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Dynamic Programming Method In Slope Stability Computations

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Background

- ***Slope stability analysis has developed primarily as a series of Limit Equilibrium Methods of Slices***
- ***Limit Equilibrium methods (LEM) of slices have two major shortcomings:***
  - ***There is uncertainty regarding the shape of the critical slip surface***
  - ***There is a disregard for the actual total stresses within the soil mass***

Fundamental Limitations with Limit Equilibrium Methods of Slices

The boundaries for a FREE BODY DIAGRAM are not known

- ***The SHAPE for the slip surface must be assumed***
- ***The LOCATION of the critical slip surface must be found by TRIAL and ERROR***

Background

- *The **shortcomings** of Limit Equilibrium Methods of Slices can largely be overcome by using a **finite element stress analysis** (FEM) in conjunction with an appropriate **optimization technique***
- *The **Dynamic Programming** technique was introduced in 1957*
- *The **Finite Element method** has been used to compute stresses in slope stability analyses since 1969 (Kulhawy)*

Objectives

- *To further develop a procedure that combines the **Dynamic Programming** technique with a **Finite Element** stress analysis for slope stability analysis*
- *To demonstrate the application of the algorithm based on the proposed procedure **(SVDynamic)***

Improvement on the Normal Stress Computations

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Assumptions

- *The slip surface is assumed to be an assemblage of linear segments*
- *The Mohr-Coulomb failure criterion is appropriate and is linear*
- *The soils behave as a linear elastic material*
- *The flow of groundwater is assumed to be independent of the stresses and obeys Darcy's law*

Hypothesis

- ***Optimization Techniques* (i.e., Dynamic Programming) can be used to find the pathway which minimizes a function of the *shear strength available* to the *actuating shear stress* within a soil mass**

***Assumption:* The stresses computed from “switching-on” gravity can be used to represent the stress state in the soil mass**

Slope Stability Analysis Using Dynamic Programming Combined with a Finite Element Stress Analysis

- Dynamic Programming (DP) optimization techniques for slope stability analysis (Spencer's Method) was introduced by Baker (1980)

- Yamagami & Ueta (1988) and Zou et al. (1995) improved on the Baker (1980) solution by coupling Dynamic Programming with a Finite Element stress analysis

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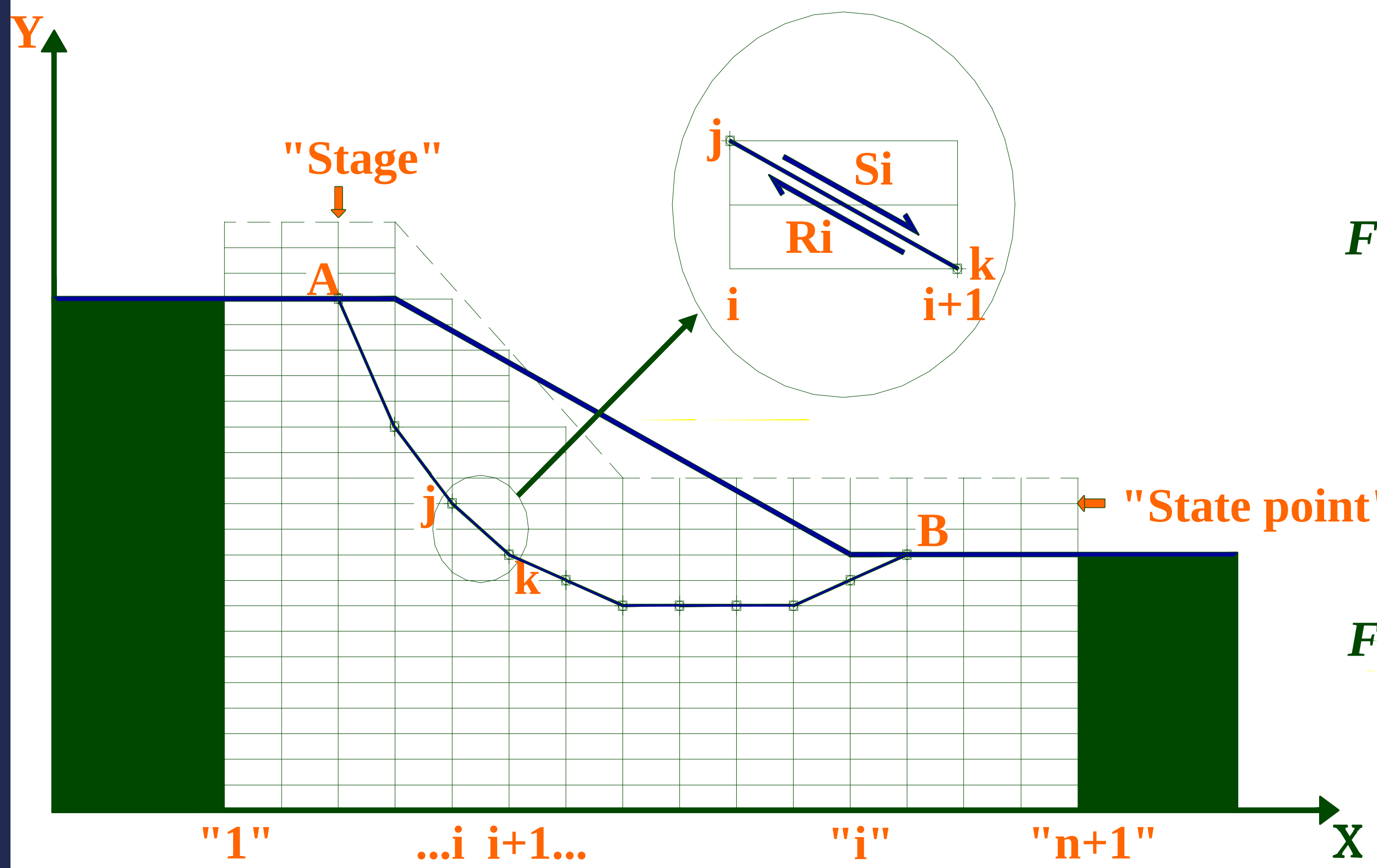
Definition of Factor of Safety

$F_s = \Sigma$ (Shear Strength) / Σ (Actuating Shear Stress)



$$F_s = \frac{\int_A^B \tau_f \, dL}{\int_A^B \tau dL}$$

$$F_s = \frac{\sum_{i=1}^{n} \tau_{fi} \Delta L_i}{\sum_{i=1}^{n} \tau_i \Delta L_i}$$

Definition of Global Factor of Safety, F_s (Past & Present)

- "That factor by which the shear strength parameters must be reduced to bring a soil mass into a state of limiting equilibrium along a slip surface"

$$\frac{c'}{F_s} \quad \text{and} \quad \frac{\tan\phi'}{F_s}$$

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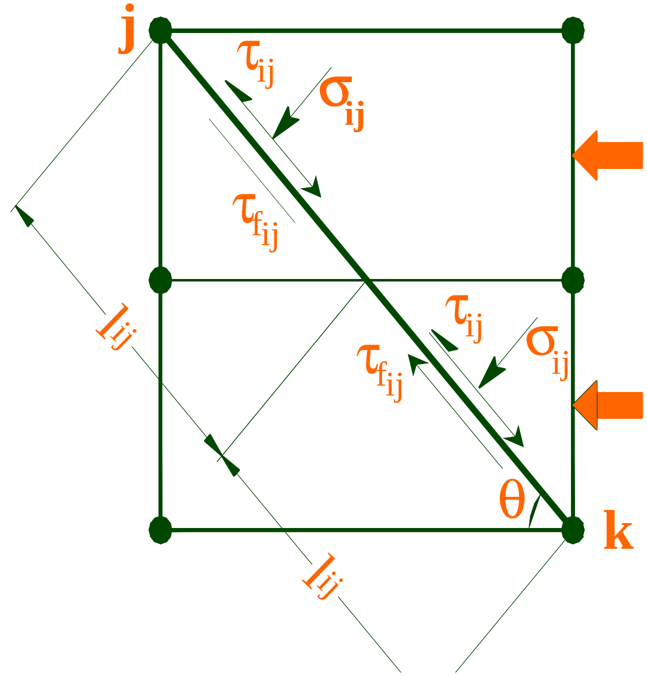
Definition of "Return Function"; G

$F_s = \Sigma$(Shear Strength) / Σ(Actuating Shear Stress)



Difficult to minimize !

$$G=\sum_{i=1}^{n}(\tau_{fi}-F_s\tau_i)\Delta L_i$$

$$G=\int_{A}^{B}(\tau_f-F_s\tau)dL$$

$$G=\sum_{i=1}^{n}(R_i-F_sS_i)$$

R = Resisting Shear Strength: S = Actuating Shear Stress

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Actuating Shear Forces and Resisting Shear

S = Actuating Shear Stress

$$S_i = \tau_i \Delta L_i = \sum_{ij=1}^{ne} S_{ij} = \sum_{ij=1}^{ne} \tau_{ij} l_{ij}$$

R = Resisting Shear Strength

$$R_i = \tau_{f_i} \Delta L_i = \sum_{ij=1}^{ne} R_{ij} = \sum_{ij=1}^{ne} \tau_{f_{ij}} l_{ij}$$

$$R_i = \sum_{ij=1}^{ne} \{c'_{ij} + (\sigma_{ij} - u_a)\tan\phi'_{ij} + (u_a - u_w)\tan\phi^b_{ij}\} l_{ij}$$

Definition of "Optimal Function": Minimum Value of "Return Function"

$$G_{min} = \min G = \min \sum_{i=1}^{n} (R_i - F_s S_i)$$

G = Return Function

Introduce an "optimal function", $H_i(j)$

$$H_{i+1}(k) = H_i(j) + G_i(j,k)$$

H = Optimal Function

where:

$H_{i+1}(k)$ *= the optimal function obtained at point {k} of stage [i+1],*

$H_i(j)$ *= the optimal function obtained at point {j} in stage [i], and*

$G_i(j,k)$ *= the return function calculated when passing from the state point {j} in stage [i] to the state point {k} in stage [i+1].*

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Boundary Conditions of "Optimal Function"

At the initial stage, (i=1) :

$$H_1(j) = 0 \qquad j = 1 \ldots NP_1$$

At the final stage, (i = n+1) :

H = Optimal Function

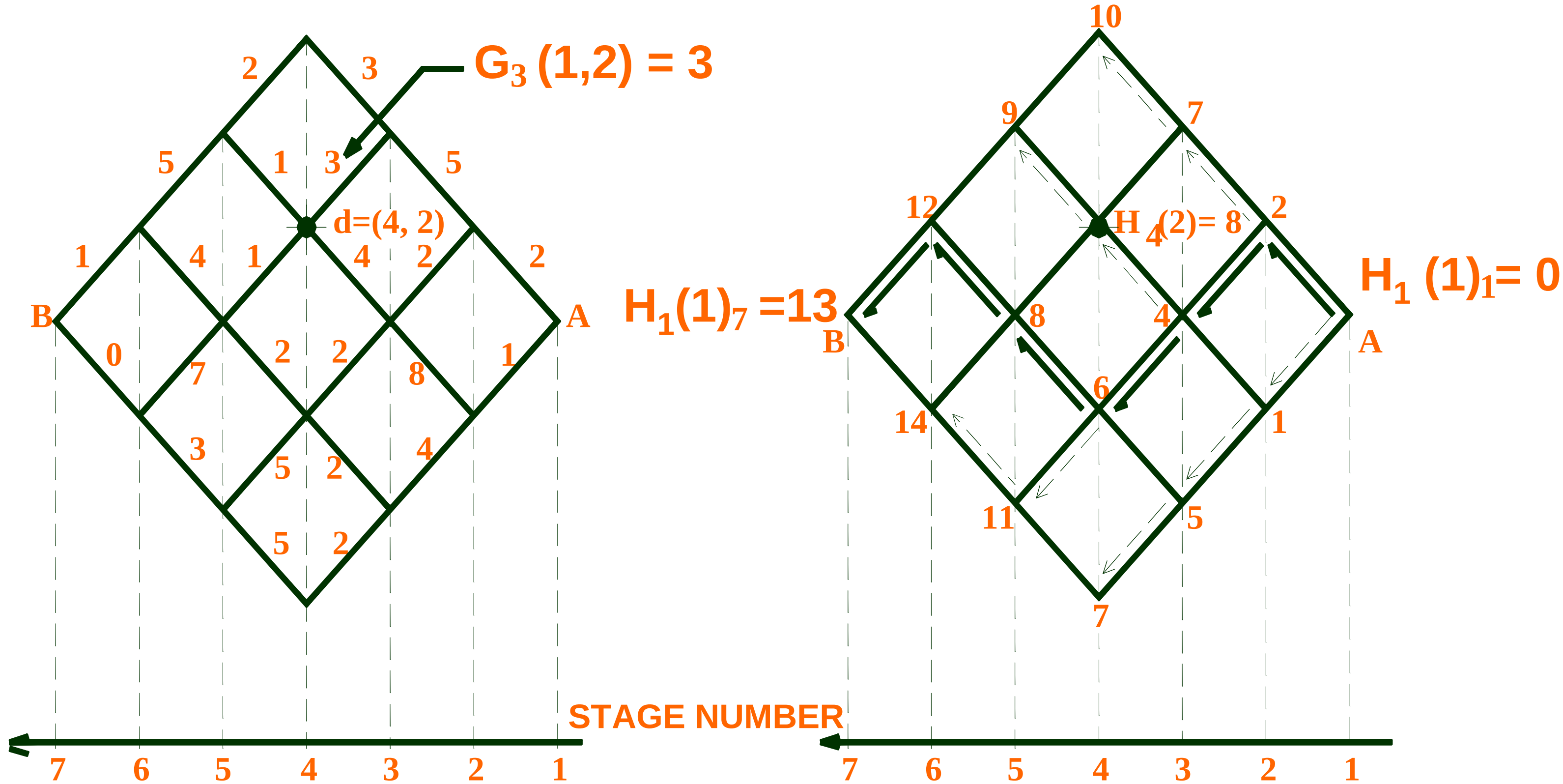
$$H_{n+1}(k) = H_n(j) + G_n(j,k)$$

$$H_{n+1}(k) = G_m = \sum_{i=1}^{n} (R_i - F_s . S_i) \qquad k = 1 \ldots NP_{n+1}$$

where:

NP_{n+1} = *the number of state points in the final stage*

The Minimum (or Optimal) Travelling Time Problem



THE MINIMUM TRAVELLING TIME PROBLEM

DYNAMIC PROGRAMMING SOLUTION

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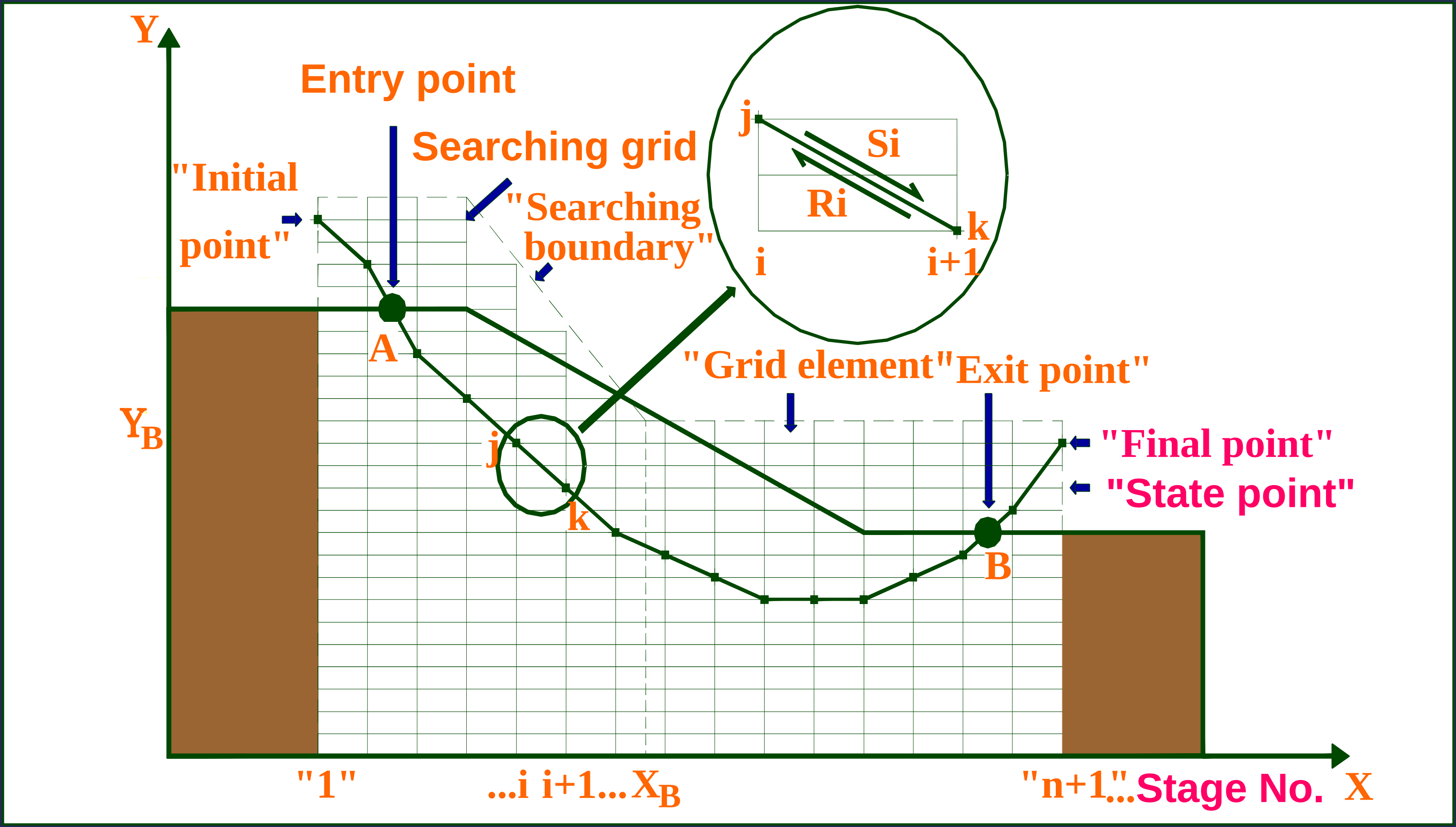
Analytical Scheme of the Dynamic Programming Method



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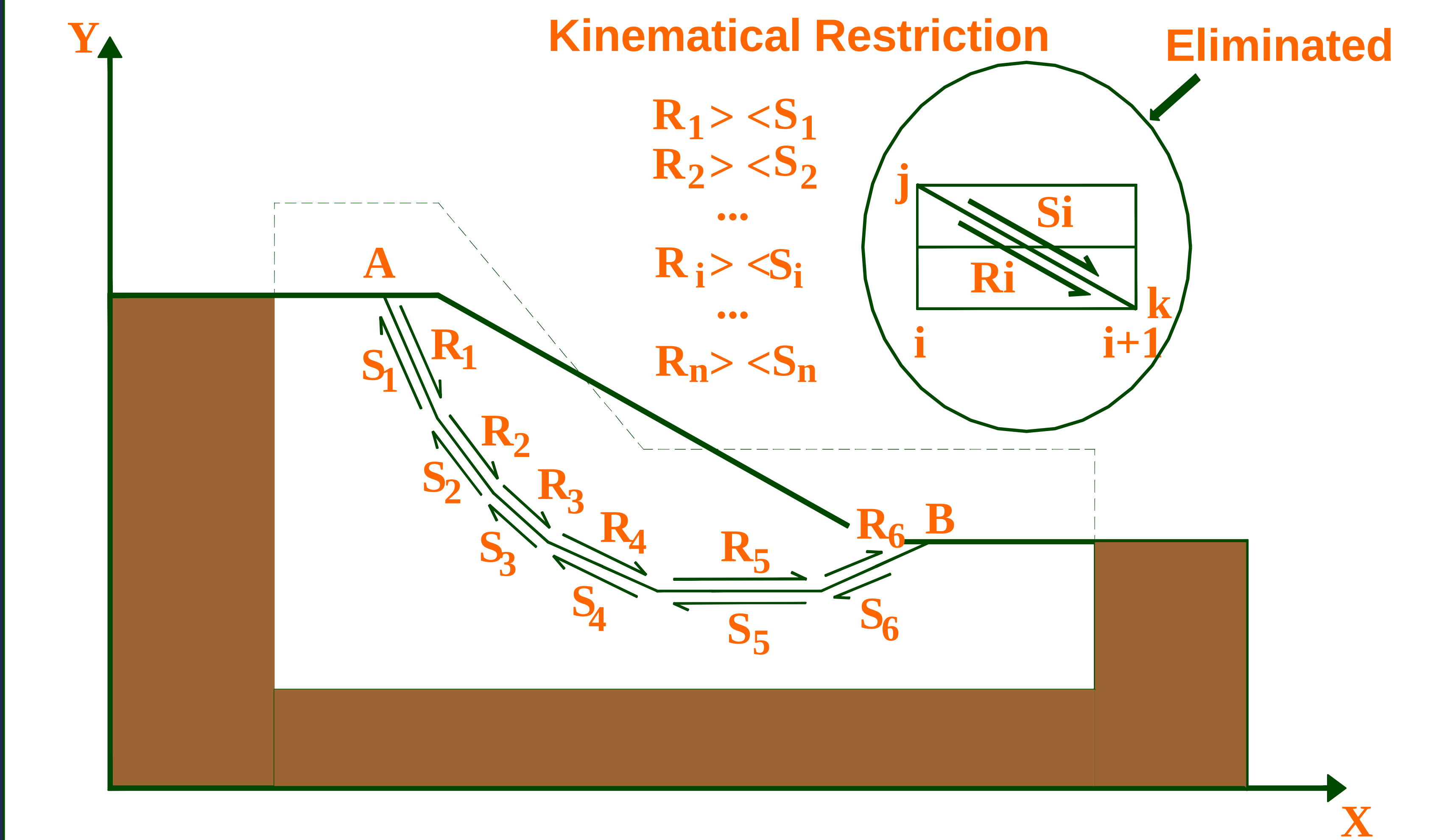
Kinematical Restriction



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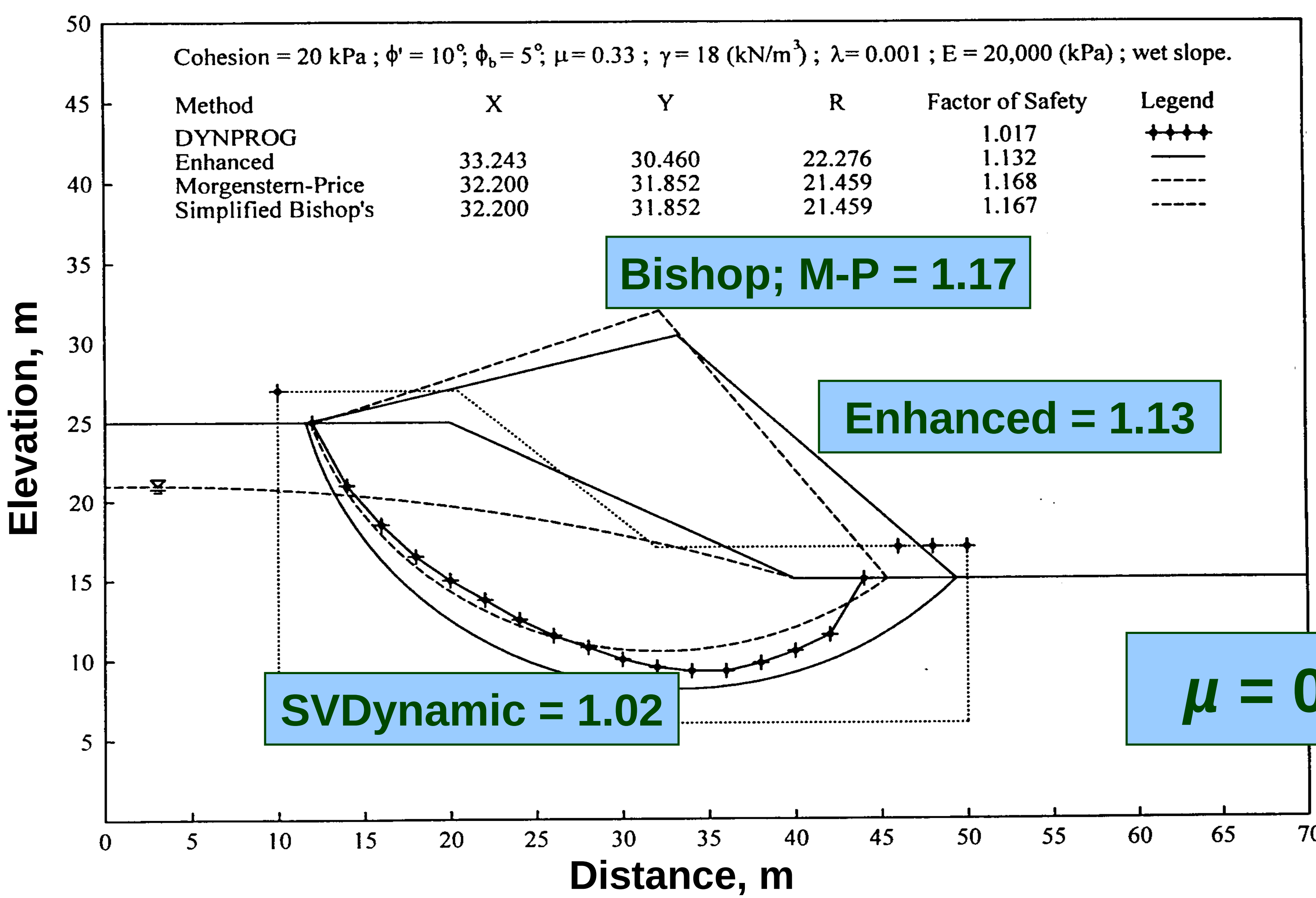
Example of a Homogeneous Slope



μ = 0.33

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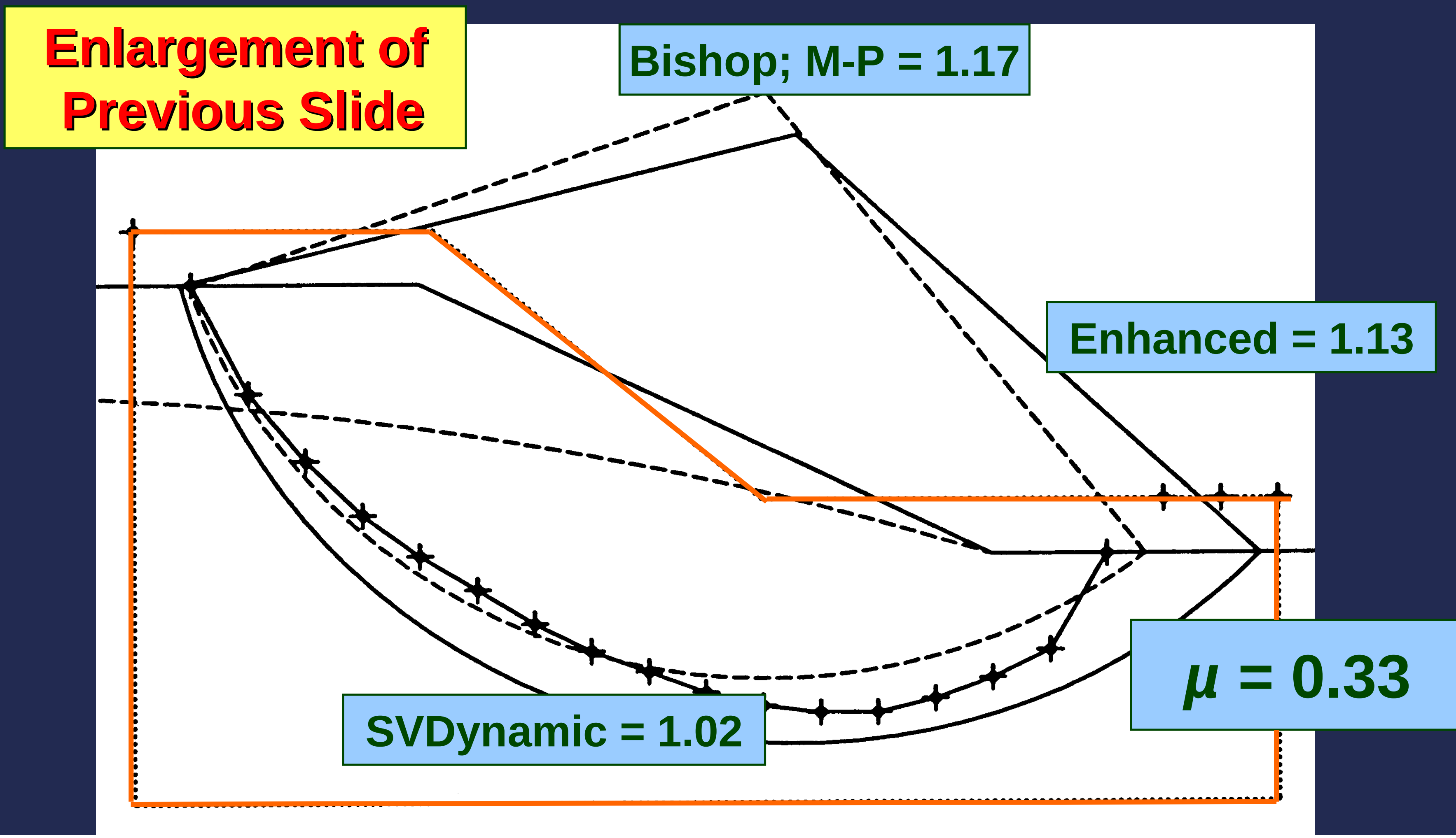
Example of a Homogeneous Slope



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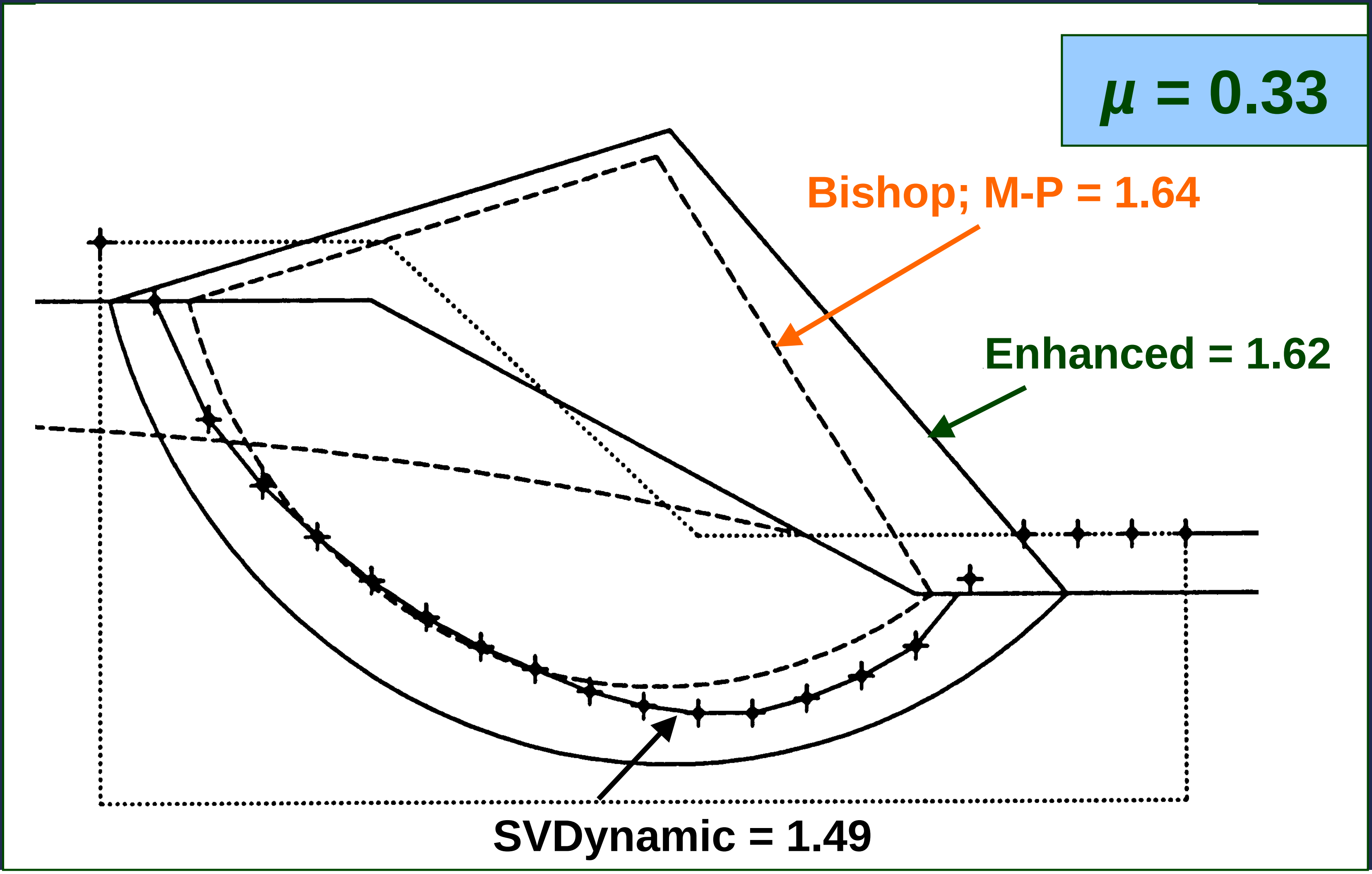
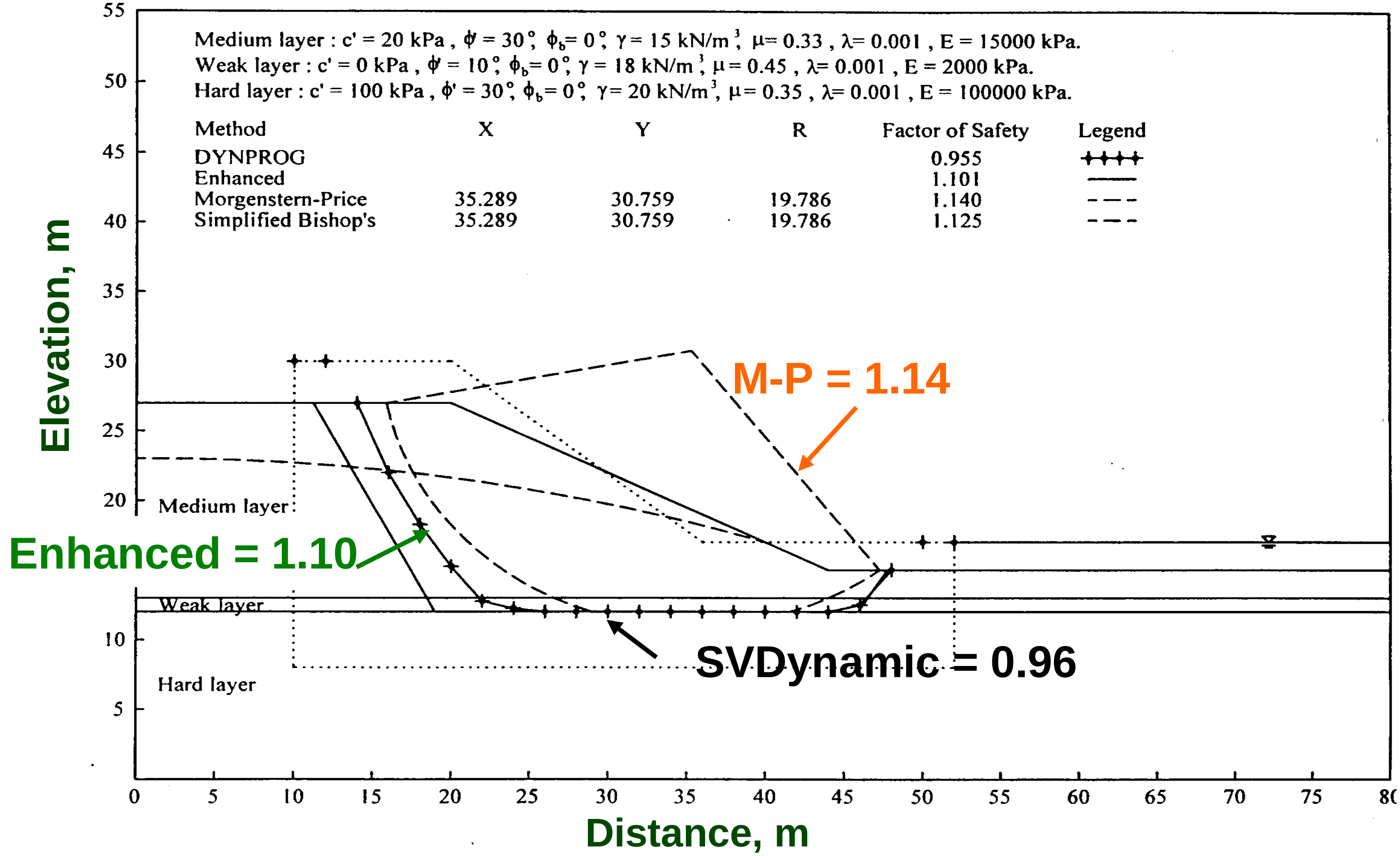
Example of a Partially Submerged Slope



Example of a Multilayered Slope



Conclusions

- *The Location of the Critical Slip Surface is a part of the overall solution*
- *The Shape of the Critical Slip Surface is part of the overall solution*
- *The shape of the Critical Slip surface can be concave or convex*
- *Complex stress-strain models for soil behaviour (e.g., elasto-plastic) can be used in the finite element method for the computation of the stress state in the soil mass*

Thank You...