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pendant upon the gradation of the expansive soil sample.

If stresses are applied to a saturated expansive soil sample in a triaxial test the stresses will be transmitted to the pore water, to the particles, and to the water in the intra layer of the expanding clay particles. c and ϕ values as determined would reflect the frictional parameters contributed by particle characteristics and structure etc, but not due to swelling pressure. In an expansive soil shear strength developed at a given depth would be a function of c, ϕ, swelling pressure and other imposed stresses.

Some studies were conducted to evaluate the effect of swelling pressure on shear strength development in an expansive soil medium. Vane shear equipment was inserted in large scale tests during compaction and preparation of the soil sample. After saturation, vane shear strength was determined without removing the overburden. The results given in Table I clearly indicate the role played by swelling pressure.

TABLE I

VARIATION OF SHEAR STRENGTH VALUES OBTAINED FROM THE VARIOUS TESTS, WITH DENSITY, SWELLING PRESSURE AND DEPTH

Density γ_d (gm/cc)	Swelling pressure Q_{sw} (kg/cm^2)	Vane shear strength s (kg/cm^2)	Shear strength evaluated from triaxial test data		Shear strength evaluated from direct shear test data		($\frac{1}{2}$xUCS)* (kg/cm^2)	Vane shear strength s_{vr}** (kg/cm^2)	Depth (cm)
			$(\frac{\sigma_1-\sigma_3}{2})_{max}$	$C_u + Q_{sw}\tan\phi_u$	τ	$C_u + Q_{sw}\tan\phi_u$			
1.24	2.60	0.525	0.650	0.586	0.50	0.49	0.27	0.20 †	41
1.36	3.15	1.025	0.975	0.900	1.06	1.05	0.60	0.652	107
1.38	3.25	1.395	1.200	1.185	1.15	1.12	0.86	0.78	137
1.385	3.28	1.442	1.413	1.400	1.18	1.167	0.865	0.84	167
1.39	3.30	1.498	1.425	1.420	1.18	1.1675	0.87	0.84	197

* Unconfined compressive strength determined in the triaxial test apparatus without applying cell pressure.

** s_{vr} corresponding to a density of 1.27 gms/cc. and a depth of 45 cms.

† Vane shear strength evaluated at various depths after removing the overburden.

D. G. Fredlund (Canada)

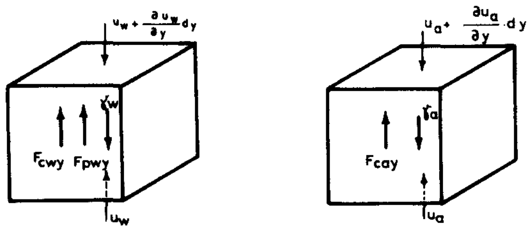
My discussion on the Second Technical Session (i.e., Flow and Shear Strength) relates primarily to three points common to most of the presented papers. First, the need for an acceptable description of the stress state of an unsaturated soil, consistent with continuum mechanics. This stress state is advocated as the basis for describing unsaturated soil behavior. Second, the need to formulate transient flow problems within the context of multiphase continuum mechanics. Third, our experience with the psychrometric technique for measuring total potential.

Generally an unsaturated soil is considered as a threephase system of air, water and soil particles. However, there is now evidence that the air-water interface (commonly referred to as the contractile skin) should be considered as an independent phase (Fredlund, 1973). Therefore, an unsaturated soil can be visualized as a mixture with two phases that come to equilibrium under applied stresses (i.e., soil particles and the contractile skin) and two phases that flow under applied pressures (i.e., the air and water).

The success of the effective stress equation for saturated soils has led research workers in a continual search for a similar equation for unsaturated soils. However, all proposed equations have met with limited success. In 1973, Fredlund theoretically proved and experimentally verified that two independent stress tensors are required to describe the stress state for the soil structure and the contractile skin in an unsaturated soil.

The theoretical derivation involves writing the force equilibrium equation for each phase of an unsaturated soil, within the context of continuum mechanics (Green and Naghdi, 1965; Truesdell, 1966; Faizullaev, 1969; and Sedov, 1971). The superposition of coincident equilibrium stress fields allows the equilibrium of each phase to be written in terms of measurable stresses. The "stress state" variables associated with each phase are obtained by extraction from the equilibrium equations.

Figure 1a shows the stresses associated with the water phase. Assuming that the contractile skin behaves as a stationary membrane, there is an interaction force between the water and the contractile skin which can be represented by a body force, F_{cw}.



a) Components of equilibrium for the pore water phase

b) Components of equilibrium for the pore air phase

FIGURE 1 COMPONENTS OF EQUILIBRIUM FOR THE FLUID PHASES OF AN UNSATURATED SOIL

There is also an interaction force between the water and the soil particles, F_{pw}. The equilibrium in the y-direction is,

$$\frac{\partial u_w}{\partial y} + \gamma_w - \frac{F_{cwy}}{n_w} - \frac{F_{pwy}}{n_w} = 0 \quad \ldots\ \ldots\ \ldots (1)$$

where u_w = pore water pressure

γ_w = unit weight of water

n_w = porosity with respect to the water phase.

Figure 1b shows the forces associated with the air phase. Let the interaction force between the air and the contractile skin be designated by the body force, F_{ca}. Summing in the y-direction gives,

$$\frac{\partial u_a}{\partial y} + \gamma_a - \frac{F_{cay}}{n_a} = 0 \quad \ldots\ \ldots\ \ldots\ \ldots (2)$$

where u_a = pore air pressure

γ_a = unit weight of air

n_a = porosity with respect to the air phase.

The surface tractions associated with the contractile skin can initially be written in a general form. The contractile skin interacts with the soil particles producing an interaction force, X_{pc}. The drag of the water and air on the contractile skin are F_{wc} and F_{ac}, respectively, summing the forces in the y-direction gives,

$$\frac{\partial \tau^c_{xy}}{\partial x} + \frac{\partial \sigma^c_y}{\partial y} + \frac{\partial \tau^c_{zy}}{\partial z} + \gamma_w + \frac{F_{wcy}}{n_c} - \frac{F_{acy}}{n_c} + \frac{X_{pcy}}{n_c} = 0 \quad \ldots\ \ldots\ \ldots (3)$$

where σ^c_y, τ^c_{xy} and τ^c_{zy} = normal and shear surface tractions.

n_c = porosity with respect to the contractile skin.

Similarly, equilibrium equations can be written for the soil particles by assuming a general stress tensor applies. Once again there is an interaction force, X_{cp}, between the contractile skin and the soil particles. In addition, there is an interaction force of the water on the soil particles, F_{wp}. In the y-direction, the equlibrium equation is,

$$\frac{\partial \tau^p_{xy}}{\partial x} + \frac{\partial \sigma^p_y}{\partial y} + \frac{\partial \tau^p_{zy}}{\partial z} + \gamma_p + \frac{F_{wpy}}{n_p} - \frac{X_{cpy}}{n_p} = 0 \quad \ldots\ \ldots (4)$$

where σ^p_y, τ^p_{xy} and τ^p_{zy} = normal and shear surface tractions associated with the soil particles.

γ_p = unit weight of the soil particles.

n_p = porosity with respect to the soil particles.

A total or overall equilibrium equation can also be written for the assembled element. In the y-direction,

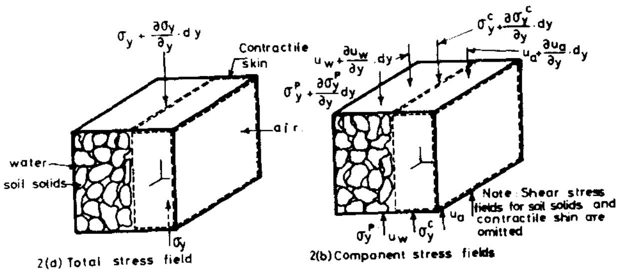
$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial y} + \gamma = 0 \quad \ldots\ \ldots\ \ldots\ \ldots (5)$$

where σ_y, τ_{xy} and τ_{zy} = total normal and shear surface tractions for the overall element.

γ= total unit weight of the assembled element.

We have written five equilibrium equations for the four-phase system and therefore one of the equations is redundant. If we omit the equilibrium equation for the soil particles, its equilibrium can be written in terms of the difference between the total stress field and the stress fields of the remaining phases. The resulting equilibrium equation for the soil particles contains the unmeasurable stress variables for the contractile skin. Likewise, if the contractile skin equilibrium equation is written in terms of the total stress field and the stress fields for the remaining phases, it contains the unmeasurable soil particle stress field. Thus, there is an interrelationship between the soil structure and the contractile skin equilibrium equations.

The above difficulty is resolved by uncoupling the unsaturated soil element into a water-soil particle multiphase and an air-contractile skin multiphase. Let us consider a fictitious element with all the air moved to one side of the element (Figure 2a) and with the total and component stress fields remaining the same (Figure 2b).



2(a) Total stress field

2(b) Component stress fields

FIGURE 2 REARRANGEMENT OF AN UNSATURATED SOIL ELEMENT

Now let us disassemble the unsaturated soil element into two equivalent elements. The first element contains only soil particles and water but has two additional forces applied to make it equivalent to the original system (Figure 3a).

The stress field $\partial T_y/\partial y$ accounts for the effect of the air and contractile skin stress fields. A body force X_{cp} must also be applied to account for the interaction between the contractile skin and the soil particles. The equilibrium equation for the soil particles can be written in terms of the above surface tractions and body forces.

$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial(\sigma_y - u_w)}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + (\gamma - \gamma_w) + \frac{F_{cwy} + F_{pwy}}{n_w} - X_{cp} - \frac{n_a \cdot \partial T_y}{\partial y} = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Using the air-contractile skin portion of the element (Figure 3b) the equilibrium equation for the contractile skin is,

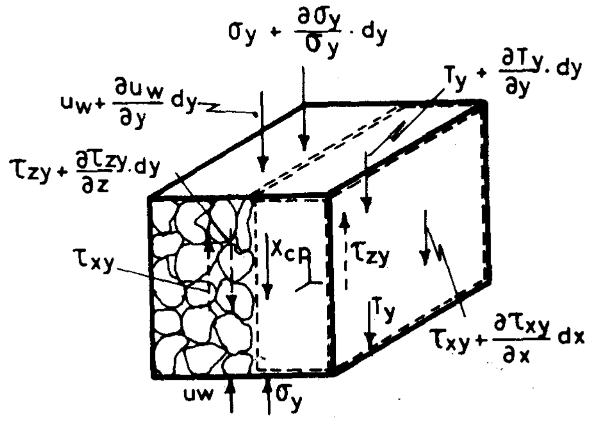
$$n_a \cdot \frac{\partial T_y}{\partial y} + X_{cp} - n_a \cdot \frac{\partial u_w}{\partial y} + n_a \cdot \frac{\partial u_a}{\partial y} + F_{cay} + n_a \cdot \gamma_a = 0 \;\ldots\ldots\ldots\ldots (7)$$

Rearranging and solving for $(n_a \cdot \frac{\partial T_y}{\partial y} + X_{cp})$

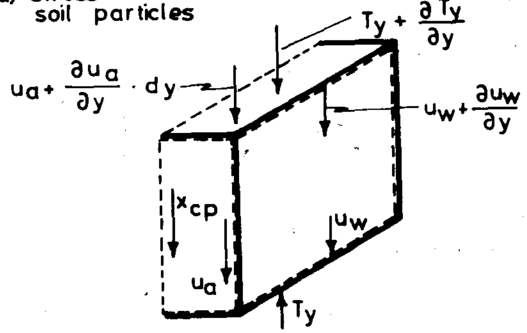
$$na. \frac{\partial T_y}{\partial y} + X_{cp} = -n_a \cdot (\frac{\partial u_a}{\partial y} - \frac{\partial u_w}{\partial y}) + F_{cay} + n_a \cdot \gamma_a \;\ldots\ldots\ldots (8)$$

Subtituting into equation (6) gives

$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial(\sigma_y - u_w)}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + (\gamma_t - \gamma_w) + \frac{F_{cwy} + F_{pwy}}{n_w} + n_a \cdot (\frac{\partial u_a}{\partial y} - \frac{\partial u_w}{\partial y}) + F_{cay} + n_a \cdot \gamma_a = 0 \;\ldots\ldots (9)$$



3 (a) Stress fields associated with the soil particles



3(b) Stress fields associated with the contractile skin

FIGURE 3 DISASSEMBLED UNSATURATED SOIL ELEMENT SHOWING MEASURABLE STRESS FIELDS

This equilibrium equation applies for both the soil particles and the contractile skin. Similar equilibrium equations can be evaluated for the x and z-directions.

An examination of the equations reveals that two normal stress state variables are involved in the equilibrium equations for the soil particles and the contractile skin. They are linked by the porosity of the air phase (i.e., volume of air relative to the total volume). The inclusion of the porosity term with the stresses is not in keeping with the usage of stress state variables in continuum mechanics. The inclusion of a soil property in a description of a stress state renders the equation a constitutive relationship (Fung, 1969). The stress state variables can be extracted from the equilibrium equations to form two independent stress tensors which apply to the soil particles and the contractile skin.

$$\begin{bmatrix} \sigma_x - u_w & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - u_w & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - u_w \end{bmatrix}$$

and

$$\begin{bmatrix} u_a - u_w & 0 & 0 \\ 0 & u_a - u_w & 0 \\ 0 & 0 & u_a - u_w \end{bmatrix}$$

The above stress tensors depict the stress state (Figure 4) and contain only quantities that can be measured. Therefore, they can readily be used in conjuction with constitutive relationships.

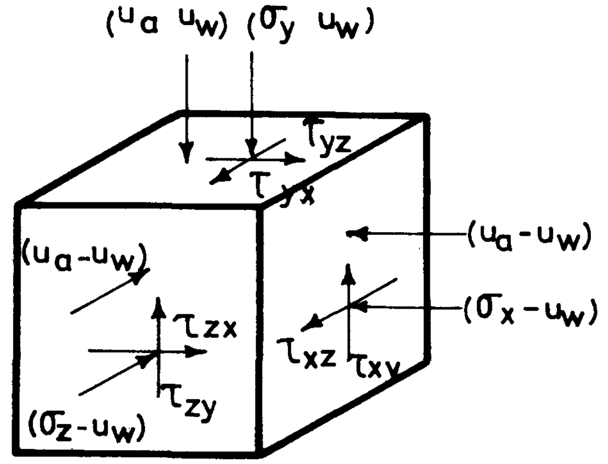


FIGURE 4 THE STRESS STATE FOR THE SOIL PARTICLES AND THE CONTRACTILE SKIN IN AN UNSATURATED SOIL.

The above analysis has considered the equilibrium equations by using the water phase as a datum. The physical assemblage of an unsaturated soil system indicates that this is a logical choice. However, the air phase could also have been used as a datum. In this case, the equilibrium equation, in the y-direction, for the soil particles and contractile skin is,

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial(\sigma_x - u_a)}{y} + \frac{\partial \tau_{zy}}{\partial z} + (\gamma - \gamma_w) + \frac{F_{cwy} + F_{pwy}}{n_a} + n_w \cdot \frac{\partial(u_a - u_w)}{\partial_y} + \frac{F_{cay}}{n_a} + n_a \cdot \gamma_a = 0 \quad \ldots(10)$$

The extracted stress tensor is,

$$\begin{bmatrix} \sigma_x - u_a & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - u_a & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - u_a \end{bmatrix}$$

and

$$\begin{bmatrix} u_a - u_w & 0 & 0 \\ 0 & u_a - u_w & 0 \\ 0 & 0 & u_a - u_w \end{bmatrix}$$

A third possible combination uses the total stress as a datum.

The above stress tensors were experimentally verified by changing the stress components of the stress state variables while keeping all stress state variables constant. The only appropriate test can be stated:

$$\Delta\sigma = \Delta u_w = \Delta u_a$$

If the overall volume change is zero and the degree of saturation change is zero, the equilibrium of the soil particles and the contractile skin has been maintained.

These tests are termed Null tests since the desired result of the changes in pressure is to not produce a process in the phase of phases under consideration. Thus, an attempt is being made to measure "no volume change" or a continuing equilibrium state.

A large number of Null tests were performed on compacted kaolin samples and a detailed analysis is presented by Fredlund (1973). Figure 5 shows a typical set of null test results. The results indicated essentially no volume change in any phase during the null tests.

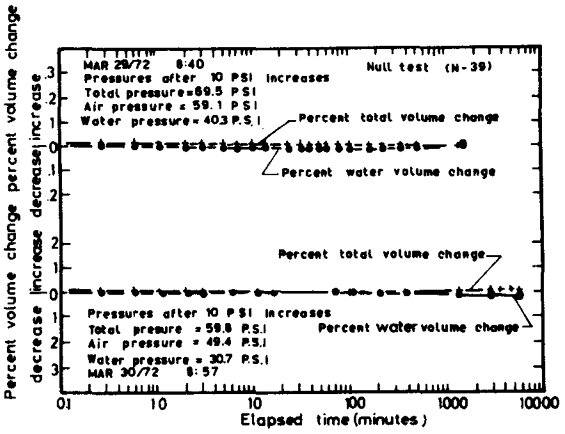


FIGURE 5 NULL TESTS (N-39) AND (N-40) ON SAMPLE NO. 31.

On the final set of tests, the overall and water volume changes were less than 1 part in 10,000 parts, even after several days duration. This corresponds to approximately two percent of the anticipated volume changes that would be associated with a change in either one of the stress state variables. The proposed stess tensors are well verified for both the soil structure and the contractile skin.

The above stress tensors assume that changes in the physico-chemical properties do not change for the processes we wish to describe. In order to facilitate physico-chemical effects, let (R - A) (i.e., the difference between the Repulsive and Attractive stresses in the adsorbed water hull) describe the state of stress in the adsorbed layer. The stress tensors can be rederived. One possible combination is:

$$\begin{bmatrix} \sigma_x - u_w - (R - A) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - u_w - (R - A) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - u_w - (R-A) \end{bmatrix}$$

and

$$\begin{bmatrix} u_a - u_w & 0 & 0 \\ 0 & u_a - u_w & 0 \\ 0 & 0 & u_a - u_w \end{bmatrix}$$

The first stress tensor is consistent with the equation experimentally verified by Balasubramanian (1972) and Chattopadhyay (1972) for saturated soils.

The magnitude of (R - A) is commonly referred to as the solute or osmotic suction while the term $(u_a - u_w)$ is referred to as the matric suction (Aitchison, 1965). The sum of the solute and matric suction is referred to as the total suction and is measurable by the psychrometric technique (Richards, 1969). Krahn and Fredlund (1972) independently measured the solute suction (using a squeezing and electrical conductivity technique), the matric suction (using a null pressure plate technique; Pufahl, 1970) and the total suction (using the psychrometric technique) on essentially identical soil samples. The results experimentally verify the above subdivision of soil suction (Figure 6).

Once the stress state variables are established, they can be used in constitutive relationships in an independent manner. In this way, the illusive, χ, β, ψ, etc., factors in previously proposed effective stress equations need not be evaluated. In fact, it is suggested that the search for one effective stress equation should be ended since the equilibrium analysis reveals that two independent stress tensors are required to describe the stress state for an unsaturated soil.

The second point I wish to discuss involves the diffusivity concept applied to flow in unsaturated soils. Common to the research literature on flow through unsaturated soils is the statement that Darcy's law must be modified prior to its application. Swartzendruber (1969) states that there must be a modification to Darcy's law that "involves the recognition that the air-filled pores will reduce the effective cross-section for liquid flow and will increase the tortuosity of the remaining liquid flow path". Therefore, the coefficient of permeability for unsaturated flow" becomes a function of water content". "Arguments of this type were first set forth by Buckingham (1907)...".

Childs and Collis-George (1948) adopted Buckingham's postulate and developed the diffusivity approach to flow in unsaturated soils. Their approach has become widely accepted in the soil science field and more recently introduced to soil mechanics (Richards, 1969).

Experiments performed by Childs and Collis-George (1950); however, show that Darcy's law does apply for unsaturated soils. They devised a method whereby the water content and the suction were maintained constant throughout a column of unsaturated soil. Various magnitudes of gradient were applied and "the rate of flow for a given degree of saturation was proportional to the potential gradient, as in the case of saturated materials".

In view of the above statements, the question arises; Why is the non linearity of the coefficient of permeability with respect to water content so important for a flow analysis in unsaturated soils when it can be successfully assumed a constant for the flow analysis of saturated soils?

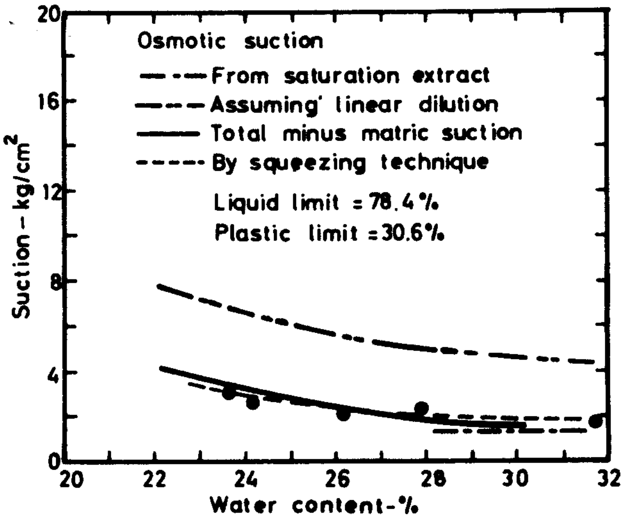


FIGURE 6 COMPARISON OF OSMOTIC SUCTIONS FOR THE REGINA CLAY

I would suggest that the necessity for having the coefficient of permeability a function of water content when analyzing unsaturated soils infers the omission of one or more elements of physics from some aspect of the analysis. First, the application of multiphase continuum mechanics shows that a continuity equation can be written for each phase of an unsaturated soil (Sedov, 1971). Where movements in one phase affects another phase, all the continuity equations must be satisfied. Second, the soil particles and the contractile skin are controlled by two stress tensors which should appear in their constitutive relationships which are introduced to the continuity equations. Third, flow in the fluid phases is basically due to the pressure gradient in that phase. In other words, flow does not bsically occur as a result of a suction or water content gradient. Rather suction comes into the formulation as one of the stress state variables for the soil structure and the contractile skin.

The author suggests that the reliability of the diffusivity formulation for flow problems be checked by comparison with a rigorous flow formulation consistent with multiphase continuum mechanics.

On the third point, I wish to briefly comment on our laboratory and field experience with the psychrometric technique. The design of the laboratory psychrometer was the same as Spanner's (1951). In the laboratory, the temperature bath was maintained within ±0.001 degree centigrade, giving a total suction measurement accuracy of approximately 0.1 kg/cm^2. In situ psychrometers designed and calibrated as outlined by Richards (1969), were installed immediately below the outside wheel path on the Regina-Lumsden highway in Saskatchewan (Bergan and Monismith, 1973). Readings during April, May and June of 1971, indicated that deviations in the output readings (0 to 3 μV) masked an accurate measurement of suctions. Laboratory matric suctions on undisturbed samples indicated low suction values.

The problems we have experienced with the psychrometric technique are similar to those reported by Baker, Kassiff and Levy (1973).

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