Deformation Characterization of Subgrade Soils for Highways and Runways in Northern Environments[1]

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Roads and runways in Northern Canada often must carry exceptionally heavy loads. In order to design for these loads, a procedure has been developed which enables the prediction of fatique life of pavements. Experimental evidence indicates that the definition of resilient deformation, which controls fatigue life of pavements, can be resolved in terms of stress state variables. The resilient modulus is defined in terms of $(\sigma_1 - \sigma_3)$, $(\sigma_3 - u_a)$, and $(u_a - u_w)$ which is applied in a theoretical analysis. Typical forms of the constitutive relationships are presented. The effect of freeze–thaw cycles does not appear to produce significant hysteresis in the constitutive relationships.

Les routes et pistes d'atterissage doivent souvent supporter des charges exceptionnellement élevées dans le nord canadien. Pour aider au dimensionnement des pavages soumis à de telles charges, une procédure a été développée qui permet la prévision de la durée de vie des pavages compte tenu du phénomène de fatique. Les observations expérimentales indiquent que la définition de la déformation de résilience, qui contrôle le développement de la fatigue des pavages, peut être exprimée en terme de paramètres de l'état de contraintes. Le module de résilience est défini en fonction de $(\sigma_1 - \sigma_3)$, $(\sigma_3 - u_a)$ et $(u_a - u_w)$ et utilisé dans une analyse théorique. Les formes types des lois constitutives sont présentées. L'effet de cycle gel–dégel ne semble pas produire de phénomène notable d'hystérésis dans les lois constitutives.

[Traduit par la Revue]

Introduction

Rapid development in Northern Canada has required the construction of roads and runways to transport raw resources out of the region. As a result, there is a necessity to be able to design facilities for heavier loads than is possible with existing conventional design procedures. During the past 6 years, the University of Saskatchewan in cooperation with the Saskatchewan Department of Highways has carried out an extensive research and development program to formulate a design procedure which will make it possible to design asphalt pavement structures for heavy axle loads and in addition be able to consider the economic consequences of increased axle loads. This procedure is now developed and is being used in the design of resource haul roads in Northern Saskatchewan.

In order to satisfactorily carry out the design, considerable knowledge is required on the deformation characteristics of subgrade soils and changes that occur throughout the year. Figure 1 illustrates the seasonal variation in Benkelman beam deflections of a section of highway between Regina and Lumsden in Saskatchewan, Canada. Bergan (1972) was able to predict the seasonal changes in the subgrade deformation characteristics of this highway by 'backing-in' on the results of Benkelman beam measurements with a theoretical stress analysis. However, this type of data is not always avail-

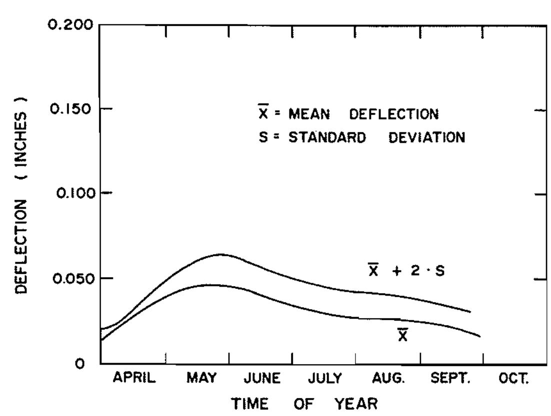

FIG. 1. Typical Benkelman beam deflections *versus* time of year for the Regina to Lumsden highway (from Bergan 1972).

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able, therefore, it would be advantageous to be able to predict the subgrade behavior from a knowledge of expected changes in the stress state variables throughout the seasons. In turn, the stress state variables would be predicted from the microclimatic environment and the geometric boundary conditions of the cross section under consideration.

The freezing and thawing of the subgrade also has an effect on its response to dynamic loading and must be given consideration in the cold environment of Canada. This paper deals with the prediction of the deformation characteristics of an unsaturated subgrade soil in terms of the associated stress state variables, giving consideration to the effects of freeze–thaw cycles.

Theory

One of the principal modes of failure in pavements is fatigue cracking from repeated flexure under dynamic loads produced by moving vehicles. During the past decade, increasing use has been made of the repeated load laboratory test for the prediction of pavement deflections (Seed *et al.* 1965). The tests measure the resilient (or elastic) strain when a sample is subjected to repeated loadings under triaxial conditions. The soil parameter evaluated is the resilient modulus.

$$[1]\qquad M_R = \frac{\sigma_d}{\epsilon} = \frac{\sigma_1 - \sigma_3}{\epsilon}$$

where M_R = modulus of resilient deformation (analogous to an elastic modulus), ϵ = resilient axial strain in the major principal stress direction, σ_d = repeatedly applied deviator stress, σ_1 = major principal stress, and σ_3 = minor principal stress.

In order to describe the resilient modulus in terms of stresses only, it is necessary to know the stress state variables associated with the prediction of strain in an unsaturated soil. Fredlund (1973) used the superposition of coincident equilibrium stress fields for each phase of an unsaturated soil in order to predict the stress state variables. The predicted stress state variables were experimentally verified by means of null-type tests. The analysis indicated that one possible set of stress state variables, in matrix form, is:

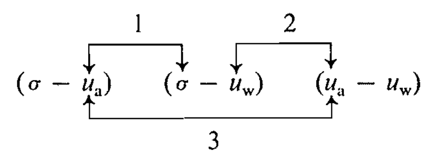
$$\begin{bmatrix} \sigma_x - u_w & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - u_w & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - u_w \end{bmatrix}$$

and

$$\begin{bmatrix} u_a - u_w & 0 & 0 \\ 0 & u_a - u_w & 0 \\ 0 & 0 & u_a - u_w \end{bmatrix}$$

where σ_x, σ_y, σ_z = total normal stress in the x, y, and z directions respectively, τ_{xy}, τ_{yz}, τ_{zx}, τ_{yx}, τ_{zy}, τ_{xz} = total shear stress, and u_a, u_w = the air and water pressures, respectively.

The stress analysis further indicates that any two of three possible forms of the stress state variables can be used.



If we assume that an unsaturated soil behaves as an isotropic linear elastic continua, the strain can be written in terms of the stress state variables (Fredlund 1973):

$$[2]\qquad \epsilon = \frac{(\sigma_1 - u_w)}{E_2} - \frac{2\mu_2}{E_2}(\sigma_3 - u_w) + \frac{(u_a - u_w)}{H_2}$$

where E_2 = the elastic modulus associated with a change in $(\sigma_1 - u_w)$, μ_2 = the corresponding Poisson's ratio, and H_2 = the elastic modulus associated with a change in $(u_a - u_w)$.

Equation [2] is of essentially the same form as that proposed by Biot (1941) and Coleman (1962).

An examination of Eqs. [2] and [1] gives an indication of the stress state variables involved. However, it must be emphasized that the combination of these equations does not uniquely describe the resilient modulus under repeated loading conditions. The dimensional-type analysis shows that:

$$[3]\qquad M_R = f[(\sigma_1 - u_w), (\sigma_3 - u_w), (u_a - u_w)]$$

The proper form of the constitutive relationship between resilient modulus and the stress state variables must be obtained through experimental analysis. Another form of Eq. [3] is:

$$[4] \quad M_R = f[(\sigma_1 - u_a), (\sigma_3 - u_a), (u_a - u_w)]$$

The form in Eq. [4] is advantageous when considering the field problem since the air pressure tends to equilibrium with the atmosphere. In other words, the air is either open to atmosphere in nature or else it diffuses through the water in order to come to equilibrium with the atmosphere.

Since $(\sigma_1 - u_a)$ is equal to $[(\sigma_1 - \sigma_3) + (\sigma_3 - u_a)]$, we can also write

$$[5] \quad M_R = f[(\sigma_1 - \sigma_3), (\sigma_3 - u_a), (u_a - u_w)]$$

In some analyses, it is sufficient to use a constant $(\sigma_1 - \sigma_3)$ stress. In this case,

$$[6] \quad M_R = f[(\sigma_3 - u_a), (u_a - u_w)]$$

and can graphically be represented as shown in Fig. 2.

There are serious technical problems with respect to measuring air and water pressures under dynamic loading conditions. All attempts to measure these pressures at the University of Saskatchewan have been unsuccessful due to the slow response of the measuring system (Fredlund and Morgenstern 1973). For this reason, it appears necessary to revert to a total

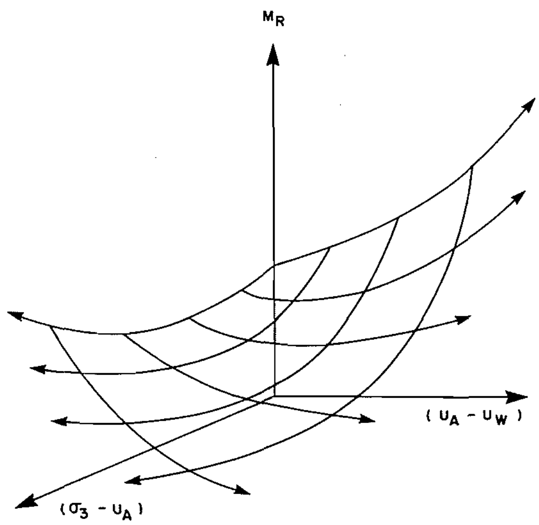


FIG. 2. Form of the constitutive surface for resilient modulus.

stress type of approximation during repeated loading.

Dehlen (1969) found that 1000 initial stress repetitions were sufficient to avoid changes in axial deflection because of sample end imperfections. MacLeod (1971) found that 50 to 100 repetitions were sufficient to establish the resilient modulus after a change in deviator stress on a sample. Culley (1971) presented typical results for the variation of resilient modulus *versus* the number of load repetitions (Fig. 3). On the basis of this evidence, it would appear satisfactory to use the resilient modulus computed after approximately 1000 repetitions, and relate it to the stress state variables just prior to loading the sample.

Effect of Freeze–Thaw Cycles

Hamilton (1966) found that samples compacted below a 90% degree of saturation shrank upon freezing while those compacted at higher water contents increased in volume. The greatest amount of shrinkage on freezing was observed for samples between 60 and 70% saturation. Upon thawing, a net increase in volume was observed. Similar observations have been reported by Mickleborough (1969) and Lidgren (1970). This indicates that a basic change is taking place as a result of freezing and thawing.

If the freezing and thawing of a soil were a nonhysteretic process, the stress state variables should be the same before and after freezing.

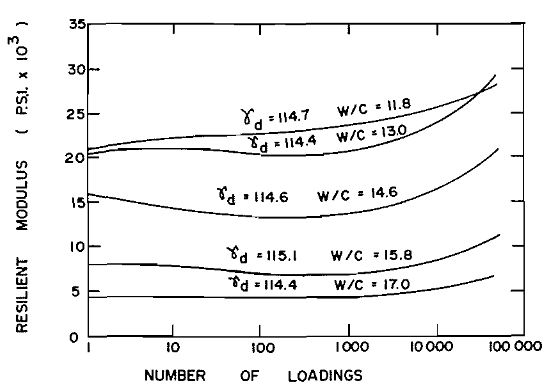


FIG. 3. Effect of water content and degree of saturation on resilient modulus *versus* number of loadings (after Culley 1971).

Since the total and air pressures are essentially the same before and after freezing, only the changes in water pressure need to be observed. Mickleborough (1970) and Bergan (1972) reported significant decreases in matric suction ($u_a - u_w$) as a result of freezing and thawing. In addition, the structure of the soil is modified as a result of freezing and thawing. Although the matric suction changes as a result of freezing and thawing, the question relevant to this paper involves the adequacy of the stress state variables to define the resilient modulus before and after freezing and thawing.

Prediction of Stress State Variable Changes

The diffusivity of air through water is such that the air phase of a soil should equalize with the atmosphere in a relatively short period of time. Therefore a knowledge of changes in the water pressure is the key to determining the changes in the stress state variables throughout the year.

A transient flow analysis such as that proposed by Richards (1965) and Nachlinger and Lytton (1969) is being considered to relate microclimatic changes with pore water pressures. As well, consideration is being given to simpler microclimatic evapotranspiration calculations to define soil moisture deficiency (Thornthwaite 1948).

Laboratory Equipment

The triaxial loading systems are designed to repeatedly apply a load to the soil, thereby attempting to simulate the stresses produced by a moving vehicle. Bergan (1972) has demonstrated that the test results do give a realistic simulation of the field case.

The loading system is operated by compressed air which applies a pneumatic pressure to a bellofram located above the main piston[2] (Fig. 4). The frequency of the loading is governed by a control timer. The loading frequency is 20 repetitions per minute with a load duration of 0.1 s, similar to that used by Seed *et al.* (1965).

The load applied to the sample is measured by a load cell in the base plate. The vertical and lateral strains are measured by linear variable differential transformers (LVDT's).

The data presented in this paper was obtained over a period of several years and during this time the equipment has undergone numerous modifications. The equipment shown in Fig. 4 is presently operating at the University of Saskatchewan.

[2]University of Saskatchewan Soils Laboratory, Saskatoon, Saskatchewan.

Soil Samples

Two types of soil have been used in the investigation: (i) a glacial till taken from the Qu'Appelle moraine in the southeast part of the Province of Saskatchewan and (ii) a highly plastic lacustrine clay from Regina, Saskatchewan. The properties are summarized in Table 1. These are the most common types of materials in this region.

Presentation of Test Results

The relationship between resilient modulus and each of the stress state variables will be examined separately. In each case it would be desirable to have all samples initially prepared to the same water content and density. However, most of the test programs were set up independent of this study and, therefore, different molding densities and water contents were used. Krahn and Fredlund (1972) showed that the effect of varying density had only a small effect on the suctions for both the till and Regina clay. These results are in agreement with the findings of Olson and Langfelder (1965) and suggest that samples with varying initial densities can be used to test the proposed hypothesis.

(a) Effect of Deviator Stress ($\sigma_1 - \sigma_3$)

Seed *et al.* (1965) showed that the magnitude of the deviator stress has a significant effect on the resilient modulus for plastic soils.

TABLE 1. Properties of soils tested

Property	Glacial till	Regina clay
Liquid limit	38.5%	78.4%
Plastic limit	16.8%	30.6%
% sand	26.8	5.6
% silt	40.0	27.2
% clay	33.3	67.2
Specific gravity	2.77	2.83
Max. std. density	106.8 p.c.f.	91.8 p.c.f.
Opt. water content	19.0%	27.8%

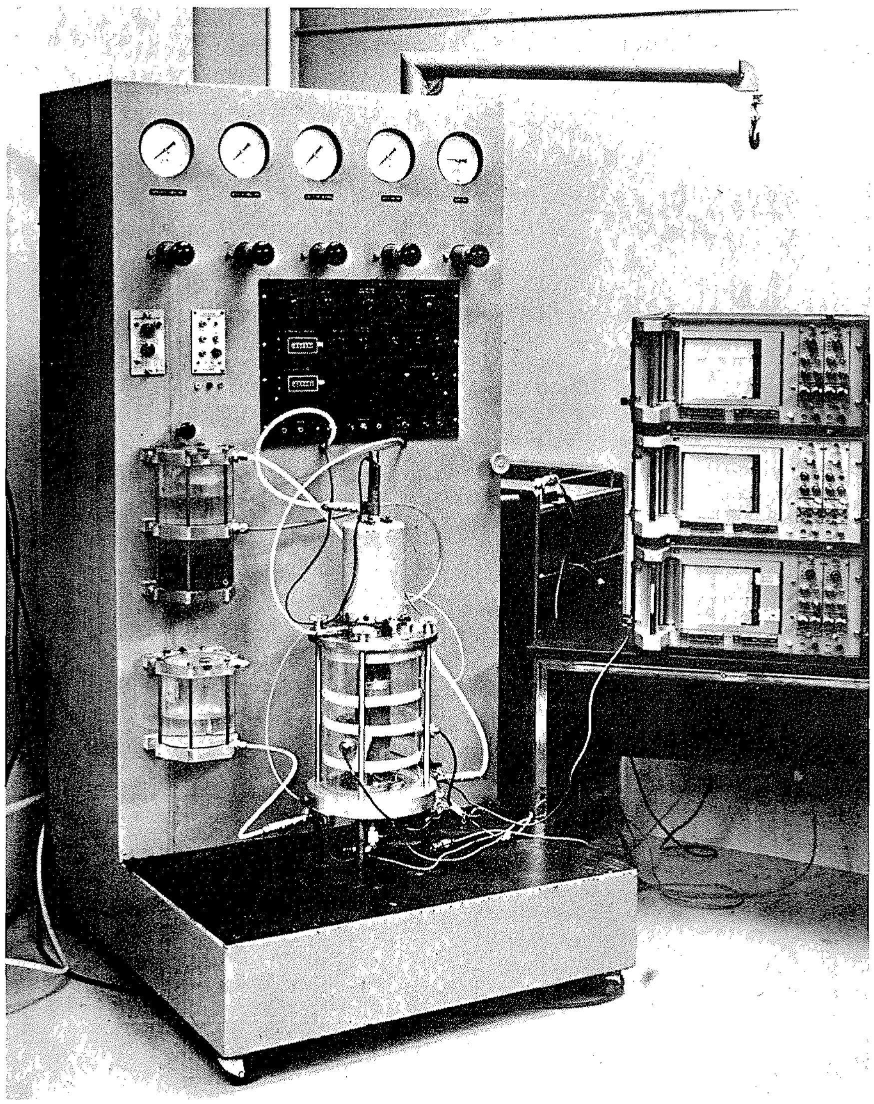


FIG. 4. Repeated loading triaxial system (University of Saskatchewan, Saskatoon).

As seen in Fig. 5, the resilient modulus decreases with increasing deviator stress in the low deviator stress range. At higher deviator stresses, the resilient modulus increases. However, the results are not too meaningful at high deviator stresses since failure conditions are approached.

Weimer (1972) and Bergan (1972) demonstrated a similar behavior for glacial till and Regina clay in the low deviator stress range (Fig. 6). The relationships are only slightly nonlinear in the stress range considered.

A deviator stress of 10 p.s.i. (0.7 kg/cm^2) has generally been used when evaluating the resilient modulus. However, a deviator stress range should be evaluated in order to duplicate the actual stress conditions that occur at the subgrade covered by a pavement structure.

(b) Effect of Confining Pressure ($\sigma_3 - u_a$)

For sands and gravels, Seed *et al.* (1965) have shown that there is a unique relationship between the resilient modulus and the confining pressure provided the applied stresses are not