

PERMEABILITY FUNCTIONS FOR UNSATURATED SOILS

By E. C. Leong¹ and H. Rahardjo²

ABSTRACT: This paper examines the three categories of permeability functions for unsaturated soils, including empirical, macroscopic, and statistical models. The theoretical backgrounds and performance of each category are examined against various experimental data. The paper also shows that it is possible to degenerate statistical models to macroscopic models and then to empirical models. A new empirical equation for the permeability function is suggested. The statistical model demonstrates good performance and can be readily applied. In some cases the performance of the statistical model can be further improved with the introduction of a correction factor.

INTRODUCTION

Many geotechnical and geoenvironmental problems involve water flow through unsaturated soils. Water coefficient of permeability in an unsaturated soil is a function of pore-water pressure or water content. Direct measurements of permeability in the laboratory can be time-consuming, especially for low water content conditions. Indirect measurements of permeability are commonly performed by establishing permeability functions through the use of the relationship between water content and pore-water pressure (i.e., soil-water characteristic curve). There are numerous permeability functions available for unsaturated soils. In general, the various permeability functions can be categorized into three groups: empirical, macroscopic, and statistical models. It is important that the assumptions and theoretical backgrounds associated with each category of function be known prior to its use in application. A review of these backgrounds is presented in the paper. The performance of each category of function is also examined against various experimental data. Advantages and limitations associated with each model are illustrated in the paper.

PERMEABILITY OF UNSATURATED SOILS

Permeability of saturated soils k_s is a function of void ratio e only. For unsaturated soils, the coefficient of permeability with respect to water k_w is a function of both void ratio e and water content w . Since void ratio e , water content w , and degree of saturation S are interrelated, k_w can be expressed as a function of any two of the three parameters, i.e.

$$k_w = f(e, w); \quad k_w = f(S, e); \quad k_w = f(S, w) \quad (1a-c)$$

If the soil structure is assumed to be incompressible, then it is possible to decouple the two parameters in (1) where the saturated permeability k_s will quantify the effect of void ratio and another function will account for the effect of water content in the soil.

Permeability measurement of unsaturated soils is a very time-consuming process. The duration of the test increases as the water content in the soil decreases. The permeability values can differ by several orders in magnitude causing direct measurement to be very difficult as there is no apparatus that can

measure such a wide range of permeability values efficiently. Permeability measurements can be performed either in the field or in the laboratory. However, field measurements are usually more variable, due partly to macroscopic features and partly from the assumptions made. In this paper only laboratory measurements are discussed.

In direct measurement there are steady-state and unsteady-state methods (Fredlund and Rahardjo 1993). In the steady-state method, a matric suction is first imposed on a soil specimen using the axis-translation technique (Hilf 1956). At equilibrium, denoted by a constant water content, a hydraulic gradient is then imposed across the soil specimen. The flow rate is measured and the permeability is obtained via Darcy's law. Using the unsteady-state method or instantaneous profile method, a cylindrical soil specimen is subjected to a continuous flow of water from one end. The hydraulic head gradient and the flow rate at various points along the specimen are computed by monitoring water content and/or pore-water pressure at these points. The associated problems of direct measurements are

1. A long time is required to complete a series of permeability measurements as the coefficient of permeability of unsaturated soils is very low, especially at high matric suction values.
2. Because of the low flow rate, the measurement of water volume change must be very accurate. Water loss from or within the apparatus and air diffusion through the water can introduce serious errors in the volume measurement.
3. In some cases an osmotic suction gradient may develop between the pore water within the soil and pure water that is used as the permeating fluid. This gradient will induce an additional osmotic flow across the specimen. The osmotic flow becomes more significant as the water content of the specimen decreases.
4. As matric suction increases, the specimen may shrink from the wall of the cell and also from the high air-entry disk. The air gap will disrupt the continuity of water flow as air is nonconductive to water flow. For the instantaneous profile method, the soil may shrink away from the instruments that are used to measure pore-water pressure changes.

In indirect measurement the water content of the soil specimen at various matric suction values is determined. The permeability is then inferred from the soil-water characteristic curve, a plot of volumetric water content and matric suction, using a statistical model. Compared with direct measurement using the steady-state method, the test duration is greatly reduced as the test only lasts up to the time when the water content in the soil specimen equilibrates with the imposed matric suction. The problems associated with the indirect measurements are

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1. Determination of the end point where the water content becomes constant at an imposed matric suction can be very difficult.
2. Similar to direct measurement, measurement of water volume changes must be very accurate. This is especially so at a very high matric suction.
3. Air diffusion through the ceramic disk reduces the accuracy of the water volume determination.
4. As most indirect measurement devices use an "open system" with the water pressure being at atmospheric, losses of water through evaporation and in some cases drying of the ceramic disk will cause a large error in the determination of water content of the soil specimen.
5. The soil specimen will shrink at a high matric suction and the volumetric water content requires the determination of the soil specimen volume. The soil volume is difficult to determine causing some errors in the soil-water characteristic curve determination.

In both methods the soil specimen can be subjected to drying and wetting processes. In the drying process the soil starts at a near saturation condition and the matric suction is gradually increased leading to a reduction in the water content in the soil specimen. Therefore, the test is termed as "drying." In the wetting process the soil specimen starts at a very low water content, or "dry," condition and the matric suction is gradually reduced causing water to be imbibed by the soil specimen. Therefore, the test is termed as "wetting." It is generally found that the drying and wetting processes exhibit hysteretic behavior.

In summary, permeability determination of unsaturated soils is a tedious and time-consuming process. In many unsaturated soil problems, permeabilities at various matric suctions are required. The relationship between permeability and matric suction is referred to as permeability function. Since matric suction and volumetric water content are related to the soil-water characteristic curve, the permeability function can also be described by a relationship between permeability and volumetric water content. Depending on the type of permeability measurements being carried out, there are three types of permeability functions that can be employed: empirical equations, macroscopic models, and statistical models. These categories of functions were suggested by Mualem (1986) as an indication of the degree of theoretical sophistication, with the statis-

tical models being the most rigorous. These categories will be discussed in detail in the following sections.

EMPIRICAL EQUATIONS

Empirical equations arise from the need for an equation to describe the variation of permeability with matric suction ψ or volumetric water content θ_w , i.e.

$$k_w = f(\psi) \text{ or } f(\theta_w) \quad (2)$$

Direct measurement of permeability is required for the determination of the empirical equation. The constants in the equations are best determined from a curve fit of the test data, however some researchers have attempted to define some of these constants (e.g., Wind 1955). A number of equations for the permeability function have been suggested and these are tabulated in Table 1. In Table 1 the permeability functions are categorized into $k(\theta_w)$ and $k(\psi)$ functions. Closer examination of the empirical equations for the permeability function in Table 1 reveals that many of them are similar in form to empirical equations suggested for the soil-water characteristic curve. The soil-water characteristic curve is a relationship between θ_w and ψ . The similarity in shape between permeability function and soil-water characteristic curve is not surprising as water only flows through the water phase in the soil. Equations for soil-water characteristic curves are juxtaposed in Table 1. Some of the equations for the soil-water characteristic curves in Table 1 were suggested by different researchers but by comparing the proposed equations for permeability and soil-water characteristic curves, it becomes evident that these researchers have actually suggested the following generalized relationship:

$$k_r = \Theta^p \quad (3)$$

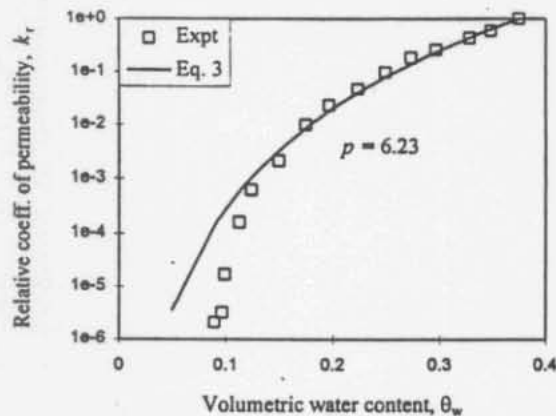
where k_r = relative coefficient of permeability or ratio of permeability k_w and saturated permeability k_s ; Θ = normalized volumetric water content or $(\theta_w - \theta_r)/(\theta_s - \theta_r)$ where the subscripts s and r denote saturated and residual, respectively; and p is a constant. The application of (3) to fit some experimental data is shown in Figs. 1 and 2. Fig. 2 illustrates the permeability corresponding to the drying and wetting curves. The exponent p has quite a wide range of values as shown in the figures. The curve fit is reasonable in all the cases except at low values of volumetric water content.

Since (3) gives a reasonable fit it will naturally follow that

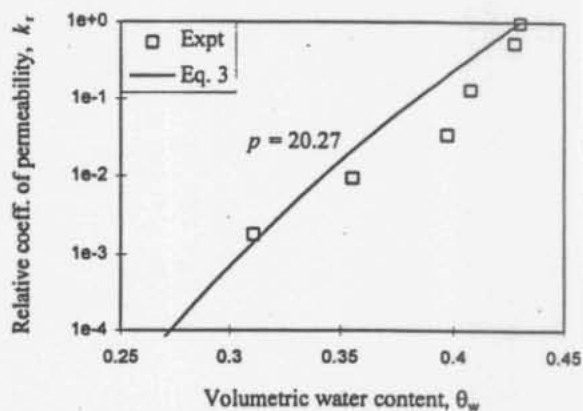
TABLE 1. Empirical Permeability Functions and Soil-Water Characteristic Curve Equations

Type (1)	Permeability functions (2)	Soil-water characteristic curve equations (3)
$k = f(\theta_w)$	$k_w = a\theta_w^b$ Gardner (1958) $k_w = k_s \left(\frac{\theta_w}{\theta_s}\right)^{2b+3}$ and $b = \frac{\Delta \log \psi}{\Delta \log \theta_w}$ Campbell (1973) Ahuja (1973, 1974) Gillham et al. (1976) Zachmann et al. (1981) Hillel (1982) $k_w = k_s \exp[b(\theta_w - \theta_s)]$ Davidson et al. (1969) Dane and Klute (1977)	$\psi = \psi_{cr} \exp[b(1 - S_r)]$ Farrel and Larson (1972)
$k = f(\psi)$	$k_w = a + b\psi$ Richards (1931) $k_w = a\psi^{-b}$ Wind (1955) Weeks and Richards (1967) Gardner (1958) Arbhahirama and Kridakorn (1968) $k_w = \frac{k_s}{1 + a \left(\frac{\psi}{\rho_w g}\right)^b}$ $k_w = a \exp(b\psi)$ Christensen (1943) $k_w = k_s$ for $\psi \leq \psi_b$ Rijtema (1965) $k_w = k_s \exp[b(\psi - \psi_b)]$ for $\psi > \psi_b$ Phillip (1986)	$\ln \psi = a + b \ln \theta_w$ Williams et al. (1983) $\Theta = \frac{1}{1 + a\psi^b}$ Gardner (1958) $\Theta = \exp\left[\frac{-(\psi - a)}{b}\right]$ McKee and Bumb (1984)

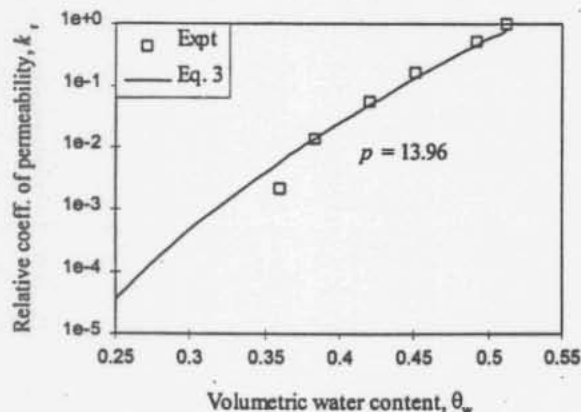
Note: In the table, a , and b are constants; k is coefficient of permeability where subscripts w and s denote unsaturated and saturated, respectively; S_r is the effective degree of saturation; ψ is the matric suction; θ_w is the volumetric water content; and Θ is the normalized volumetric water content.



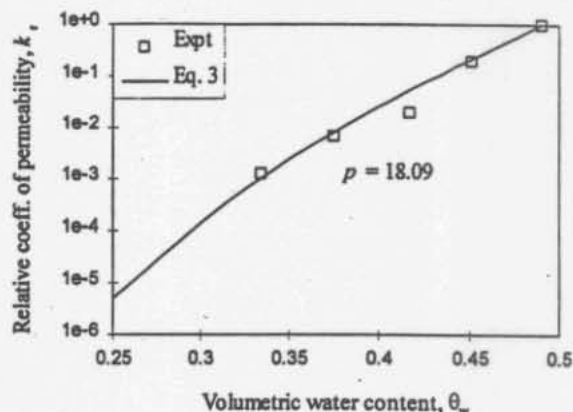
(a) Lakeland sand



(b) Drummer soil 0-30 cm depth



(c) Drummer soil 30-75 cm depth



(d) Drummer soil 75-90 cm depth

FIG. 1. Best Fitted Permeability Function $k(\theta_w)$ Using (3) [Data from Elzeftawy and Cartwright (1981)]

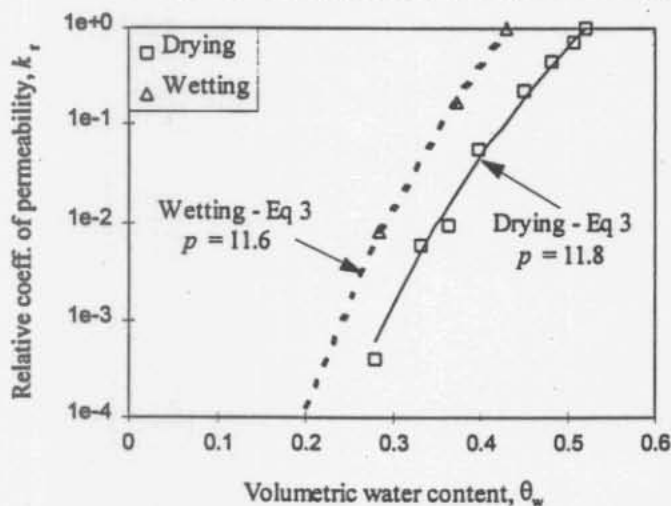


FIG. 2. Best Fitted Permeability Function $k(\theta_w)$ Using (3) for Guelph Loam (Brooks and Corey 1964)

if Θ is expressed as a function of the matric suction ψ then k_r can also be expressed as a function of the matric suction ψ . Leong and Rahardjo (1997) in the companion paper have made a critical evaluation of the $\Theta = \psi$ relationships and have shown that the following equation suggested by Fredlund and Xing (1994) fits the experimental soil-water characteristic curves well and is robust:

$$\Theta = C(\psi) \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^b \right] \right\}^c} \quad (4)$$

Good but use then say C(ψ)=1

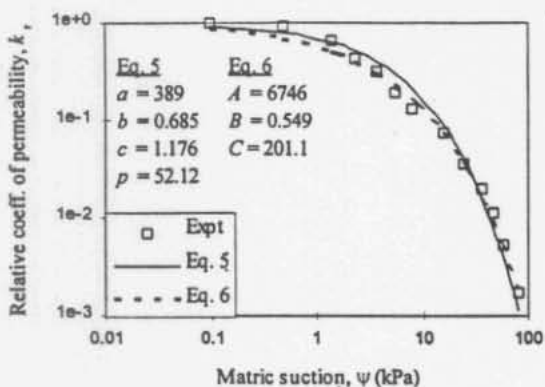
In (4), a , b , and c are constants with a having the same unit as matric suction ψ and $C(\psi)$ is a correction factor. Leong and Rahardjo (1997) have further illustrated that (4) is very robust when $C(\psi) = 1$ and recommended that (4) be used with $C(\psi) = 1$.

If k_r is a power function of Θ then

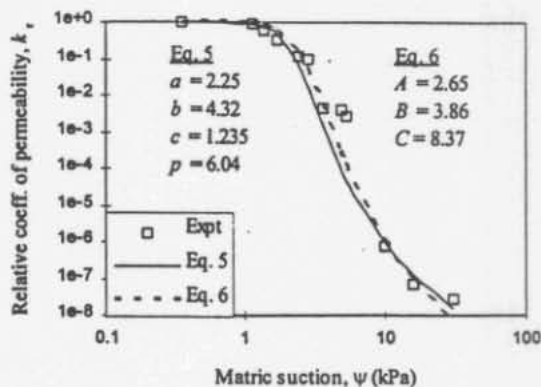
$$k_r = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^b \right] \right\}^{cp}} \quad (5)$$

where the exponent c in (4) now becomes $c' = cp$. Seven experimental data sets having soil-water characteristic and permeability data are used to assess (5). The soil-water characteristic curves and details of determination of the constants are given by Leong and Rahardjo (1997). The constants a , b , and c in the soil-water characteristic curve (4), were first determined as in Leong and Rahardjo (1997). Subsequently p was determined by curve fitting the permeability data. These best fitted curves are shown by the solid lines in Figs. 3 and 4 together with the values of a , b , and c , and the exponent p . Fig. 4 shows permeability values corresponding to the drying and wetting processes. The curves given by (5) show a good fit to the data.

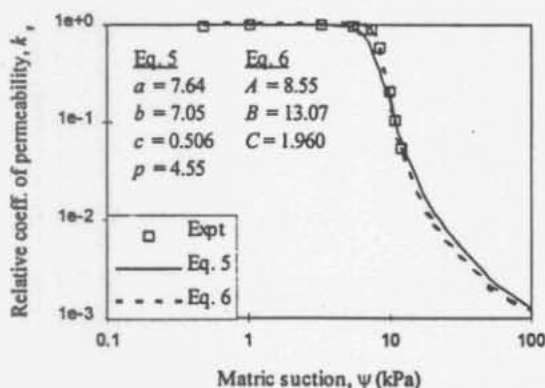
The discussion thus far implied that it will be very attractive if the exponent p is well established for a given soil. As a result, the k_w can then be determined indirectly from the soil-water characteristic curve. This procedure will considerably reduce the effort in determining k_w . However, if k_w has been determined directly with respect to ψ , then the writers proposed the following equation where constants A , B , and C are determined by curve fitting:



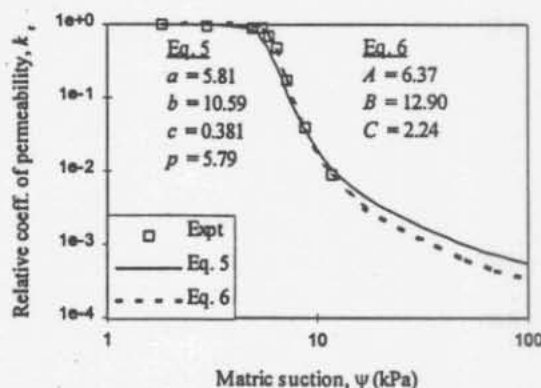
(a) Beit Netofa Clay



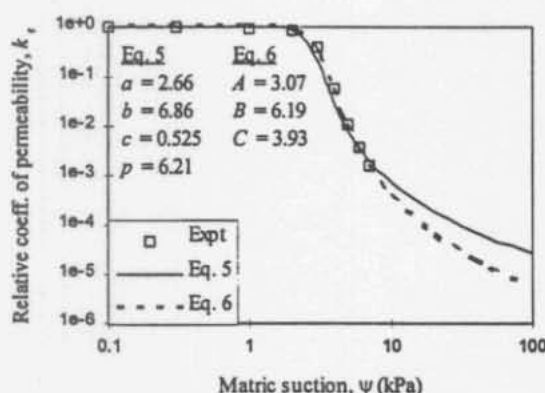
(b) Rehovot sand



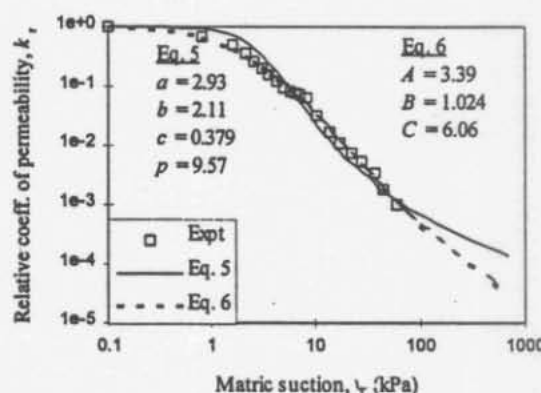
(c) Touchet silt loam (GE3)



(d) Columbia sandy loam



(e) Superstition sand



(f) Yolo light clay

FIG. 3. Best Fitted Permeability Function $k(\psi)$ Using (5) and (6) for Several Soil Types [Data from Moore (1939), Richards (1952), Brooks and Corey (1964), Rubin et al. (1964), and Rawitz (1965)]

$$k_r = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{A} \right)^b \right] \right\}^c} \quad (6)$$

The curves given by (6) are shown in Figs. 3 and 4 as dotted lines. The values of the constants A , B , and C are also shown in the figures. It can be observed that the fit in all cases is better than that given by (5).

The empirical permeability functions given by (3) and (6) are very useful for soils with enough permeability data where representative values of p or A , B , and C can be obtained.

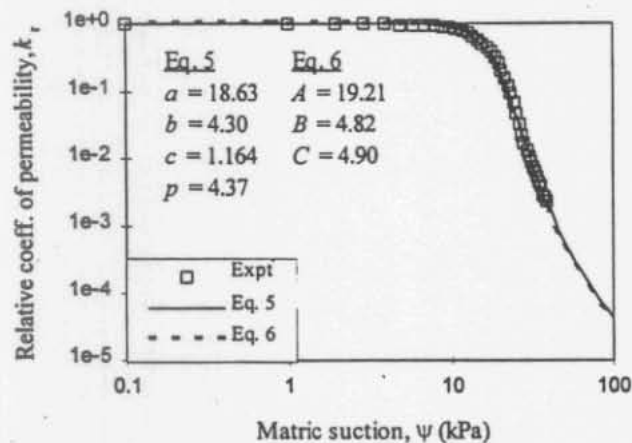
MACROSCOPIC MODELS

The objective of the macroscopic models is to derive an analytical expression for the permeability function (Mualem 1986). Common to these models is the first assumption of

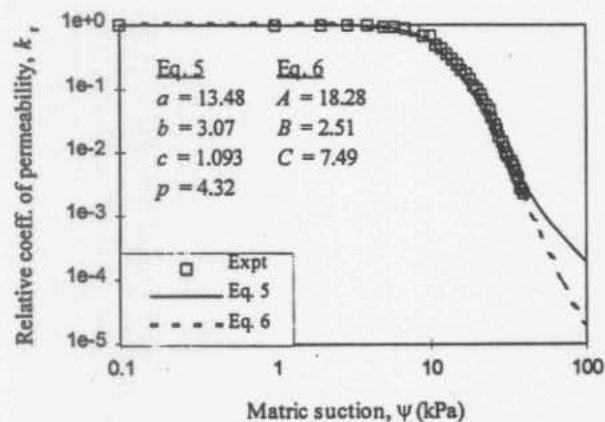
similarity between laminar flow (microscopic level) to flow in porous media (macroscopic level). The flow is then solved for a simple laminar flow system to interrelate the macroscopic variables of average flow velocity, hydraulic gradient, permeability, and hydraulic radius. A direct analogy of these variables to the corresponding variables for a soil-water-air system is then made. Because of the simplifying assumptions made, all the microscopic models have the following general form:

$$k_r = S_e^\delta \quad (7)$$

where S_e = effective degree of saturation defined as $S_e = (S - S_r)/(1 - S_r)$; S and S_r = degree of saturation and residual degree of saturation, respectively; and the exponent δ is a constant. The value of δ varies depending on the assumptions made: Averjanov (1950) suggested $\delta = 3.5$, Yuster (1951) suggested $\delta = 2$, Irmay (1954) suggested $\delta = 3$, and Corey (1954) suggested $\delta = 4$. The main criticism of the macroscopic models



(a) Relative coefficient of permeability for drying process.



(b) Relative coefficient of permeability for wetting process.

FIG. 4. Best Fitted Permeability Function $k(\psi)$ Using (5) and (6) for Mine Tailings [Data from Gonzalez and Adams (1980)].

is that the models neglect the effect of pore-size distribution (Childs and Collis-George 1950; Brooks and Corey 1964). Brooks and Corey (1964) showed that the exponent δ is 3 for a soil with a uniform pore-size distribution and that in general $\delta = (2 + 3\lambda)/\lambda$ where λ is a pore-size distribution index and is positive. Mualem (1976), on the basis of data from 50 soils, suggested that $\delta = 3 - 2m$ where m is a soil-water parameter being positive for granular material and negative for unstructured soils of fine texture. Mualem found that δ ranges from 2.5 to 24.5.

S_e is equivalent to Θ and therefore (7) is equivalent to (3) where $\delta = p$. Therefore, it shows that the empirical permeability equation given in (3) has some theoretical basis and following Mualem the range of p is 2.5 to 24.5. From Figs. 1-4 it can be observed that p falls within this range for all the data except for Beit Netofa clay where $p = 52.12$.

STATISTICAL MODELS

The statistical models are the most rigorous models for permeability functions. In these models the coefficient of permeability k_w is derived from the soil-water characteristic curve. The methodology of the statistical models are based on three assumptions (Mualem 1986):

1. The porous medium consists of a set of randomly distributed interconnected pores characterized by a pore radius r and its statistical distribution is given by $f(r)$. The areal pore distribution is the same for all cross sections and is equal to $f(r)$.

2. The Hagen-Poiseuille equation given by

$$\bar{v} = -\left(\frac{r^2 g}{Cv}\right) \left(\frac{d\phi}{dx}\right) \quad (8)$$

is valid, where \bar{v} = average flow velocity; $(d\phi/dx)$ = hydraulic gradient; r = hydraulic radius; v = kinematic coefficient of viscosity; C = shape constant of the flow system; and g = gravitational constant. Eq. (8) is used to estimate the permeability of a pore channel and the total permeability is determined by integration over the contributions from the filled pores.

3. The soil-water characteristic curve is considered analogous to the pore-size distribution function using Kelvin's capillary law.

Mualem (1986) has reviewed statistical models and concluded that the statistical models can be represented by three general formulas

$$k_r = S_e^n \frac{\int_0^{\theta_w} \frac{d\theta_w}{\psi^{2+m}}}{\int_0^{\theta_s} \frac{d\theta_w}{\psi^{2+m}}}; \quad k_r = S_e^n \left(\frac{\int_0^{\theta_w} \frac{d\theta_w}{\psi^{1+m}}}{\int_0^{\theta_s} \frac{d\theta_w}{\psi^{1+m}}} \right)^2 \quad (9, 10)$$

$$k_r = S_e^n \frac{\int_0^{\theta_w} \frac{(\theta_w - \vartheta)}{\psi^{2+m}} d\vartheta}{\int_0^{\theta_s} \frac{(\theta_w - \vartheta)}{\psi^{2+m}} d\vartheta} \quad (11)$$

where n and m are constants; and ϑ is a dummy variable of integration.

The form given by (9) was suggested by Gates and Leitz (1950), Fatt and Dykstra (1951), and Burdine (1953). In Gates and Leitz's study, $n = m = 0$; in Fatt and Dykstra's work, $n = 0$ and m varies with soil type; in Burdine's work, $n = 2$ and $m = 0$. The form given by (10) was suggested by Mualem (1976), where n was defined as 0.5 and $m = 0$.

Eq. 11 originated from the work of Childs and Collis-George (1950), who investigated the influence of random distribution of pores on the coefficient of permeability. To study this they consider the probability when two sections of a porous medium are randomly connected such that larger pores of radius r in one section are connected to smaller pores of radius ρ in the other section. The probability $prob(r, \rho)$ is given by

$$prob(r, \rho) = f(r)f(\rho) dr d\rho \quad (12)$$

To make the computation simple, two assumptions were made: the resistance to flow is from the smaller radius ρ of the connected cross-section, and there is only one connection between pores. Therefore making use of (8) and (12), the discharge flow dq of the pores contributed by the pair of r and ρ is

$$dq = M prob(r, \rho) \rho^2 \nabla \phi \quad (13)$$

where M is a constant accounting for geometry and fluid properties. By integrating (13) over the filled pores and applying Darcy's law, the following expression for permeability is obtained:

$$k_w(\theta_w) = M \left[\int_{\rho=R_{min}}^{\rho=R(\theta_w)} \int_{r=R(\theta_w)}^{\rho=R(\theta_w)} \rho^2 f(\rho) f(r) dr d\rho + \int_{\rho=R_{min}}^{\rho=R(\theta_w)} \int_{r=R(\theta_w)}^{\rho=R(\theta_w)} r^2 f(r) f(\rho) d\rho dr \right] \quad (14)$$

Using (14), k_w can be computed for any given θ_w using the soil-water characteristic curve. Childs and Collis-George

(1950) have suggested transforming the soil-water characteristic curve $\theta_w(\psi)$ into a $\theta_w(r)$ curve through Kelvin's capillary law and then carrying out the integration of (14). This computational procedure is tedious. Marshall (1958) improved the procedure by suggesting the use of equal water content intervals. This procedure led to the following expression:

$$k_w(\theta_w) = \frac{T_s^2}{2\rho_w g \mu} \frac{n^2}{m^2} \sum_{i=1}^l \frac{2(l-i)-1}{\psi_i^2} \quad (15)$$

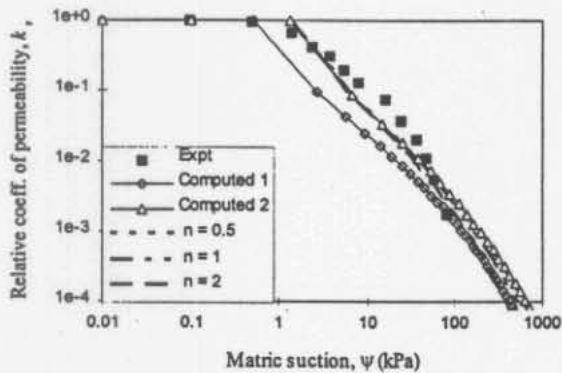
where T_s = surface tension of water; ρ_w = density of water; μ = dynamic viscosity of water; n = porosity of soil; and m ($=\theta_w/\Delta\theta_w$) = total number of intervals; l ($=\theta_w/\Delta\theta_w$) = number of intervals corresponding to θ_w ; and ψ_i = matric suction corresponding to the midpoint of the i th interval of the soil-water characteristic curve. Marshall used (15) mainly for computing the saturated coefficient of permeability. Kunze et al. (1968) made further modification of (15) and applied it to the computation of coefficient of permeability for unsaturated soils.

Nielsen et al. (1960) found that the computation of permeability is significantly improved if an adjusting factor is used to match the computed to the measured coefficients of permeability at saturation. The resulting relative permeability function can then be written as

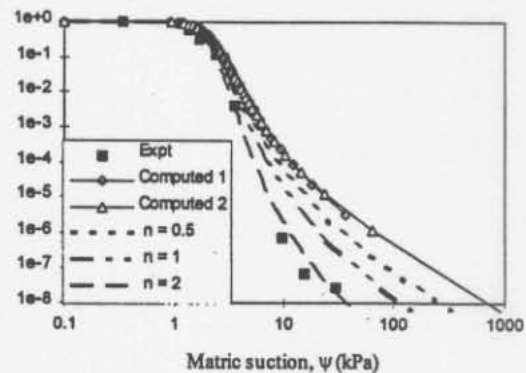
$$k_r(\theta_w) = \frac{\sum_{i=1}^l \frac{2(l-i)-1}{\psi_i^2}}{\sum_{i=1}^m \frac{2(m-i)-1}{\psi_i^2}} \quad (16)$$

Mualem (1974, 1976) showed that the analytical form of (16) is

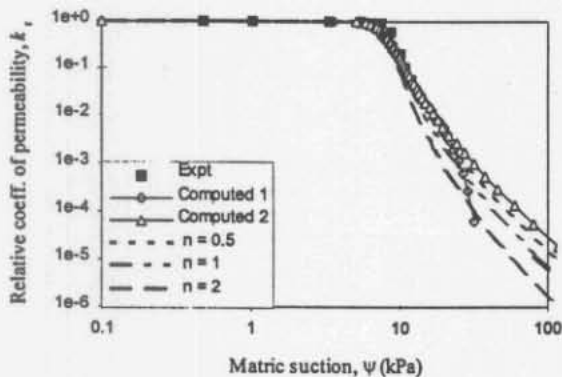
$$k_r(\theta_w) = \frac{\int_0^{\theta_w} \frac{(\theta_w - \theta)}{\psi^2} d\theta}{\int_0^{\theta_s} \frac{(\theta_w - \theta)}{\psi^2} d\theta} \quad (17)$$



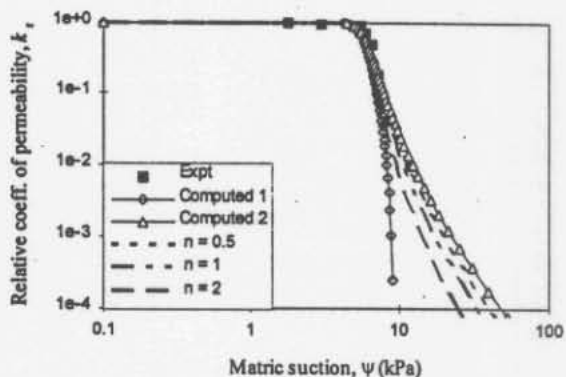
(a) Beit Netofa Clay



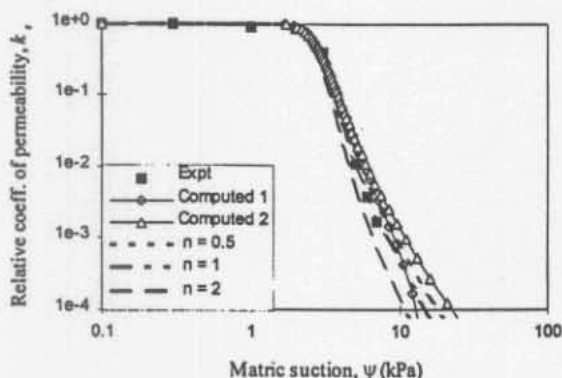
(b) Rehovot sand



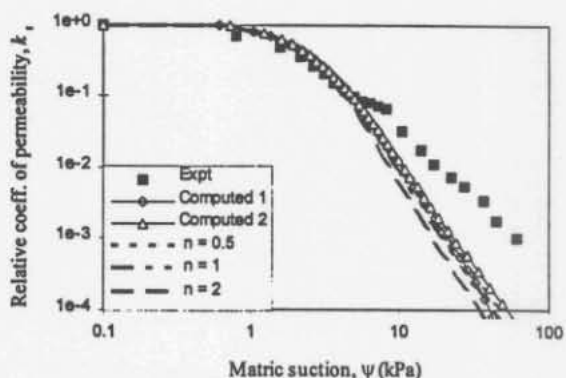
(c) Touchet silt loam (GE3)



(d) Columbia sandy loam



(e) Superstition sand



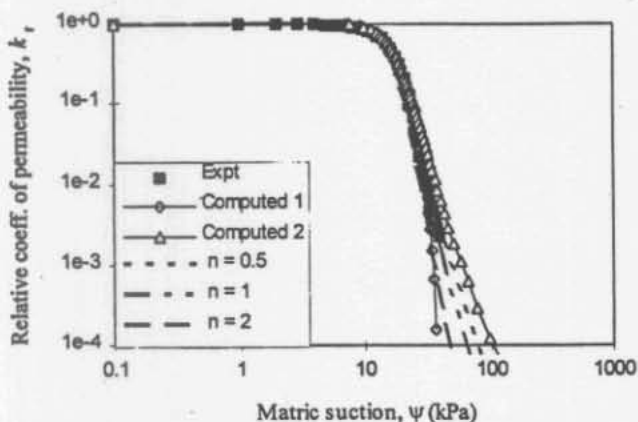
(f) Yolo light clay

FIG. 5. Permeability Function $k(\psi)$ from Statistical Model for Several Soil Types [Data from Moore (1939), Richards (1952), Brooks and Corey (1964), Rubin et al. (1964), and Rawitz (1965)]

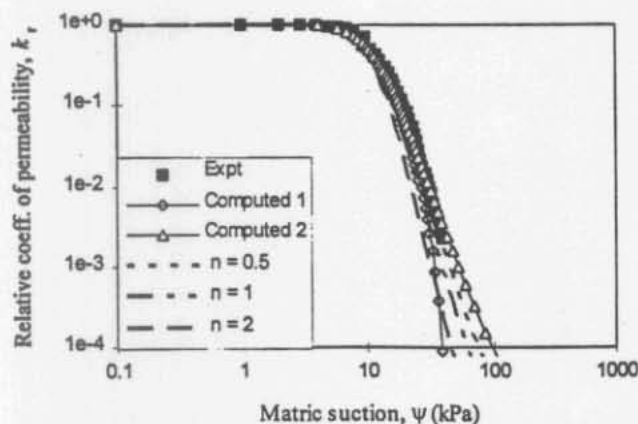
which is the form given by (11) with $n = m = 0$.

The term S_r^n in (9)–(11) is a correction factor suggested by a number of investigators to improve the prediction of the coefficient of permeability from the statistical models. This factor is supposed to account for the tortuosity. Burdine (1953) used $n = 2$ in (9), Millington and Quirk (1961) suggested $n = 4/3$, whereas Kunze et al. (1968) and Jackson (1972) recommended that $n = 1$ for the Childs and Collis-George model, and Mualem (1976) suggested $n = 0.5$ for (10). If assumption is made with regards to the other term on the right-hand side of (9)–(11), such as by Brooks and Corey (1964) that the ratio of the integrals in (9) may be approximated by S_r^{2n} , the macroscopic model of (7) is obtained.

The statistical model given by (16) in the summation form or (17) in the integral form is very popular. The computed coefficient of permeability for this statistical model showed good agreement with the measured coefficient of permeability (van Genuchten 1980; Fredlund et al. 1994) without the use of the correction factor S_r^n . This model will be investigated in depth in this paper. It is also a more common practice to carry out the summation or integration from the lower limit θ_L , which is the lowest volumetric water content of the experimental soil-water characteristic curve, to the upper limit θ_u . Kunze et al. (1968) investigated this effect and concluded that more accurate predictions of the coefficient of permeability are obtained if the whole soil-water characteristic curve is used; that is, from the residual volumetric water content θ_r to the saturated volumetric water content θ_s . However, in many soil-water characteristic determinations, θ_r is not usually measured. This effect is also investigated in this paper. The soil-water characteristic curve described by (4) is used in the statistical

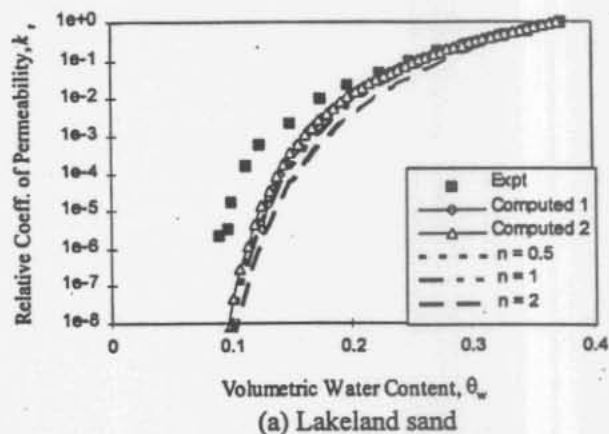


(a) Relative coefficient of permeability for drying process.

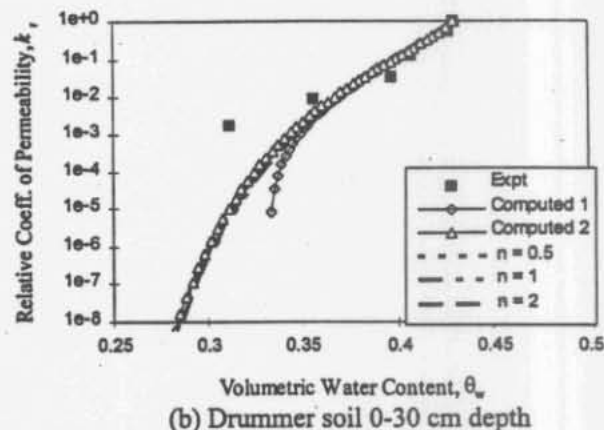


(b) Relative coefficient of permeability for wetting process.

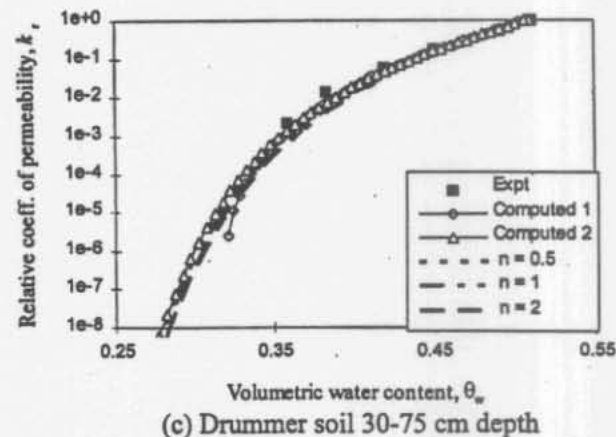
FIG. 6. Permeability Function $k(\psi)$ from Statistical Model for Mine Tailings [Data from Gonzalez and Adams (1980)]



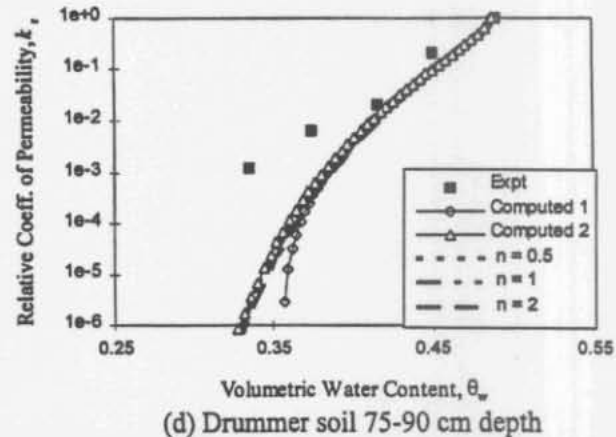
(a) Lakeland sand



(b) Drummer soil 0-30 cm depth

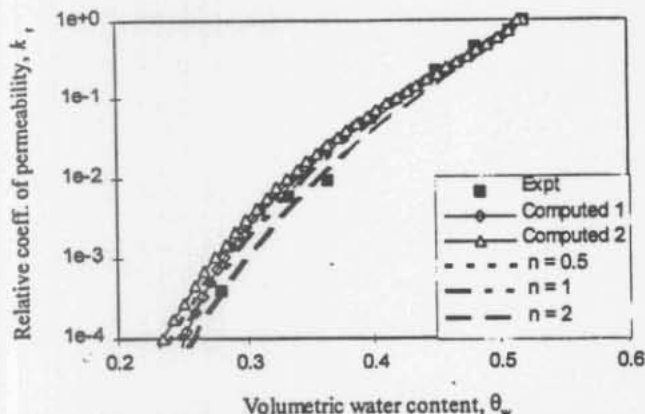


(c) Drummer soil 30-75 cm depth

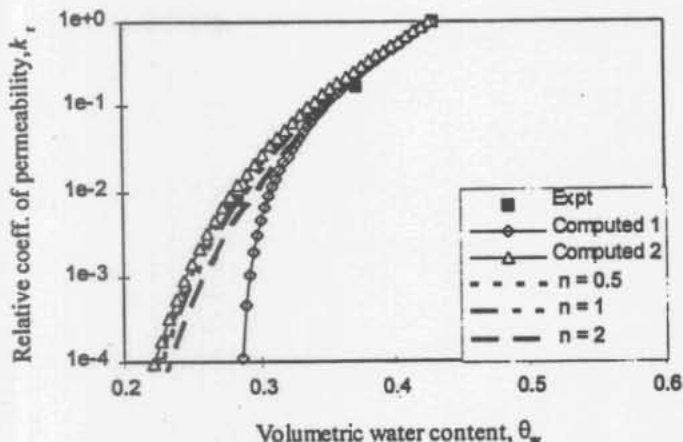


(d) Drummer soil 75-90 cm depth

FIG. 7. Permeability Function $k(\theta_w)$ from Statistical Model [Data from Elzeftawy and Cartwright (1981)]



(a) Relative coefficient of permeability for drying process



(a) Relative coefficient of permeability for wetting process

FIG. 8. Permeability Function $k(\theta_w)$ from Statistical Model for Guelph Loam (Brooks and Corey 1964)

model. Fredlund et al. (1994) suggested performing the integration of (17) along the soil suction axis such as

$$k_r(\psi) = \frac{\int_{\psi}^{\psi_{aw}} \frac{\theta_w(\theta) - \theta_w(\psi)}{\theta^2} \theta'_w(\theta) d\theta}{\int_{\psi_{aw}}^{\psi_{aw}} \frac{\theta_w(\theta) - \theta_w(\psi)}{\theta^2} \theta'_w(\theta) d\theta} \quad (18)$$

where θ'_w is the derivative of (4) and ψ_{aw} is the air-entry value. The problem with this transformation is that one of the limits of integration is the air-entry value ψ_{aw} that is not well defined in most soil-water characteristic curves. In this paper, therefore, it is suggested that integration be performed along the volumetric water content axis as in (17) where the lower limit of integration is taken as θ' that has a value corresponding to $\psi = 10^6$ kPa. The effect of using θ_L and θ' on the computed coefficient permeability is shown in Figs. 5–8. The relative coefficient of permeability curves corresponding to the computations using θ_L and θ' are denoted as "Computed 1" and "Computed 2," respectively. Figs. 5 and 6 show the relative coefficient of permeability k_r as a function of matric suction ψ , while Figs. 7 and 8 show k_r as a function of volumetric water content θ_w . Furthermore, Figs. 6 and 8 show k_r values corresponding to the drying and wetting processes. As can be observed from the figures, the computed coefficient of permeability using θ' is correspondingly higher than the computed coefficient of permeability using θ_L . More noticeable is that k_r computed using θ_L , Computed 1, tends to decrease more rapidly at higher values of ψ or lower values of θ_w . The comparison between k_r computed using θ' and the experimental

values is reasonably good in all cases except for Rehovot sand [Fig. 5(b)], Yolo light clay [Fig. 5(f)], Drummer soil 0–30 cm depth [Fig. 7(b)], and Drummer soil 75–90 cm depth [Fig. 7(d)]. The experimental values of k_r for Yolo light clay show a distinct kink from 4 kPa onwards and are correspondingly higher than the computed values. The experimentally measured values from matric suction of 4 kPa onwards may be erroneous. As for Drummer soil 0–30 cm depth and Drummer soil 75–90 cm depth, deviation of the measured values from the computed values (Computed 2) occur at lower values of θ_w . No satisfactory explanation can be offered for the deviation except that direct measurements of permeability at lower water contents are always more difficult and subject to larger experimental errors.

The effect of introducing the correction factor S_r^n into (17) can be shown as follows: the curves in Figs. 5–8 for computed k_r values using θ' as denoted by "Computed 2" refer to the condition when $n = 0$ for the correction factor S_r^n . The computed k_r values using θ' and by incorporating S_r^n with values of n equal to 0.5, 1, and 2 are also shown in Figs. 5–8. These values of n have been suggested by past researchers. The effect of increasing the value of the exponent n in S_r^n is to decrease the computed k_r values at higher matric suction or lower volumetric water content. In some cases the use of S_r^n improves the estimate of coefficient of permeability. Notably, improvements of the fitting is obtained for Rehovot sand using $n = 2$ [Fig. 5(b)], Superstition sand using $n = 1$ [Fig. 5(e)], and Guelph loam using $n = 1$ (Fig. 8). A value of $n = 2$ gives the lower limit for the estimated coefficient of permeability.

CONCLUSIONS

Coefficient of permeability is required for many geotechnical applications. However, such measurements for unsaturated soil are especially time-consuming and tedious. Measurement of permeability for unsaturated soil at low water content is not easy to perform and demands a highly accurate means of water volume change determination. The coefficient of permeability for unsaturated soils may be determined directly or indirectly. Depending on the availability of data, permeability functions for a soil can be obtained using empirical equations, macroscopic models, or statistical models. The degree of sophistication increases from the empirical equations to the statistical models. By making simplifying assumptions, it has been illustrated that the statistical models can be degenerated into the macroscopic models and then to the empirical equations. If a database of coefficient of permeability for a local soil is available, it may be more expedient to use empirical equations for the permeability functions [(3) and (6)]. It has been illustrated that if the exponent p in (3) is known for a given soil, the coefficient of permeability can be obtained indirectly from the soil-water characteristic curve. The writers suggest the use of a new empirical equation for the permeability function as given in (6). Eq. (6) gives a good fit to all the experimental data illustrated in the paper. For the statistical model, the effect of using θ_L , the lowest value of volumetric water content of the experimental soil-water characteristic curve, and θ' , the volumetric water content corresponding to the matric suction of 10^6 kPa, has been investigated in this paper. A better estimate of coefficient of permeability will be obtained when θ' is used instead of θ_L . The effect of using the correction factor S_r^n for n values of 0.5, 1, and 2 was also investigated. An improved agreement between the computed and measured coefficients can be obtained by varying the n values in some cases. It appears that the n values to be used in the correction factor S_r^n ranges from 0 to 2. Unless confirmed by direct measurement of permeability, using $n = 2$ in the correction factor S_r^n will give the lower limit of the estimated coefficient of permeability.

APPENDIX. REFERENCES

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