Application of dynamic programming to evaluate the slope stability of a vertical extension to a balefill

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The slope-stability of a proposed vertical extension of a balefill was investigated in the present study, in an attempt to determine a geotechnically conservative design, compliant with New Jersey Department of Environmental Protection regulations, to maximize the utilization of unclaimed disposal capacity. Conventional geotechnical analytical methods are generally limited to well-defined failure modes, which may not occur in landfills or balefills due to the presence of preferential slip surfaces. In addition, these models assume an a priori stress distribution to solve essentially indeterminate problems. In this work, a different approach has been applied, which avoids several of the drawbacks of conventional methods. Specifically, the analysis was performed in a two-stage process: (a) calculation of stress distribution, and (b) application of an optimization technique to identify the most probable failure surface. The stress analysis was performed using a finite element formulation and the location of the failure surface was located by dynamic programming optimization method. A sensitivity analysis was performed to evaluate the effect of the various waste strength parameters of the underlying mathematical model on the results, namely the factor of safety of the landfill. Although this study focuses on the stability investigation of an expanded balefill, the methodology presented can easily be applied to general geotechnical investigations.

Keywords: waste landfills, balefills, waste strength parameters, slope stability, finite element simulation, dynamic programming

Introduction

To optimize the available permitted site life, landfill engineers and operators may choose to adopt one or more of the following technologies and methods into their operations: alternative daily cover materials, intense compaction efforts, implementation of bioreactor technology in lined cell areas, landfill mining for disposal capacity recovery, including resource recovery from the incoming waste stream, or site operation as a ‘balefill’. In a balefill, the incoming waste stream is compacted into rectangular bales prior to landfilling. Balefills are operated separately from landfills, where the incoming waste is usually compacted in situ by heavy equipment. Despite the fact that the majority of landfills do not operate as balefills and that there exist conflicting reports with regard to the efficacy of balefills, numerous owners/operators worldwide have weighed the pros and cons and operate many sites as balefills.

The use of bales allows for the construction of a more uniform waste body, generally resulting in higher waste densities, reduction of the time required placing waste through the use of specialized equipment at the working face, and thus optimization of the waste placement processes. Balefills are generally associated with lower leachate strengths, and exhibit more homogeneous consolidation characteristics. The enhanced and more uniform consolidation of balefills combined with the increasing need for landfill disposal capacity encourages the consideration of ‘piggy-backing’, namely the placement of additional waste on top of exhausted waste bodies. Baldasano et al. (2003) have reported that rectangular, plastic-wrapped bales are more cost efficient than conventional landfills.

Placement of new landfill cells on top of existing, older ones requires careful consideration of the stability of the
proposed resulting waste body. Stability analyses of landfills are complex, as the stress–strain relationship of municipal solid waste has not been resolved due to various uncertainties and difficulties. In addition, waste strength parameters are widely influenced by a plethora of variables. As a result, literature values vary widely, reflecting the influence of test methods and sample size among others. Waste strength parameters have been found to depend on a wide range of physical, biological, and operational factors, such as waste composition, liquid content variations, age, degree of degradation, location within the landfill, initial in-place compaction, and testing protocol and technique (Kölsch 1992).

Advanced landfilling techniques, such as baling, introduce additional complexities at bale interfaces and lifts. Moreover, conventional, well-defined failure surfaces may not develop within the waste mass due to the presence of preferential failure surfaces, defined by liner systems, lifts, and abandoned work faces. Therefore, it is essential to demonstrate the impact of waste strength parameters on the slope stability of balefills, while concurrently taking into account the potential irregular geometry of the failure surface.

The objective of this paper is a quantitative evaluation of the geotechnical stability of a proposed vertical expansion for an existing balefill in New Jersey, USA. In a preliminary, unpublished feasibility study the gain in disposal capacity for several expansion alternatives were evaluated, considering cost, constructability, usability and geotechnical stability. Based upon these results the preferred configuration was selected (Figure 1), and a detailed analysis was conducted. The analysis included a review of applicable strength models for landfilled waste and a novel approach to evaluate the geotechnical stability of complex geometries.

**Background**

Municipal waste, like soil, is essentially a particulate material, thus, most researchers have attempted to quantify its mechanical behaviour employing the principles of conventional soil mechanics. During the last 25 years researchers have employed conventional and unorthodox techniques to investigate the strain–stress relationship of landfilled, domestic waste (Gabro & Valero 1995; Grisolia et al. 1995; Machado et al. 2002; Hudson et al. 2004; Krase & Dinkler 2005). Other researchers have conducted back-calculations of waste shear strengths from mass waste instabilities (e.g., Kavazanjian 1999; Eid et al. 2000; Stark et al. 2009), and these results compared favourably to small scale testing (Singh & Murphy 1990). In addition, several studies have concluded that landfill slope instability occurs most often when excess liquid is present and is not due to poor slope stability design. The majority of the related research was conducted on non-baled waste. So far, only a limited amount of research has focused on baled waste, in general, investigating the effect of baling on landfill gas (LFG) production rate and yield, and waste degradation at small scales (e.g. Robles-Martínéz & Goudon 1999, 2000), as well as economical issues (Baldasano et al. 2003).

Soil particle incompressibility is a fundamental assumption of soil mechanics, and although waste actually behaves like particulate material, the applicability of the traditional analytical tools cannot be implicitly accepted (Powrie et al. 1999). Kölsch (1995) has proposed that the strength of a waste arises partly from inter-particle friction and partly from the reinforcing effects of sheet-like materials within the waste; affecting the internal angle of friction and tensile strength, respectively. The frictional component would be expected to increase with increasing normal effective stress as shown in Figure 2.

Similarly, the reinforcement component of the waste will increase, due to the increasing normal effective stress, but upon tearing of the reinforcing materials (‘yield point’) this strength component is lost. For this reason, Kölsch (1995) postulates a bilinear failure envelope on a graph of shear stress against normal effective stress, similar to the one obtained for a reinforced granular soil, as demonstrated by Gray & Ohashi (1983) and shown in Figure 3. At low normal effective stresses, the slope of the failure envelope is steeper, representing the effects of both friction and reinforcement. When the tensile strength of the reinforcing elements is reached, this component of the overall strength is lost and...
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the slope of the failure envelope changes abruptly to the apparent angle of friction of the waste. In a detailed study by Zekkos (2005), it has been proven that the increased shear strength observed after direct shear and tri-axial compression tests of large-scale reconstituted waste specimens were attributed to the reinforcing effect of fibrous material in the waste.

Nevertheless, Jessberger et al. (1995) argue against a bilinear failure envelope. The authors advocate that the various reinforcing elements that can be found in a waste body, have different tensile strengths and different stress-strain properties. A clear breakpoint, as present in the case of a granular soil reinforced by strands of a particular material, would therefore not be expected. Consequently, they suggest a ‘multi’-linear failure envelope that would degenerate to a conventional linear model.

In addition, Powrie et al. (1999) caution against the application of ‘conventional’ strength models to waste, raising concerns about deformability and compressibility. In particular, the solid fraction of the waste is highly compressible, necessitating alternative methods to calculate the effective stresses governing volume change and overall compressibility. As an alternative, it is recommended that geotechnical analyses involving waste should be based on collapse limit state analysis, mobilized at certain permissible limiting shear strength.

However, if waste could be shown to conform to a failure law based on the linear Mohr–Coulomb shear strength model applicable for soils, the whole array of traditional limit state analytical tools would be available for landfill geotechnical calculations. The aforementioned literature review has depicted that most researchers have accepted this requirement, and adopted it as the underlying assumption to their work. The result of this pragmatic decision is that widely available geotechnical techniques, algorithms and software become available to analyse and predict landfill geotechnical behaviour.

The use of conventional methods for slope modelling is considered appropriate, provided that site conditions, waste types, moisture content and other pertinent information have been appropriately considered in the analysis.
Conventionally, methods employed to evaluate slope stability are based on the limit equilibrium method (LEM) as applied to a series of slices that are located above a slip surface of assumed shape and size. As the method is inherently indeterminate, LEMs generally introduce assumptions regarding the relationship between interslice stresses to eliminate this deficiency. As a result, the assumed approximate stress distribution may differ significantly from the actual stresses encountered within the soil. In a slope stability analysis involving the limit equilibrium methods of slices, the minimum factor of safety is found by trial-and-error of slip surfaces of predetermined shapes and sizes acting at different locations.

These assumptions with respect to the interslice force function in LEMs are not required when a finite element (FE) stress analysis is performed to obtain the normal and shear stresses acting within the soil mass. Furthermore, assumptions regarding the uncertainty of the shape and size of the critical slip surfaces can be omitted when an appropriate optimization technique is integrated into the analysis. The dynamic programming (DP) method for slope stability analysis has not been widely used in engineering practice, primarily because of the complexity of the formulation and the lack of verification of the computed results.

Baker (1980) introduced an optimization procedure that utilized the algorithm of the dynamic programming method to determine critical slip surfaces of embankments. In that approach, the associated safety factors were calculated using Spencer’s (1967) methods of slices. Yamagami & Ueta (1988) enhanced Baker’s approach by combining the DP method with a FE stress analysis to more accurately calculate the factor of safety. The critical slip surface was assumed to consist of a chain of linear segments connecting two state points located in successive stages. Zou et al. (1995) proposed an improvement to the aforementioned method, by allowing the critical slip surface to contain a segment connecting two state points located in the same stage.

In general, dynamic search methods can be more efficient than conventional ones due to the fact that a large number of potential slip surfaces is generated and examined, thus, leading to a higher probability in locating the most critical slip surface having the minimum factor of safety. In order to increase the efficiency of these approaches and avoid local optima, random search techniques have also been implemented. Furthermore, DP can overcome any limitations concerning the geometry, the mechanical properties and any other special conditions of the problem at hand. Recent developments in computing techniques and advances in technology have made the FE/DP approach more attractive and more easily available as an alternative tool to conventional methods that can be efficiently applied in many geotechnical engineering applications (Yamagami & Jiang 1997; Pham & Fredlund 2003; Alkasawneh et al. 2008).

Case study
The active phase of Landfill A is operated as a balefill, after earlier phases have been operated as a conventional landfill. After 2 years of operations the side slopes of the current cell have been found to be inclined at 6 : 1 (horizontal : vertical), significantly flatter than the designed and permitted slope of 3 : 1. As the unclaimed airspace, that is, disposal capacity, comprises a significant volume and thus economic resource, the operator retained the corresponding author’s company to evaluate alternatives that would optimize slope reclamation while ensuring site safety and slope stability as pertaining to the site’s permit requirements. Specifically, under static conditions, slopes should be designed for a factor of safety (FS) of at least 1.5; that is, FS ≥ 1.5.

In an initial investigation (not presented in this study) the slope stability and the gain in airspace were estimated for three different alternatives. In alternative 1, the additional waste body would be placed starting at the first benchroad up to the final height at a slope of 3 : 1, with the exception of a second benchroad. For alternative 2, the preferred configuration, a clay toe berm has been added to increase the vertical stresses at the toe of the slope, thus increasing the resistance to shear failure at that location in the subsurface (as shown in Figure 1). For alternative 3, the toe of the additional waste has been moved up-slope to coincide with the top of the clay toe berm. Table 1 summarizes the volume and weight gains of each one of the alternatives per unit length slope of the landfill. The present investigation has been focused on the second alternative, evaluating a range of waste strength parameters. The calculations have been performed in a two-phased approach: (1) finite element stress analysis of the slope, and (2) determination of the critical slip surface using a dynamic programming optimization approach. The results of this investigation are presented in the following sections.

Slope stability analysis
The DP method coupled with an FE stress analysis provides a more complete solution for the analysis of slope stability, as the technique overcomes the principal limitations associated with LEM. The advantage of the DP approach is that the critical slip surface can be irregular in shape and is determined as part of the slope stability solution, whereas with conventional methods the failure mode needs to be anticipated by the engineer. More complex stress and strain behaviours of

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Volume (m³/m)</th>
<th>Weight (t/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Waste</td>
<td>Clay</td>
</tr>
<tr>
<td>1</td>
<td>2775</td>
<td>–</td>
</tr>
<tr>
<td>2*</td>
<td>2768</td>
<td>205</td>
</tr>
<tr>
<td>3</td>
<td>3260</td>
<td>192</td>
</tr>
</tbody>
</table>

*Selected configuration.
soils, such as nonlinear, elasto-plastic models, can easily be accommodated by the FE stress analysis, and subsequently analysed for slope stability using the DP method. The combination results in a versatile analytical tool, suited for complex slope geometries and soil materials. The disadvantage of the DP approach is, however, that more variables need to be specified for the analysis, specifically the Poisson ratio and the elastic moduli of the materials involved, which may not be readily available.

For the analysis of this rather complex geotechnical application, the geotechnical software package developed by SoilVision® Systems Ltd. has been used. This software includes an FE soil stress analysis program (SVSolid, version 4.5.4) and the slope stability optimization model SVDynamic (version 1.16). The calculations are performed in two steps: (1) stress analysis, and (2) critical slip surface calculation. The program has been found to compare favourably against conventional slope stability analysis algorithms (Pham 2002; Pham & Fredlund 2003).

Typically, slope stability problems are solved for a two-dimensional vertical cross-section of unit thickness, for which the static equilibrium equations consider plane-strain conditions. SVSolid is a numerical analysis package, which is capable of solving the stress–strain problem using the finite element method. SVDynamic employs the dynamic programming technique (Bellman 1957) to find the pathway, comprised of linear segments, that minimizes the factor of safety of the slope. The iterative algorithm introduces the return function, \( G \), defined for each line segment \( i \) connecting two mesh points, which is substituted into the objective function to yield the optimal, that is, the minimal value for the safety factor. For this purpose, the model domain is discretized into a separate, rectangular grid of ‘stages’ and ‘states’ as depicted in Figure 4. The calculations are performed for an initial value of \( FS \), and are repeated until the difference between subsequent values is smaller than a preset threshold value. The algorithm does not include any implicit assumptions regarding the shape of the slip surface, and only requires the definition of a search box that forces the search routine to route the slip surface through the soil.

### Conceptual model

Differences in waste age will invariably result in contrasts in waste properties between the original and supplemental waste bodies, producing two distinct waste bodies separated by a potential slip surface. The interface is likely to exhibit little resistance to shear forces, with friction being the main failure-resisting mechanism. A number of assumptions have been introduced to assure consistency between the conceptual model and the numerical one. Some originate in the practice of waste management and the implementation of regulatory guidelines, whereas others are necessary to conform to generally accepted geotechnical assumptions.

- Stresses and strains within the soil/waste mass are assumed to be linearly related.
- The waste body is unsaturated, thus \( u = 0 \) throughout the model domain, where \( u \) denotes the hydrostatic pressure.
- All bales, old and new, are placed in an interlocking three-dimensional pattern.
- The overburden on bales causes them to deform, resulting in an increase in the coefficient of friction at their interfaces.
- As a result of waste degradation, interfaces between bales become less distinct, resulting in a more homogeneous waste mass, and development of cohesion along the interfaces of the bales.

![Fig. 4: Search for the critical slip surface (AB) based on the dynamic programming method (reproduced from Pham, H.T.V. & Fredlund, D.G. (2003) The application of dynamic programming to slope stability analysis. Canadian Geotechnical Journal. 40, 830–847. © 2008 NRC Canada or its licensors. Reproduced with permission.)](image-url)
• Placement of the additional waste is assumed to occur sufficiently fast, thus permitting consideration of its properties to be homogeneous.
• Similar considerations allow the existing waste body to be assumed homogeneous.
• A ‘vertical’ sand layer separates the new from the old waste.
• Lifts are separated by drainage sand layers.
• All alternatives are considered with the final cover in place.

Soil and waste strength properties
The peer-reviewed and professional literature have provided ample guidance on determining ranges for the cohesion and friction angle of solid waste. However, at present, only limited information on the numerical values for the elastic parameters, that is, the shear modulus ($G$), Young’s modulus ($E$) and Poisson’s ratio ($\nu$), have been published. These parameters quantify the response of a material to a change in stress, and are related to each other by:

$$G = \frac{E}{2(1+\nu)}$$  \hspace{1cm} (1)

Generally, laboratory testing of elastic parameters on disturbed and small-scale samples does not provide reliable results (Dixon & Jones 2005). However, in situ measurements of the shear and compression wave propagation velocities, which are related to the elastic properties, in downhole geophysical investigations have been shown to be consistent. Employing this methodology, Sharma et al. (1990) determined the Poisson’s ratio to be 0.49, close to the theoretical maximum of 0.5. In their calculation of the shear modulus, Sharma et al. advise that simultaneous measurements of the unit weight should be taken, and report a value of $G = 29$ MPa as representative for municipal solid waste. Zekkos (2005) performed cyclic triaxial and monotonic triaxial tests to determine the Poisson ratio of solid waste. The results indicate a Poisson’s ratio of about 0.3–0.35 for specimens not containing fibrous materials. As the amount of fibrous materials increases the total unit weight of the waste specimen decreases, and Poisson’s ratio is significantly lowered to values ranging from 0.3 to possibly as low as zero. Machado et al. (2002) developed a constitutive model for municipal solid waste and determined the Poisson’s ratio and the shear modulus to be 0.36 and 97 MPa, respectively.

Waste unit weight (density), $\gamma$, is a dynamic parameter that depends on the methodology employed for its determination (Zekkos et al. 2006). It has been reported to increase over time as the organic fraction decomposes and the waste body consolidates, and is highly dependent on the initial compaction effort invested during waste placement. The weight of the incoming waste typically is recorded and the surface elevation of the site is surveyed annually. The data were used to calculate the in situ waste density, corrected for the volume (weight) of the sand used for intermediate and drainage layers, as presented in Table 2.

Kavazanjian (2001) determined the in situ waste unit weight at two landfills in southern California and found the actual unit weight values to be significantly higher than those reported by site operators. The author also stresses the failure in literature to distinguish between the weight of the waste and the combined weight of the waste, soil and liquids in the control volume. Furthermore, reports of the ratio of refuse to soil in a unit volume generally fail to specify whether the cited ratio was established on a weight or volume basis. Common operator practice is to report a volume ratio, which has occasionally been interpreted by engineers as representing the total weight of soil and refuse in a unit volume, and as the weight-based ratio of refuse to soil. In addition, the presence of absorbed liquids is often underestimated due to the heterogeneity and spatial variability in the composition and properties of the waste body, introducing uncertainties in wet or bioreactor landfills, thus possibly underestimating the landfill stability in the course of analysis. Solid waste densities in bioreactors and leachate recirculation landfills are likely to be higher than even the corrected estimates, and values sometime approach or exceed 20.0 kN/m$^3$ (Kavazanjian 2001).

In this work, a linear stress–strain model describing the waste strength characteristics has been used. In a large number of papers and publications, researchers and engineers have reported widely varying values for waste density, cohesion and the internal friction angle. By and large, the majority evaluated in situ compacted waste placed and sampled under diverse, not necessarily comparable conditions. Table 3 presents characteristic waste strength parameters as reported in a representative sample of publications. As the amount of data for baled waste properties is limited (Oweis & Khera 1990), the more general range based on data presented in Table 3 has been used. In addition, a sensitivity analysis of the slip surface location has been performed for the values indicated in Figure 5. Parameter values for the soils, namely sub-stratum, clay for the toe berm, drainage layers, and final cover soil, were taken from laboratory results determined for actual soil samples submitted for Landfill ‘A’.

### Numerical investigation
A significant number of simulations, consisting of FE and DP analyses, have been performed to assess the effect of waste strength parameters on slope failure modes and associated FS values. To reduce the number of variables of the problem at hand, strength parameters for the underlying substratum

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight (kN/m$^3$)</td>
<td>7.36</td>
<td>7.58</td>
<td>7.19</td>
<td>7.38</td>
<td>7.81</td>
<td>8.33</td>
<td>8.56</td>
<td>7.87</td>
<td>8.33</td>
</tr>
</tbody>
</table>

Table 2: In-situ, net waste unit weights.
have been kept constant throughout the simulations. The initial choice of strength parameter values for the existing and additional waste were guided by literature values. Additional strength parameter combinations were analysed to obtain a more complete picture describing the transition from one failure mode to another. For each parameter combination a stress analysis has been performed, followed by a DP analysis to determine the slip surface, namely the failure mode and the associated FS.

The FE mesh consisted of 427,091 nodes and 213,336 elements, while for the DP analysis, the FE grid was interpolated onto a Cartesian grid with \( dx = dy = 1.0 \) m. The DP grid is divided into ‘Stages’ along the \( x \)-axis and ‘States’ on the \( y \)-axis as depicted in Figure 4. The search box was extended to the model boundaries, except at the soil surface. The soil surface was initially intercepted below the second bench road (Figure 1), to force the search algorithm to penetrate the slope. For each case, multiple DP analyses were performed differing in the location of the intercept of the search box with the soil surface. Only slip surfaces that were not collinear with segments of the search box, and whose FS was independent of the location of the intercept were considered valid.

Table 3: Characteristic waste strength parameters.

<table>
<thead>
<tr>
<th>Description</th>
<th>Cohesion (kPa)</th>
<th>Friction angle (°)</th>
<th>Test method</th>
<th>Remarks</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compacted waste</td>
<td>10.0</td>
<td>20.0</td>
<td></td>
<td></td>
<td>Kölsch (1995)</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>25.0–36.0</td>
<td>Back calculations</td>
<td></td>
<td>Oweis &amp; Khera (1990)</td>
</tr>
<tr>
<td></td>
<td>25.0</td>
<td>35.0</td>
<td>Direct shear</td>
<td></td>
<td>Eid et al. (2000)</td>
</tr>
<tr>
<td></td>
<td>43.0</td>
<td>31.0</td>
<td>Direct simple shear</td>
<td></td>
<td>Kavazanjian et al. (1999)</td>
</tr>
<tr>
<td></td>
<td>16.3–29.9</td>
<td>33.0–59.0</td>
<td>Direct simple shear</td>
<td></td>
<td>Kavazanjian et al. (1999)</td>
</tr>
<tr>
<td></td>
<td>15.0 (friction)</td>
<td>35.0 (tensile)</td>
<td>Direct shear</td>
<td>Fresh waste</td>
<td>Kölsch (1995)</td>
</tr>
<tr>
<td></td>
<td>22.0 (friction)</td>
<td></td>
<td>Calculated</td>
<td>Old waste</td>
<td>Kölsch (1995)</td>
</tr>
<tr>
<td></td>
<td>15.0 (friction)</td>
<td></td>
<td>DIN 4017</td>
<td>Fresh waste</td>
<td>Spillman (1980)</td>
</tr>
<tr>
<td></td>
<td>18.0–19.0 (friction)</td>
<td></td>
<td>Triaxial</td>
<td>Older waste</td>
<td>Jessberger et al. (1991)</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>( \Phi = 36.0 – 5 \log(\sigma_n/\sigma_p) )</td>
<td>Direct shear</td>
<td></td>
<td>Zekkos (2005)</td>
</tr>
<tr>
<td></td>
<td>0.0–23.5</td>
<td>21.0–42.0</td>
<td>Triaxial</td>
<td></td>
<td>Dixon &amp; Jones (2005)</td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>15.0–25.0</td>
<td>Back calculations</td>
<td></td>
<td>Dixon &amp; Jones (2005)</td>
</tr>
<tr>
<td>Baled waste</td>
<td>62.2</td>
<td>15.0–25.0</td>
<td></td>
<td></td>
<td>Oweis &amp; Khera (1990)</td>
</tr>
</tbody>
</table>

Fig. 5. Classification of principal failure modes (vertical exaggeration factor: 2).
Slip surfaces in homogeneous slopes are commonly idealized as being circular, rather than of more complex cross-sections. The presence of soil layers of contrasting strength parameters may channel the failure surface along the interface, resulting in translational or compound slips, depending on slope geometry, soil properties and the application of external forces. To facilitate the discussion, the resulting slip surfaces have been categorized into four groups, based on the location and shape of the slip surface, as (a) surficial failure, (b) contained failure, (c) shallow failure, and (d) deep failure (Figure 5). Two subtypes of surficial failures were identified, namely (1) the long and (2) the short ones.

Surficial failures occur when the contrast between the existing and additional wastes is sufficiently high, to prevent the slip surface from entering the existing waste body. The slip surface generally runs parallel to the interface between the waste bodies, thus resulting in a translational slip. Similarly, in contained failures, the strength of the flexible liner is sufficiently high to contain the slip surface within. In shallow failures, the shear strength of the liner is insufficient to contain the failed waste body and is sheared at the bottom on the containment slope. While the shape of the slip surface more closely resembles that of a non-circular failure, the failure mechanism is actually more closely related to that of a translational slip. The slip surface exits the native soil outside the landfill, essentially creating a non-circular failure, in which the liner system intercepts and directs the slip surface. Finally, in a deep failure, the slip surface shears through the waste body and liner system, and exits through the native soil outside the landfill. This failure most closely resembles a circular slip surface.

The relative waste properties were defined as the ratio of the strength parameters between the existing and additional waste, namely the cohesion, \( c_r \), and internal angle of friction, \( \phi_r \), namely

\[
\frac{c_r}{c_{new}} = \frac{c_{old}}{c_{new}}, \quad \text{and} \quad \frac{\phi_r}{\phi_{new}} = \frac{\phi_{old}}{\phi_{new}}
\]

where ‘old’ and ‘new’ denote the existing and additional waste body, respectively. For each simulation the failure mode was determined, and plotted at the coordinates \((c_r, \phi_r)\) on a log-log scale, with symbol size being proportional to the factor of safety as depicted in Figure 6.

In general, the FS increases more rapidly along the \( \phi_r \)-axis than along the horizontal, indicating the larger contribution of an enhanced contrast in the internal angle of friction over that in cohesion. The largest increases in FS are obtained along the diagonal ranging across all failure modes, rather than along the axes involving only two failure modes. The resulting FS above the line \( \phi_r = 1 \) are significantly larger than those below that line, further indicating the relative importance of the friction angle over cohesion.

Failure modes were found to be roughly clustered in the four quadrants defined by the axes \( c_r = 1 \) and \( \phi_r = 1 \), specifically the major failures modes, deep (d) and surficial (a), are concentrated in the first and second quadrant, respectively. The minor failure modes, contained (b) and shallow (c), are concentrated in the third and final quadrant in somewhat irregular regions. The transition between regions is gradual, and can partially be attributed to a lack in resolution introduced during mesh generation, the non-linear nature of the underlying calculations, and the somewhat subjective classifications of the resulting slip surfaces.

Surficial (a) and contained (b) failures are closely related, as they rely on a sharp contrast in waste strength properties. Such failures are relatively common in landfills, as site operation, for example, compaction, installation of intermediate layers, or the presence of a saturated waste layer, may contribute to its occurrence. Surficial failures are also known to occur in steep waste slopes (Koerner & Soong 1999); however this aspect has not been addressed in this paper. Deep (d) failures are indicative of a foundation collapse, and should not be attributed to the waste body, as the substratum under the landfill was inadequate to support the load, and the liner system could not withstand the shear forces acting on it. These results qualify Kölsch’s (1995) statement, that a slip surface along the waste/liner interface would be the worst-case scenario for waste bodies exhibiting high internal strengths, if the subsoil is substantially more resilient than the waste.

A sensitivity analysis has been performed to investigate the impact of several basic parameters in the proposed FE/DP approach. Systematically varying the values of Poisson’s ratio, Young’s modulus and other parameters associated with the FE analysis, while maintaining waste strength parameter values constant, has not been found to affect the shape and location of the slip surface. The numerical results indicate however, that the underlying stress field and associated strains are affected to a certain extent.

US Federal (CFR 40 Parts 257 and 258, RCRA Subtitle D) and New Jersey State regulations (N.J.A.C. 7:26-2A.7(b) et seq.) require the static factor of safety against slope failure to be at least 1.5. The resulting factor of safety indicate that waste

![Fig. 6: Failure mode as function of relative waste strength parameter values. Note that symbol size of each failure mode is proportional to the factor of safety for results with FS < 1.5, only the symbol outline has been plotted. Parameters appearing in the figure are defined in Section 5, Results and Discussion.](image-url)
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...strength parameter combinations resulting in short surficial or contained failure modes would fail the regulatory requirements. The range of factor of safety values for the three remaining failure modes, long surficial, shallow and deep, is sufficiently high for them to be considered inherently stable. Figure 6 indicates that all combinations within the marked region result in stable slopes, with the exception for short, surficial failures. However, decreasing the slope between the toe berm and the bench road should easily compensate for this kind of failure. A waste body more resilient to failure than the subsurface is characterized by strength parameters mapped in the third quadrant. While subsurface conditions are site-specific, it should be noted that these failures occur only if the cohesive strength and the friction angle of the existing waste body is less than that of the additional waste. This condition is unlikely to occur in the field, as more decomposed waste is likely to exhibit increased density, cohesion and friction (see Table 4).

Conclusions
The lack of reliable ways to determine and assign waste strength properties is complicating the geotechnical analysis of solid waste landfills. Conventional strength models and algorithms are not necessarily applicable to waste, significantly impacting analytical efforts. Under the conventional limit equilibrium method, the engineer selects a slip surface and assumes an inter slice stress distribution before the factor of safety can be determined. The procedure is repeated for a sufficiently large number of slip surfaces and material strength parameters combinations until the engineer is satisfied to have found the critical failure mode.

For the preceding analysis a two-step process has been employed, which initially determines the stress distribution within the waste body, and then determines the slip surface by employing the DP techniques. This process requires more resources than conventional analyses but allows consideration of more complex stress-strain relation than conventional approaches and geometries. Landfill design requirements include several preferential slip surfaces; for example, the drainage layer and intermittent soil cover, necessitating a more flexible description of the geometry. However, the added flexibility in the choices of stress models, and the ability to review and calibrate stress distributions, mitigate the increased resource requirements. Neither methodology can be endorsed without detailed knowledge of its intended application; the approach described in this paper is more generalized and applicable for complex scenarios. It can be calibrated to field data and yields additional information, not available from the conventional method. However, the final selection rests with the experienced professional.

SVDynamic, the software used for the analysis, employs separate grids for the FE and DP analysis, respectively. The stress field is interpolated from the triangular FE grid onto a rectangular DP grid. This step introduces inaccuracies, ‘smearing’, especially at well defined material or geometric transitions; that is ‘smearing’; for example, the liner, drainage layer, cover soil layers, whose vertical dimension is significantly less than the DP grid size. Optimization of the interpolation process may avoid shifts in slip surfaces that are evident in Figure 5. Use of separate grids for FE and DP analysis and the need to interpolate between them may cause the slip surfaces to be shifted with respect to material boundaries as prescribed in the FE mesh. This may result in the slip surface being shifted. To validate the proposed approach and to determine macro-scale geotechnical parameters, the authors propose to apply this approach to solve the inverse problem of failed landfill slopes.

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