

## SLOPE STABILITY OVERVIEW

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The following document is intended to be a brief overview of the application of slope stability analysis in current geotechnical engineering practice. A brief summary of the historical landmarks in analysis methods is also presented.

### BASIC THEORY

The principles of slope stability have been developed over the past seventy years and provide a set of soil mechanics principles from which to approach practical problems. Although the mechanism of slope failure in heap leaching may be difficult to predict, the principles used in a standard of practice examination are relatively straightforward.

An analysis of slope stability begins with the hypothesis that the stability of a slope is the result of downward or motivating forces (i.e., gravitational) and resisting (or upward) forces. These forces act in equal and opposite directions as can be seen in Figure 1. The resisting forces must be greater than the motivating forces in order for a slope to be stable. The relative stability of a slope (or how stable it is at any given time) is typically conveyed by geotechnical engineers through a Factor of Safety  $F_s$  defined as follows:

$$F_s = \frac{\sum R}{\sum M}$$

The equation states that the factor of safety is the ratio between the forces/moments resisting (R) movement and the forces/moments motivating (M) movement. When the factor of safety is equal to 1.0 a slope has just reached failure conditions. If the factor of safety falls below 1.0 then a failure is imminent, or has already occurred. Factors of safety in the range of 1.3 to 1.5 are considered reasonably safe in many design scenarios. However, the actual factor of safety used in design is influenced by the risk involved as well as the certainty with which other variables are known.

Downward or motivating (M) forces include the following:

- *Gravitational weight of soil:* The downward force of gravity is the primary force tending to cause movement. This force changes during a heap leaching operation as mass removed from the material affects the force of gravity.

Also, changes due to compression of the material change the gravitational force.

- *Gravitational weight of water:* Changes in the degree of saturation of the soil change and also the specific gravity of the pore fluid may not remain equal to that of water.

Upward or resisting (R) forces include the following:

- *Frictional strength of material:* Likely decreases somewhat as the heap leach material leaches and loses mass but there may also be an increase as the material progressively compresses due to its self weight and the stacking procedure.
- *Cohesional strength of material:* Cohesional strength is primarily associated with the clay-sized fraction. Agglomeration bonds could likely be lumped in as cohesion.
- *Soil suction:* Contribution of this component is related to the saturation level. Completely saturating the soil would eliminate this contribution due to soil suction. Loss of suction is a common "trigger" for landslides worldwide.

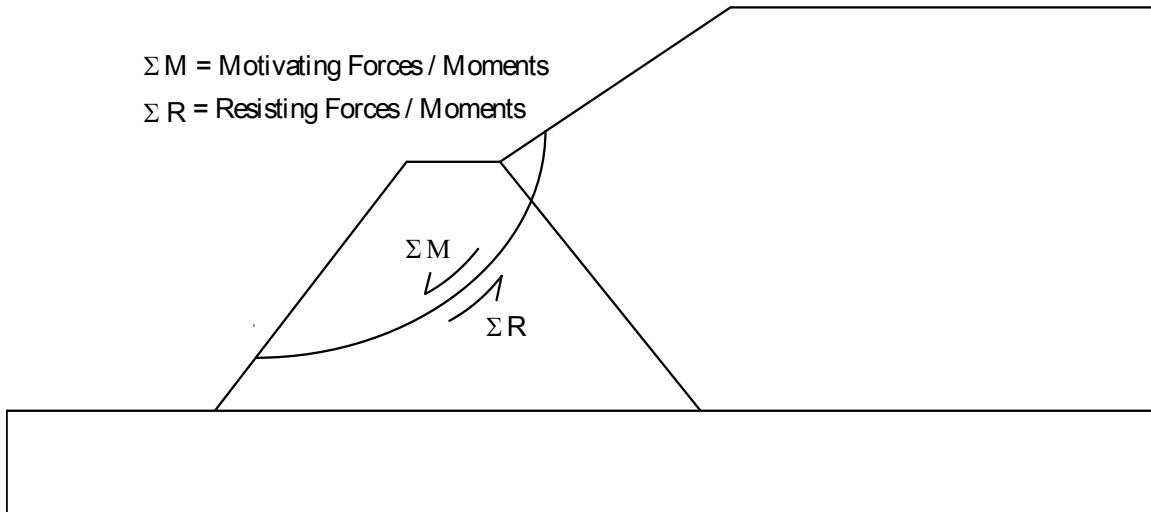
Each of the resisting forces mentioned contributes to the total shear strength of the soil through the following equation:

$$\tau = c' + (\sigma_n - u_a) \tan(\phi') + (u_a - u_w) \tan \phi^b$$

If it is assumed that air pressure is atmospheric or negligible then the above equation reduces to the following:

$$\tau = c' + (\sigma_n) \tan(\phi') + (-u_w) \tan \phi^b$$

where:  $\tau$  = shear strength of the soil material,  
 $\sigma_n$  = normal total stress,  
 $\phi'$  = effective friction angle,  
 $u_w$  = pore-water pressure (negative for suction),  
 $\phi^b$  = friction angle associated with soil suction.



**Figure 1: Illustration of the motivating and resisting forces/moments involved in a slope stability analysis**

## HISTORY OF ANALYTICAL METHODS

The basis for most methods of slope stability analysis can be traced back to 1922, when the Geotechnical Commission was appointed by the Swedish State Railways to investigate solutions following a costly slope failure. The method presented (Statens Järnvägers, 1922) has become known as the Swedish Slip Circle Method. This method assumes the slide occurs along a circular arc.

Fellenius (1927, 1936) developed this method further, creating a method known as the Ordinary Method of Slices, or Fellenius' Method. In any method of slices, the soil mass above the failure surface is subdivided into vertical slices, and the stability is calculated for each individual slice. Fellenius' Method simplifies the equation by assuming that the forces acting on the sides of each slice cancel each other. While this enables a solution to be determined, the assumption is not completely correct, and leads to low values for the computed Factor of Safety.

Using a similar approach, Bishop (1955) refined the method of slices technique by accounting for the interslice normal forces, thus calculating the Factor of Safety with increased accuracy. The method of slices he developed is known as the Simplified Bishop Method (Bishop 1955). However, Bishop's Method still does not satisfy all the conditions of static equilibrium (i.e., summation of horizontal

forces is missing); therefore, it is an 'incomplete equilibrium method'.

In 1967, Spencer developed a complete equilibrium method known as Spencer's Method, which satisfies both force and moment equilibrium forces. As a result, the Factor of Safety calculated by this method should be more precise. Spencer's Method can also be adapted for use with non-circular slip surfaces, which is useful because many slides do not have circular failure surfaces.

Other methods of slices for calculating stability for non-circular failure surfaces include Janbu's Rigorous Method and Janbu's Simplified Method (Janbu 1954). His rigorous method accounts for the interslice forces; his simplified method assumes these forces are zero, but includes a correction factor to compensate for the interslice forces. An alternative method for analyzing slides with a non-circular failure path was developed by Morgenstern and Price in 1965. This method satisfies all equations of statical equilibrium, and is known as the Morgenstern-Price Method.

A general, comprehensive framework for limit equilibrium methods of slices is the General Limit Equilibrium method developed by Fredlund et al. (1981). This methodology can be used for analyses in both circular and non-circular slip surfaces.



Two summaries of current limit equilibrium methods may be seen in the following two tables.

**Table 1: Summary of Common Methods of Analysis (TRB Special Report, 1996)**

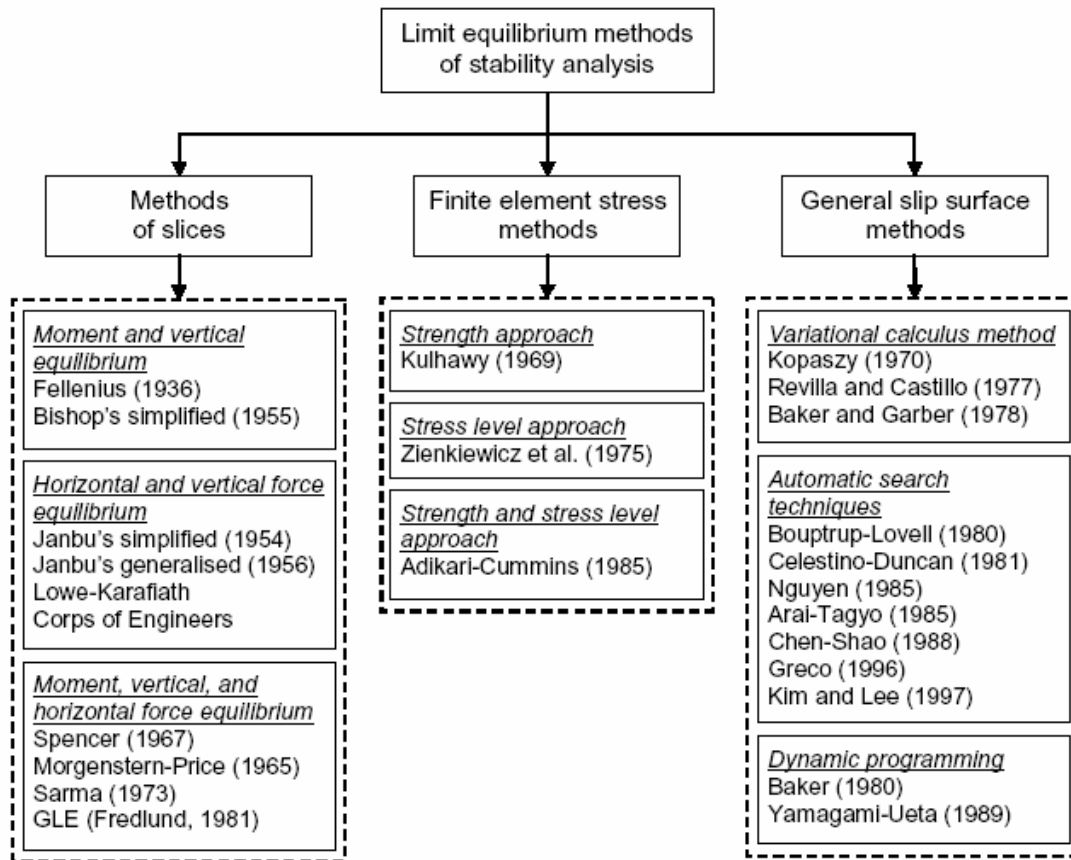
Method Name:	Developed By:	Limitations, Assumptions, and Equilibrium Conditions Satisfied
Ordinary Method of Slices	Fellenius (1927)	Factors of safety low - inaccurate for flat slopes with high pore pressures; only for circular slip surfaces; assumes that normal force on the base of each slice is $W \cos \alpha$ ; one equation (moment equilibrium of entire mass), one unknown (factor of safety)
Bishop's modified method	Bishop (1955)	Accurate method; only for circular slip surfaces; satisfies vertical equilibrium and overall moment equilibrium; assumes side forces on slices are horizontal; $N + 1$ equations and unknowns
Force equilibrium methods		Satisfy force equilibrium; applicable to any shape of slip surface; assume side force inclinations, which may be the same for all slices or may vary from slice to slice; small side force inclinations result in values of $F_s$ less than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns
Janbu's simplified method	Janbu (1968)	Force equilibrium method; applicable to any shape of slip surface; assumes side forces are horizontal (same for all slices); factors of safety are usually considerably lower than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns
Modified Swedish method	U.S. Army Corps of Engineers (1970)	Force equilibrium method, applicable to any shape of slip surface; assumes side force inclinations are equal to the inclination of the slope (same for all slices); factors of safety are often considerably higher than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns
Lowe and Karafiath's method	Lowe and Karafiath (1960)	Generally most accurate of the force equilibrium methods; applicable to any shape of slip surface; assumes side force inclinations are average of slope surface and slip surface (varying from slice to slice); satisfies vertical and horizontal force equilibrium; $2N$ equations and unknowns
Janbu's generalized procedure of slices	Janbu (1968)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes heights of side forces above base of slice (varying from slice to slice); more frequent numerical convergence problems than some other methods; accurate method; $3N$ equations and unknowns
Spencer's method	Spencer (1967)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that inclinations of side forces are the same for every slice; side force inclination is calculated in the process of solution so that all conditions of equilibrium are satisfied; accurate method; $3N$ equations and unknowns
Morgenstern and Price's method	Morgenstern and Price (1965)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that inclinations of side forces follow a prescribed pattern, called $f(x)$ ; side force inclinations can be the same or can vary from slice to slice; side force inclinations are calculated in the process of solution so that all conditions of equilibrium are satisfied; accurate method; $3N$ equations and unknowns
Sarma's method	Sarma (1973)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that magnitudes of vertical side forces follow prescribed patterns; calculates horizontal acceleration for barely stable equilibrium; by prefactoring strengths and iterating to find the value of the prefactor that results in zero horizontal acceleration for barely stable equilibrium, the value of the conventional factor of safety can be determined; $3N$ equations, $3N$ unknowns

**Table 2: Summary of Common Methods of Analysis (Abramson et al., 2002)**

<b>Method Name:</b>	<b>Developed By:</b>	<b>Description of Method:</b>
Ordinary Method of Slices (OMS)	Fellenius (1927, 1936)	This method neglects all interslice forces and fails to satisfy force equilibrium for the slide mass as well as for individual slices. However, this is one of the simplest procedures based on the method of slices.
Bishop's Simplified Method	Bishop (1955)	Bishop assumes that all interslice shear forces are zero, reducing the number of unknowns by $(n - 1)$ . This leaves $(4n - 1)$ unknowns, leaving the solution overdetermined as horizontal force equilibrium will not be satisfied for one slice.
Janbu's Simplified Method	Janbu (1954a, 1954b, 1973)	Janbu also assumes zero interslice shear forces, reducing the number of unknowns to $(4n - 1)$ . This leads to an overdetermined solution that will not completely satisfy moment equilibrium conditions. However, Janbu presented a correction factor, $f_0$ , to account for this inadequacy.
Lowe and Karafiath's Method	Lowe and Karafiath (1960)	Lowe and Karafiath assume that the interslice forces are inclined at an angle equal to the average of the ground surface and slice base angles. This simplification leaves $(4n - 1)$ unknowns and fails to satisfy moment equilibrium.
Corps of Engineers Method	Corps of Engineers (1970)	The Corps of Engineers approach considers the inclination of the interslice force as either (1) parallel to ground surface, or (2) equal to the average slope angle between the left and right end-points of the failure surface. The approach is similar to the one proposed by Lowe and Karafiath (1960) and presents an overdetermined system where moment equilibrium is not satisfied for all slices.
Spencer's Method	Spencer (1967, 1973)	Spencer proposes a method that rigorously satisfies static equilibrium by assuming that the resultant interslice force has a constant, but unknown, inclination. These $(n - 1)$ assumptions again reduce the number of unknowns to $(4n - 1)$ , but the unknown inclination is an additional component that subsequently increases the number of unknowns to match the required $4n$ equations.
Bishop's Rigorous Method	Bishop (1955)	Bishop assumes $(n - 1)$ interslice shear forces to calculate an $F_s$ . Since this assumption leaves $(4n - 1)$ unknowns, moment equilibrium cannot be directly satisfied for all slices. However, Bishop introduces an additional unknown by suggesting that there exists a unique distribution of the interslice resultant force, out of a possible infinite number, that will rigorously satisfy the equilibrium equations.
Janbu's Generalized Method	Janbu (1954a, 1954b, 1973)	Janbu assumes a location of the thrust line, thereby reducing the number of unknowns to $(4n - 1)$ . Sarma (1979) points out that the position of the normal stress on the last (uppermost) slice is not used and hence moment equilibrium is not satisfied for this last slice. However, similar to the rigorous Bishop method, Janbu's generalized method also suggests that the actual location of the thrust line is an additional unknown, and thus equilibrium can be satisfied rigorously if the assumption selects the correct thrust line.
Sarma's Method	Sarma (1973)	Sarma uses the method of slices to calculate the magnitude of a horizontal seismic coefficient needed to bring the failure mass into a state of limiting equilibrium. This allows the procedure to develop a relationship between the seismic coefficient and the presumed $F_s$ . The static $F_s$ will then correspond to the case of a zero seismic coefficient. Sarma uses an interslice force distribution function (similar to Morgenstern and Price, 1965) and the value of the seismic coefficient can be calculated directly for the presumed $F_s$ . All equilibrium conditions are satisfied by this method. However, it should be noted that the critical surface corresponding to the static $F_s$ (for a zero seismic coefficient) will often be different than the surface determined using the more conventional approach where the $F_s$ is treated as an unknown.
Morgenstern-Price Method	Morgenstern and Price (1965)	Morgenstern and Price proposed a method that is similar to Spencer's method, except that the inclination of the interslice resultant force is assumed to vary according to a "portion" of an arbitrary function. This additional "portion" of a selected function introduces an additional unknown, leaving $4n$ unknowns and $4n$ equations.

The various methods used to approach a slope stability analysis can be organized as presented in Figure 2. It can be seen that most methods make primary use of the method of slices, of finite element stress methods, or of slightly different methods such as variational calculus. Certain methods also make

use of combinations of these approaches. For example, the Kulhawy (1969) approach uses stresses at the base of each slice but still makes use of the method of slices for calculation of the factor of safety.



**Figure 2: Classification of Slope Stability Methodologies (Gitirana, 2005)**

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