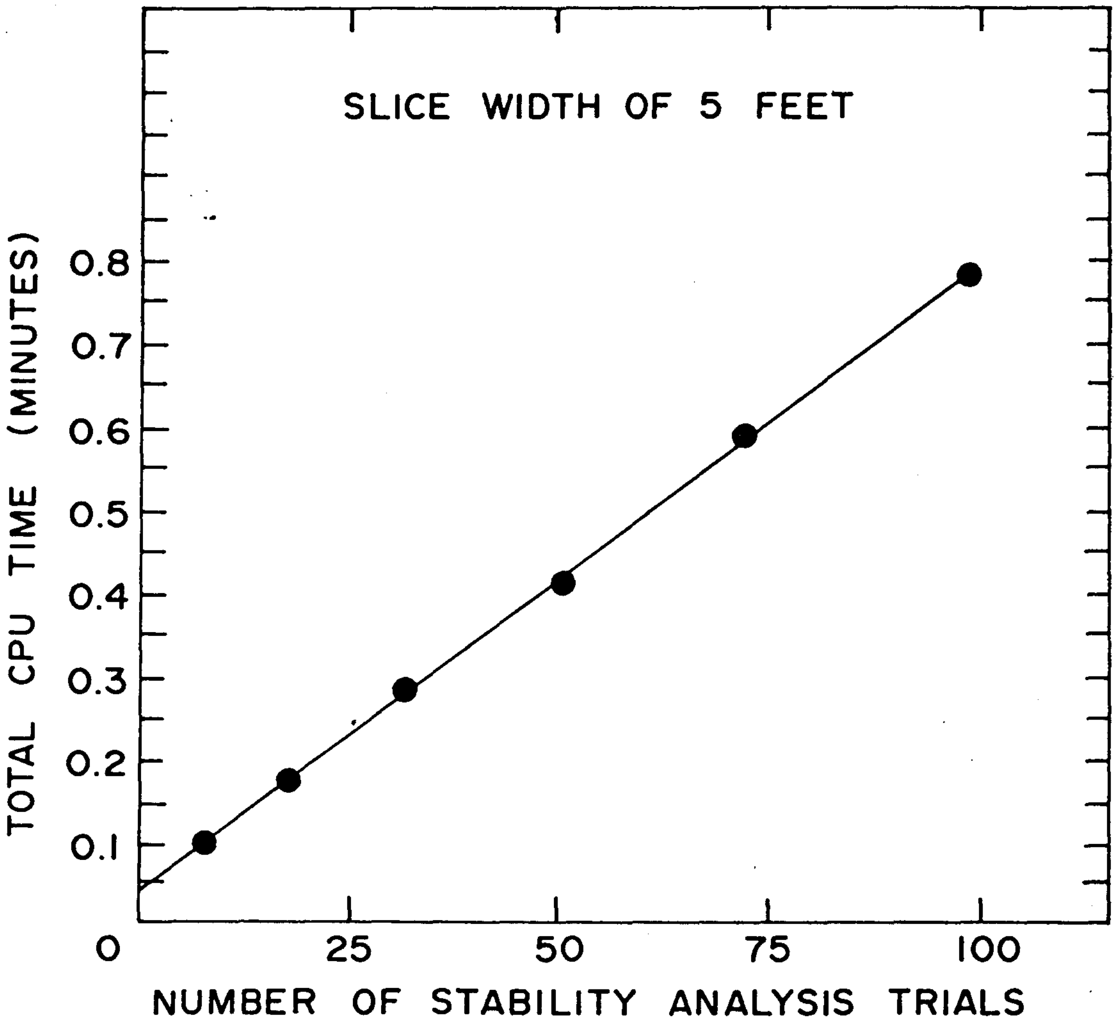


FIG. 21 EFFECT OF NUMBER OF STABILITY ANALYSIS TRIALS PERFORMED ON TOTAL CPU TIME (SIMPLIFIED BISHOPS METHOD)

The same was also shown to be true in answers obtained from The University of Alberta Morgenstern-Price and the SLOPE programs. These results have greatly increased the confidence in this program and its suitability for distribution. All the examples and case histories used in this paper can now be used as "benchmark examples" for future program verification studies.

The computing cost of time evaluation has made it possible to suggest means of minimizing computing costs.

Acknowledgements

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LIST OF RESEARCH PAPERS

	Report No.
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On Total, Matric and Osmotic Suction by J. Krahn and D.G. Fredlund, Soil Sciences Journal, Volume 114, No. 5, November, 1972	RP - 2
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