INFILTRATION-RUNOFF BOUNDARY CONDITIONS IN SEEPAGE ANALYSIS

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ABSTRACT
Infiltration and runoff must be taken into account in most seepage problems involving soil-atmosphere interaction. Empirical and semi-empirical functions have been replaced by more rigorous solutions, based on partial differential equations governing water flow and based on soil-atmosphere coupling equations. However, the strong nonlinearities associated with such problems often pose a numerical challenge. This paper presents how infiltration-runoff problems can be analysed from a theoretical and practical standpoint. The performance of two available commercial software packages is examined using three illustrative seepage problems.

RÉSUMÉ
L'infiltration et le ruissellement doivent être considérés dans la plupart des problèmes d'infiltration impliquant l'interaction de sol et d'atmosphère. Des fonctions empiriques et semi-empiriques ont été remplacées par des solutions plus rigoureuses, basées sur des équations différentielles partielles régissant l'écoulement de l'eau et basées sur des équations d'accouplement de sol et d'atmosphère. Cependant, les non-linéarités fortes liées à de tels problèmes posent souvent un défi numérique. Cet article présente comment des problèmes d'infiltration et de ruissellement peuvent être analysés d'un point de vue théorique et pratique. L'exécution de deux logiciels commerciaux disponibles est examinée en utilisant trois problèmes d'illustration d'infiltration.

1. INTRODUCTION

Numerous practical problems in geotechnical, geoenvironmental engineering and geo-hydrology require the computation of infiltration and runoff. The assessment of slope hazards, design of dams, mining waste cover systems, collapse, heave, and other problems, require consideration of the influence of weather conditions on soil water content and pore-water pressures.

Advances in computational geotechnics have allowed the use of partial differential equation (PDEs) solvers in routine engineering. PDEs can be established for the flow of moisture within saturated-unsaturated soils. These PDEs pose a challenge to numerical solutions due to the strong nonlinearities associated with the soil properties. Infiltration-runoff boundary conditions produce additional difficulties, due to sharp wetting fronts that accentuate these nonlinearities.

Several approaches are available for the analysis of infiltration-runoff problems within the framework of partial differential equation solutions. This paper discusses some of the available formulations for runoff-infiltration analysis and presents three cases demonstrating the application of these solutions to hypothetical and real soil conditions.

2. LITERATURE REVIEW

Infiltration is the process whereby water enters the soil, generally through the soil-atmosphere boundary. The sources of water available for infiltration can be from rain, snow melt, and irrigation. In the hydrological cycle, part of the water from precipitation may infiltrate the soil, be intercepted or become runoff.

Infiltration produces a downward flux that changes the water content and the pore-water pressure gradients with depth. Horton (1933) showed that during a period of constant precipitation, the rate of infiltration decreases with time. Horton (1933) also showed that there is a limiting curve that gives the maximum infiltration rate with time, assuming that there is always water available to be infiltrated (see Fig. 1).

If the amount of precipitation minus interception is higher than the maximum rate obtained from the limiting curve, runoff takes place. When there is plenty of water available for infiltration, the infiltration rate follows the limiting function, until a constant rate is reached. The constant rate is called the *infiltration capacity*. Rubin et al. (1964) cited by Freeze and Cherry (1979) showed that the infiltration capacity is equal to the saturated coefficient of permeability, k^w_{sat}.

There are several theories for the computation of the amount of infiltration and runoff. Two main approaches are available, the first using empirical equations and the second using the PDEs governing saturated/unsaturated water flow. The most common empirical equations were proposed by Kostiakov (1932) and Horton (1933). The equation proposed by Kostiakov (1932) establishes a relationship between the infiltration rate and time using the following function:

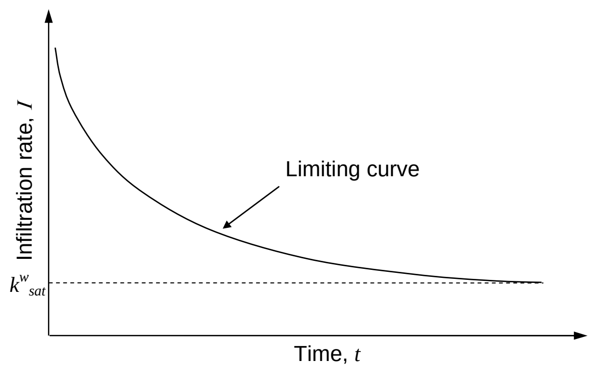

Figure 1 Infiltration rate versus time (Horton, 1933 and Koorevaar et al., 1983).

$$I = I_0 t^{-\alpha} \quad [1]$$

where I = infiltration rate; I_0 = the initial infiltration rate; α = empirical constant experimentally determined for a site of interest.

Horton (1933) proposed a popular empirical equation that is based on three parameters. The equation is as follows:

$$I = f_c + (f_0 - f_c)e^{-\beta t} \quad [2]$$

where f_c = infiltration capacity; f_0 = the maximum infiltration rate; β = rate of decrease of the infiltration rate.

Equations 1 and 2 merely attempt to reproduce the shape and main features identified in the limiting infiltration curve (Fig. 1). The equations proposed by Green and Ampt (1911) and by Philip (1957) incorporate elements of physics into the infiltration rate equations. The Green and Ampt (1911) model uses Darcy's equation and is based on infiltration into deep homogeneous reservoirs with a homogeneous initial water content distribution. A well defined wetting front is assumed with the soil saturated beyond the wetting front:

$$I = k_{sat}^{w}\left[1 + \frac{(n-\theta_i)S_f}{D}\right] \quad [3]$$

where n = soil porosity; θ_i = the initial volumetric water content; S_f = the soil suction at the wetting front; D = depth infiltrated.

Philip (1957) proposed an analytical solution for the water flow equation that is based on a finite difference solution. The equation for the infiltration rate is as follows:

$$I = \frac{C}{2}t^{1/2} + F \quad [4]$$

where C = the field capacity; F = a constant related to the gravity driving mechanism.

The infiltration rate can also be computed in a rigorous manner by modelling the internal moisture flow combined with boundary conditions that represent the precipitation and drainage conditions. The one-dimensional PDE governing liquid and water vapour flow can be written as follows (Gitirana Jr., 2005):

$$\frac{\partial}{\partial y}\left[\left(\frac{k^w + k^v}{\gamma_w}\right)\frac{\partial u_w}{\partial y} + k^w - \frac{k^v}{\gamma_w}\left(\frac{u_w}{T + 273.15}\right)\frac{\partial T}{\partial y}\right] = \frac{\partial (nS)}{\partial t} \quad [5]$$

where k^w = hydraulic conductivity; k^v = vapour conductivity; γ_w = unit weight of water; u_w = pore-water pressure; T = temperature; n = porosity; S = degree of saturation.

Freeze and Cherry (1979) present analyses based on Richard's equation (similar to Eq. 5) using a finite difference solution. The results are excellent and in qualitative agreement with the shape of the limiting curve presented in Fig. 1. This approach results in a comprehensive analysis that is able to handle infiltration and runoff conditions. However, the manner in which the precipitation and runoff boundary conditions are posed have a significant influence on the results.

3. THEORY FOR INFILTRATION AND RUNOFF BOUNDARY CONDITIONS

The net soil-atmosphere moisture flux is a function of some of the key components of the hydrology cycle; namely, precipitation, actual evaporation, and run-off. The net soil-atmosphere flux may result in either infiltration (positive flux) or exfiltration (negative flux). The net soil-atmosphere moisture flux can be determined by the following water balance equation:

$$NF = P\cos\alpha - AE - R \quad [6]$$

where NF = the net moisture flux; P = precipitation; α = ground surface slope; AE = actual evaporation; R = runoff.

The terms in Eq. 6 are illustrated in Fig. 2. The net moisture flux, NF, corresponds to a natural (i.e., flux) boundary condition. The amount of precipitation, P, is a "known" input. The term $\cos\alpha$ is based on the assumption that precipitation falls in a vertical trajectory and is typically measured on a horizontal surface. The terms AE and R are a function of weather parameters and the soil suction at the soil-atmosphere boundary.

Actual evaporation can be computed based on the potential evaporation and a limiting function (Wilson et al., 1997). Runoff must be computed in an interactive manner. If the embankment being analysed has an efficient drainage system, any runoff water will be removed from the ground surface. In this case, the amount of net moisture flux, NF, should not produce pore-water pressures higher than zero at ground surface. The following set of equations can be used to represent this condition (Gitirana Jr., 2005):

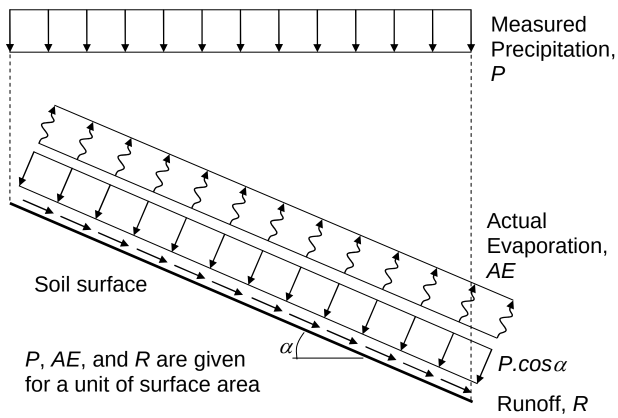

Figure 2 Soil-atmosphere moisture flux components.

$$NF = \begin{cases} P\cos\alpha - AE & : \text{if } P\cos\alpha - AE > 0 \text{ and } u_{ws} < 0 \\ EF(0 - u_{ws}) & : \text{if } P\cos\alpha - AE > 0 \text{ and } u_{ws} \geq 0 \\ P\cos\alpha - AE & : \text{if } P\cos\alpha - AE \leq 0 \end{cases} \quad [7]$$

where u_{ws} = pore-water pressure at the surface; *EF* = a large number.

Runoff may take place when the value of ($P\cos\alpha$ – *AE)* is larger then the saturated hydraulic conductivity. The amount of runoff corresponds to the difference between the water available, ($P\cos\alpha$ – *AE)*, and the amount of infiltration computed using Eq. 7.

If the multiplier *EF* tends to infinity, the area flux boundary condition $NF = EF(0 - u_{ws})$ becomes mathematically equivalent to an essential (i.e., node value) boundary condition, $u_w = 0$. Therefore, the boundary condition from Eq. 7 is an alternative to switching to an essential boundary condition when the pore-water pressure at the soil surface becomes positive.

The approach based on switching boundary conditions often results in numerical oscillations due to instantaneous changes on node values. These instantaneous changes do not represent real-world conditions. Instantaneous changes in node values require mesh refinements that should theoretically be infinitesimal (Gitirana Jr., 2005, Nelson, 2005). Therefore, Eq. 7 appears to impose a more realistic condition.

4. METHODOLOGY

Two software packages were used in this study. SVFlux (SoilVision Ltd., 2005) is a seepage analysis package capable of solving 2D, and 3D seepage problems under steady-state and transient conditions. One-dimensional mesh capabilities have been recently implemented, but were not used in this study. SVFlux uses FlexPDE (PDE Solutions Inc., 2005) as a PDE solver engine. Liquid water and water vapour flow are considered in the PDEs adopted, according to the formulation presented in Gitirana Jr. (2005). Soil-atmosphere interaction is reproduced using Eq. 7. SVFlux makes use of an automatic adaptive mesh technique. The adaptive mesh refinement algorithm is tuned to be sensitive to high gradients in pore-water pressure, hydraulic conductivity, or any other variable that needs to be solved with a desired accuracy.

Vadose/W (Geo-Slope International, 2005) is capable of solving seepage problems where soil-atmosphere interaction is of interest. Vadose/W is primarily a 2D package but can be used for 1D problems through the use of appropriate boundary conditions. Vadose/W has a special feature that allows the accumulation of runoff in surface depressions and subsequent infiltration. The mesh construction in Vadose/W is manual and the mesh remains fixed throughout the analysis.

Vadose/W uses a technique for computing infiltration and runoff were "natural" and "essential" boundary conditions are switched based on the soil conditions at the ground surface. The ground surface is treated as a "potential seepage surface". The instantaneous application of "essential" boundary conditions that are not continuous may result in numerical oscillations. However, the small difference between pore-water pressures at the surface and instantaneously imposed conditions is expected to minimize such oscillations. Such differences can be kept small by using small time steps.

Three problems were investigated as part of this study. First, a simple problem with constant rainfall on a 1D column was analyzed. Precipitation was varied in order to investigate the computations of infiltration and runoff for increasingly high precipitations. Significantly high gradients were expected in this problem, resulting in demanding computations.

Next, two problems are analyzed that involved alternating weather conditions. The fluxes past soil covers associated with the reclamation of mine sites are presented.

5. ANALYSIS OF HYPOTHETICAL CONSTANT RAINFALL CONDITIONS

The analysis of a 10 m high soil column subjected to precipitation under isothermal conditions is first presented. Figure 3 presents the problem geometry adopted and finite element meshes generated. The horizontal dimension of the soil column is irrelevant as the flow pattern is one-dimensional. The mesh manually constructed in Vadose/W consists of 408 eight-node isoparametric elements. The mesh automatically generated by FlexPDE/SVFlux has a number of elements that varies according to the degree of nonlinearity of the problem at a given time step. As many as 500 elements were required for an error tolerance of 0.1%. The initial pore-water pressure distribution corresponds to a hydrostatic pore-water pressure distribution (Fig. 3e), with the water table located at the bottom of the column (elevation = 0 m).

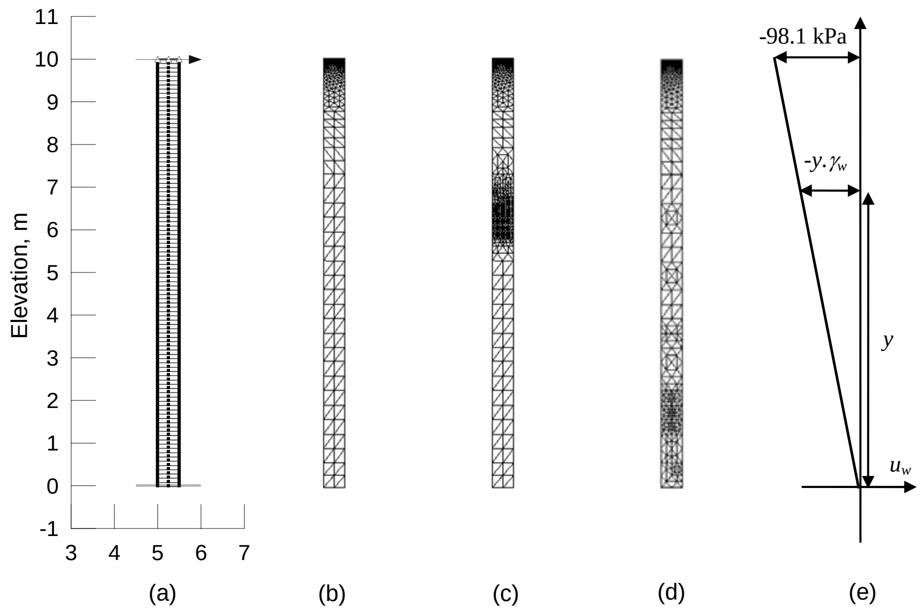

Figure 3 Problem 1: geometry, mesh, and boundary conditions. (a) Vadose/W mesh; (b) SVFlux mesh for t = 0 day; (c) SVFlux mesh for t = 3 days; (d) SVFlux mesh for t = 7 days; and (e) initial pore-water pressure.

A "no flow" natural boundary condition was applied at the lateral and lower boundaries of the SVFlux and Vadose/W meshes. The boundary condition applied to the upper boundary of the SVFlux mesh corresponds to the conditional soil-atmosphere natural boundary equation (Eq. 7). The boundary condition available in Vadose/W that best represents the precipitation/runoff conditions analyzed herein corresponds to the "climate boundary" option. Different amounts of precipitation were selected in order to produce different amounts of infiltration and runoff. The precipitation rates applied at the upper boundary were equal to $1\times k^w_{sat}$, $1.5\times k^w_{sat}$, $2\times k^w_{sat}$, $4\times k^w_{sat}$, and $10\times k^w_{sat}$. These amounts of precipitation were applied during a period of 7 days.

Figure 4 presents the soil properties assumed. The saturated hydraulic conductivity of 1×10^{-6} m/s corresponds to a moderate permeability soil, typical of silts and certain loams. Figure 4 shows that the vapour conductivity, k^v, of this soil is negligible for soil suctions lower than 7000 kPa. Therefore, though the vapour flow is included in SVFlux, it should not affect the results because the soil suction is expected to be lower than 100 kPa at all time steps. This combination of initial and boundary conditions and hydraulic properties is expected to produce runoff during higher precipitation rates.

Figures 5 and 6 present the computed infiltration rates corresponding to various precipitation rates. The precipitation rates correspond to 86.4, 129.4, 172.8, 345.6, and 864 mm/day, respectively. It can be observed in Fig. 5 that the results obtained using SVFlux agree qualitatively with the experimental limiting curve proposed by Horton (1933). The higher the precipitation rate, the sooner runoff commences. Runoff occurred for all precipitation rates with exception of the precipitation rate corresponding to $1\times k^w_{sat}$. This is as expected. The amount of runoff can be calculated by taking the difference between the precipitation and the computed infiltration.

The infiltration rates computed using SVFlux and Vadose/W were in close agreement for precipitation rates up to $2\times k^w_{sat}$. Somewhat different results were obtained for higher precipitation rates, during the first few hours. Vadose/W does not appear to be capable of responding to the high pore-water pressure gradients that occur during the beginning of the wetting process. SVFlux was able to track the high pressure gradients through use of the adaptive mesh technique. As a result, lower infiltration rates were computed by Vadose/W and the shape of the limiting infiltration curve presented by Horton (1933) could not be closely simulated. The results come into closer agreement after t = 1 day. A relatively dense mesh was used in Vadose/W. Further mesh refinements were also used with relatively little improvement.

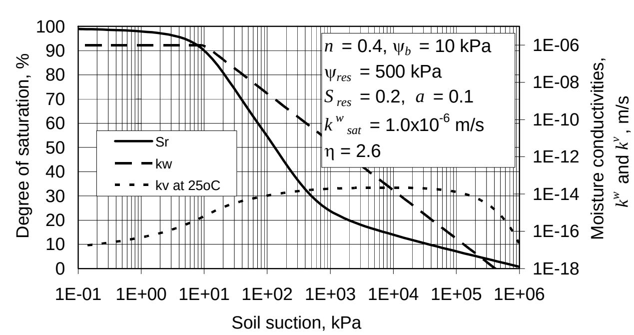

Figure 4 Hydraulic properties used in the verification of the moisture flow model for infiltration/runoff conditions.

Figures 7 and 8 present the pore-water pressure profiles at several time steps for precipitation rates of 1×k^w_{sat} and 4×k^w_{sat}, respectively. Sharp infiltration fronts were obtained for all three cases, as expected. The higher the precipitation rate the deeper the wetting front, at any given time step. For a precipitation rate equal to the saturated hydraulic conductivity the matric suction at the soil surface remained equal to the matric suction at the break point of the k^w function (i.e., 10 kPa).

Higher precipitation rates produced larger pore-water pressures at the soil surface, eventually reaching zero and resulting in runoff. Figure 7 shows that pore-water pressures at the soil surface reached zero at approximately 4 days. The water table at the bottom of the soil column was unaffected by precipitation rates of 1×k^w_{sat}, 1.5×k^w_{sat}, but moved upwards for higher precipitation rates (see Fig. 8, t = 7 days).

Good agreement was observed between the pore-water pressure profiles obtained using SVFlux and Vadose/W, for precipitation rates lower or equal than 1.5×k^w_{sat}. A poorer simulation of the infiltration profile was obtained when using Vadose/W for high precipitation rates. The wetting front lagged behind the front obtained using SVFlux, as can be seen in Fig. 8. The lag in the position of the wetting front calculated using Vadose/W was proportional to the error in the amount of infiltration occurring during the first few hours for precipitation rates higher or equal than 4×k^w_{sat}, (See Figs. 5 and 6).

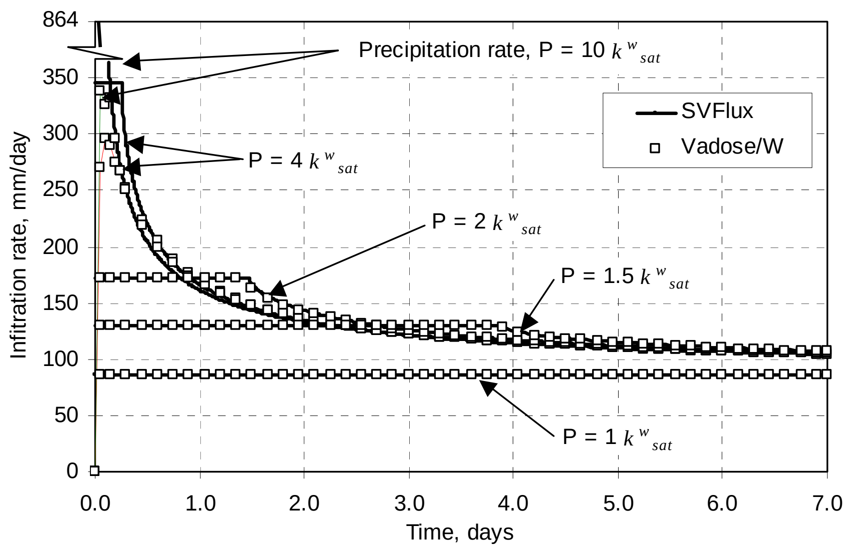

Figure 5 Verification of the moisture flow model for infiltration/runoff conditions: comparison of the infiltration rates for different precipitation rates.

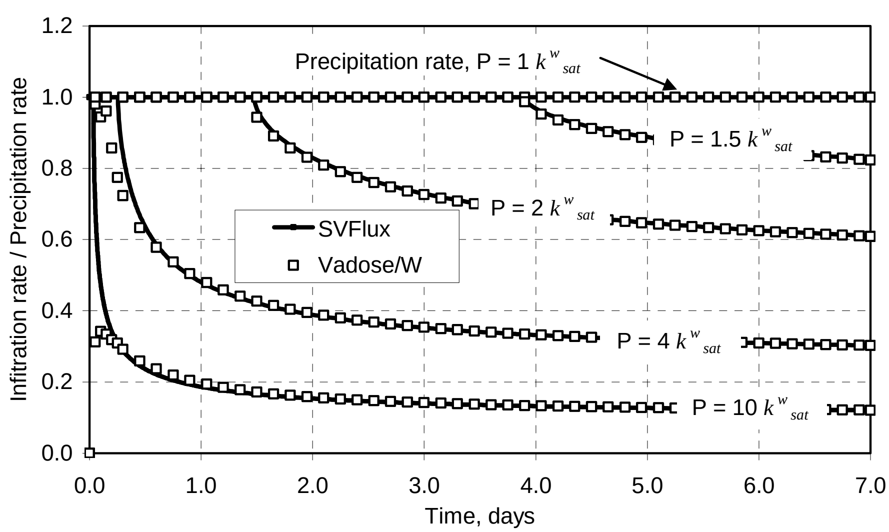

Figure 6 Verification of the moisture flow model for infiltration/runoff conditions: comparison of the infiltration rates / precipitation for different precipitation rates.

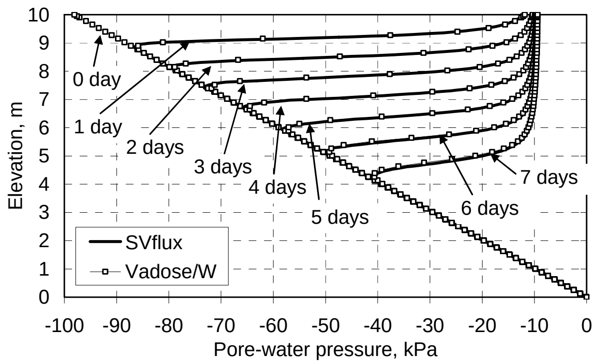

Figure 7 Infiltration profiles, Precipitation = k^w_{sat}.

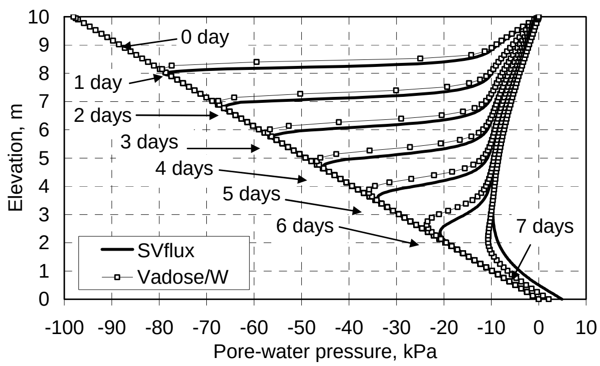

Figure 8 Infiltration profiles, Precipitation = 10 k^w_{sat}.

6 ANALYSIS OF TWO SOIL COVER CONFIGURATIONS

Analysis of infiltration past soil covers is a common type of analysis used in the long-term reclamation of mine sites. 1D or 2D finite element seepage software packages are typically used for this type of analysis. The final proposed design is often based on the results of the finite element analysis of the system. Calculation of actual evaporation rates and runoff becomes significant in the long-term fluxes through cover systems.

Two problems were used to illustrate the impact of varying methods of calculating runoff and infiltration on calculations performed over the course of one year. In the examples presented in this paper the input data was kept as much the same as possible between SVFlux and Vadose/W. The exact same temperature, relative humidity, precipitation, and potential evaporation datasets were used in both software packages. Identical soil property data was also used in each software package.

Applied precipitation levels in the first 14 days of analysis are abnormally high due to the occurrence of snowmelt during this time. The daily precipitation values were assumed to be concentrated at a period of eight hours.

The first soil column is comprised of a Till cover 1 m thick with k^w_{sat} = 9×10^{-2} m/day. The van Genuchten and Mualem representation of the unsaturated hydraulic conductivity curve was used for all soils. Beneath the cover there were two separate layers of tailings each 2.5 m thick with varying hydraulic parameters of 9×10^{-1} m/day and 9×10^{-2} m/day respectively.

Results of the analysis of column 1 are presented in Figure 9. From these results it can be seen that this scenario accentuates the differences in runoff calculation. Vadose/W overestimates runoff values, as previously observed. As a result, less water entered the system than what was predicted using SVFlux. The reduced amount of water entering the system results in higher soil suctions, thereby producing lower actual evaporation rates.

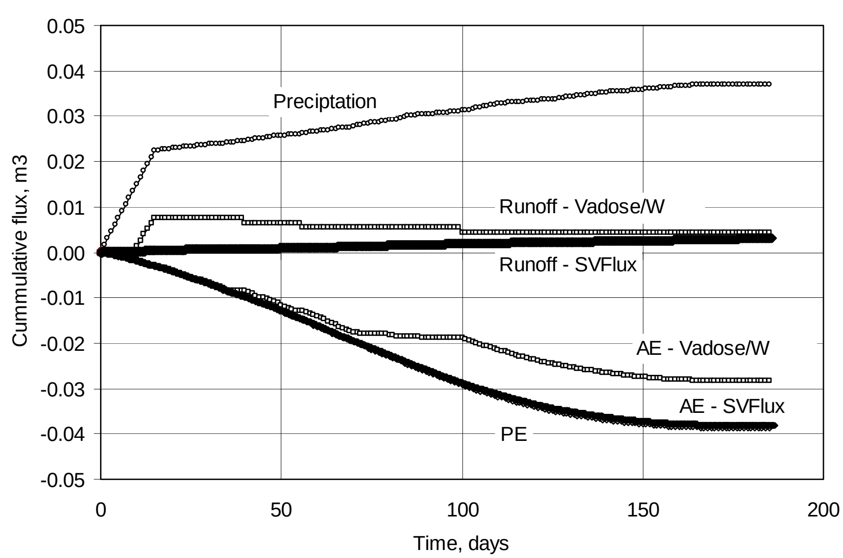

Figure 9 Runoff and AE predictions for column 1.

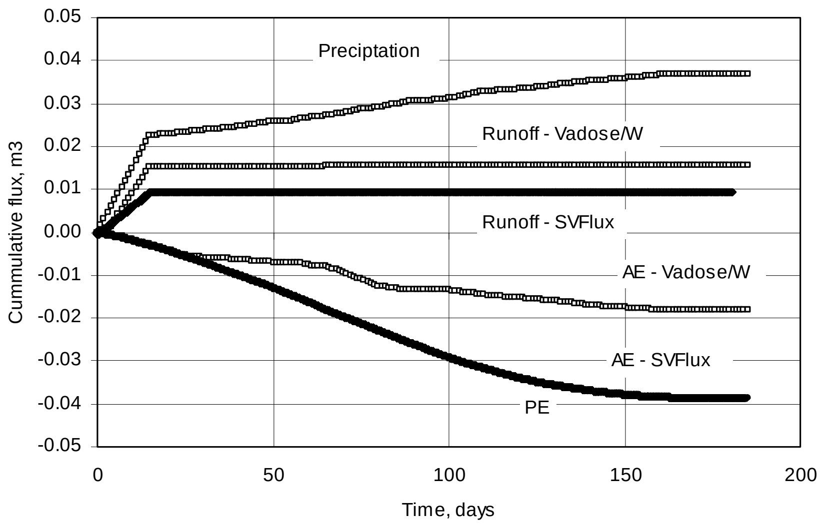

Figure 10 Runoff and AE predictions for column 2.

In the second column the same soil geometry was used but the conductivity of the top layer was decreased to 9×10^{-3} m/day. This should have the effect of increasing the total runoff calculation. The results for the second column may be seen in Figure 10. Significantly greater quantities of runoff were predicted using both software packages. It can also be seen that Vadose/W again presents higher runoff rates with the resulting decrease in actual evaporation.

Mass balance checking was performed on all SVFlux runs and all scenarios solved with a total mass balance error of less than 5%.

Figures 11 and 12 present a closer looks at infiltration and runoff predictions using Vadose/W and SVFlux. The manner how precipitation is represented by each package is somewhat different. Vadose/W uses a sinusoid whose total area is equal to the amount of daily precipitation. SVFlux uses a step-function whose area also corresponds to the daily amount of precipitation.

As precipitation progresses, both packages predict high rates of infiltration, close to the total amount of precipitation. However, the amount of precipitation predicted by Vadose/W is slightly lower. The lower values of infiltration predicted by Vadose/W are in agreement with the observation from Example 1. Runoff progressively increases until a peak value is achieved.

There are differences in the way that the thermal characteristics of the soil are considered in both software packages. SVFlux 5.55 does not couple the thermal fluxes within the soil while Vadose/W does. Thermal effects may be negligible for short-term analyses of up to 100 days (Gitirana Jr., 2005). However, thermal fluxes accumulated over long periods of time may become significant. Water migration due to thermal fluxes is not expected to be significant for the two soil-cover systems presented herein due to the relatively short period of time analysed.

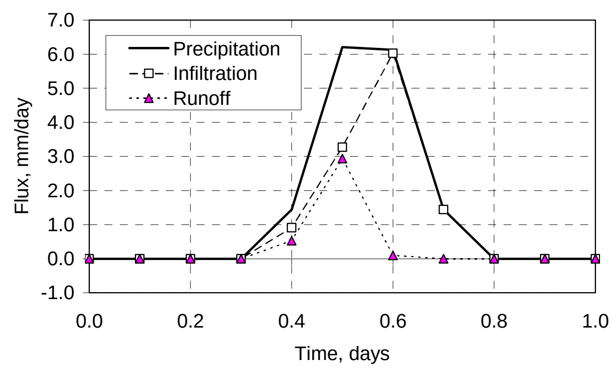

Figure 11 Vadose/W predictions for column 1, day 1.

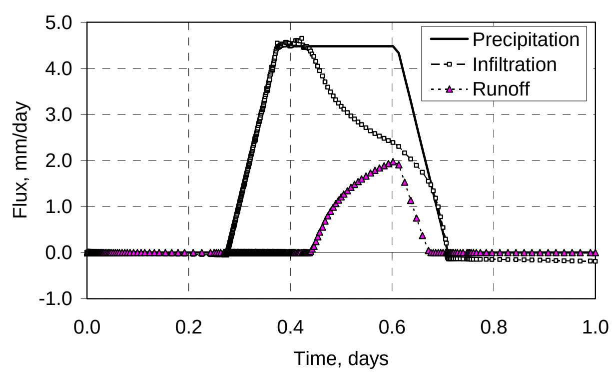

Figure 12 SVFlux predictions for column 1, day 1.

7. CONCLUDING REMARKS

This paper presented a method of verification of the validity of runoff and infiltration calculations using finite element seepage software packages. It can be seen from the results presented that the calculation of runoff is sensitive to the method of time-stepping and the grid refinement at the upper boundary. The propagation of small runoff calculation errors can lead to significant differences in results when averaged over the period of months or years. If sufficient time-stepping is not selected then the net long-term results will be an over-estimation of the runoff. An overestimation of runoff then results in less water entering the model and infiltrating past a cover design which then leads to an over-idealized cover design. In the field of cover design there should be particular attention given to the influence of runoff calculations for long-term cover performance modeling.

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