PREDICTION OF HEAVE IN UNSATURATED SOILS

by

D. G. Fredlund
Associate Professor
Civil Engineering Department
University of Saskatchewan
Saskatoon, Saskatchewan, Canada

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INTRODUCTION

Practicing engineers are well aware that many of the problems they encounter involve unsaturated soils. In addition to unsaturated, compacted soils, there are large portions of the land surface of the earth subjected to desiccating influences that leave the upper portion of the profile cracked and unsaturated. Light structures placed on these soil profiles generally suffer distress due to the subsequent expansion of the desiccated soil.

Numerous procedures have been developed to enable the prediction of the amount of heave (Jennings, 1969). Each procedure has been developed and tested within one geographical region and little attempt has been made to embrace the various methods within one consistent theoretical context.

This paper considers the heaving prediction analysis within the context of multiphase continuum mechanics. The basic steps are:

i) the establishment of the stress state variables
ii) the description of suitable deformation state variables
iii) the proposal of suitable constitutive relationships and the experimental verification of their uniqueness.

On the basis of the above steps, a formulation for the one-dimensional heaving analysis is proposed.

THEORETICAL CONSIDERATIONS

Element Description

Generally an unsaturated soil is considered to be a three-phase system; however, the independent properties and continuous bounding surfaces of the air-water interphase (i.e. contractile skin) require its consideration as a fourth phase (Davies and Rideal, 1963). An element of unsaturated soil can therefore be considered as a mixture with two phases that come to equilibrium under applied stresses (i.e. soil particles and the contractile skin) and two phases that flow under applied pressures (i.e. the air and water).

Stress State Variables

Several effective stress equations have been proposed for unsaturated soils (Cooling, 1961). However, they have lacked theoretical justification and have not proven satisfactory in practice. Fredlund (1973a, 1973b) used the principle of superposition of coincident equilibrium stress fields to predict the stress state variables associated with the soil particles and the contractile skin. The analysis indicated that any two of three possible normal stress variables can be used to define the stress state.
Possible combinations are:

i) $(\sigma - u_a)$	ii) $(\sigma - u_w)$	iii) $(\sigma - u_w)$
and	and	and
$(u_a - u_w)$	$(u_a - u_w)$	$(\sigma - u_a)$

where

σ = total normal stress

u_a = air pressure

u_w = water pressure

Null type experiments (i.e. $\Delta\sigma = \Delta u_a = u_w$) showed extremely good verification of the theoretically proposed stress state variables (Fredlund, 1973a).

Deformation State Variables

The variables required for mapping the relative phase movements in an element can be derived from the continuity requirements for a multiphase continua (Fredlund, 1973a, 1974). If the soil particles are assumed incompressible and the volume change of the contractile skin assumed internal to the element, the volumetric requirement for an element reduces to,

$$\frac{\Delta V}{V} = \frac{\Delta V_w}{V} + \frac{\Delta V_a}{V} \tag{1}$$

where

V = total volume of the element

V_w = volume of water in the element

V_a = volume of air in the element

The measurement or prediction of any two of the above volumes allows a complete monitoring of the volume weight relationships. The amount of air and water in the element can be represented respectively by one volumetric deformation state variable (i.e. θ_w and θ_a, respectively). The overall element (i.e. soil structure) requires a tensor composed of normal and shear deformation state variables.

Constitutive Relationships

The independently derived stress and deformation state variables are linked by suitable constitutive relationships. These are proposed from a semi-empirical standpoint and must then be experimentally checked for uniqueness.

The constitutive relationship for the soil structure is modelled in an incremental sense as a simple, linear, elastic, isotropic material. Selecting $(\sigma - u_w)$ and $(u_a - u_w)$ as the stress state variables, the normal strain in the x-direction, ε_x is,

$$\varepsilon_x = \frac{(\sigma_x - u_w)}{E_1} - \frac{\mu_1}{E_1} \cdot (\sigma_y + \sigma_z - 2 \cdot u_w) + \frac{(u_a - u_w)}{H_1} \qquad (2)$$

where E_1 = an elastic modulus with respect to a change in $(\sigma_x - u_w)$

μ_1 = poisson's ratio with respect to relative changes in $(\sigma_y - u_w)$ and $(\sigma_x - u_w)$

H_1 = an elastic modulus with respect to a change in $(u_a - u_w)$

Similar equations can be written for the y and z directions. The equations are essentially the same as those proposed by Biot (1941) and Coleman (1962).

The linear combination of state variables indicates that the volumetric representation of either the air or water phase must also be mapped. Using the water phase,

$$\theta_w = \frac{(\sigma_x + \sigma_y + \sigma_z - 3 \cdot u_w)}{3H_1'} + \frac{(u_a - u_w)}{R_1} \qquad (3)$$

where R_1 = a water volumetric modulus associated with a change in $(u_a - u_w)$.

H_1' = a water volumetric modulus associated with a change in $(\sigma - u_w)$. Introducing the assumption of a potential energy for the soil, Biot,(loc cit) proved that

$$H_1' = H_1.$$

FORMULATION FOR ONE DIMENSIONAL HEAVING

Let us consider the example of a granular fill and asphalt surface placed immediately over a desiccated, unsaturated soil (Figure 1).

Boundary Conditions

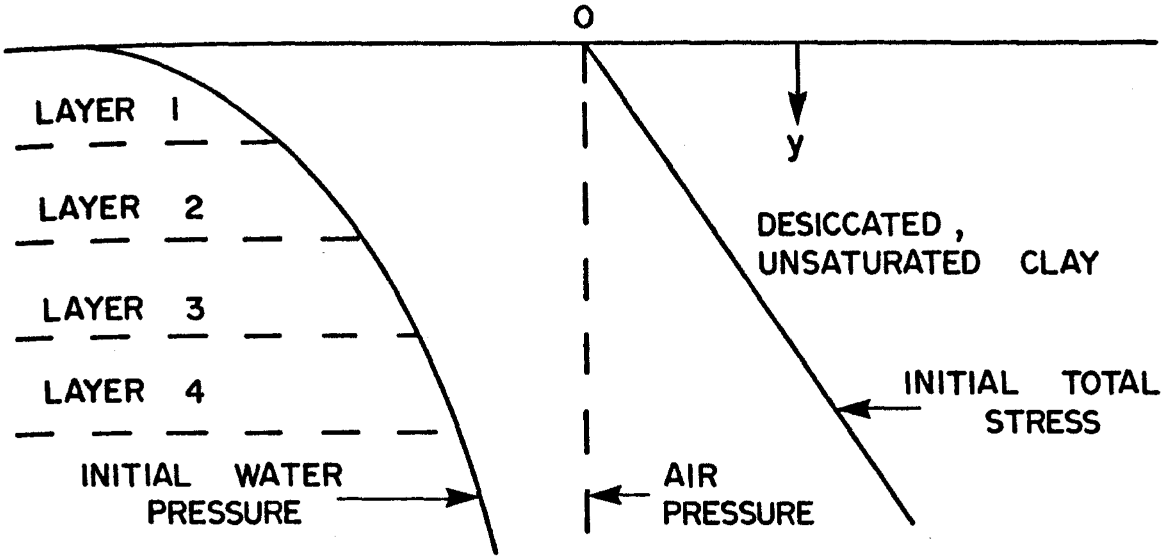
At the time of construction, the soil has a tension in the pore water while the air phase is approximately equal to atmospheric pressure. For design purpose, let us assume that eventually the pore water pressure comes to equilibrium with a water table at ground surface and the matric suction {i.e. $(u_a - u_w)$}goes to zero.

Any analysis for the prediction of total heave has three aspects:

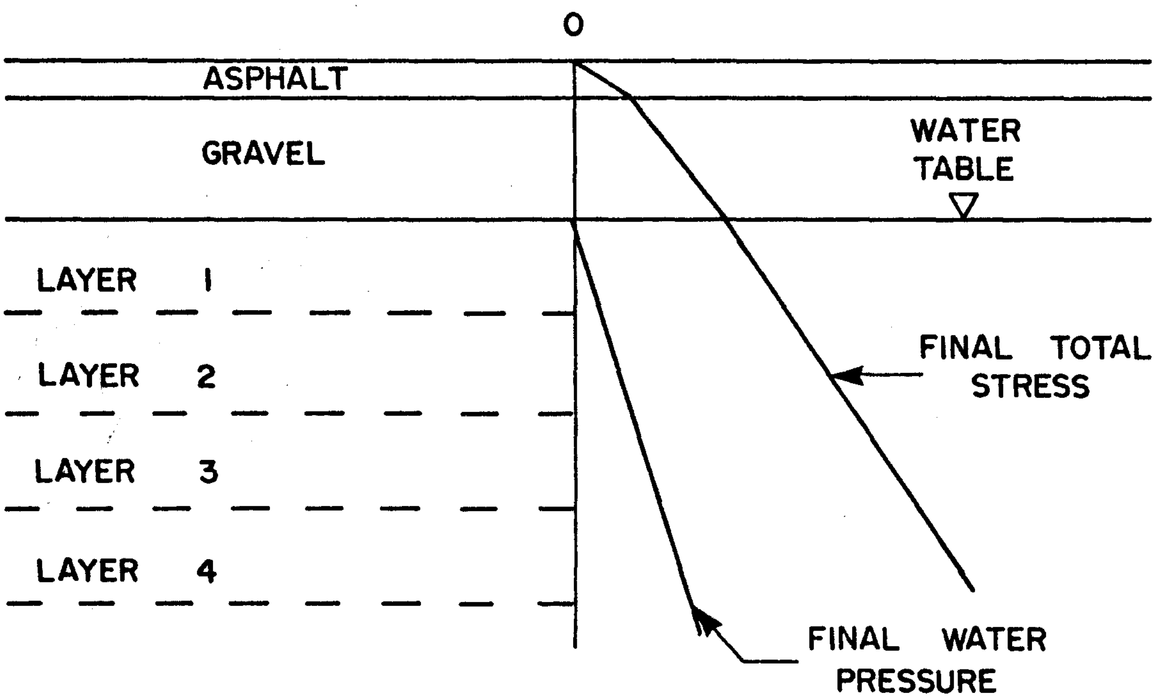
i) the initial stress boundary conditions. This primarily involves the determination of the initial pore water tension.

ii) the final stress boundary conditions. Possible final conditions depend on studies of equilibrium conditions in various geographical environments. Figure 1 indicates one possibility.

iii) the constitutive equations relating the initial and final stress condit The latter part of this paper is concerned with the verification of the uniqueness of the proposed constitutive equations.



1 a.) INITIAL BOUNDARY CONDITIONS



1 b.) FINAL BOUNDARY CONDITIONS

FIGURE 1 INITIAL AND FINAL BOUNDARY CONDITIONS FOR HEAVE ANALYSIS

Derivation for One-Dimensional Heave

The strain in the soil structure in the y-direction is,

$$\varepsilon_y = \frac{(\sigma_y - u_w)}{E_1} - \frac{\mu_1}{E_1}(\sigma_x + \sigma_z - 2 \cdot u_w) + \frac{(u_a - u_w)}{H_1} \tag{4}$$

In the one-dimensional case, the strains in the x and z-directions are zero while the changes in their total stresses are equal. Therefore, the $(\sigma_x - u_w)$ and $(\sigma_z - u_w)$ terms can be written;

$$(\sigma_x - u_w) = (\sigma_z - u_w) = \frac{\mu_1}{(1 - \mu_1)} \cdot (\sigma_y - u_w) - \frac{E_1}{(1 - \mu_1) \cdot H_1} \cdot (u_a - u_w) \tag{5}$$

Substituting equation (5) into (4) gives,

$$\varepsilon_y = m^s_{v1} \cdot d(\sigma_y - u_w) + m^s_{h1} \cdot d(u_a - u_w) \tag{6}$$

where $m^s_{v1} = \left[\dfrac{(1 + \mu_1)(1 - 2 \cdot \mu_1)}{E_1 \cdot (1 - \mu_1)}\right]$

$m^s_{h1} = \left[\dfrac{(1 - \mu_1)}{H_1 \cdot (1 + \mu_1)}\right]$

The amount of heave associated with each arbitrary layer (i.e. the i-th layer) below the gravel, is computed as the strain ε_{yi}, multiplied by the thickness of the layer h_i (Table I).
For the first layer,

$$\Delta h_1 = h_1 \cdot \left[m^s_{v1} \cdot d(\sigma_y - u_w) + m^s_{h1} \cdot d(u_a - u_w)\right] \tag{7}$$

The stress state variable changes and the corresponding compressibilities vary from one layer to the next since the equation is applied in an incremental manner. The total heave is equal to the sum of the heaves for each layer.

Along with the changes in total volume or heave, changes in the volume of water within the element can be traced. Thereby, changes in all volume weight variables can be computed. Since the soil is assumed isotropic and changes in σ_x equal changes in σ_z, equation (3) can be written,

$$\theta_w = \frac{(\sigma_y - u_w) + 2(\sigma_x - u_w)}{3 \cdot H_1} + \frac{(u_a - u_w)}{R_1} \tag{8}$$

TABLE I

Calculations for the Prediction of Heave

Layer No.	Layer Thickness (meters)	Change in $(\sigma_y - u_w)$ kN/m^2	$m^s_{v1} \times 10^{-4}$ (m^2/kN)	Change in $(u_a - u_w)$ kN/m^2	$m^s_{h1} \times 10^{-4}$ (m^2/kN)	Heave (meters)
1	0.5	427.	+ 6.02	414.	-.33 - 2.00	0.087
2	0.5	228.	+ 5.28	207.	.33 - 1.76	0.042
3	0.6	90.	+ 3.73	69.	.33 - 1.24	0.015
4	0.8	32.	+ 3.49	10.	-.33 - 1.16	0.008

Total Heave <u>0.152</u>

Substituting (5) into (8) gives,

$$\theta_w = m_{v1}^w \cdot d(\sigma_y - u_w) + m_{h1}^w \cdot d(u_a - u_w) \tag{9}$$

where $m_{v1}^w = \dfrac{1}{3 \cdot H_1 \cdot (1 - \mu_1)}$

$$m_{h1}^w = \frac{3 \cdot (1 - \mu_1) H_1^2 - E_1 \cdot R_1}{3 \cdot H_1^2 \cdot R_1 (1 - \mu_1)}$$

UNIQUENESS OF THE CONSTITUTIVE SURFACES

Two constitutive surfaces are required for a complete monitoring of volume change associated with an unsaturated soil (Figure 2). Their uniqueness can be explored; first, in terms of small deviations from a stress point and second, in terms of larger stress state variable changes and reversals of stress (Fredlund, 1973a, 1974.)

The simplest test for uniqueness near a stress point involves moving in two directions and then predicting the path for movement in a third direction (Figure 3). The predicted movement can be checked experimentally by changing the stress state by the appropriate amount.

Tests were performed on a one-dimensional oedometer modified to facilitate the control of total, air and water pressures and the measurement of total and water volume changes, (Fredlund (1963a). Undisturbed samples of a highly plastic clay from Regina, Saskatchewan, Canada were used for the test. The liquid limit was 75%, the plastic limit was 26% and the initial water content was 20.0%. Sixty five percent of the particles are clay size, rich in montmorillonite. Typical results for the soil structure deformation are shown in Figure 4. The high degree of correlation demonstrates the uniqueness of the constitutive surface.

Correlation between the predicted and measured water phase volume changes did not demonstrate as high a degree of correlation as did the soil structure results (Figure 5). The poorer correlation was primarily attributed to the difficulties associated with accurately measuring minute water volume changes over long periods of time. As the system of measuring the water volume change was improved, the degree of correlation greatly improved.

In general, the results do indicate satisfactory uniqueness of the proposed constitutive relationships. It is beyond the scope of this paper to deal with large stress state variable changes and hysteresis effects. However, the proposed procedure for heave prediction provides the necessary theoretical context providing there is monotonic deformation of each phase.

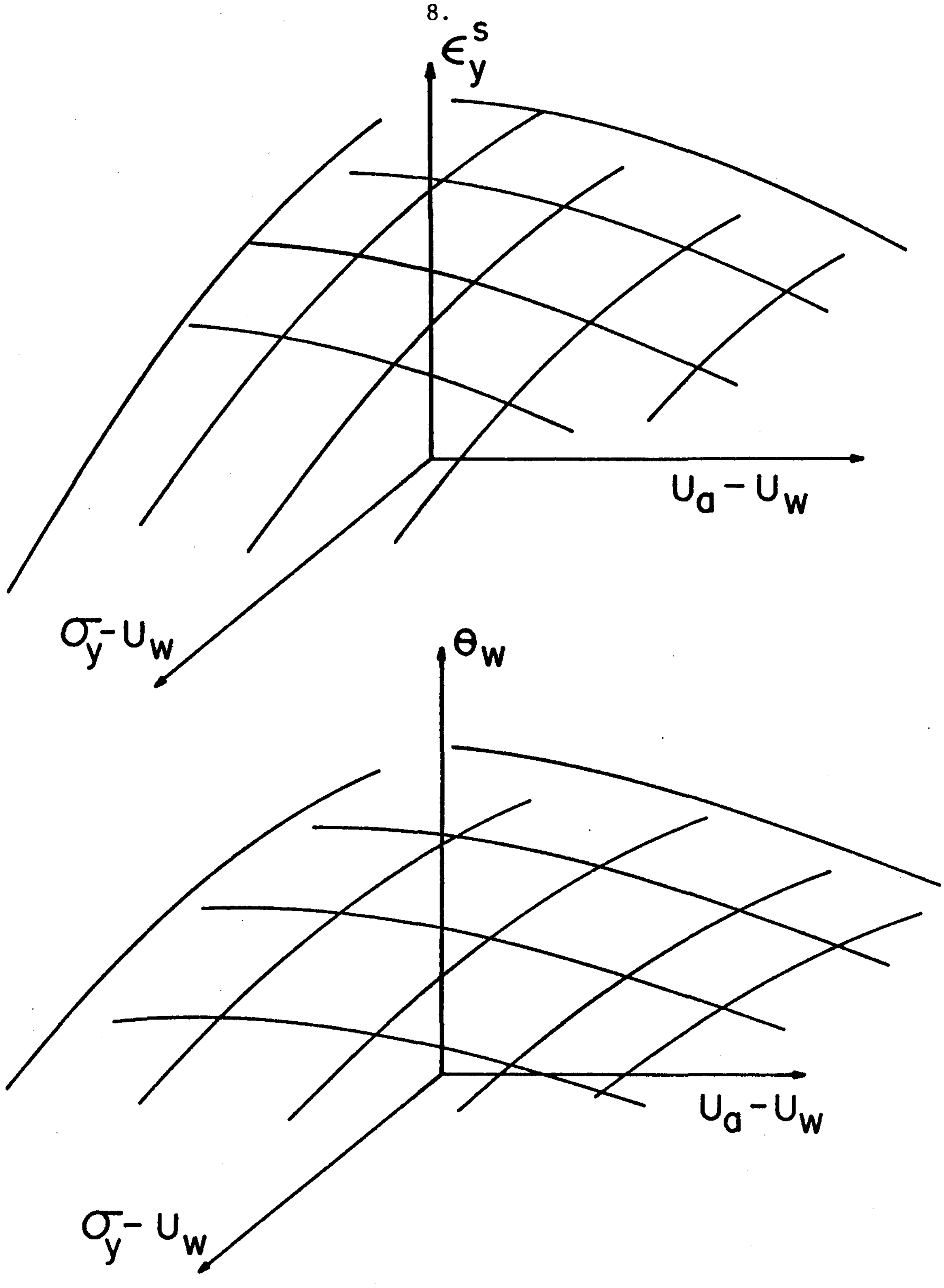


FIG. 2 CONSTITUTIVE SURFACES FOR THE SOIL STRUCTURE AND THE WATER PHASE.

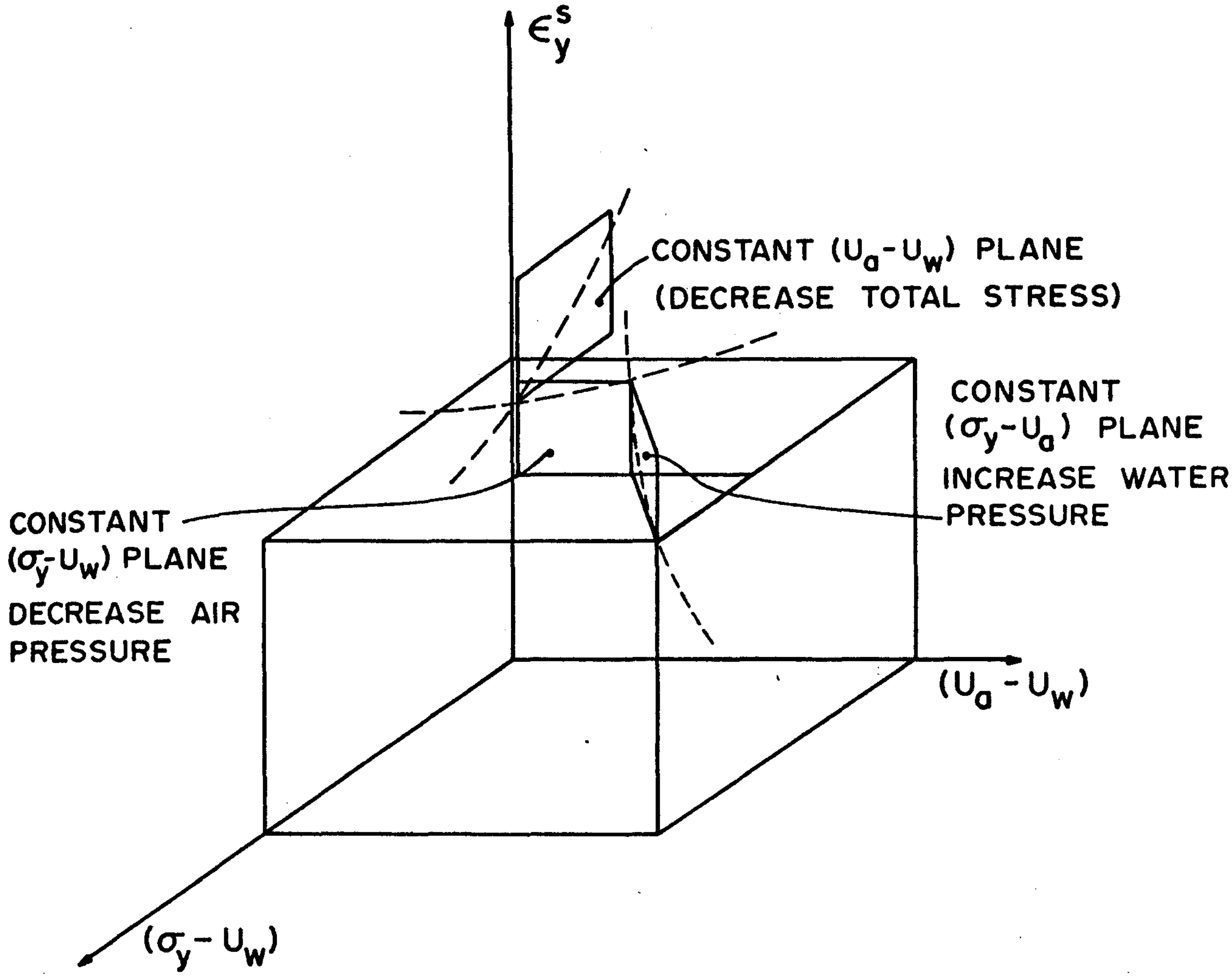


FIG. 3 STRESS PATH ADHERED TO FOR THE SOIL STRUCTURE DURING THE UNIQUENESS TESTS AT A POINT.

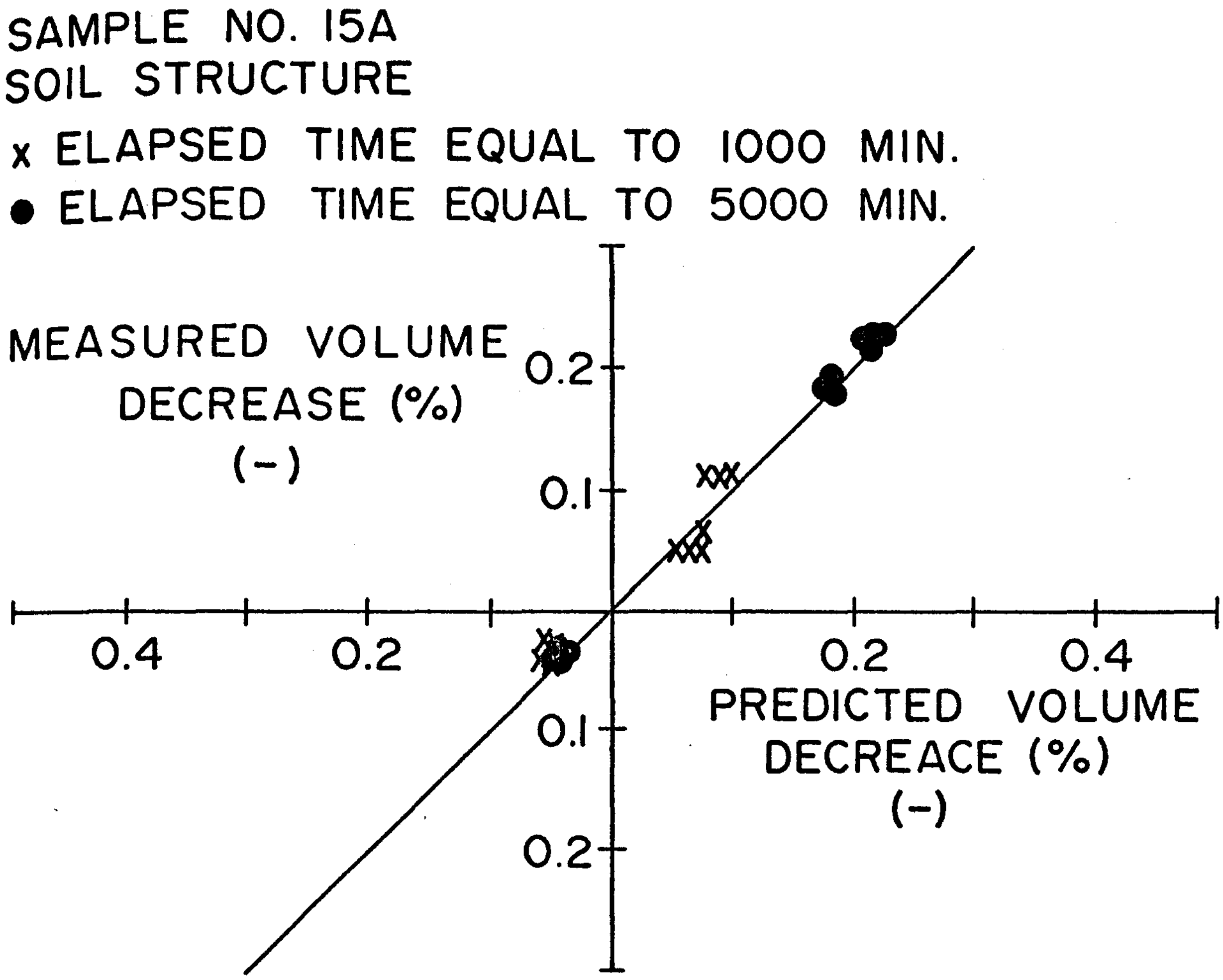


FIG.4 COMPARISON OF THE PREDICTED AND MEASURED SOIL STRUCTURE DEFORMATIONS FOR SAMPLE NO. 15A (REGINA CLAY).

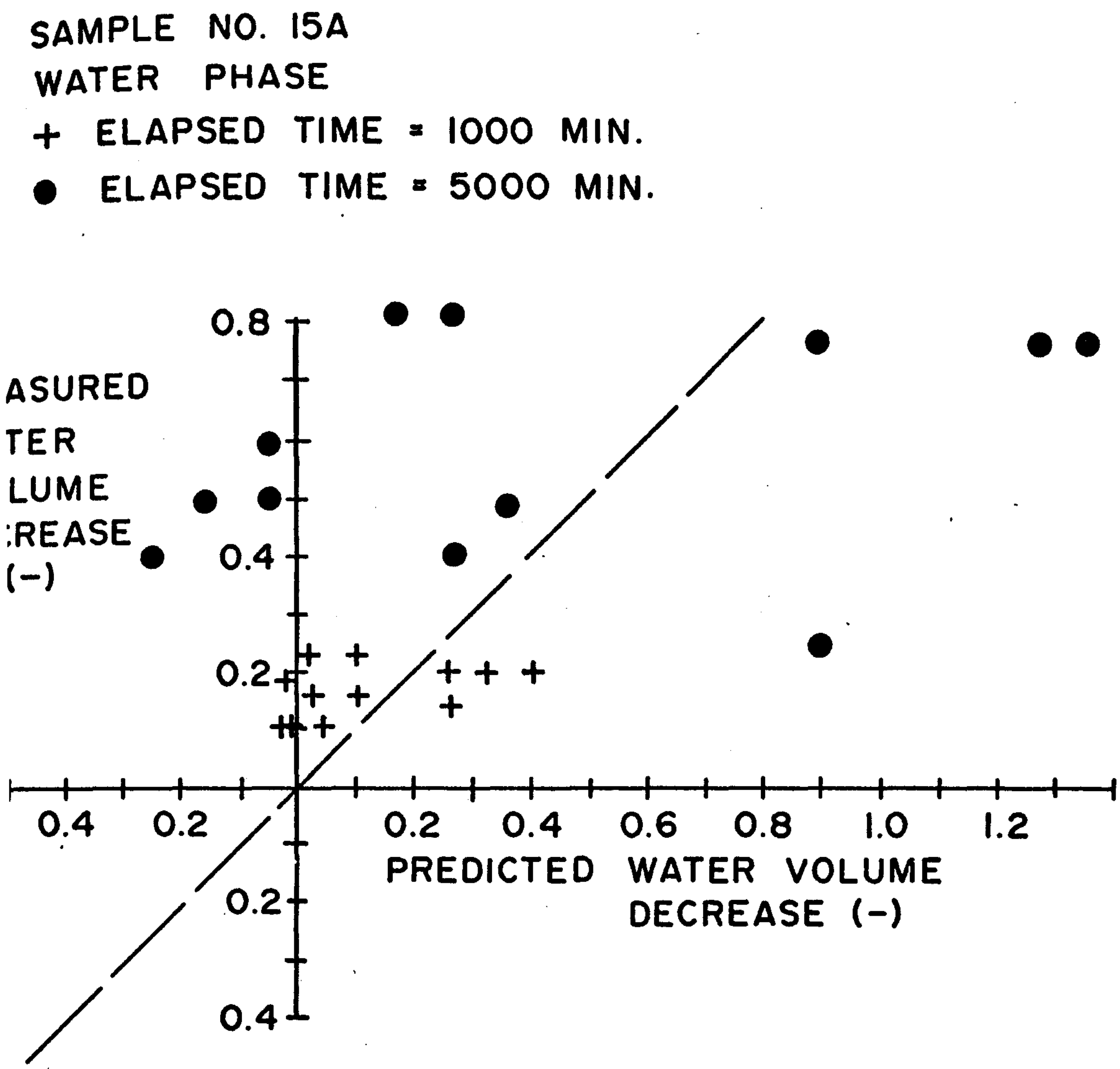


FIG. 5 COMPARISON OF PREDICTED AND MEASURED WATER VOLUME CHANGES FOR SAMPLE NO. 15A (REGINA CLAY).

REFERENCES

BIOT, M.A. (1941), "General Theory of Three-Dimensional Consolidation", Journal of Applied Physics, Volume 12, February.

COLEMAN, J.D. (1962), "Stress/Strain Relations for Partly Saturated Soil", Correspondence in Geotechnique, Volume 12, No. 4, pp. 348-350.

COOLING, L.F. (1961), "Discussion of the 'Pore Pressure and Suction in Soils' Conference", Butterworths, London.

DAVIES, J.T. and E.K. RIDEAL (1963), "Interfacial Phenomena", Second Edition, Academic Press, New York.

FREDLUND, D.G. (1973a), "Volume Change Behavior of Unsaturated Soils", Ph.D. Dissertation, University of Alberta, Edmonton, Alberta.

FREDLUND, D.G. (1973b) "Discussion of the Second Technical Session, Division Two (Flow and Shear Strength)" Preceedings of the Third International Conference on Expansive Soils, Volume II, Jerusalem Academic Press, Haifa, Israel.

FREDLUND, D.G. (1974), "The Prediction of Amount and Rate of Heave likely to be Experienced in Engineering Construction on Expansive Soils", Second International Conference on Expansive Clays, College Station, Texas, U.S.A.

LIST OF RESEARCH PAPERS

	Report No.
Geotechnical Analysis of Pleistocene Deposits in Southern Saskatchewan by E.K. Sauer, Presented to the 26th Annual Canadian Geotechnical Conference, Toronto, October, 1973	RP - 1
On Total, Matric and Osmotic Suction by J. Krahn and D.G. Fredlund, Soil Sciences Journal, Volume 114, No. 5, November, 1972.	RP - 2
Some Fatigue Considerations in the Design of Asphalt Concrete Pavements by A.T. Bergan and R.W. Culley, Symposium on Frost Action Roads, Report II, Oslo, Norway, October, 1973.	RP - 3
Characterization of Freeze-Thaw Effects on Subgrade Soils by A.T. Bergan and D.G. Fredlund, Symposium on Frost Action Roads, Report II, Oslo, Norway October, 1973.	RP - 4
Pressure Response Below High Air Entry Discs by D.G. Fredlund and N.R. Morgenstern, Third International Research and Engineering Conference on Expansive Soils, Haifa, Israel, August, 1973.	RP - 5
Moving Grain in the 70's by Gordon A. Sparks, Annual Conference of the Roads and Transportation Association of Canada, Halifax, Canada, October, 1973.	RP - 6
Optimal Spacing of Country Elevators in Western Canada by Gordon A. Sparks, Annual Conference of the Transportation Research Forum, Cleveland, U.S.A. October, 1973.	RP - 7
Frost Action on Roads - O.E.C.D. Symposium A Summary Prepared for North American Practice by R.W. Culley & A.T. Bergan, presented to the Roads and Transportation Association of Canada, Annual Conference, Toronto, Sept. 1974	RP - 8
Terrain Evaluation of the Dempster Highway Across the Eagle Plain and Along the Richardson Mountains, Yukon Territory, by N.W. Richardson and E.K. Sauer, Presented at the 27th Annual Geotechnical Conference, November, 1974, Edmonton, Alberta.	RP - 9
Engineering Approach to Soil Continua by D.G. Fredlund, Presented at the Second Symposium on Solid Mechanics, McMaster University, Hamilton, Ontario, June, 1974.	RP - 10
A Comprehensive and Flexible Slope Stability Program, by D.G. Fredlund, Presented at the Roads and Transportation Association of Canada Meeting, Calgary, Alberta, September, 1975.	RP - 11

Prediction of Heave In Unsaturated Soils
by D.G. Fredlund, Presented at the Fifth
Regional Conference, Bangdore, India,
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